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Resonance catalyzed *CP* asymmetries in $D \to P\ell^+\ell^-$

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The recently observed increase of direct *CP* asymmetry in charm meson nonleptonic decays is difficult to explain within the standard model. If this effect is induced by new physics, this might be investigated in other charm processes. We propose to investigate new *CP*-violating effects in rare decays $D \to P\ell^+\ell^-$, which arise due to the interference of the resonant part of the long-distance contribution and the new physics affected short-distance contribution. Performing a model-independent analysis, we identify as appropriate observables the differential direct *CP* asymmetry and partial decay width *CP* asymmetry. We find that in the most promising decays $D^+ \to \pi^+\ell^+\ell^-$ and $D_s^+ \to K^+\ell^+\ell^-$ the "peak-symmetric" and "peak-antisymmetric" *CP* asymmetries are strongly phase dependent and can be of the order 1% and 10%, respectively.

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I. INTRODUCTION

In the last two decades chances to observe new physics in charm processes were considered to be very small. In the case of flavor changing neutral current processes the Glashow-Iliopoulos-Maiani mechanism plays a significant role, leading to cancellations of contributions of s and d quarks, while intermediate b quark contribution is suppressed by V_{ub} element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. However, this has changed at the end of last year when the LHCb experiment reported a nonvanishing direct *CP* asymmetry in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ [1] also confirmed by the CDF experiment [2]. The lack of appropriate theoretical tools to handle long-distance dynamics in these processes is even more pronounced than in the case of B mesons due to abundance of charmless resonances with the masses close to the masses of charm mesons. Many papers investigated whether this result can be accommodated within the standard model (SM) or is it new physics (NP) that causes such an effect. The measured difference between the *CP* asymmetry in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ is a factor 5–10 larger than expected in the SM and eventually can be a result of nonperturbative QCD dynamics as pointed out in Refs. [3–10]. Model-independent studies [11,12] indicated that among operators describing NP effect, the most likely candidate is the effective $\Delta C = 1$ chromomagnetic dipole operator. In order to distinguish between SM or NP scenarios as an explanation of the observed phenomena it is crucial to investigate experimentally and theoretically all possible processes in which the same operator might contribute. Recently the effects of the same kind of new physics have been explored in radiative [13] and inclusive charm decays with a lepton pair in the final state [14]. In Ref. [13] it was found that NP induces an enhancement of the matrix elements of the electromagnetic dipole operators leading to *CP* asymmetries of the order of few percent.

In addition to radiative weak decays, charm meson decays to a light meson and leptonic pair might serve as a testing ground for *CP*-violating new physics contributions. As in other weak decays of charm mesons the long-distance dynamics dominates the decay widths of $D \to P\ell^+\ell^-$ [15–17] and it requires a special task to find the appropriate variables containing mainly short-distance contributions. In this study we investigate partial decay width *CP* asymmetry in the case of $D \to P\ell^+\ell^-$ decay. The short-distance dynamics is described by effective operators $\mathcal{O}_7$, $\mathcal{O}_9$, and $\mathcal{O}_{10}$ of which the electromagnetic dipole operator $\mathcal{O}_7$ carries a *CP*-odd phase of beyond the SM origin, developed due to mixing under QCD renormalization with the chromomagnetic operator. In this paper we investigate impact of this mixing on the $D \to P\ell^+\ell^-$ decay dynamics. The paper is organized as follows: Section II contains the description of the short-distance contributions and hadronic form factors; Sec. III is devoted to the long-distance dynamics. In Sec. IV we present the partial-width asymmetry. We summarize our findings in Sec. V.

II. EFFECTIVE HAMILTONIAN AND SHORT-DISTANCE AMPLITUDE

The dynamics of $c \to u\ell^+\ell^-$ decay on scale $\sim m_c$ is defined by the effective Hamiltonian [11,15]

$$\mathcal{H}_{\rm eff} = \lambda_d \mathcal{H}^d + \lambda_s \mathcal{H}^s + \lambda_b \mathcal{H}^{\rm peng}, \qquad (1)$$

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where the CKM weights are $\lambda_i = V_{ci}^* V_{ui}$. For the first two generations we have the current-current operators

$$\mathcal{H}^{q=d,s} = -\frac{4G_F}{\sqrt{2}}(C_1\mathcal{O}_1^q + C_2\mathcal{O}_2^q),$$
$$\mathcal{O}_1^q = (\bar{q}_L^\alpha \gamma^\mu c_L^\alpha)(\bar{u}_L^\beta \gamma_\mu q_L^\beta), \qquad (2)$$
$$\mathcal{O}_2^q = (\bar{q}_L^\alpha \gamma^\mu c_L^\beta)(\bar{u}_L^\beta \gamma_\mu q_L^\alpha),$$

with color indices α, β. The effects of b quark and heavier particles are contained within the set operators of dimension 5 and 6

$$\mathcal{H}^{\text{peng}} = -\frac{4G_F}{\sqrt{2}} \sum_{i=3,\ldots,10} C_i \mathcal{O}_i, \qquad (3)$$

where electromagnetic (chromomagnetic) penguins and electroweak penguins or boxes with leptons are

$$\mathcal{O}_7 = \frac{e m_c}{(4\pi)^2} \bar{u}\sigma_{\mu\nu} P_R c F^{\mu\nu},$$
$$\mathcal{O}_8 = \frac{g m_c}{(4\pi)^2} \bar{u}\sigma_{\mu\nu} P_R c G^{\mu\nu},$$
$$\mathcal{O}_9 = \frac{e^2}{(4\pi)^2} (\bar{u}\gamma^\mu P_L c)(\bar{\ell}\gamma_\mu \ell), \qquad (4)$$
$$\mathcal{O}_{10} = \frac{e^2}{(4\pi)^2} (\bar{u}\gamma^\mu P_L c)(\bar{\ell}\gamma_\mu \gamma_5 \ell).$$

The complete set of QCD penguin operators $\mathcal{O}_{3,\ldots,6}$ can be found in Refs. [15,18]. Decay width spectrum of $c \to u\ell^+\ell^-$ is dominated by the two light generations' effective Hamiltonians $\mathcal{H}^{d,s}$ and is exactly CP even when $\lambda_d + \lambda_s = 0$ holds. Only when we include the third generation we get a possibility of having a nonvanishing imaginary part: $\text{Im}(\lambda_b/\lambda_d) = -\text{Im}(\lambda_s/\lambda_d)$. However, the CP-violating parts of the amplitude are suppressed by a tiny factor $\lambda_b/\lambda_d \sim 10^{-3}$ with respect to the CP-conserving ones and only tiny effects of CP violation are expected. On the other hand, too large direct CP is measured in singly Cabibbo suppressed decays $D^0 \to \pi\pi$, KK. Should this enhancement be due to new physics, one can most naturally satisfy other flavor constraints by assigning a NP contribution to the chromomagnetic operator $\mathcal{O}_8$ at some high scale above m_t [11]. In this case one must also get $C_7(m_c)$ that carries related new physics CP phase due to mixing of $\mathcal{O}_8$ into $\mathcal{O}_7$ under QCD renormalization. We shall consider the range proposed in [13],

$$|\text{Im}[\lambda_b C_7(m_c)]| = (0.2\text{–}0.8) \times 10^{-2}, \qquad (5)$$

where the authors used this particular value to estimate the size of direct CP violation in $D \to P\gamma$ decays. This approach was further scrutinized recently in Ref. [19].

We define the short-distance amplitude as the one coming from operators $\mathcal{O}_7$, $\mathcal{O}_9$, and $\mathcal{O}_{10}$ (they do not contain, apart from c and u fields, any colored degrees of freedom). While their contribution to the decay width is negligible in the resonance-dominated regions due to small CKM elements, possible imaginary parts of Wilson coefficients may generate direct CP violation via interference with the CP-even long-distance amplitude (that we define below). In light of the above discussion we will assume that in the short-distance (SD) amplitude only $\mathcal{O}_7$ carries a CP-violating phase. Relevant SD amplitude of $D \to \pi\ell^+\ell^-$, where $\ell = e, \mu$, is then

$$\mathcal{A}_{\text{SD}}^{\text{CPV}} = -\frac{i\sqrt{2}G_F\alpha}{\pi}\lambda_b C_7(m_c) \times \frac{m_c}{m_D + m_\pi} f_T(q^2)\bar{u}(k_-)\not{p}v(k_+), \qquad (6)$$

where p is momentum of the D meson and $q = k_- + k_+$ is momentum of the lepton pair. The form factors for $D \to \pi$ transition via vector current and electromagnetic dipole operators are defined as customary:

$$\langle \pi(p')|\bar{u}\gamma_\mu c|D(p)\rangle = \left[(p+p')_\mu - \frac{m_D^2 - m_\pi^2}{q^2} q_\mu\right] F_1(q^2) + \frac{m_D^2 - m_\pi^2}{q^2} q_\mu F_0(q^2),$$
$$\langle \pi(p')|\bar{u}\sigma_{\mu\nu} c|D(p)\rangle = -i(p_\mu p'_\nu - p_\nu p'_\mu)\frac{2f_T(q^2)}{m_D + m_\pi}, \qquad (7)$$

with $q^2 = (p - p')^2$.

Parameterization of the tensor form factor

The lattice QCD calculations of the form factors for the semileptonic $D \to \pi$ transitions are rather well known (see e.g., Ref. [20]) and their analysis are based on the use of z parametrization [21,22]. The z parametrization of the $D \to P$ form factors in practical use is often replaced by the Bečirević-Kaidalov parametrization [23] (as in Refs. [24,25]). Quenched lattice QCD results exist for $F_{1,0}$ as well as for the tensor form factor [26,27] and are presented in the Bečirević-Kaidalov parameterization:

$$F_1(q^2) = \frac{F_1(0)}{\left(1 - \frac{q^2}{m_{D^*}^2}\right)\left(1 - a\frac{q^2}{m_{D^*}^2}\right)}, \qquad F_0(q^2) = \frac{F_1(0)}{1 - \frac{1}{b}\frac{q^2}{m_{D^*}^2}}, \qquad (8)$$

$$F_1(0) = 0.57(6), \qquad a = 0.18(17), \qquad b = 1.27(17). \qquad (9)$$

For $f_T(q^2;\mu)$ it has recently been noted that in the high q^2 region the $B \to K$ matrix elements are well described by the nearest pole ansatz for form factors F_1 and f_T (see Appendix A of Ref. [28]). Analogously we expect a dominance of the D^* resonance for $F_1(q^2)$ and $f_T(q^2;\mu)$ close to the zero-recoil point and consequently the ratio of the two form factors becomes a constant. The following scale invariant function:

$$\tilde{f}_T(q^2) \equiv \frac{m_{D^*}}{m_D + m_\pi} \frac{f^V_{D^*}}{f^T_{D^*}(\mu)} f_T(q^2;\mu) \tag{10}$$

approaches $F_1(q^2)$ at large q^2. Here $f^V_{D^*}$ and $f^T_{D^*}(\mu)$ are the decay constants of D^* via the vector and tensor currents, respectively. A fit of the lattice data [26,27] to the Bečirević-Kaidalov shape

$$\begin{aligned} \tilde{f}_T(q^2) &= \frac{\tilde{f}_T(0)}{\left(1-\frac{q^2}{m^2_{D^*}}\right)\left(1-a_T\frac{q^2}{m^2_{D^*}}\right)}, \\ \tilde{f}_T(0) &= 0.56(5), \\ a_T &= 0.18(16), \end{aligned} \tag{11}$$

tells us that within the errors the form factor is single pole-like. Extrapolation to the low q^2 region, which is more relevant for our discussion, gives $f_T(q^2;\mu)/F_1(q^2)|_{q^2=0} = 0.83 \pm 0.19$ that is marginally compatible with the results expected in the large energy effective theory limit, where one expects the same ratio to be $1 + m_\pi/m_D = 1.07$ [29,30]. The ratio of tensor and vector decay constants, needed in formula (10) at the charm scale, is

$$\frac{f^T_{D^*}(\mu = 2\ \text{GeV})}{f^V_{D^*}} = 0.82(3). \tag{12}$$

III. LONG-DISTANCE AMPLITUDE

Close to the ϕ resonant peak the long-distance amplitude is, to a good approximation, driven by nonfactorizable contributions of four-quark operators in $\mathcal{H}^s$. The width of ϕ resonance is very narrow ($\Gamma_\phi/m_\phi \approx 4\times 10^{-3}$) and well separated from other vector resonances in the q^2 spectrum of $D \to P\ell^+\ell^-$. Relying on the vector meson dominance hypothesis the q^2 dependence of the decay spectrum close to the resonant peak follows the Breit-Wigner shape [15–17]

$$\begin{aligned} &\mathcal{A}^\phi_{\text{LD}}[D\to\pi\phi\to\pi\ell^-\ell^+] \\ &\quad = \frac{iG_F}{\sqrt{2}}\lambda_s \frac{8\pi\alpha}{3} a_\phi e^{i\delta_\phi} \frac{m_\phi \Gamma_\phi}{q^2 - m^2_\phi + i m_\phi \Gamma_\phi} \bar{u}(k_-) \not{p} v(k_+). \end{aligned} \tag{13}$$

Here we use $\alpha = 1/137$ in the leading order in electromagnetic interaction.

The long-distance amplitude is also affected by nonfactorizable effects of four-quark operators $\mathcal{O}_{3\text{–}6}$ and by the gluonic penguin operator $\mathcal{O}_8$. Whereas the former have only tiny *CP* violation and are suppressed with λ_b/λ_s compared to (13), the $\mathcal{O}_8$ contribution can be important for the results of this study since NP *CP*-odd phases present in Wilson coefficients C_7 and C_8 are closely related. Opposed to the $\mathcal{O}_7$ mediated amplitude with a single photon exchange the $\mathcal{O}_8$ amplitude necessarily involves a strong loop suppression factor of the order $\alpha_s(\mu = m_c)/\pi$ and is therefore subdominant in this perturbative picture. However, in the full nonperturbative treatment we cannot exclude an order of magnitude enhancement of amplitude with $\mathcal{O}_8$ insertion.[1] In this work we will neglect such contributions and therefore our conclusions will be quantitatively valid provided there is no nonperturbative enhancement of the $\mathcal{O}_8$ amplitude.

Finite width of the resonance generates a q^2-dependent strong phase that varies across the peak. We have also introduced the strong phase on peak, δ_ϕ, and the normalization a_ϕ that are both assumed to be independent of q^2. Parameter a_ϕ is real and can be fixed from measured branching fractions of $D \to \pi\phi$ and $\phi \to \ell^+\ell^-$ decays [17]. For definiteness we will focus on the $\ell = \mu$ decay modes. From the Particle Data Group compilation we read [36]

$$\begin{aligned} \text{Br}(D^+ \to \phi\pi^+) &= (2.65 \pm 0.09) \times 10^{-3}, \\ \text{Br}(\phi \to \mu^+\mu^-) &= (0.287 \pm 0.019) \times 10^{-3}, \end{aligned} \tag{14}$$

and when we take into account the small width of ϕ

$$\begin{aligned} &\text{Br}(D^+ \to \pi^+\phi(\to \mu^+\mu^-)) \\ &\qquad \approx \text{Br}(D^+ \to \phi\pi^+) \times \text{Br}(\phi \to \mu^+\mu^-), \end{aligned} \tag{15}$$

we find from Eq. (13)

$$a_\phi = 1.23 \pm 0.05. \tag{16}$$

IV. DIRECT *CP* ASYMMETRY

The direct *CP* violation in the resonant region is driven by the interference between the *CP*-odd imaginary part of the SD amplitude and the long-distance (LD) amplitude. The pair of *CP*-conjugated amplitudes read

$$\begin{aligned} \mathcal{A}(D^+ \to \pi^+\ell^+\ell^-) &= \mathcal{A}^\phi_{\text{LD}} + \mathcal{A}^{\text{CPV}}_{\text{SD}}, \\ \bar{\mathcal{A}}(D^- \to \pi^-\ell^+\ell^-) &= \mathcal{A}^\phi_{\text{LD}} + \bar{\mathcal{A}}^{\text{CPV}}_{\text{SD}}. \end{aligned} \tag{17}$$

In principle the short-distance amplitude contains a strong phase that can be rotated away because the overall phase of the total amplitude is irrelevant. The *CP*-odd part of the LD amplitude is proportional to the imaginary part of the relevant CKM factor λ_s that can be safely neglected and accordingly we have put $\mathcal{A}^\phi_{\text{LD}} = \bar{\mathcal{A}}^\phi_{\text{LD}}$. Then the differential direct *CP* violation reads

[1] Analogous nonfactorizable amplitudes of $\mathcal{O}_8$ in B physics have been studied in the framework of QCD factorization [31,32] in $B \to V^*\ell^+\ell^-$ decay modes [33–35].

$$a_{CP}\left(\sqrt{q^2}\right) \equiv \frac{|\mathcal{A}|^2 - |\bar{\mathcal{A}}|^2}{|\mathcal{A}|^2 + |\bar{\mathcal{A}}|^2} = \frac{-3}{2\pi^2}\frac{f_T(q^2)}{a_\phi}\frac{m_c}{m_D + m_\pi} \times \mathrm{Im}\left[\frac{\lambda_b}{\lambda_s}C_7\right]\left[\cos\delta_\phi - \frac{q^2 - m_\phi^2}{m_\phi \Gamma_\phi}\sin\delta_\phi\right]. \tag{18}$$

The imaginary part in the above expression can be approximated as $\mathrm{Im}[\lambda_b C_7]/\mathrm{Re}\lambda_s$. When considering numerics in what follows we will set $\mathrm{Im}[\lambda_b C_7]$ to the benchmark value of 0.8×10^{-2} in order to illustrate largest possible *CP* effect. Relative importance of the $\cos\delta_\phi$ and $\sin\delta_\phi$ for representative choices of δ_ϕ is shown on the upper plot in Fig. 1. The linearly rising behavior of the $\sin\delta_\phi$-driven term of the asymmetry is compensated by a rapid drop of the resonant amplitude (13) that severely diminishes the number of experimental events as we move several Γ_ϕ away from $m_{\ell\ell} = m_\phi$. Both effects are included in the effective experimental sensitivity that also takes into account the rate of events in the considered kinematical region and is shown on the bottom plot of Fig. 1. There we plot $a_{CP}(m_{\ell\ell})$, weighted by the differential branching ratio, a combined quantity that scales as $\sim \mathcal{A}_{\mathrm{LD}}^{\phi}\,\mathrm{Im}\mathcal{A}_{\mathrm{SD}}^{\mathrm{CPV}}$. These sensitivity curves expose entirely different behavior than $a_{CP}(m_{\ell\ell})$. If the phase δ_ϕ is close to 0 or π one finds the best sensitivity close to the peak. On the contrary, for $\delta_\phi \sim \pm\pi/2$, the *CP* asymmetry is an odd function with respect to the resonant peak position and is maximal when we are slightly off the peak. Therefore, experiment collecting events in a symmetric bin around $m_{\ell\ell} = m_\phi$ would be unable to observe *CP* asymmetry for maximal phase $\delta \sim \pm\pi/2$.

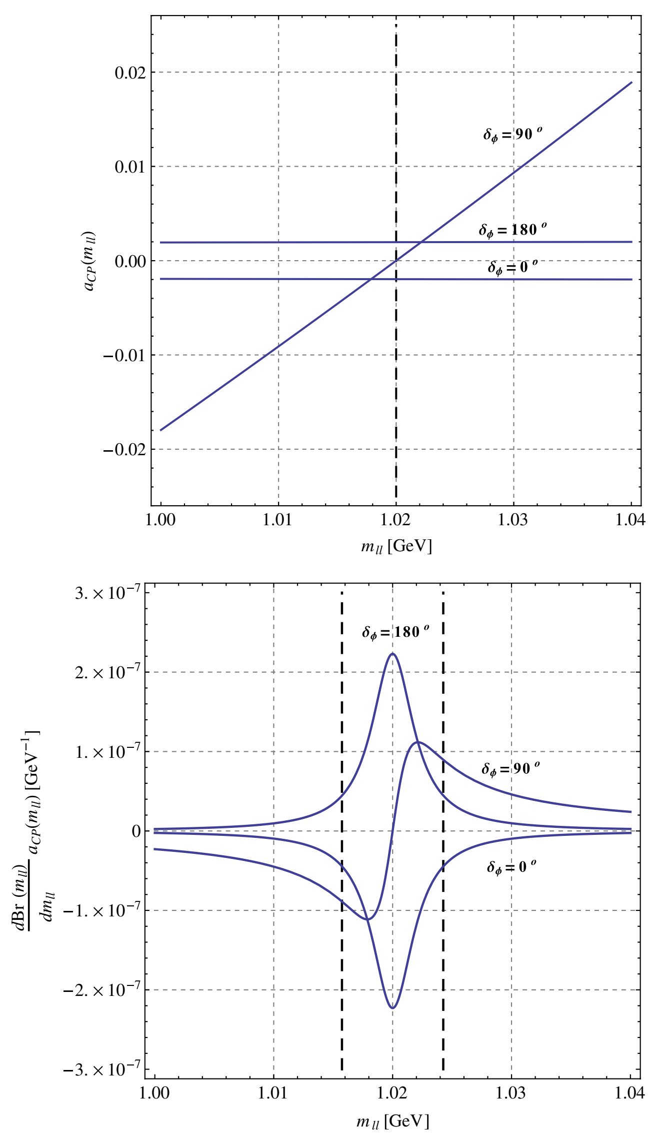

FIG. 1 (color online). Top: *CP* asymmetry $a_{CP}(m_{\ell\ell})$ around the ϕ resonance (dashed vertical line) for representative values of strong phase $\delta_\phi = 0, \pi/2, \pi$. Bottom: $(d\mathrm{Br}/dm_{\ell\ell})a_{CP}(m_{\ell\ell})$, the measure of sensitivity to direct *CP* violation. Dashed vertical lines at $m_{\ell\ell} = m_\phi \pm \Gamma_\phi$ denote the width of the resonance.

A. Partial-width *CP* asymmetries

In order to keep the experimental search as general as possible one should use appropriate search strategies to address the two limiting possibilities, i.e., $\delta_\phi = 0, \pi$ and $\delta_\phi = \pm\pi/2$. First, let us define a *CP* asymmetry of a partial width in the range $m_1 < m_{\ell\ell} < m_1$:

$$A_{CP}(m_1, m_2) = \frac{\Gamma(m_1 < m_{\ell\ell} < m_2) - \bar{\Gamma}(m_1 < m_{\ell\ell} < m_2)}{\Gamma(m_1 < m_{\ell\ell} < m_2) + \bar{\Gamma}(m_1 < m_{\ell\ell} < m_2)}, \tag{19}$$

where Γ and $\bar{\Gamma}$ denote partial decay widths of D^+ and D^- decays, respectively, to $\pi^\pm\mu^+\mu^-$. A_{CP} is related to the differential asymmetry $a_{CP}(\sqrt{q^2})$ as

$$A_{CP}(m_1, m_2) = \frac{\int_{m_1^2}^{m_2^2} dq^2 R(q^2) a_{CP}\left(\sqrt{q^2}\right)}{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}} dq^2 R(q^2)}, \tag{20}$$

where

$$R(q^2) = \frac{1}{(q^2 - m_\phi^2)^2 + m_\phi^2\Gamma_\phi^2} \times \int_{s_{\mathrm{min}}(q^2)}^{s_{\mathrm{max}}(q^2)} ds \sum_{s_+, s_-} |\bar{u}^{(s_-)}(k_-)\not{p}v^{(s_+)}(k_+)|^2 \tag{21}$$

involves the resonant shape and the integral of the lepton trace over the Dalitz variable $s \equiv (p' + k_-)^2$ whose kinematical limits read

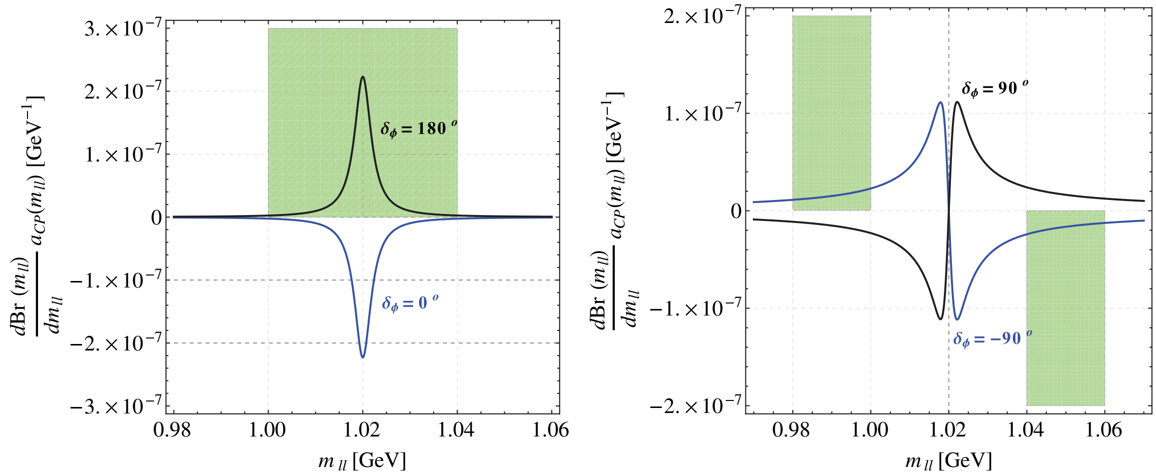

FIG. 2 (color online). Left: Asymmetry $a_{CP}(m_{\ell\ell})$ weighted by $d\mathrm{Br}/dm_{\ell\ell}$ in the case when dominated by the $\cos\delta_\phi$ term. The shaded region denotes the defining bin for asymmetry C^{ϕ}_{CP}. Right: $a_{CP}(m_{\ell\ell})$ when dominated by $\sin\delta_\phi$. Shown are also the two bins where the asymmetry S^{ϕ}_{CP} is defined as the difference of A_{CP} in the two bins.

$$s_{\max/\min}(q^2) = \frac{(m_D^2 - m_\pi^2)^2}{4q^2} - \frac{\left(q^2\sqrt{1-\frac{4m_\mu^2}{q^2}} \mp \lambda^{1/2}(q^2, m_D^2, m_\pi^2)\right)^2}{4q^2},$$
$$\lambda(x,y,z) = (x+y+z)^2 - 4(xy+yz+zx). \tag{22}$$

The $D^+ \to \pi^+ e^+ e^-$ decay mode been searched for by the CLEO experiment [37] where signal in a bin around the ϕ resonance was observed. The following partial branching ratio was reported:

$$\mathrm{Br}(D^+ \to \pi^+ e^+ e^-)_{|m_{ee}-m_\phi|\leq 20\ \mathrm{MeV}} = (1.7 \pm 1.4 \pm 0.1) \times 10^{-6}, \tag{23}$$

in a bin up covering the region $\sim 5\Gamma_\phi$ to the left and right from the nominal position of the ϕ resonance. We define the asymmetry on the same bin for the $\pi^+\mu^+\mu^-$ final state as

$$C^{\phi}_{CP} \equiv A_{CP}(m_\phi - 20\ \mathrm{MeV}, m_\phi + 20\ \mathrm{MeV}). \tag{24}$$

The asymmetry C^{ϕ}_{CP} is most sensitive to the $\cos\delta_\phi$ term in Eq. (18) and is therefore optimized for cases when $\delta_\phi \sim 0$

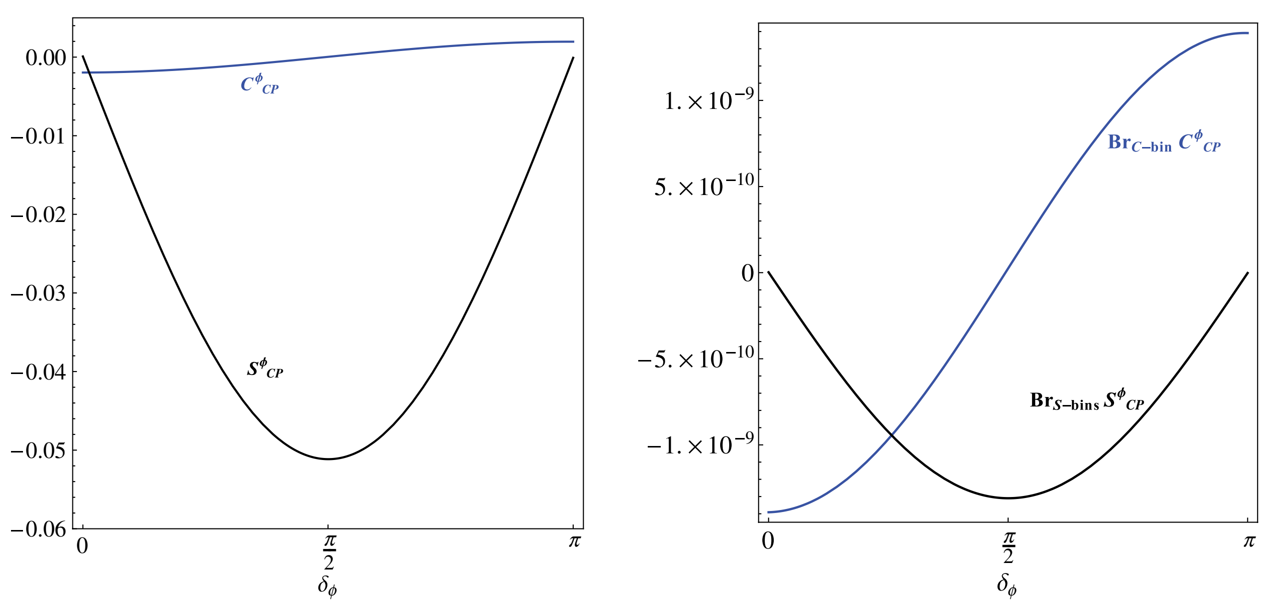

FIG. 3 (color online). Partial-width asymmetries of $D \to \pi^+\ell^+\ell^-$ decay. Left: Asymmetries C^{ϕ}_{CP} and S^{ϕ}_{CP} for $\mathrm{Im}C_7 = 0.8 \times 10^{-2}$ and their dependence on δ_ϕ. Right: Asymmetries rescaled by the branching ratios in the corresponding bins, thus representing effective sensitivity to direct *CP* violation.

TABLE I. Values of $D \to \pi^+\mu^+\mu^-$ CP asymmetries C_{CP}^{ϕ} and S_{CP}^{ϕ} for representative values of δ_ϕ. The last two columns show effective sensitivity.

δ_ϕ	$C_{CP}^{\phi} \times 10^2$	$S_{CP}^{\phi} \times 10^2$	Br(C-bin)$C_{CP}^{\phi} \times 10^7$	Br(S-bin)$S_{CP}^{\phi} \times 10^7$
$0,\ \pi$	∓ 0.20	± 0.008	∓ 0.014	$\pm 2 \times 10^{-5}$
$\pm\pi/2$	± 0.003	∓ 5.1	$\pm 2.4 \times 10^{-4}$	∓ 0.013

or $\delta_\phi \sim \pi$. Its sensitivity would decrease if we approached $\delta_\phi \sim \pm\pi/2$, since the $a_{CP}(m_{\ell\ell})$ would be asymmetric in $(m_{\ell\ell} - m_\phi)$ in this case. For that very region of δ_ϕ we find the following observable with good sensitivity to direct CP violation:

$$S_{CP}^{\phi} \equiv A_{CP}(m_\phi - 40\ \text{MeV}, m_\phi - 20\ \text{MeV}) - A_{CP}(m_\phi + 20\ \text{MeV}, m_\phi + 40\ \text{MeV}). \quad (25)$$

The bins where the partial-width CP asymmetries C_{CP}^{ϕ} and S_{CP}^{ϕ} are defined are shown in Fig. 2 together with $a_{CP}(m_{\ell\ell})$.

B. Case study for C_{CP}^{ϕ} and S_{CP}^{ϕ}

The asymmetry S_{CP}^{ϕ} can be an order of magnitude bigger than C_{CP}^{ϕ} (see Fig. 3, left). However, when we rescale the asymmetries by the branching ratios in the bins where these asymmetries defined, namely by 7.1×10^{-7} for C_{CP}^{ϕ} and 6.7×10^{-8} for S_{CP}^{ϕ}, we find evenly distributed sensitivity to direct CP violation over the entire range of δ_ϕ. Also in the transient regions between the regimes where either $\cos\delta_\phi$ or $\sin\delta_\phi$ terms dominate the sensitivity does not decrease significantly. Numerical values of the central values are summarized in Table I, whereas the errors coming dominantly from parameter a_ϕ (16) and the form factor f_T (11) are estimated to be of the order 20%.

C. Comment on $D_s \to \phi K^+ \to K^+\ell^+\ell^-$

The same type of asymmetries can be defined for the decay mode of D_s meson via the ϕ resonance to final state $K^+\ell^+\ell^-$. The resonant amplitude is described by an analogous expression to (13) and is parameterized by real a'_ϕ and δ'_ϕ. The branching ratio

$$\text{Br}(D_s^+ \to \phi K^+) = (1.8 \pm 0.4) \times 10^{-4}, \quad (26)$$

obtained from $\text{Br}(D_s^+ \to \phi(\to K^+K^-)K^+) = (9.0 \pm 2.1) \times 10^{-5}$ and $\text{Br}(\phi \to K^+K^-) = 0.489 \pm 0.005$ [36], is an order of magnitude smaller than the corresponding $\text{Br}(D^+ \to \phi\pi^+)$. By employing the narrow width approximation the value we find is $a'_\phi = 0.49$ with $\sim 10\%$ error. On the other hand, the short-distance amplitude remains of the same order of magnitude as in the $D^+ \to \pi^+\mu^+\mu^-$ case. We neglect the $SU(3)$-breaking corrections to the form factor and use $f_T(q^2)$ as given in (11) adjusted by $m_\pi \to m_K$. The asymmetries $C_{CP}^{\phi\prime}$ and $S_{CP}^{\phi\prime}$ are larger, whereas the experimental sensitivity is weaker due to smaller branching fractions, as shown in Fig. 4 and Table II.

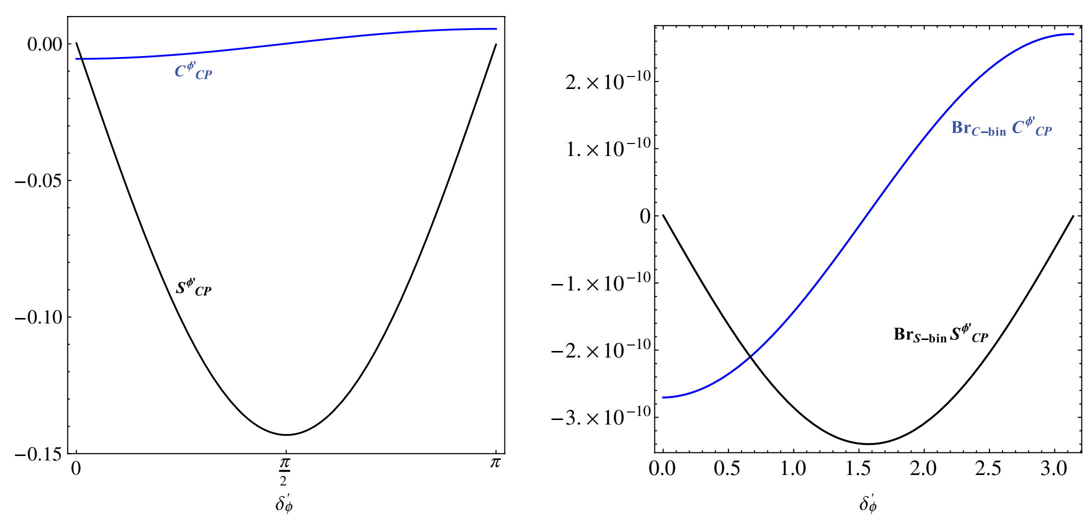

FIG. 4 (color online). Partial-width asymmetries of $D_s \to K^+\ell^+\ell^-$ decay. Left: Asymmetries $C_{CP}^{\phi\prime}$ and $S_{CP}^{\phi\prime}$ for $\text{Im}C_7 = 0.8 \times 10^{-2}$ and their dependence on δ'_ϕ. Right: Asymmetries rescaled by the branching ratios in the corresponding bins, thus representing effective sensitivity to direct CP violation.

TABLE II. Values of $D_s \to K^+\mu^+\mu^-$ CP asymmetries $C_{CP}^{\phi\prime}$ and $S_{CP}^{\phi\prime}$ for representative values of δ'_ϕ. The last two columns show effective sensitivity.

δ'_ϕ	$C_{CP}^{\phi\prime} \times 10^2$	$S_{CP}^{\phi\prime} \times 10^2$	Br(C-bin)$C_{CP}^{\phi\prime} \times 10^7$	Br(S-bin)$S_{CP}^{\phi\prime} \times 10^7$
$0,\ \pi$	∓ 0.55	± 0.024	∓ 0.0027	$\pm 1 \times 10^{-5}$
$\pm\pi/2$	± 0.008	∓ 14	$\pm 4 \times 10^{-5}$	∓ 0.007

V. SUMMARY

In this article we have studied CP asymmetries of rare decays $D^+ \to \pi^+\mu^+\mu^-$ and $D_s \to K^+\mu^+\mu^-$ defined close to the ϕ resonance that couples to the lepton pair. These asymmetries can be generated by imaginary parts of Wilson coefficients in the effective Hamiltonian for $c \to u\ell^+\ell^-$ processes. We have limited the discussion to the electromagnetic dipole coefficient C_7 which can carry a large CP-odd imaginary part, if the direct CP violation in singly Cabibbo suppressed decays $D \to \pi\pi, KK$ is to be explained by NP contribution to the chromomagnetic operator $\mathcal{O}_8$.

We have focused on the CP asymmetry around the ϕ resonant peak in the spectrum of dilepton invariant mass. The approximate description of the resonant amplitude by means of the Breit-Wigner ansatz with two additional parameters is expected to dominate over all other CP-conserving contributions. Possible long-distance CP-violating contributions of chromomagnetic operator have been neglected in this work. We have fixed one of the resonance parameters from the known resonant branching fractions of $D_{(s)} \to \phi(\to \mu^+\mu^-)P$, while the remaining parameter is an unknown CP-even strong phase δ_ϕ. The resonant amplitude in addition generates a phase that depends on the dilepton invariant mass. The hadronic dynamics of the short-distance part of the amplitude is contained in a tensor form factor f_T that has been calculated in quenched lattice simulations of QCD.

The interference term between the resonant and the short-distance amplitude that drives the direct CP asymmetry depends decisively on the particular value of the strong phase. Namely, for large strong phase δ_ϕ, i.e., close to either $+\pi/2$ or $-\pi/2$, the CP asymmetry would vanish should the experimental bin enclose the ϕ peak symmetrically. Conversely, the same CP asymmetry would be most sensitive when the strong phase was either close to 0 or π. In order to cover experimentally the whole range of strong phase values we have devised two asymmetries that are maximally sensitive either to peak-symmetric or peak-antisymmetric CP violation. Taking 0.008 for the imaginary part of $V_{cb}^* V_{ub} C_7$, the two asymmetries can take values of the order 10% for $\delta_\phi = \pm\pi/2$ or of the order 0.1%–1% for $\delta_\phi = 0,\ \pi$. When we multiply the asymmetries by the partial branching fractions in the corresponding bins, the two asymmetries provide an almost even sensitivity for all values of the strong phase. For the $D \to \pi^+\mu^+\mu^-$ thus defined sensitivity amounts to $\sim 1 \times 10^{-9}$ and $\sim 3 \times 10^{-10}$ for $D_s \to K^+\mu^+\mu^-$, bearing in mind that CP asymmetry and experimental sensitivity are proportional to the imaginary part of C_7.

With the CP observables presented in this article one is able to answer the question whether the effective coefficients C_7, C_8 carry large imaginary parts as expected in NP models which contribute to the short-distance penguins. Should the two asymmetries be measured as predicted in the paper, we would know that the exotic phase in C_8 is also responsible for direct CP violation measured in $D \to \pi^+\pi^-$, K^+K^- decays.

On the contrary, if no enhancement of CP violation is observed in $D^+ \to \pi^+\ell^+\ell^-$ then one cannot judge whether CP violation in $D \to \pi\pi, KK$ is entirely due to SM dynamics or not. However, knowing in this case that C_7 and C_8 carry no exotic phases, it is hard to conceive a new physics model that would affect only operators $\mathcal{O}_{1\ldots6}$. Not seeing any CP asymmetry in $D^+ \to \pi^+\ell^+\ell^-$ around the ϕ peak would merely hint at SM explanation of the observed CP violation in $D \to \pi\pi, KK$.

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