# $\boldsymbol{A}_{4}$ flavor symmetry model for Dirac neutrinos and sizable $\boldsymbol{U}_{\boldsymbol{e} 3}$ 

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#### Abstract

Models based on flavor symmetries are the most often studied approaches to explain the unexpected structure of lepton mixing. In many flavor symmetry groups a product of two triplet representations contains a symmetric and an antisymmetric contraction to a triplet. If this product of two triplets corresponds to a Majorana mass term, then the antisymmetric part vanishes, and in economic models tribimaximal mixing is achieved. If neutrinos are Dirac particles, the antisymmetric part is, however, present and leads to deviations from tribimaximal mixing, in particular, nonzero $U_{e 3}$. Thus, the nonvanishing value of $U_{e 3}$ and the nature of the neutrino are connected. We illustrate this with a model based on $A_{4}$ within the framework of a neutrinophilic two Higgs doublet scenario.


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## I. INTRODUCTION

Fermions of the Standard Model (SM) have an interesting property: They mix among each other. While describing mixing is straightforward, explaining the observed values is not possible in the SM. Determining the theory behind fermion mixing is therefore one of the most pressing issues in current particle physics.

In particular, the unlikely and peculiar mixing structure of the leptons, that has been determined experimentally in the past two decades, made the situation more puzzling. For quite some time, the so-called tribimaximal mixing (TBM) scheme was considered to be an excellent description of the leptonic mixing matrix [1]:

$$
U_{\mathrm{TBM}}=\left(\begin{array}{ccc}
\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0  \tag{1}\\
-\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\
-\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}
\end{array}\right)
$$

Apparently, some symmetry input is required to generate such a mixing pattern. These flavor symmetries assume that the left- and right-handed leptons, as well as new particles, transform as irreducible representations of some (typically discrete) symmetry group. In the vast majority of theoretical approaches to Eq. (1), the rotation group of the tetrahedron, $A_{4}$, is applied. This symmetry was first proposed in Ref. [2]. Surprisingly, it was possible to construct rather economic and straightforward models [3]. The group $A_{4}$ is a natural choice for the flavor symmetry, since it is the smallest group with a threedimensional representation 3. Thus, the three generations of left-handed weak lepton doublets could be unified and identified with the 3. Furthermore, $A_{4}$ has three onedimensional irreducible representations $\mathbf{1}, \mathbf{1}^{\prime}$, and $\mathbf{1}^{\prime \prime}$,

[^0]which can be identified with the three right-handed charged lepton singlets. We note that, to the best of our knowledge, all of the literally hundreds of flavor symmetry models, be it with $A_{4}$ or any other group, were exclusively assuming Majorana neutrinos (see, for instance, Ref. [4] for a list and classification of $A_{4}$ models). This assumption can be tested only in experiments looking for neutrinoless double beta decay, $(A, Z) \rightarrow(A, Z+2)+2 e^{-}$, and currently various collaborations are performing searches for this process [5]. The question on whether neutrinos are Dirac or Majorana particles is one of the most interesting and important ones of the field. In this paper, we will assume neutrinos to be Dirac particles. Hence, in the absence of any other lepton number violating physics, there will be no neutrinoless double beta decay.

A recent observation that cast some doubt on tribimaximal mixing was the largish value of the mixing matrix element $\left|U_{e 3}\right| \simeq 0.16$, that has been determined by reactor neutrino experiments [6]. Also, the best-fit points [7-9] of $U_{e 2}$ and $U_{\mu 3}$ deviate, though less strongly, from the predictions of Eq. (1). Generating sizable corrections to the mixing scheme is possible but requires, for instance, large higher dimensional contributions to the models or large neutrino masses and/or $\tan \beta$ in renormalization group effects. Here care has to be taken to guarantee that $\delta U_{e 3}>$ $\delta U_{e 2, \mu 2}$. Of course, it is also possible to construct models that give mixing schemes different from TBM, in particular, with initially nonzero $U_{e 3}$. However, those models are typically less economic than the ones leading to Eq. (1), as they usually involve much larger groups; see, for instance, Ref. [10]. In this work, we wish to keep the minimality of typical $A_{4}$ models and introduce a new way to generate nonzero $U_{e 3}$ and other deviations from TBM.

A typical ingredient in flavor symmetry models that use a group involving irreducible triplet representations is that there will be a coupling of three triplets, $\mathbf{3} \times \mathbf{3} \times \mathbf{3}$. Here two of the triplets are left- and/or right-handed fermions, and the third triplet is a set of scalar flavon fields or Higgs doublets. In many phenomenologically interesting flavor
symmetries [e.g., $A_{4}, T^{\prime}, \Delta\left(3 n^{2}\right)$ and $\Delta\left(6 n^{2}\right)$ [11]], the tensor product of two triplets contains both symmetric and antisymmetric contractions to a triplet:

$$
\begin{equation*}
\mathbf{3} \times \mathbf{3}=\mathbf{3}_{\mathrm{s}}+\mathbf{3}_{\mathrm{a}}+\cdots \tag{2}
\end{equation*}
$$

Here $\mathbf{3}_{\mathrm{s}}$ and $\mathbf{3}_{\mathrm{a}}$ are a symmetric and an antisymmetric combination, respectively, of the components in the two multiplied triplets [12]. The remaining terms in Eq. (2) depend on the group and in $A_{4}$ are given by $\mathbf{1}+\mathbf{1}^{\prime}+\mathbf{1}^{\prime \prime}$. If the mass term that is constructed from Eq. (2) is a Majorana mass term-i.e., the two triplets are left- or right-handed neutrinos-then the antisymmetric combination vanishes. In these theories, TBM is then eventually achieved.

Our observation is the following: If neutrinos are Dirac instead of Majorana particles, and their mass term depends on a product of two triplets of fermions, where one is leftand the other right-handed, then the antisymmetric $\mathbf{3}_{\mathrm{a}}$ term will not vanish, in general. Hence, with essentially the same particle content and transformation properties under the flavor symmetry group, deviations from tribimaximal mixing will arise, in particular, nonzero $U_{e 3}$. Thus, the nature of the neutrino and the nonvanishing value of $U_{e 3}$ are linked. This provides a new way to accommodate nonzero $U_{e 3}$ in flavor symmetry models while keeping the economic structure of typical models.

We note that the property given in Eq. (2) does not hold solely for $A_{4}$, but also for other popular groups. Moreover, the idea we propose can also be applied to any mixing scheme other than TBM. The example that we will give to illustrate our observation will for definiteness be in the framework of $A_{4}$ and tribimaximal mixing.

We also need to specify the mechanism that guarantees the Dirac nature of the neutrinos. Usually, the flavor symmetry models available in the literature assume Majorana neutrinos and generate Majorana masses either via an effective operator or within the type I or II seesaw mechanism. Our choice is the "neutrinophilic" two Higgs doublet model for Dirac neutrinos as considered in Refs. [13-15], in which a second Higgs doublet is introduced which exclusively couples to neutrinos. The smallness of their masses is explained by the small vacuum expectation value of this doublet. A consistent two Higgs doublet framework that incorporates such a small vacuum expectation value requires soft breaking of the underlying symmetry by a bilinear term that couples the two Higgs doublet. This general idea was first introduced in Ref. [16]. We note that this type of model is not in conflict with the recently observed [17] new particle at the Large Hadron Collider and could, in fact, as any two Higgs doublet model [18], be used to explain an excessive decay rate of that particle into two photons (if the rather mild preference for this remains with more data). In the next sections, we will first demonstrate how a typical $A_{4}$ model for Majorana neutrinos works, before modifying it to the Dirac neutrino
case, realized in the framework of the neutrinophilic two Higgs doublet model.

## II. MAJORANA MODEL EXAMPLE

In order to illustrate the situation in an easier manner, we start with a brief review of an economic and minimal $A_{4}$ "role model," in which the full symmetry group is given by $G_{S M} \otimes A_{4} \otimes Z_{3}\left(G_{S M}\right.$ being the Standard Model gauge group) and neutrinos are assumed to be Majorana particles. Here an additional cyclic $Z_{3}$ is added in order to disentangle the flavons for the charged lepton and neutrino sectors. Similar to the model addressed in Ref. [19], apart from the SM particle content we introduce three righthanded neutrinos assigned to the three-dimensional representation of $A_{4}$, together with three sets of flavon fields $\varphi$, $\varphi^{\prime}$, and $\xi$ (see Table I for details of the particle assignments). The invariant Lagrangian at leading order can be written as

$$
\begin{align*}
\mathcal{L}= & \frac{y_{e}}{\Lambda}(\varphi \bar{\ell})_{1} H e_{R}+\frac{y_{\mu}}{\Lambda}(\varphi \bar{\ell})_{1^{\prime}} H \mu_{R}+\frac{y_{\tau}}{\Lambda}(\varphi \bar{\ell})_{1^{\prime \prime}} H \tau_{R} \\
& +y_{D}\left(\bar{\ell} \nu_{R}\right)_{1} \tilde{H}+x_{A} \xi\left(\overline{\nu_{R}^{c}} \nu_{R}\right)_{1}+x_{B} \varphi^{\prime}\left(\overline{\nu_{R}^{c}} \nu_{R}\right)_{3_{\mathrm{s}}} \tag{3}
\end{align*}
$$

where $\tilde{H} \equiv i \tau_{2} H^{*}$ and the well-known tensor product rules of $A_{4}$ can be read from, e.g., Ref. [20] and are given for completeness also in the Appendix. The notation in Eq. (3) is as usual; $(\varphi \bar{\ell})_{1^{\prime}}$ denotes the part of the triplet product between $\varphi$ and $\bar{\ell}$ that transforms as $\mathbf{1}^{\prime}$. Following Ref. [19], we assume that the flavon fields develop a vacuum expectation value (VEV) along the directions

$$
\langle\varphi\rangle=\left(\begin{array}{lll}
v & v & v
\end{array}\right)^{T}, \quad\left\langle\varphi^{\prime}\right\rangle=\left(\begin{array}{lll}
0 & v^{\prime} & 0
\end{array}\right)^{T}, \quad\langle\xi\rangle=u
$$

See, e.g., Ref. [19] for the techniques to achieve this VEV alignment in a natural way. We assume in what follows that the usual mechanisms to guarantee such alignment are at work. After the breaking of the flavor and the electroweak symmetries, the charged leptons develop a mass term

$$
\frac{\langle H\rangle}{\Lambda}\left(\overline{e_{L}}, \overline{\mu_{L}}, \overline{\tau_{L}}\right)\left(\begin{array}{ccc}
y_{e} v & y_{\mu} v & y_{\tau} v  \tag{4}\\
y_{e} v & \omega y_{\mu} v & \omega^{2} y_{\tau} v \\
y_{e} v & \omega^{2} y_{\mu} v & \omega y_{\tau} v
\end{array}\right)\left(\begin{array}{c}
e_{R} \\
\mu_{R} \\
\tau_{R}
\end{array}\right)
$$

and can be diagonalized by using a biunitary transformation $V_{L}^{\dagger} M_{\ell} V_{R}=\operatorname{diag}\left(m_{e}, m_{\mu}, m_{\tau}\right)$. Here $\quad m_{f}=\sqrt{3} y_{f}\langle H\rangle v / \Lambda$ (for $f=e, \mu, \tau$ ) are the charged-lepton masses and

TABLE I. Particle assignments for the Majorana neutrino model. The additional $Z_{3}$ symmetry decouples the charged lepton and neutrino sectors, and $\omega=e^{i 2 \pi / 3}$ is the complex cube root of unity.

| Field | $\ell$ | $e_{R}$ | $\mu_{R}$ | $\tau_{R}$ | $H$ | $\varphi$ | $\varphi^{\prime}$ | $\xi$ | $\nu_{R}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A_{4}$ | $\mathbf{3}$ | $\mathbf{1}$ | $\mathbf{1}^{\prime \prime}$ | $\mathbf{1}^{\prime}$ | $\mathbf{1}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{1}$ | $\mathbf{3}$ |
| $Z_{3}$ | $\omega$ | $\omega^{2}$ | $\omega^{2}$ | $\omega^{2}$ | 1 | 1 | $\omega^{2}$ | $\omega^{2}$ | $\omega^{2}$ |

$$
V_{L}=\frac{1}{\sqrt{3}}\left(\begin{array}{ccc}
1 & 1 & 1  \tag{5}\\
1 & \omega & \omega^{2} \\
1 & \omega^{2} & \omega
\end{array}\right)
$$

The Dirac neutrino mass matrix is simply proportional to the unit matrix: $M_{D}=y_{D}\langle H\rangle \operatorname{diag}(1,1,1)$. The righthanded neutrino mass matrix is found to be

$$
M_{R}=\left(\begin{array}{ccc}
x_{A} u & 0 & x_{B} v^{\prime}  \tag{6}\\
0 & x_{A} u & 0 \\
x_{B} v^{\prime} & 0 & x_{A} u
\end{array}\right) .
$$

Finally, the mass matrix for the light neutrinos is obtained by using the standard seesaw formula $M_{\nu}=M_{D} M_{R}^{-1} M_{D}^{T}$, leading to

$$
M_{\nu}=\frac{y_{D}^{2}\langle H\rangle^{2}}{x_{A}^{2} u^{2}-x_{B}^{2} v^{\prime 2}}\left(\begin{array}{ccc}
x_{A} u & 0 & -x_{B} \boldsymbol{v}^{\prime}  \tag{7}\\
0 & \frac{x_{A}^{2} u^{2}-x_{B}^{2} v^{\prime 2}}{x_{A} u} & 0 \\
-x_{B} v^{\prime} & 0 & x_{A} u
\end{array}\right),
$$

where we omit the minus sign for simplicity. $M_{\nu}$ is easily diagonalized by

$$
V_{\nu}=\frac{1}{\sqrt{2}}\left(\begin{array}{ccc}
1 & 0 & -1  \tag{8}\\
0 & \sqrt{2} & 0 \\
1 & 0 & 1
\end{array}\right)
$$

and the leptonic flavor mixing matrix stems from the mismatch between $V_{L}$ and $V_{\nu}$ :

$$
U=V_{L}^{\dagger} V_{\nu}=\left(\begin{array}{ccc}
\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0  \tag{9}\\
\frac{1+\omega}{\sqrt{6}} & \frac{\omega^{2}}{\sqrt{3}} & \frac{\omega-1}{\sqrt{6}} \\
\frac{1+\omega^{2}}{\sqrt{6}} & \frac{\omega}{\sqrt{3}} & \frac{\omega^{2}-1}{\sqrt{6}}
\end{array}\right)
$$

Thus, TBM is obtained, up to irrelevant phases.
We stress here that in Eq. (7) the terms proportional to $x_{B}$ stem from the triple-triplet product $\nu_{R} \times \nu_{R} \times \varphi^{\prime}$, where due to the Majorana nature of the $\nu_{R}$ only the symmetric contribution of the $\nu_{R} \times \nu_{R}$ tensor product survives.

## III. $\boldsymbol{A}_{\mathbf{4}}$ SYMMETRY IN THE $\boldsymbol{\nu} \mathbf{2 H D M}$

In the Dirac neutrino case, we work in the $\nu 2$ HDM, or neutrinophilic two Higgs doublet model. Here, an additional $S U(2)$ doublet $H_{\nu}$-with the same quantum numbers as the SM Higgs doublet $H$-is introduced. The flavon content is the same as in the previous Majorana neutrino model, except that the $Z_{3}$ charges of $\varphi, \xi$, and the righthanded neutrinos are modified. The economic structure of minimal $A_{4}$ models is therefore preserved. Furthermore, a global $U(1)$ symmetry under which the new Higgs doublet $H_{\nu}$ and $\nu_{R}$ carry charge +1 while all the other SM fields are uncharged is imposed. This $U(1)$ symmetry is needed to forbid Majorana mass terms for the right-handed neutrinos and enforces a Yukawa coupling structure in which

TABLE II. Particle assignments of the Dirac flavor $A_{4}$ model.

| Field | $\ell$ | $e_{R}$ | $\mu_{R}$ | $\tau_{R}$ | $H$ | $H_{\nu}$ | $\varphi$ | $\varphi^{\prime}$ | $\xi$ | $\nu_{R}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A_{4}$ | $\mathbf{3}$ | $\mathbf{1}$ | $\mathbf{1}^{\prime \prime}$ | $\mathbf{1}^{\prime}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{1}$ | $\mathbf{3}$ |
| $U(1)$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| $Z_{3}$ | $\omega$ | $\omega^{2}$ | $\omega^{2}$ | $\omega^{2}$ | 1 | 1 | 1 | $\omega$ | $\omega$ | $\omega$ |

only $H_{\nu}$ couples to right-handed neutrinos. We refer the readers to Ref. [15] for a detailed description of the constraints and phenomenology of the $\nu 2 \mathrm{HDM}$. It is enough for our purposes to know that a consistent and allowed framework is possible, in which the VEV of the doublet responsible for neutrino Dirac masses $v_{\nu}=\mathcal{O}(\mathrm{eV})$ is small. Therefore, the smallness of neutrino Dirac masses is explained by the small VEV and not by tiny Yukawa couplings.

The particle content and their assignments under $A_{4}, Z_{3}$, and $U(1)$ are summarized in Table II. The invariant Lagrangian now reads

$$
\begin{align*}
\mathcal{L}= & \frac{y_{e}}{\Lambda}(\varphi \bar{\ell})_{1} H e_{R}+\frac{y_{\mu}}{\Lambda}(\varphi \bar{\ell})_{1^{\prime}} H \mu_{R}+\frac{y_{\tau}}{\Lambda}(\varphi \bar{\ell})_{1^{\prime \prime}} H \tau_{R} \\
& +\frac{y_{s}}{\Lambda}\left(\varphi^{\prime} \bar{\ell}\right)_{3_{\mathrm{s}}} \tilde{H}_{\nu} \nu_{R}+\frac{y_{a}}{\Lambda}\left(\varphi^{\prime} \bar{\ell}\right)_{3_{\mathrm{a}}} \tilde{H}_{\nu} \nu_{R}+\frac{y_{x}}{\Lambda}\left(\bar{\ell} \tilde{H}_{\nu} \nu_{R}\right)_{1} \xi \tag{10}
\end{align*}
$$

We should note that in the general neutrinophilic $\nu 2$ HDM the small VEV $v_{\nu}$ is generated by introducing an explicit and soft $U(1)$ breaking term $m_{12}^{2} H^{\dagger} H_{\nu}$ in the Higgs potential [15] (first proposed in a Majorana neutrino model in Ref. [16]). By attributing $U(1)$ quantum numbers to $H_{\nu}$ and $\nu_{R}$, and by breaking it softly only by the $m_{12}^{2} H^{\dagger} H_{\nu}$ term, one has actually imposed a residual symmetry in the Lagrangian, namely, $U(1)$ lepton number, thus avoiding a Majorana mass term.

It is important to stress that, since neutrinos are Dirac particles, the neutrino mass term is from the triple-triplet product $\varphi^{\prime} \times \bar{\ell} \times \nu_{R}$. With the property in Eq. (2), it is clear that the triplet product of $\bar{\ell}$ and $\nu_{R}$ contains a symmetric and an antisymmetric part. For Majorana neutrinoscf. Eq. (3)-the mass term would depend on $\varphi^{\prime} \times \nu_{R} \times \nu_{R}$, not containing an antisymmetric term. As we will see, the antisymmetric part of the Dirac mass term in Eq. (10) is, in essentially the same model as the one leading to Eq. (3), responsible for deviations from TBM, in particular, nonzero $U_{e 3}$.

Taking the same VEV alignments as before, the charged lepton sector is identical to the previous model. The neutrino mass matrix is given by

$$
M_{\nu}=\frac{\boldsymbol{v}_{\nu}}{\Lambda}\left(\begin{array}{ccc}
y_{x} u & 0 & \left(y_{s}+y_{a}\right) v^{\prime}  \tag{11}\\
0 & y_{x} u & 0 \\
\left(y_{s}-y_{a}\right) \boldsymbol{v}^{\prime} & 0 & y_{x} u
\end{array}\right),
$$

where $v_{\nu}=\left\langle H_{\nu}\right\rangle$ is the VEV of the Higgs doublet responsible for Dirac neutrino masses. Note that $M_{\nu}$ is not symmetric. In
particular, the terms proportional to $y_{a}$ stem from the antisymmetric part of the product of the two $A_{4}$ triplets $\ell$ and $\nu_{R}$. Furthermore, the matrix elements in $M_{\nu}$ are, in general, complex. One can, however, take $y_{x}$ to be real without loss of generality. The physically relevant part of the neutrino mass matrix can be expressed as the Hermitian matrix $\mathcal{H}=M_{\nu} M_{\nu}^{\dagger}$, satisfying the relation $V_{\nu}^{\dagger} \mathcal{H} V_{\nu}=$ $\operatorname{diag}\left(m_{1}^{2}, m_{2}^{2}, m_{3}^{2}\right)$, with $m_{i}$ being the neutrino masses. Explicitly, one has

$$
\mathcal{H}=\left(\begin{array}{ccc}
|a|^{2}+b^{2} & 0 & b\left(a+c^{*}\right)  \tag{12}\\
0 & b^{2} & 0 \\
b\left(a^{*}+c\right) & 0 & |c|^{2}+b^{2}
\end{array}\right),
$$

where $a=\left(y_{s}+y_{a}\right) \boldsymbol{v}^{\prime} \boldsymbol{v}_{\nu} / \Lambda, \quad b=y_{x} u v_{\nu} / \Lambda$, and $c=$ $\left(y_{s}-y_{a}\right) v^{\prime} \boldsymbol{v}_{\nu} / \Lambda$. It can be diagonalized by

$$
R_{13}(\theta, \phi)=\left(\begin{array}{ccc}
\cos \theta & 0 & \sin \theta e^{-i \phi}  \tag{13}\\
0 & 1 & 0 \\
-\sin \theta e^{i \phi} & 0 & \cos \theta
\end{array}\right)
$$

where the rotation angle and phase are given by

$$
\sin 2 \theta=\frac{2\left|b\left(a+c^{*}\right)\right|}{m_{0}^{2}}, \quad \tan \phi=\frac{\operatorname{Im}\left(a+c^{*}\right)}{\operatorname{Re}\left(a+c^{*}\right)}
$$

with $m_{0}^{2}=\sqrt{\left(|a|^{2}-|c|^{2}\right)^{2}+4 b^{2}\left|a+c^{*}\right|^{2}}$. The neutrino masses are

$$
\begin{align*}
& m_{1}^{2}=\frac{1}{2}\left(|a|^{2}+|c|^{2}\right)+b^{2}-\frac{1}{2} m_{0}^{2}, \\
& m_{2}^{2}=b^{2},  \tag{14}\\
& m_{3}^{2}=\frac{1}{2}\left(|a|^{2}+|c|^{2}\right)+b^{2}+\frac{1}{2} m_{0}^{2} .
\end{align*}
$$

An interesting relation can be inferred, namely, $\Delta m_{23}^{2}+$ $\Delta m_{21}^{2}=-\left(|a|^{2}+|c|^{2}\right)<0$. An immediate consequence is that $m_{2}$ cannot be larger than $m_{3}$, and the inverted neutrino mass ordering is therefore not allowed in the model. The leptonic flavor mixing matrix is given by $U=V_{L}^{\dagger} R_{13}(\theta, \phi)$ and reads

$$
U=\left(\begin{array}{ccc}
\frac{c_{\theta}-s_{\theta} e^{i \phi}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{c_{\theta}+s_{\theta} e^{-i \phi}}{\sqrt{3}}  \tag{15}\\
\frac{c_{\theta}-s_{\theta} e^{i \phi} \omega}{\sqrt{3}} & \frac{\omega^{2}}{\sqrt{3}} & \frac{s_{\theta} e^{-i \phi}+c_{\theta} \omega}{\sqrt{3}} \\
\frac{c_{\theta}-s_{\theta} e^{i \phi} \omega^{2}}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} & \frac{s_{\theta} e^{-i \phi}+c_{\theta} \omega^{2}}{\sqrt{3}}
\end{array}\right) .
$$

It contains only two real parameters. This is to be compared to three experimentally measured neutrino mixing angles and one Dirac $C P$ phase. The antisymmetric contribution proportional to $y_{a}$ in the Dirac mass matrix (11) is crucial for deviations from TBM: In the limit $y_{a}=0$, which implies $a=c$, it follows that $c_{\theta}+s_{\theta} e^{-i \phi}=0$, and tribimaximal mixing is reproduced. Therefore, the antisymmetric entry $y_{a}$ in the Dirac neutrino mass matrix, that has its origin in the Dirac nature of the neutrinos, causes the deviations from
tribimaximal mixing, thereby linking the nature of the neutrino with nonvanishing $U_{e 3}$.

Compared to the exact TBM mixing pattern, the absolute values of the second column of $U$ remain to be $(\sqrt{1 / 3}, \sqrt{1 / 3}, \sqrt{1 / 3})^{T}$, which leads to the following wellknown relations [21]:

$$
\begin{align*}
\sin ^{2} \theta_{12} & =\frac{1}{3} \frac{1}{1-\left|U_{e 3}\right|^{2}} \\
\cos \delta \tan 2 \theta_{23} & =\frac{1-2\left|U_{e 3}\right|^{2}}{\left|U_{e 3}\right| \sqrt{2-3\left|U_{e 3}\right|^{2}}} . \tag{16}
\end{align*}
$$

In our case the Jarlskog invariant is

$$
\begin{equation*}
\mathcal{J}=\operatorname{Im}\left\{U_{e 1} U_{\mu 2} U_{e 2}^{*} U_{\mu 1}^{*}\right\}=-\frac{1}{6 \sqrt{3}} \cos 2 \theta \tag{17}
\end{equation*}
$$

It is worthwhile to notice that $\mathcal{J}$ is independent of the phase $\phi$.

We proceed with numerical illustrations of the Dirac neutrino model. Scanning values of the parameters $a, b$, and $c$ and comparing them to the $3 \sigma$ ranges of the oscillation parameters from Forero et al. [8], we obtain the plots in Fig. 1. Here $\phi_{a}$ and $\phi_{c}$ are the phases of $a$ and $c$, respectively. One reads from the plots that the neutrino mass spectrum tends to be hierarchical, i.e., $|b|=m_{2} \sim$ [ $0.015,0.035] \mathrm{eV}$. Since the charged component of the second Higgs doublet mediates lepton flavor violating processes, we also show the branching ratio of $\mu \rightarrow e \gamma$ :

$$
\begin{equation*}
\mathrm{BR}(\mu \rightarrow e \gamma)=\frac{\alpha}{96 \pi} \frac{\left|\mathcal{H}_{e \mu}\right|^{2}}{8 G_{F}^{2} M_{H^{+}}^{4} v_{\nu}^{4}} \tag{18}
\end{equation*}
$$

versus the Jarlskog invariant $\mathcal{J}$, which is proportional to the imaginary part of $\mathcal{H}_{e \mu} \mathcal{H}_{\mu \tau} \mathcal{H}_{\tau e}$. We have taken $M_{H^{+}}=100 \mathrm{GeV}$ (red, upper points) and $M_{H^{+}}=$ 150 GeV (green, lower points) as two examples, together with $v_{\nu}=4 \mathrm{eV}$. The current upper bound on the branching ratio, $\mathrm{BR}(\mu \rightarrow e \gamma)<2.4 \times 10^{-12}$ at $90 \%$ C.L. [22], is also indicated on the plot by using a black line, and a possible future limit of $2 \times 10^{-13}$ is also indicated. We have nothing to add to the study of the usual Higgs phenomenology of the $\nu 2$ HDM [15]; the decay $\mu \rightarrow e \gamma$ is the only interesting place where some nontrivial correlation exists.

In order to demonstrate the predictive power of our model for the neutrino mixing parameters, we compare (following the strategy of Ref. [23]) the model predictions to the experimental data with a $\chi^{2}$ function:

$$
\begin{equation*}
\chi^{2}=\sum_{i} \frac{\left(\rho_{i}-\rho_{i}^{0}\right)^{2}}{\sigma_{i}^{2}} \tag{19}
\end{equation*}
$$

Here $\rho^{0}$ represents the data of the $i$ th experimental observable (taken from Ref. [8]), $\sigma_{i}$ the corresponding $1 \sigma$ absolute error, and $\rho_{i}$ the prediction of the model. In Fig. 2, we present the allowed region of the mixing angles and the


FIG. 1 (color online). Parameter values in Eq. (12) that reproduce the allowed $3 \sigma$ ranges of the neutrino parameters. The right plot in the lower row gives the correlation between leptonic $C P$ violation and the decay $\mu \rightarrow e \gamma$.

Dirac $C P$ phase at $1 \sigma, 2 \sigma$, and $3 \sigma$ C.L., defined as the contours in $\Delta \chi^{2}$ for 2 degrees of freedom with respect to the $\chi^{2}$ minimum $\left(\chi_{\min }^{2} \simeq 1.7\right)$.

We find that the best-fit value $\sin ^{2} \theta_{12}=0.342$ slightly deviates from its $1 \sigma$ experimental interval, while
$\sin ^{2} \theta_{23}=0.428$ agrees very well the current global fit value. Note that the parameter space below the thin line in the left plot of Fig. 2 is not allowed by the model itself, due to the correlation between $\theta_{12}$ and $\theta_{23}$. One reads from the right panel of Fig. 2 that the Dirac $C P$ phase


FIG. 2 (color online). Allowed region of the physical observables at $1 \sigma, 2 \sigma$, and $3 \sigma$ C.L.
is constrained to be between $-0.5 \pi$ and $0.5 \pi$, while the best-fit values for $\delta$ and $\sin ^{2} \theta_{13}$ are around $\pm 0.27 \pi$ and 0.024 . The contour is symmetric with respect to $\delta=0$.

## IV. CONCLUSIONS

Nonzero $U_{e 3}$ seems to make flavor symmetry models less economic, both in size of the symmetry group as well as in particle content. We have presented here a new method to accommodate nonzero $U_{e 3}$ (and other deviations from tribimaximal mixing) that keeps the minimality of typical models that were constructed to produce tribimaximal mixing. Our idea takes into account that the product of two triplets contains an antisymmetric term, which vanishes for Majorana neutrinos. In the case of Dirac neutrinos it remains and thus creates necessary deviations from tribimaximal mixing. This is not limited to the particular mixing scheme (tribimaximal mixing) or the flavor group $\left(A_{4}\right)$ that we used, or to the particular framework guaranteeing the Dirac nature (a neutrinophilic two Higgs doublet model), but can be applied also to many other cases. Conceptually, our observation links the nature of the neutrino with the nonvanishing value of $U_{e 3}$.

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## APPENDIX: $\boldsymbol{A}_{4}$ TENSOR PRODUCTS

The basic tensor products of $A_{4}$, which we apply here, are given by [20]

$$
\begin{array}{rlrl}
\mathbf{3} \times \mathbf{3} & =\mathbf{3}_{\mathrm{a}}+\mathbf{3}_{\mathrm{s}}+\mathbf{1}+\mathbf{1}^{\prime}+\mathbf{1}^{\prime \prime}, & \mathbf{1} \times \mathbf{1}=\mathbf{1} \\
\mathbf{1}^{\prime} \times \mathbf{1}^{\prime} & =\mathbf{1}^{\prime \prime}, & \mathbf{1}^{\prime \prime} \times \mathbf{1}^{\prime \prime}=\mathbf{1}^{\prime}, & \mathbf{1}^{\prime} \times \mathbf{1}^{\prime \prime}=\mathbf{1} \tag{A1}
\end{array}
$$

where (with $\omega=e^{i 2 \pi / 3}$ )

$$
\begin{align*}
(\mathbf{3} \times \mathbf{3})_{3_{\mathrm{s}}} & =\left(\begin{array}{lll}
x_{2} y_{3}+x_{3} y_{2}, & x_{3} y_{1}+x_{1} y_{3}, & x_{1} y_{2}+x_{2} y_{1}
\end{array}\right) \\
(\mathbf{3} \times \mathbf{3})_{3_{\mathrm{a}}} & =\left(\begin{array}{lll}
x_{2} y_{3}-x_{3} y_{2}, & x_{3} y_{1}-x_{1} y_{3}, & x_{1} y_{2}-x_{2} y_{1}
\end{array}\right) \\
(\mathbf{3} \times \mathbf{3})_{1} & =x_{1} y_{1}+x_{2} y_{2}+x_{3} y_{3} \\
(\mathbf{3} \times \mathbf{3})_{1^{\prime}} & =x_{1} y_{1}+\omega x_{2} y_{2}+\omega^{2} x_{3} y_{3} \\
(\mathbf{3} \times \mathbf{3})_{1^{\prime \prime}} & =x_{1} y_{1}+\omega^{2} x_{2} y_{2}+\omega x_{3} y_{3} . \tag{A2}
\end{align*}
$$

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