

Decay of the pseudoscalar glueball into scalar and pseudoscalar mesonsWalaa I. Eshraim,¹ Stanislaus Janowski,¹ Francesco Giacosa,¹ and Dirk H. Rischke^{1,2}¹*Institute for Theoretical Physics, Goethe University, Max-von-Laue-Strasse 1, D-60438 Frankfurt am Main, Germany*²*Frankfurt Institute for Advanced Studies, Goethe University, Ruth-Moufang-Strasse 1, D-60438 Frankfurt am Main, Germany*

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We study a chiral Lagrangian that describes the two- and three-body decays of a pseudoscalar glueball into scalar and pseudoscalar mesons. The various branching ratios are a parameter-free prediction of our approach. We compute the decay channels for a pseudoscalar glueball with a mass of 2.6 GeV, as predicted by lattice QCD in the quenched approximation, which is in the reach of the PANDA experiment at the upcoming Facility for Antiproton and Ion Research. For completeness, we also repeat the calculation for a glueball mass of 2.37 GeV that corresponds to the mass of the resonance $X(2370)$ measured in the BESIII experiment.

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I. INTRODUCTION

The fundamental symmetry underlying quantum chromodynamics (QCD), the theory of strong interactions, is the exact local $SU(3)_c$ color symmetry. As a consequence of the non-Abelian nature of this symmetry, the gauge fields of QCD, the gluons, are colored objects and therefore interact strongly with each other. Because of confinement, one expects that gluons can also form colorless, or “white,” states which are called glueballs.

The first calculations of glueball masses were based on the bag-model approach [1]. Later on, the rapid improvement of lattice QCD allowed for precise simulations of Yang-Mills theory, leading to a determination of the full glueball spectrum [2]. However, in full QCD (i.e., gluons plus quarks), the mixing of glueball and quark-antiquark configurations with the same quantum number occurs, rendering the identification of the resonances listed in the Particle Data Group [3] more difficult. The search for states that are (predominantly) glueball represents an active experimental and theoretical area of research; see Ref. [4] and references therein. The reason for these efforts is that a better understanding of the glueball properties would represent an important step in the comprehension of the nonperturbative behavior of QCD. However, although up to now some glueball candidates exist (see below), no state that is (predominantly) glueball has been unambiguously identified.

In general, a glueball state should fulfill two properties regarding its decays: it exhibits “flavor blindness,” because the gluons couple with the same strength to all quark flavors, and it is narrow, because QCD in the large- N_c limit shows that all glueball decay widths scale as N_c^{-2} , which should be compared to the N_c^{-1} scaling law for a quark-antiquark state. The lightest glueball state predicted by lattice QCD simulations is a scalar-isoscalar state ($J^{PC} = 0^{++}$) with a mass of about 1.7 GeV [2]. The resonance $f_0(1500)$ shows a flavor-blind decay pattern and is narrow, thus representing a good candidate for a state that is (predominantly) a scalar glueball. Also the resonance $f_0(1710)$ is a glueball

candidate because its mass is very close to lattice QCD predictions, and it is copiously produced in the gluon-rich decay of the J/ψ meson. Both scenarios have been investigated in a variety of works, e.g., Refs. [5–9] and references therein, in which mixing patterns involving the scalar resonances $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ are considered. In particular, in Ref. [9] the decays of the J/ψ have been included in a phenomenological fit and both assignments turn out to be consistent, but slightly favor a predominant gluonic amount in $f_0(1500)$.

The second lightest lattice-predicted glueball state has tensor quantum numbers ($J^{PC} = 2^{++}$) and a mass of about 2.2 GeV; a good candidate could be the very narrow resonance $f_J(2200)$ [10,11], if the total spin of the latter will be experimentally confirmed to be $J = 2$.

The third least massive glueball predicted by lattice QCD (in the quenched approximation) has pseudoscalar quantum numbers ($J^{PC} = 0^{-+}$) and a mass of about 2.6 GeV. Quite remarkably, most theoretical works investigating the pseudoscalar glueball did not take into account this prediction of Yang-Mills lattice studies but concentrated their search around 1.5 GeV in connection with the isoscalar-pseudoscalar resonances $\eta(1295)$, $\eta(1405)$, and $\eta(1475)$. A candidate for a predominantly light pseudoscalar glueball is the middle-lying state $\eta(1405)$ due to the fact that it is largely produced in (gluon-rich) J/ψ radiative decays and is missing in $\gamma\gamma$ reactions [12]. In this framework the resonances $\eta(1295)$ and $\eta(1475)$ represent radial excitations of the resonances η and η' . Indeed, in relation to η and η' , a lot of work has been done in determining the gluonic amount of their wave functions. The KLOE Collaboration found that the pseudoscalar glueball fraction in the mixing of the pseudoscalar-isoscalar states η and η' can be large ($\sim 14\%$) [13], but the theoretical work of Ref. [14] found that the glueball amount in η and η' is compatible with zero (see, however, also Ref. [15]).

In this work we study the decay properties of a pseudoscalar glueball state whose mass lies, in agreement with lattice QCD, between 2 and 3 GeV. Following Ref. [16] we

write down an effective chiral Lagrangian that couples the pseudoscalar glueball field (denoted as \tilde{G}) to scalar and pseudoscalar mesons. We can thus evaluate the widths for the decays $\tilde{G} \rightarrow PPP$ and $\tilde{G} \rightarrow PS$, where P and S stand for pseudoscalar and scalar quark-antiquark states. The pseudoscalar state P refers to the well-known light pseudoscalars $\{\pi, K, \eta, \eta'\}$, while the scalar state S refers to the quark-antiquark nonet of scalars above 1 GeV: $\{a_0(1450), K_0^*(1430), f_0(1370), f_0(1500) \text{ or } f_0(1710)\}$. The reason for the latter assignment is a growing consensus that the chiral partners of the pseudoscalar states should not be identified with the resonances below 1 GeV; see Refs. [6,17,18] for results within the so-called extended linear sigma model and also other theoretical works in Ref. [5,19–21] (and references therein).

The chiral Lagrangian that we construct contains one unknown coupling constant which cannot be determined without experimental data. However, the branching ratios can be unambiguously calculated and may represent a useful guideline for experimental search of the pseudoscalar glueball in the energy region between 2 and 3 GeV. In this respect, the planned PANDA experiment at the Facility for Antiproton and Ion Research [22] will be capable of scanning the mass region above 2.5 GeV. The experiment is based on proton-antiproton scattering, thus the pseudoscalar glueball \tilde{G} can be directly produced as an intermediate state. We shall therefore present our results for the branching ratios for a putative pseudoscalar glueball with a mass of 2.6 GeV.

On the other hand, it is also possible that the pseudoscalar glueball \tilde{G} has a mass that is a bit lower than the lattice QCD prediction and that it has been already observed in the BESIII experiment where pseudoscalar resonances have been investigated in J/ψ decays [23]. In particular, the resonance $X(2370)$, which has been clearly observed in the $\pi^+\pi^-\eta'$ channel, represents a good candidate, because it is quite narrow (~ 80 MeV) and its mass lies just below the lattice QCD prediction. For this reason we repeat our calculation for a pseudoscalar glueball mass of 2.37 GeV and thus make predictions for the resonance $X(2370)$, which can be tested in the near future.

This paper is organized as follows. In Sec. II we present the effective Lagrangian coupling the pseudoscalar glueball to scalar and pseudoscalar quark-antiquark degrees of freedom, and we calculate the branching ratios for the decays into PPP and SP . In Sec. III we present our conclusions and an outlook.

II. THE EFFECTIVE LAGRANGIAN

Following Ref. [16] we introduce a chiral Lagrangian that couples the pseudoscalar glueball $\tilde{G} \equiv |gg\rangle$ with quantum numbers $J^{PC} = 0^{-+}$ to scalar and pseudoscalar mesons

$$\mathcal{L}_{\tilde{G}}^{\text{int}} = ic_{\tilde{G}\Phi} \tilde{G} (\det \Phi - \det \Phi^\dagger), \quad (1)$$

where $c_{\tilde{G}\Phi}$ is a coupling constant,

$$\Phi = (S^a + iP^a)t^a \quad (2)$$

represents the multiplet of scalar and pseudoscalar quark-antiquark states, and t^a are the generators of the group $U(N_f)$. In this work we consider the case $N_f = 3$, and the explicit representation of the scalar and pseudoscalar mesons reads [18,24]

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_S^+ + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_S^0 + iK^0 \\ K_S^- + iK^- & \bar{K}_S^0 + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}. \quad (3)$$

Under $U_L(3) \times U_R(3)$ chiral transformations the multiplet Φ transforms as $\Phi \rightarrow U_L \Phi U_R^\dagger$ where U_L and U_R are $U(3)$ matrices. The determinant of Φ is invariant under $SU(3)_L \times SU(3)_R$ but not under $U(1)_A$. On the other hand, the pseudoscalar glueball field \tilde{G} is invariant under $U(3)_L \times U(3)_R$ transformations. Under parity, $\Phi \rightarrow \Phi^\dagger$ and $\tilde{G} \rightarrow -\tilde{G}$, thus the effective Lagrangian of Eq. (1) is invariant under $SU(3)_L \times SU(3)_R$ and under parity. Notice that Eq. (1) is not invariant under $U_A(1)$, in agreement with the so-called axial anomaly in the isoscalar-pseudoscalar sector. The rest of the mesonic Lagrangian, which describes the interactions of Φ and also includes (axial-)vector degrees of freedom, is presented in Part 1 of the Appendix. For more details, see Refs. [18,24,25].

The assignment of the quark-antiquark fields in this paper is as follows. (i) In the pseudoscalar sector the fields $\tilde{\pi}$ and K represent the pions or the kaons, respectively [3]. The bare fields $\eta_N \equiv |\bar{u}u + \bar{d}d\rangle/\sqrt{2}$ and $\eta_S \equiv |\bar{s}s\rangle$ are the nonstrange and strange contributions of the physical states η and η' [3]:

$$\begin{aligned} \eta &= \eta_N \cos \varphi + \eta_S \sin \varphi, \\ \eta' &= -\eta_N \sin \varphi + \eta_S \cos \varphi, \end{aligned} \quad (4)$$

where $\varphi \simeq -44.6^\circ$ is the mixing angle [24]. Using other values for the mixing angle, e.g., $\varphi = -36^\circ$ [26] or $\varphi = -41.4^\circ$, as determined by the KLOE Collaboration [13], affects the presented results only marginally. (ii) In the scalar sector we assign the field \tilde{a}_0 to the physical isotriplet state $a_0(1450)$ and the scalar kaon fields K_S to the resonance $K_0^*(1430)$. As a first approximation, the nonstrange bare field $\sigma_N \equiv |\bar{u}u + \bar{d}d\rangle/\sqrt{2}$ is assigned to the physical isoscalar resonance $f_0(1370)$ and the bare field $\sigma_S \equiv |\bar{s}s\rangle$ is assigned either to $f_0(1710)$ or to $f_0(1500)$. In a more complete framework, σ_N , σ_S and a bare scalar glueball field G mix and generate the physical resonances $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$; see the discussion below.

In order to evaluate the decays of the pseudoscalar glueball \tilde{G} we have to take into account that the spontaneous breaking of chiral symmetry takes place, which implies the need of shifting the scalar-isoscalar fields by their vacuum expectation values ϕ_N and ϕ_S ,

$$\sigma_N \rightarrow \sigma_N + \phi_N \quad \text{and} \quad \sigma_S \rightarrow \sigma_S + \phi_S. \quad (5)$$

In addition, when (axial-)vector mesons are present in the Lagrangian, one also has to “shift” the axial-vector fields and to define the wave-function renormalization constants of the pseudoscalar fields:

$$\tilde{\pi} \rightarrow Z_\pi \tilde{\pi}, \quad K^i \rightarrow Z_K K^i, \quad \eta_j \rightarrow Z_{\eta_j} \eta_j, \quad (6)$$

where $i = 1, 2, 3, 4$ runs over the four kaonic fields and $j = N, S$. The numerical values of the renormalization constants are $Z_\pi = 1.709$, $Z_K = 1.604$, $Z_{K_S} = 1.001$, $Z_{\eta_N} = Z_\pi$, and $Z_{\eta_S} = 1.539$ [24]. Moreover, the condensates ϕ_N and ϕ_S read

$$\begin{aligned} \phi_N &= Z_\pi f_\pi = 0.158 \text{ GeV}, \\ \phi_S &= \frac{2Z_K f_K - \phi_N}{\sqrt{2}} = 0.138 \text{ GeV}, \end{aligned} \quad (7)$$

where the standard values $f_\pi = 0.0922 \text{ GeV}$ and $f_K = 0.110 \text{ GeV}$ have been used [3]. Once the operations in Eqs. (5) and (6) have been performed, the Lagrangian in Eq. (1) contains the relevant tree-level vertices for the decay processes of \tilde{G} ; see the Appendix (Part 2).

The branching ratios of \tilde{G} for the decays into three pseudoscalar mesons are reported in Table I for both choices of the pseudoscalar masses, 2.6 and 2.37 GeV (relevant for PANDA and BESIII experiments, respectively). The branching ratios are presented relative to the total decay width of the pseudoscalar glueball $\Gamma_{\tilde{G}}^{\text{tot}}$. (For details of the calculation of the three-body decay we refer to Part 3 of the Appendix.)

Next we turn to the decay process $\tilde{G} \rightarrow PS$. The results, for both choices of $M_{\tilde{G}}$, are reported in Table II for the cases in which the bare resonance σ_S is assigned to $f_0(1710)$ or to $f_0(1500)$.

Concerning the decays involving scalar-isoscalar mesons, one should go beyond the results of Table II by including the full mixing pattern above 1 GeV, in which the resonances $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ are mixed states of the bare quark-antiquark contributions

TABLE I. Branching ratios for the decay of the pseudoscalar glueball \tilde{G} into three pseudoscalar mesons.

Quantity	Case (i):	Case (ii):
	$M_{\tilde{G}} = 2.6 \text{ GeV}$	$M_{\tilde{G}} = 2.37 \text{ GeV}$
$\Gamma_{\tilde{G} \rightarrow KK\eta} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.049	0.043
$\Gamma_{\tilde{G} \rightarrow KK\eta'} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.019	0.011
$\Gamma_{\tilde{G} \rightarrow \eta\eta\eta} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.016	0.013
$\Gamma_{\tilde{G} \rightarrow \eta\eta\eta'} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.0017	0.00082
$\Gamma_{\tilde{G} \rightarrow \eta\eta'\eta'} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.00013	0
$\Gamma_{\tilde{G} \rightarrow KK\pi} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.47	0.47
$\Gamma_{\tilde{G} \rightarrow \eta\pi\pi} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.16	0.17
$\Gamma_{\tilde{G} \rightarrow \eta'\pi\pi} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.095	0.090

TABLE II. Branching ratios for the decay of the pseudoscalar glueball \tilde{G} into a scalar and a pseudoscalar meson. In the last two rows σ_S is assigned to $f_0(1710)$ or to $f_0(1500)$ (values in the parentheses).

Quantity	Case (i):	Case (ii):
	$M_{\tilde{G}} = 2.6 \text{ GeV}$	$M_{\tilde{G}} = 2.37 \text{ GeV}$
$\Gamma_{\tilde{G} \rightarrow KK_S} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.060	0.070
$\Gamma_{\tilde{G} \rightarrow a_0\pi} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.083	0.10
$\Gamma_{\tilde{G} \rightarrow \eta\sigma_N} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.0000026	0.0000030
$\Gamma_{\tilde{G} \rightarrow \eta'\sigma_N} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.039	0.026
$\Gamma_{\tilde{G} \rightarrow \eta\sigma_S} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.012 (0.015)	0.0094 (0.017)
$\Gamma_{\tilde{G} \rightarrow \eta'\sigma_S} / \Gamma_{\tilde{G}}^{\text{tot}}$	0 (0.0082)	0 (0)

$\sigma_N \equiv |\bar{u}u + \bar{d}d\rangle/\sqrt{2}$ and σ_S and a bare scalar glueball field G . This mixing is described by an orthogonal (3×3) matrix [5–9]. In view of the fact that a complete evaluation of this mixing in the framework of our chiral approach has not yet been done, we use the two solutions for the mixing matrix of Ref. [7] and the solution of Ref. [8] in order to evaluate the decays of the pseudoscalar glueball into the three scalar-isoscalar resonances $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$. In all three solutions $f_0(1370)$ is predominantly described by the bare configuration $\sigma_N \equiv |\bar{u}u + \bar{d}d\rangle/\sqrt{2}$, but the assignments for the other resonances vary: in the first solution of Ref. [7] the resonance $f_0(1500)$ is predominantly gluonic, while in the second solution of Ref. [7] and the solution of Ref. [8] the resonance $f_0(1710)$ has the largest gluonic content. The results for the decay of the pseudoscalar glueball into scalar-isoscalar resonances are reported in Table III.

In Fig. 1 we show the behavior of the total decay width $\Gamma_{\tilde{G}}^{\text{tot}} = \Gamma_{\tilde{G} \rightarrow PPP} + \Gamma_{\tilde{G} \rightarrow PS}$ as function of the coupling constant $c_{\tilde{G}\Phi}$ for both choices of the pseudoscalar glueball mass. (We assume here that other decay channels, such as decays into vector mesons or baryons are negligible.) In the case of $M_{\tilde{G}} = 2.6 \text{ GeV}$, one expects from large- N_c considerations that the total decay width $\Gamma_{\tilde{G}}^{\text{tot}} \lesssim 100 \text{ MeV}$. In fact, as discussed in the Introduction, the scalar glueball candidate $f_0(1500)$ is roughly 100 MeV broad and the tensor candidate $f_J(2220)$ is even narrower. In the present work, the condition $\Gamma_{\tilde{G}}^{\text{tot}} \lesssim 100 \text{ MeV}$ implies that $c_{\tilde{G}\Phi} \lesssim 5$. Moreover, in the case of $M_{\tilde{G}} = 2.37 \text{ GeV}$ in which the identification $\tilde{G} \equiv X(2370)$ has been made, we can indeed use the experimental knowledge on the full decay width ($\Gamma_{X(2370)} = 83 \pm 17 \text{ MeV}$ [23]) to determine the coupling constant to be $c_{\tilde{G}\Phi} = 4.48 \pm 0.46$. (However, we also refer to the recent work of Ref. [27], where the possibility of a broad pseudoscalar glueball is discussed.)

Some comments are in order:

- (i) The results depend only slightly on the glueball mass, thus the two columns of Tables I and II are similar. It turns out that the channel $KK\pi$ is the dominant one (almost 50%). Also the $\eta\pi\pi$ and

TABLE III. Branching ratios for the decays of the pseudoscalar glueball \tilde{G} into η and η' , respectively, and one of the scalar-isoscalar states: $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ by using three different mixing scenarios of these scalar-isoscalar states reported in Refs. [7,8]. The mass of the pseudoscalar glueball is $M_{\tilde{G}} = 2.6$ GeV and $M_{\tilde{G}} = 2.37$ GeV (values in the brackets), respectively.

Quantity	Solution 1 of Ref. [7]	Solution 2 of Ref. [7]	Solution of Ref. [8]
$\Gamma_{\tilde{G} \rightarrow \eta f_0(1370)} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.00093 (0.0011)	0.00058 (0.00068)	0.0044 (0.0052)
$\Gamma_{\tilde{G} \rightarrow \eta f_0(1500)} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.000046 (0.000051)	0.0082 (0.0090)	0.011 (0.012)
$\Gamma_{\tilde{G} \rightarrow \eta f_0(1710)} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.011 (0.0089)	0.0053 (0.0042)	0.00037 (0.00029)
$\Gamma_{\tilde{G} \rightarrow \eta' f_0(1370)} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.038 (0.026)	0.033 (0.022)	0.043 (0.029)
$\Gamma_{\tilde{G} \rightarrow \eta' f_0(1500)} / \Gamma_{\tilde{G}}^{\text{tot}}$	0.0062 (0)	0.00020 (0)	0.00013 (0)
$\Gamma_{\tilde{G} \rightarrow \eta' f_0(1710)} / \Gamma_{\tilde{G}}^{\text{tot}}$	0 (0)	0 (0)	0 (0)

$\eta' \pi \pi$ channels are sizable. On the contrary, the two-body decays are subdominant and reach only 20% of the full mesonic decay width.

- (ii) The decay of the pseudoscalar glueball into three pions vanishes:

$$\Gamma_{\tilde{G} \rightarrow \pi \pi \pi} = 0. \quad (8)$$

This result represents a further testable prediction of our approach.

- (iii) The decays of the pseudoscalar glueball into a scalar-isoscalar meson amount only to 5% of the total decay width. Moreover, the mixing pattern in the scalar-isoscalar sector has a negligible influence on the total decay width of \tilde{G} . Nevertheless, in the future it may represent an interesting and additional test for scalar-isoscalar states.
- (iv) Once the shifts of the scalar fields have been performed, there are also bilinear mixing terms of the form $\tilde{G} \eta_N$ and $\tilde{G} \eta_S$ that lead to a nondiagonal mass matrix. In principle, one should take these terms into account, in addition to the already mentioned

$\eta_N \eta_S$ mixing, and solve a three-state mixing problem in order to determine the masses of the pseudoscalar particles. This will also affect the calculation of the decay widths. However, due to the large mass difference of the bare glueball fields \tilde{G} to the other quark-antiquark pseudoscalar fields, the mixing of \tilde{G} turns out to be very small in the present work, and can be safely neglected. For instance, it turns out that the mass of the mixed state which is predominantly glueball is (at most) just 0.002 GeV larger than the bare mass $M_{\tilde{G}} = 2.6$ GeV.

- (v) If a standard linear sigma model without (axial-) vector mesons is studied, the replacements $Z_\pi = Z_K = Z_{\eta_N} = Z_{\eta_S} = 1$ need to be performed. Most of the results of the branching ratios for the three-body decay are qualitatively, but not quantitatively, similar to the values of Table I (variations of about 25%–30%). However, the branching ratios for the two-body decay change sizably with respect to the results of Table II. This fact shows once more that the inclusion of (axial-)vector degrees of freedom has sizable effects also concerning the decays of the pseudoscalar glueball.
- (vi) In principle, the three-body final states for the decays shown in Table I can also be reached through a sequential decay from the two-body final states shown in Table II, where the scalar particle S further decays into PP , for instance, $K_0^*(1430) \rightarrow K\pi$. There are then two possible decay amplitudes, one from the direct three-body decay and one from the sequential decay, which have to be added coherently before taking the modulus square to obtain the total three-body decay width. Summing the results shown in Tables I and II gives a first estimate (which neglects interference terms) for the magnitude of the total three-body decay width. We have verified that the correction from the interference term to this total three-body decay width in a given channel is at most of the order of 10% for $M_{\tilde{G}} = 2.6$ GeV and 15% for $M_{\tilde{G}} = 2.37$ GeV. For a full

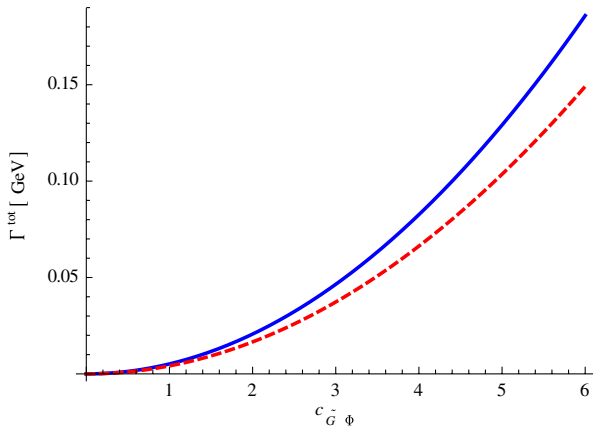


FIG. 1 (color online). Solid (blue) line: Total decay width of the pseudoscalar glueball with the bare mass $M_{\tilde{G}} = 2.6$ GeV as function of the coupling $c_{\tilde{G}\phi}$. Dashed (red) line: Same curve for $M_{\tilde{G}} = 2.37$ GeV.

understanding of the contribution of the various decay amplitudes to the final three-body state, one needs to perform a detailed study of the Dalitz plot for the three-body decay.

III. CONCLUSIONS AND OUTLOOK

In this work we have presented a chirally invariant effective Lagrangian describing the interaction of the pseudoscalar glueball with scalar and pseudoscalar mesons for the three-flavor case $N_f = 3$. We have studied the decays of the pseudoscalar glueball into three pseudoscalar quark-antiquark fields and into a scalar and pseudoscalar quark-antiquark field.

The branching ratios are parameter free once the mass of the glueball has been fixed. We have considered two possibilities. (i) In agreement with lattice QCD in the quenched approximation we have chosen $M_{\bar{G}} = 2.6 \text{ GeV}$. The existence and the decay properties of such a hypothetical pseudoscalar resonance can be tested in the upcoming PANDA experiment [22]. (ii) We assumed that the resonance $X(2370)$, measured in the experiment BESIII, is (predominantly) a pseudoscalar glueball state, and thus we have also used a mass of 2.37 GeV [23]. The results for both possibilities have been summarized in Tables I and II: we predict that $KK\pi$ is the dominant decay channel, followed by (almost equally large) $\eta\pi\pi$ and $\eta'\pi\pi$ decay channels. On the contrary, the decay into three pions is predicted to vanish. In the case of BESIII, by measuring the branching ratio for other decay channels than the measured $\eta'\pi\pi$, one could ascertain if $X(2370)$ is (predominantly) a

pseudoscalar glueball. In the case of PANDA, our results may represent a useful guideline for the search of the pseudoscalar glueball.

Future studies should consider possible mixing of the pseudoscalar glueball with charmonia states and an improved description of the scalar-isoscalar sector. New lattice results for the pseudoscalar glueball mass, which go beyond the quenched approximation and include the effect of dynamical fermions, would be very useful for model building. Moreover, the mechanism of the glueball production via proton-antiproton annihilation using the so-called mirror assignment [28] represents an interesting outlook [29].

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APPENDIX: DETAILS OF THE CALCULATION

1. The full mesonic Lagrangian

The chirally invariant $U(N_f)_L \times U(N_f)_R$ Lagrangian for the low-lying mesonic states with (pseudo)scalar and (axial-)vector quantum numbers has the form

$$\begin{aligned} \mathcal{L}_{\text{mes}} = & \text{Tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 - \frac{1}{4} \text{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] \\ & + \text{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \text{Tr}[H(\Phi + \Phi^\dagger)] + c_1(\det \Phi - \det \Phi^\dagger)^2 + i\frac{g_2}{2}\{\text{Tr}(L_{\mu\nu}[L^\mu, L^\nu]) \\ & + \text{Tr}(R_{\mu\nu}[R^\mu, R^\nu])\} + \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(L_\mu^2 + R_\mu^2) + h_2 \text{Tr}[|L_\mu \Phi|^2 + |\Phi R_\mu|^2] + 2h_3 \text{Tr}(L_\mu \Phi R^\mu \Phi^\dagger), \quad (\text{A1}) \end{aligned}$$

where

$$L_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N^\mu + \rho^{\mu 0}}{\sqrt{2}} + \frac{f_{1N}^\mu + a_1^{\mu 0}}{\sqrt{2}} & \rho^{\mu+} + a_1^{\mu+} & K^{*\mu+} + K_1^{\mu+} \\ \rho^{\mu-} + a_1^{\mu-} & \frac{\omega_N^\mu - \rho^{\mu 0}}{\sqrt{2}} + \frac{f_{1N}^\mu - a_1^{\mu 0}}{\sqrt{2}} & K^{*\mu 0} + K_1^{\mu 0} \\ K^{*\mu-} + K_1^{\mu-} & \bar{K}^{*\mu 0} + \bar{K}_1^{\mu 0} & \omega_S^\mu + f_{1S}^\mu \end{pmatrix}$$

and

$$R_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N^\mu + \rho^{\mu 0}}{\sqrt{2}} - \frac{f_{1N}^\mu + a_1^{\mu 0}}{\sqrt{2}} & \rho^{\mu+} - a_1^{\mu+} & K^{*\mu+} - K_1^{\mu+} \\ \rho^{\mu-} - a_1^{\mu-} & \frac{\omega_N^\mu - \rho^{\mu 0}}{\sqrt{2}} - \frac{f_{1N}^\mu - a_1^{\mu 0}}{\sqrt{2}} & K^{*\mu 0} - K_1^{\mu 0} \\ K^{*\mu-} - K_1^{\mu-} & \bar{K}^{*\mu 0} - \bar{K}_1^{\mu 0} & \omega_S^\mu - f_{1S}^\mu \end{pmatrix}.$$

For details see Refs. [6,18,24,25]. In the present context we are interested in the wave-function renormalization constants Z_i introduced in Eq. (6). Their explicit expressions read [24,25]

$$Z_\pi = Z_{\eta_N} = \frac{m_{a_1}}{\sqrt{m_{a_1}^2 - g_1^2 \phi_N^2}}, \quad Z_K = \frac{2m_{K_1}}{\sqrt{4m_{K_1}^2 - g_1^2(\phi_N + \sqrt{2}\phi_S)^2}}, \quad (\text{A2})$$

$$Z_{K_S} = \frac{2m_{K^*}}{\sqrt{4m_{K^*}^2 - g_1^2(\phi_N - \sqrt{2}\phi_S)^2}}, \quad Z_{\eta_S} = \frac{m_{f_{1S}}}{\sqrt{m_{f_{1S}}^2 - 2g_1^2 \phi_S^2}}. \quad (\text{A3})$$

2. Explicit form of the Lagrangian in Eq. (1)

After performing the field transformations in Eqs. (5) and (6), the effective Lagrangian (1) takes the form

$$\begin{aligned} \mathcal{L}_{\tilde{G}}^{\text{int}} = & \frac{c_{\tilde{G}\Phi}}{2\sqrt{2}} \tilde{G} (\sqrt{2}Z_{K_S}Z_K a_0^0 K_S^0 \bar{K}^0 + \sqrt{2}Z_K Z_{K_S} a_0^0 K^0 \bar{K}_S^0 - 2Z_{K_S}Z_K a_0^+ K_S^0 K^- - 2Z_{K_S}Z_K a_0^+ K_S^- K^0 - 2Z_{K_S}Z_K a_0^- \bar{K}_S^0 K^+ \\ & - \sqrt{2}Z_{K_S}Z_K a_0^0 K_S^- K^+ - \sqrt{2}Z_K^2 Z_{\eta_N} K^0 \bar{K}^0 \eta_N + \sqrt{2}Z_{K_S}^2 Z_{\eta_N} K_S^0 \bar{K}_S^0 \eta_N - \sqrt{2}Z_K^2 Z_{\eta_N} K^- K^+ \eta_N + Z_{\eta_S} a_0^{02} \eta_S \\ & + 2Z_{\eta_S} a_0^- a_0^+ \eta_S + Z_{\eta_N}^2 Z_{\eta_S} \eta_N^2 \eta_S - \sqrt{2}Z_K^2 Z_\pi K^0 \bar{K}^0 \pi^0 + \sqrt{2}Z_{K_S}^2 Z_\pi K_S^0 \bar{K}_S^0 \pi^0 + \sqrt{2}Z_K^2 Z_\pi K^- K^+ \pi^0 - Z_{\eta_S} Z_\pi^2 \eta_S \pi^{02} \\ & + 2Z_K^2 Z_\pi \bar{K}^0 K^+ \pi^- + 2Z_K^2 Z_\pi K^0 K^- \pi^+ - 2Z_{K_S}^2 Z_\pi K_S^0 K_S^- \pi^+ - 2Z_{\eta_S} Z_\pi^2 \eta_S \pi^- \pi^+ - 2Z_{K_S}Z_K a_0^- K_S^+ \bar{K}^0 \\ & + \sqrt{2}Z_{K_S}^2 Z_{\eta_N} K_S^+ K_S^- \eta_N - \sqrt{2}Z_K^2 Z_\pi K_S^+ K_S^- \pi^0 - 2Z_{K_S}^2 Z_\pi K_S^+ \bar{K}_S^0 \pi^- - \sqrt{2}Z_{K_S}Z_K a_0^0 K_S^+ K^- + \sqrt{2}Z_K Z_{K_S} K^- K_S^+ \phi_N \\ & + \sqrt{2}Z_K Z_{K_S} K^- K_S^+ \sigma_N + \sqrt{2}Z_{K_S}Z_K K_S^0 \bar{K}^0 \phi_N + \sqrt{2}Z_{K_S}Z_K K_S^0 \bar{K}^0 \sigma_N + \sqrt{2}Z_K Z_{K_S} K^0 \bar{K}_S^0 \phi_N + \sqrt{2}Z_K Z_{K_S} K^0 \bar{K}_S^0 \sigma_N \\ & + \sqrt{2}Z_{K_S}Z_K K_S^- K^+ \phi_N + \sqrt{2}Z_{K_S}Z_K K_S^- K^+ \sigma_N - Z_{\eta_S} \eta_S \phi_N^2 - Z_{\eta_S} \eta_S \sigma_N^2 - 2Z_{\eta_S} \eta_S \phi_N \sigma_N + 2Z_\pi a_0^0 \pi^0 \phi_S \\ & + 2Z_\pi a_0^0 \pi^0 \sigma_S + 2Z_\pi a_0^+ \pi^- \phi_S + 2Z_\pi a_0^+ \pi^- \sigma_S + 2Z_\pi a_0^- \pi^+ \phi_S + 2Z_\pi a_0^- \pi^+ \sigma_S - 2Z_{\eta_N} \eta_N \phi_N \phi_S \\ & - 2Z_{\eta_N} \eta_N \phi_N \sigma_S - 2Z_{\eta_N} \eta_N \sigma_N \phi_S - 2Z_{\eta_N} \eta_N \sigma_N \sigma_S). \end{aligned} \quad (\text{A4})$$

The latter expression is used to determine the coupling of the field \tilde{G} to scalar and pseudoscalar mesons.

3. Three-body decay

For completeness we report the explicit expression for the three-body decay width for the process $\tilde{G} \rightarrow P_1 P_2 P_3$ [3]:

$$\Gamma_{\tilde{G} \rightarrow P_1 P_2 P_3} = \frac{s_{\tilde{G} \rightarrow P_1 P_2 P_3}}{32(2\pi)^3 M_{\tilde{G}}^3} \int_{(m_1+m_2)^2}^{(M_{\tilde{G}}-m_3)^2} dm_{12}^2 \int_{(m_{23})_{\min}}^{(m_{23})_{\max}} |-i\mathcal{M}_{\tilde{G} \rightarrow P_1 P_2 P_3}|^2 dm_{23}^2,$$

where

$$(m_{23})_{\min} = (E_2^* + E_3^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} + \sqrt{E_3^{*2} - m_3^2} \right)^2, \quad (\text{A5})$$

$$(m_{23})_{\max} = (E_2^* + E_3^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} - \sqrt{E_3^{*2} - m_3^2} \right)^2, \quad (\text{A6})$$

and

$$E_2^* = \frac{m_{12}^2 - m_1^2 + m_2^2}{2m_{12}}, \quad E_3^* = \frac{M_{\tilde{G}}^2 - m_{12}^2 - m_3^2}{2m_{12}}. \quad (\text{A7})$$

The quantities m_1 , m_2 , m_3 refer to the masses of the three pseudoscalar states P_1 , P_2 , and P_3 , $\mathcal{M}_{\tilde{G} \rightarrow P_1 P_2 P_3}$ is the corresponding tree-level decay amplitude, and $s_{\tilde{G} \rightarrow P_1 P_2 P_3}$ is a symmetrization factor (it equals 1 if all P_1 , P_2 , and P_3 are different, it equals 2 for two identical particles in the final state, and it equals 6 for three identical particles in the final state). For instance, in the case $\tilde{G} \rightarrow K^- K^+ \pi^0$ one has $|-i\mathcal{M}_{\tilde{G} \rightarrow K^- K^+ \pi^0}|^2 = \frac{1}{4} c_{\tilde{G}\Phi}^2 Z_K^4 Z_\pi^2$, $m_1 = m_2 = m_K = 0.494$ GeV, $m_3 = m_{\pi^0} = 0.135$ GeV, and $M_{\tilde{G}} = 2.6$ GeV. Then,

$$\Gamma_{\tilde{G} \rightarrow K^- K^+ \pi^0} = 0.00041 c_{\tilde{G}\Phi}^2 [\text{GeV}]. \quad (\text{A8})$$

The full decay width into the channel $KK\pi$ results from the sum

$$\Gamma_{\tilde{G} \rightarrow KK\pi} = \Gamma_{\tilde{G} \rightarrow K^- K^+ \pi^0} + \Gamma_{\tilde{G} \rightarrow K^0 \bar{K}^0 \pi^0} + \Gamma_{\tilde{G} \rightarrow \bar{K}^0 K^+ \pi^-} + \Gamma_{\tilde{G} \rightarrow K^0 K^- \pi^+} = 6\Gamma_{\tilde{G} \rightarrow K^- K^+ \pi^0}. \quad (\text{A9})$$

The other decay channels can be calculated in a similar way.

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