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Properties of *CP* violation in neutrino-antineutrino oscillations

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If the massive neutrinos are the Majorana particles, the unavoidable question of how to pin down the Majorana *CP*-violating phases will eventually become relevant to future neutrino experiments. We argue that a study of neutrino-antineutrino oscillations will greatly help in this regard, although the issue remains purely academic at present. In this work, we first derive the probabilities and *CP*-violating asymmetries of neutrino-antineutrino oscillations in the three-flavor framework, and then illustrate their properties in two special cases: the normal neutrino mass hierarchy with $m_1 = 0$ and the inverted neutrino mass hierarchy with $m_3 = 0$. We demonstrate the significant contributions of the Majorana phases to the *CP*-violating asymmetries, even in the absence of the Dirac phase.

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If the massive neutrinos are the Majorana particles, then a neutrino flavor ν_α can in principle oscillate into an antineutrino flavor $\bar{\nu}_\beta$ (for $\alpha, \beta = e, \mu, \tau$). The intriguing idea of neutrino-antineutrino oscillations was first proposed by Pontecorvo in 1957 [1], but it has been regarded as unrealistic because such lepton-number-violating processes are formidably suppressed by the factors m_i^2/E^2 with $m_i \lesssim 1$ eV (for $i = 1, 2, 3$) being the neutrino masses and E being the neutrino beam energy [2]. Taking the reactor antineutrino experiment, for example, one has $E \sim \mathcal{O}(1)$ MeV and thus $m_i^2/E^2 \lesssim 10^{-12}$, implying that the probability of $\bar{\nu}_e \to \nu_e$ oscillations is too small to be observable. That is why only the phenomena of neutrino-neutrino and antineutrino-antineutrino oscillations, which are lepton number conserving and do not involve the helicity suppression factors m_i^2/E^2, have so far been observed in solar, atmospheric, reactor, and accelerator experiments [3]. If the Majorana nature of the massive neutrinos is identified someday, will it be likely to detect neutrino-antineutrino oscillations in a realistic experiment?

The answer to this question seems to be quite pessimistic today, but it might not really be hopeless in the future. The history of neutrino physics is full of surprises in making the impossible possible. Let us mention a naive idea. To enhance the helicity suppression factors m_i^2/E^2, one may consider inventing some new techniques and producing a sufficiently low-energy neutrino (or antineutrino) beam. For instance, the possibility of producing a Mössbauer electron antineutrino beam with $E = 18.6$ keV[1] [5] and using it to carry out a $\bar{\nu}_e \to \bar{\nu}_e$ oscillation experiment has been discussed [6]. If the $\bar{\nu}_e \to \nu_e$ oscillation is taken into account in this case, the helicity suppression can be improved by a factor of $\mathcal{O}(10^4)$ as compared with the case of the aforementioned reactor antineutrinos.

It is theoretically interesting to study the properties of neutrino-antineutrino oscillations even in a Gedanken experiment, because they may help us understand some salient properties of the Majorana neutrinos. This kind of study has been done in the literature [2,7], but in most cases only two species of neutrinos and antineutrinos were taken into account.

In the present work, we shall first derive the probabilities of neutrino-antineutrino oscillations within the standard three-flavor framework, and then discuss the generic properties of *CP* violation in them. To illustrate, we shall focus on the *CP*-violating effects in neutrino-antineutrino oscillations by considering two special cases of the neutrino mass spectrum: (a) the normal hierarchy with $m_1 = 0$ and (b) the inverted hierarchy with $m_3 = 0$. We demonstrate the importance of the Majorana phases in generating the *CP*-violating asymmetries, even when the Dirac phase is absent. Our analytical results can easily be generalized to accommodate the light or heavy sterile Majorana neutrinos and antineutrinos.

Let us begin with the standard form of leptonic weak charged-current interactions:

$$\mathcal{L}_{\rm cc} = -\frac{g}{\sqrt{2}}\left[\overline{(e \quad \mu \quad \tau)_{\rm L}}\gamma^\mu U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W^-_\mu + \overline{(\nu_1 \quad \nu_2 \quad \nu_3)_{\rm L}}\gamma^\mu U^\dagger \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{\rm L} W^+_\mu\right], \qquad (1)$$

in which U is the 3×3 Pontecorvo-Maki-Nakagawa-Sakata (PMNS) flavor mixing matrix [8]. Now we consider

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[1]The Mössbauer electron antineutrinos are the 18.6 keV $\bar{\nu}_e$ events emitted from the bound-state beta decay of ^{3}H to ^{3}He [4], and they can be resonantly captured in the reverse bound-state process in which ^{3}He is converted into ^{3}H.

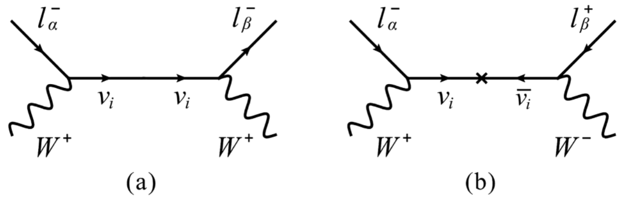

FIG. 1. Feynman diagrams for (a) neutrino-neutrino and (b) neutrino-antineutrino oscillations, where "×" stands for the chirality flip in the neutrino propagator which is proportional to the mass m_i of the Majorana neutrino $\nu_i = \bar{\nu}_i$. The initial (ν_α) and final (ν_β or $\bar{\nu}_\beta$) neutrino flavor eigenstates are produced and detected via the weak charged-current interactions, respectively.

$\nu_\alpha \to \nu_\beta$ and $\nu_\alpha \to \bar{\nu}_\beta$ oscillations (for $\alpha, \beta = e, \mu, \tau$), whose typical Feynman diagrams are illustrated in Fig. 1. It is clear that the $\nu_\alpha \to \nu_\beta$ oscillations are lepton number conserving and can take place no matter whether the massive neutrinos are the Dirac or Majorana particles. In contrast, the $\nu_\alpha \to \bar{\nu}_\beta$ oscillations are lepton number violating and cannot take place unless the massive neutrinos are the Majorana particles. Focusing on the oscillation $\nu_\alpha \to \bar{\nu}_\beta$ and its CP-conjugate process $\bar{\nu}_\alpha \to \nu_\beta$, one may write out their amplitudes as follows [2,7]:[2]

$$
\begin{aligned}
A(\nu_\alpha \to \bar{\nu}_\beta) &= \sum_i \left[U^*_{\alpha i} U^*_{\beta i} \frac{m_i}{E} \exp\left(-\mathrm{i}\frac{m_i^2}{2E}L\right)\right] K, \\
A(\bar{\nu}_\alpha \to \nu_\beta) &= \sum_i \left[U_{\alpha i} U_{\beta i} \frac{m_i}{E} \exp\left(-\mathrm{i}\frac{m_i^2}{2E}L\right)\right] \bar{K},
\end{aligned}
\tag{2}
$$

where m_i is the mass of the neutrino mass eigenstate ν_i, E denotes the neutrino (or antineutrino) beam energy, L is the baseline length, and K and $\bar{K}$ stand for the kinematical factors which are independent of the index i (and satisfy $|K| = |\bar{K}|$). The helicity suppression in the transition between ν_i and $\bar{\nu}_i$ is described by m_i/E, which is absent for normal neutrino-neutrino or antineutrino-antineutrino oscillations.

Equation (2) allows us to calculate the probabilities of neutrino-antineutrino oscillations $P(\nu_\alpha \to \bar{\nu}_\beta) \equiv |A(\nu_\alpha \to \bar{\nu}_\beta)|^2$ and $P(\bar{\nu}_\alpha \to \nu_\beta) \equiv |A(\bar{\nu}_\alpha \to \nu_\beta)|^2$. After a straightforward exercise, we arrive at

$$
\begin{aligned}
P(\nu_\alpha \to \bar{\nu}_\beta) &= \frac{|K|^2}{E^2}\left[|\langle m\rangle_{\alpha\beta}|^2 - 4\sum_{i<j} m_i m_j \,\mathrm{Re}(U_{\alpha i}U_{\beta i}U^*_{\alpha j}U^*_{\beta j}) \sin^2\frac{\Delta m^2_{ji}L}{4E} + 2\sum_{i<j} m_i m_j \,\mathrm{Im}(U_{\alpha i}U_{\beta i}U^*_{\alpha j}U^*_{\beta j}) \sin\frac{\Delta m^2_{ji}L}{2E}\right], \\
P(\bar{\nu}_\alpha \to \nu_\beta) &= \frac{|\bar{K}|^2}{E^2}\left[|\langle m\rangle_{\alpha\beta}|^2 - 4\sum_{i<j} m_i m_j \,\mathrm{Re}(U_{\alpha i}U_{\beta i}U^*_{\alpha j}U^*_{\beta j}) \sin^2\frac{\Delta m^2_{ji}L}{4E} - 2\sum_{i<j} m_i m_j \,\mathrm{Im}(U_{\alpha i}U_{\beta i}U^*_{\alpha j}U^*_{\beta j}) \sin\frac{\Delta m^2_{ji}L}{2E}\right],
\end{aligned}
\tag{3}
$$

in which $\Delta m^2_{ji} \equiv m_j^2 - m_i^2$, and the effective mass term $\langle m\rangle_{\alpha\beta}$ is defined as

$$
\langle m\rangle_{\alpha\beta} \equiv \sum_i m_i U_{\alpha i} U_{\beta i} \equiv M_{\alpha\beta}, \tag{4}
$$

which is simply the (α, β) element of the Majorana neutrino mass matrix $M = U\hat{M}U^T$ with $\hat{M} \equiv \mathrm{Diag}\{m_1, m_2, m_3\}$ in the flavor basis where the charged-lepton mass matrix is diagonal [10]. The CPT invariance assures that $P(\nu_\alpha \to \bar{\nu}_\beta) = P(\nu_\beta \to \bar{\nu}_\alpha)$ and $P(\bar{\nu}_\alpha \to \nu_\beta) = P(\bar{\nu}_\beta \to \nu_\alpha)$ hold. The CP-violating asymmetry between $\nu_\alpha \to \bar{\nu}_\beta$ and $\bar{\nu}_\alpha \to \nu_\beta$ oscillations turns out to be

$$
\mathcal{A}_{\alpha\beta} \equiv \frac{P(\nu_\alpha \to \bar{\nu}_\beta) - P(\bar{\nu}_\alpha \to \nu_\beta)}{P(\nu_\alpha \to \bar{\nu}_\beta) + P(\bar{\nu}_\alpha \to \nu_\beta)} = \frac{2\sum_{i<j} m_i m_j \,\mathrm{Im}(U_{\alpha i}U_{\beta i}U^*_{\alpha j}U^*_{\beta j}) \sin\frac{\Delta m^2_{ji}L}{2E}}{|\langle m\rangle_{\alpha\beta}|^2 - 4\sum_{i<j} m_i m_j \,\mathrm{Re}(U_{\alpha i}U_{\beta i}U^*_{\alpha j}U^*_{\beta j}) \sin^2\frac{\Delta m^2_{ji}L}{4E}}, \tag{5}
$$

which is no longer suppressed by m_i^2/E^2. Of course, $\mathcal{A}_{\alpha\beta} = \mathcal{A}_{\beta\alpha}$ holds too. Hence, only six of the nine CP-violating asymmetries are independent. Equations (3) and (5) allow us to look at the salient features of neutrino-antineutrino oscillations and CP violation in them. Some discussions are in order.

(a) *The zero-distance effect*. Taking $L = 0$, one obtains

$$
P(\nu_\alpha \to \bar{\nu}_\beta) = P(\bar{\nu}_\alpha \to \nu_\beta) = \frac{|K|^2}{E^2}|\langle m\rangle_{\alpha\beta}|^2, \tag{6}
$$

which is CP conserving (i.e., $\mathcal{A}_{\alpha\beta} = 0$ at $L = 0$). Given $\alpha = \beta = e$, for example, the above probabilities are actually determined by the effective mass term $|\langle m\rangle_{ee}|$ of the neutrinoless double beta decay. A measurement of the latter will therefore provide a meaningful constraint on the oscillation between

[2]Here we do not consider the details on the production of ν_α (or $\bar{\nu}_\alpha$) and the detection of $\bar{\nu}_\beta$ (or ν_β), and thus it is possible to factorize the amplitudes of $\nu_\alpha \to \bar{\nu}_\beta$ and $\bar{\nu}_\alpha \to \nu_\beta$ as in Eq. (2) [9].

electron neutrinos and electron antineutrinos. Of course, the zero-distance effect in Eq. (6) is extremely suppressed due to $E \gg |\langle m \rangle_{\alpha\beta}|$ in practice. Note that $P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = \delta_{\alpha\beta}$ holds at $L = 0$ in normal neutrino-neutrino or antineutrino-antineutrino oscillations, provided U is unitary.

(b) *CP violation in* $\nu_\alpha \to \bar{\nu}_\alpha$ *oscillations*. We find that Eq. (3) will not be much simplified even if $\alpha = \beta$ is taken, and the CP-violating term will not disappear in this case. The point is simply that the $\nu_\alpha \to \bar{\nu}_\alpha$ oscillation is actually a kind of "appearance" process, different from the normal $\nu_\alpha \to \nu_\alpha$ and $\bar{\nu}_\alpha \to \bar{\nu}_\alpha$ oscillations which belong to the "disappearance" processes. In this flavor-unchanging case,

$$\mathcal{A}_{\alpha\alpha} = \frac{2\sum\limits_{i<j} m_i m_j \mathrm{Im}(U_{\alpha i}^2 U_{\alpha j}^{*2}) \sin\frac{\Delta m_{ji}^2 L}{2E}}{|\langle m \rangle_{\alpha\alpha}|^2 - 4\sum\limits_{i<j} m_i m_j \mathrm{Re}(U_{\alpha i}^2 U_{\alpha j}^{*2}) \sin^2\frac{\Delta m_{ji}^2 L}{4E}}. \tag{7}$$

Of course, $\mathcal{A}_{\alpha\alpha}$ (or more generally, $\mathcal{A}_{\alpha\beta}$) may vanish on the "finely tuned" points with $\Delta m_{ji}^2 L/(2E) = \pi$, 2π, 3π, and so on. But such special points can only be chosen, in principle, for a monochromatic neutrino or antineutrino beam [7].

(c) *The Majorana CP-violating phases*. As shown in Eqs. (3) or (5), the effects of CP violation in neutrino-antineutrino oscillations are measured by $\mathrm{Im}(U_{\alpha i} U_{\beta i} U_{\alpha j}^* U_{\beta j}^*)$, which would vanish if the PMNS matrix U were real. The combination $U_{\alpha i} U_{\beta i} U_{\alpha j}^* U_{\beta j}^*$ is invariant under a redefinition of the phases of three charged-lepton fields, but it is sensitive to the rephasing of the neutrino fields.[3] Hence, the Majorana CP-violating phases of U must play an important role in neutrino-antineutrino oscillations via $\mathrm{Im}(U_{\alpha i} U_{\beta i} U_{\alpha j}^* U_{\beta j}^*)$, even if $\alpha = \beta$ is taken. This observation motivates us to ask a meaningful question: what can we do about the Majorana CP-violating phases after the Majorana nature of the massive neutrinos is identified via a measurement of the neutrinoless double beta decay [12] and the Dirac CP-violating phase is determined through a delicate long-baseline experiment of neutrino oscillations in the foreseeable future? The experiment of neutrino-antineutrino oscillations is apparently a possible step towards pinning down or constraining the Majorana CP-violating phases, although it presents a considerable challenge. Is there a better way out?

To see the properties of CP violation (or equivalently, the roles of the Majorana phases) in neutrino-antineutrino oscillations in a simpler and clearer way, let us take two phenomenologically allowed limits of the neutrino mass spectrum for illustration.

(1) *A special normal mass hierarchy with* $m_1 = 0$. In this case the 3×3 PMNS matrix U can be parametrized in terms of three mixing angles (θ_{12}, θ_{13}, θ_{23}) and two CP-violating phases (δ, σ) [13]:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{8}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ (for $ij = 12, 13, 23$). A global analysis of the available neutrino oscillation data [14] points to $\theta_{12} \simeq 33.4°$, $\theta_{13} \simeq 8.66°$, and $\theta_{23} \simeq 40.0°$, but δ is essentially unrestricted. In addition, $m_2 = \sqrt{\Delta m_{21}^2} \simeq 8.66 \times 10^{-3}$ eV and $m_3 = \sqrt{\Delta m_{31}^2} \simeq 4.97 \times 10^{-2}$ eV are obtained by using the typical inputs $\Delta m_{21}^2 \simeq 7.50 \times 10^{-5}$ eV^2 and $\Delta m_{31}^2 \simeq 2.47 \times 10^{-3}$ eV^2 [14]. Both δ and σ enter the CP-violating asymmetry $\mathcal{A}_{\alpha\beta}$, which is now simplified to

$$\begin{aligned}\mathcal{A}_{\alpha\beta} &= \frac{2m_2 m_3 \,\mathrm{Im}(U_{\alpha 2} U_{\beta 2} U_{\alpha 3}^* U_{\beta 3}^*) \sin\frac{\Delta m_{32}^2 L}{2E}}{|m_2 U_{\alpha 2} U_{\beta 2} + m_3 U_{\alpha 3} U_{\beta 3}|^2 - 4m_2 m_3 \,\mathrm{Re}(U_{\alpha 2} U_{\beta 2} U_{\alpha 3}^* U_{\beta 3}^*) \sin^2\frac{\Delta m_{32}^2 L}{4E}} \\ &= \frac{2\,\mathrm{Im}(U_{\alpha 2} U_{\beta 2} U_{\alpha 3}^* U_{\beta 3}^*) \sin\frac{\Delta m_{32}^2 L}{2E}}{\left|\sqrt{\frac{m_2}{m_3}} U_{\alpha 2} U_{\beta 2} + \sqrt{\frac{m_3}{m_2}} U_{\alpha 3} U_{\beta 3}\right|^2 - 4\,\mathrm{Re}(U_{\alpha 2} U_{\beta 2} U_{\alpha 3}^* U_{\beta 3}^*) \sin^2\frac{\Delta m_{32}^2 L}{4E}}.\end{aligned} \tag{9}$$

[3] In comparison, the strength of CP violation in normal neutrino-neutrino or antineutrino-antineutrino oscillations is determined by $\mathrm{Im}(U_{\alpha i} U_{\beta j} U_{\alpha j}^* U_{\beta i}^*)$ [11], which is absolutely rephasing invariant. In other words, it is impossible to probe the Majorana nature of the massive neutrinos (or antineutrinos) through the $\nu_\alpha \to \nu_\beta$ (or $\bar{\nu}_\alpha \to \bar{\nu}_\beta$) oscillations.

TABLE I. The CP-violating asymmetry of neutrino-antineutrino oscillations in two special cases: (1) the normal neutrino mass hierarchy with $m_1 = 0$ and $\Delta m^2_{32} L/(2E) = \pi/2$, together with the typical inputs $\theta_{12} \simeq 33.4°$, $\theta_{13} \simeq 8.66°$, $\theta_{23} \simeq 40.0°$, $\Delta m^2_{21} \simeq 7.50 \times 10^{-5}$ eV^2, and $\Delta m^2_{31} \simeq 2.47 \times 10^{-3}$ eV^2; (2) the inverted neutrino mass hierarchy with $m_3 = 0$ and $\Delta m^2_{21} L/(2E) = \pi/2$, together with the typical inputs $\theta_{12} \simeq 33.4°$, $\theta_{13} \simeq 8.66°$, $\theta_{23} \simeq 50.4°$, $\Delta m^2_{21} \simeq 7.50 \times 10^{-5}$ eV^2, and $\Delta m^2_{32} \simeq -2.43 \times 10^{-3}$ eV^2. The typical values of the CP-violating phases δ and σ are taken below.

Normal hierarchy	$\delta = 0$ and $\sigma = \pi/4$	$\delta = \pi/2$ and $\sigma = \pi/4$
$\mathcal{A}_{ee}$	+0.74	−0.74
$\mathcal{A}_{e\mu}$	+0.87	+0.075
$\mathcal{A}_{e\tau}$	−0.80	+0.088
$\mathcal{A}_{\mu\mu}$	+0.29	+0.34
$\mathcal{A}_{\mu\tau}$	−0.25	−0.25
$\mathcal{A}_{\tau\tau}$	+0.22	+0.17
Inverted hierarchy	$\delta = 0$ and $\sigma = \pi/4$	$\delta = \pi/2$ and $\sigma = \pi/4$
$\mathcal{A}_{ee}$	−0.73	−0.73
$\mathcal{A}_{e\mu}$	+0.91	+0.92
$\mathcal{A}_{e\tau}$	+0.96	+0.96
$\mathcal{A}_{\mu\mu}$	−1.00	−0.54
$\mathcal{A}_{\mu\tau}$	−0.80	−0.75
$\mathcal{A}_{\tau\tau}$	−0.46	−0.64

We see that the ratio $\sqrt{m_2/m_3} \simeq 0.42$ or its reciprocal may have a greater or lesser effect on the magnitude of $\mathcal{A}_{\alpha\beta}$. The latter also depends on Δm^2_{32} via its oscillating term.

(2) *A special inverted mass hierarchy with* $m_3 = 0$. In this case the 3×3 PMNS matrix U can also be parametrized as in Eq. (8) with a single Majorana CP-violating phase σ, and the present global fit yields $\theta_{12} \simeq 33.4°$, $\theta_{13} \simeq 8.66°$, and $\theta_{23} \simeq 50.4°$ [14]. Furthermore, we obtain $m_1 = \sqrt{-\Delta m^2_{21} - \Delta m^2_{32}} \simeq 4.85 \times 10^{-2}$ eV and $m_2 = \sqrt{-\Delta m^2_{32}} \simeq 4.93 \times 10^{-2}$ eV by using the typical inputs $\Delta m^2_{21} \simeq 7.50 \times 10^{-5}$ eV^2 and $\Delta m^2_{32} \simeq -2.43 \times 10^{-3}$ eV^2 [14]. The CP-violating asymmetry $\mathcal{A}_{\alpha\beta}$ turns out to be

$$\begin{aligned}\mathcal{A}_{\alpha\beta} &= \frac{2 m_1 m_2 \,\mathrm{Im}(U_{\alpha 1} U_{\beta 1} U^*_{\alpha 2} U^*_{\beta 2}) \sin \frac{\Delta m^2_{21} L}{2E}}{|m_1 U_{\alpha 1} U_{\beta 1} + m_2 U_{\alpha 2} U_{\beta 2}|^2 - 4 m_1 m_2 \,\mathrm{Re}(U_{\alpha 1} U_{\beta 1} U^*_{\alpha 2} U^*_{\beta 2}) \sin^2 \frac{\Delta m^2_{21} L}{4E}} \\ &\simeq \frac{2 \,\mathrm{Im}(U_{\alpha 1} U_{\beta 1} U^*_{\alpha 2} U^*_{\beta 2}) \sin \frac{\Delta m^2_{21} L}{2E}}{|U_{\alpha 1} U_{\beta 1} + U_{\alpha 2} U_{\beta 2}|^2 - 4 \,\mathrm{Re}(U_{\alpha 1} U_{\beta 1} U^*_{\alpha 2} U^*_{\beta 2}) \sin^2 \frac{\Delta m^2_{21} L}{4E}},\end{aligned} \tag{10}$$

where $m_1 \simeq m_2$ has been adopted in obtaining the approximate result. One can see that the magnitude of $\mathcal{A}_{\alpha\beta}$ is essentially independent of the absolute neutrino masses m_1 and m_2 in this special case, although it relies on Δm^2_{21} via the oscillating term.

To illustrate the magnitude of $\mathcal{A}_{\alpha\beta}$, one may simplify its expression by taking $\Delta m^2_{32} L/(2E) = \pi/2$ in Eq. (9) or taking $\Delta m^2_{21} L/(2E) = \pi/2$ in Eq. (10). In either case, it is now possible to get a ballpark feeling about the size of $\mathcal{A}_{\alpha\beta}$ if the values of the CP-violating phases δ and σ are input. For simplicity, we fix $\sigma = \pi/4$ and take $\delta = 0$ or $\pi/2$. The numerical results of $\mathcal{A}_{\alpha\beta}$ are then listed in Table I.[4] We see that there can be quite sizable CP-violating effects in neutrino-antineutrino oscillations, and they may simply arise from the Majorana CP-violating phase(s) even if the Dirac CP-violating phase δ is switched off (or the flavor mixing angle θ_{13} is switched off).

In addition to the above two special cases, the three neutrinos may also have a nearly degenerate mass spectrum [15]. Namely, $m_1 \approx m_2 \approx m_3$, but the exact equality is forbidden because it is in conflict with the neutrino oscillation data. In this interesting case, $m_i \approx m_j$ can be factored out on the right-hand side of Eq. (3) and thus the CP-violating asymmetry

[4]For the inverted hierarchy with $m_3 = 0$, $\delta = 0$, and $\sigma = \pi/4$, the result $\mathcal{A}_{\mu\mu} \simeq -1.00$ in Table I is a consequence of $\mathrm{Re}(U^2_{\mu 1} U^{*2}_{\mu 2}) = 0$, $\mathrm{Im}(U^2_{\mu 1} U^{*2}_{\mu 2}) \simeq -|U_{\mu 1}|^4$, and $|U^2_{\mu 1} + U^2_{\mu 2}|^2 \simeq |U_{\mu 1}|^4$, because of the special values of the input parameters.

$\mathcal{A}_{\alpha\beta}$ in Eq. (5) is simplified to

$$\mathcal{A}_{\alpha\beta} \approx \frac{2\sum_{i<j} \mathrm{Im}(U_{\alpha i}U_{\beta i}U^*_{\alpha j}U^*_{\beta j}) \sin\frac{\Delta m^2_{ji}L}{2E}}{|\sum_i U_{\alpha i}U_{\beta i}|^2 - 4\sum_{i<j} \mathrm{Re}(U_{\alpha i}U_{\beta i}U^*_{\alpha j}U^*_{\beta j})\sin^2\frac{\Delta m^2_{ji}L}{4E}}, \tag{11}$$

which is independent of the absolute neutrino masses. The values of $\mathcal{A}_{\alpha\beta}$ may be sensitive to the sign of Δm^2_{31} (or Δm^2_{32}) through the sum of three oscillating terms in the numerator of $\mathcal{A}_{\alpha\beta}$.

Note that a complete or partial degeneracy of three neutrino masses is sometimes taken to reveal the distinct properties of flavor mixing and *CP* violation for the Majorana neutrinos [16]. A systematic analysis [17] shows that the PMNS matrix U can in general be simplified if both the neutrino mass degeneracy and the Majorana phase degeneracy, which are both conceptually interesting, are assumed. Given $m_1 = m_2 = m_3$, for example, Eq. (3) is simplified to

$$P(\nu_\alpha \to \bar{\nu}_\beta) = P(\bar{\nu}_\alpha \to \nu_\beta) = \frac{|K|^2}{E^2} m_1^2 \left|\sum_i U_{\alpha i}U_{\beta i}\right|^2. \tag{12}$$

This result is very similar to the zero-distance effect given in Eq. (6). Of course, $\mathcal{A}_{\alpha\beta} = 0$ holds in this special case, although there are still nontrivial *CP*-violating phases in U. If the Majorana phases of three neutrinos were exactly degenerate (i.e., $\phi_1 = \phi_2 = \phi_3$ with ϕ_i being associated with the neutrino mass eigenstate ν_i), we would be able to rotate away all of them from the PMNS matrix U. In this case, the *CP*-violating asymmetry $\mathcal{A}_{\alpha\beta}$ is only dependent on the Dirac phase δ. This point can be clearly seen from the combination $U_{\alpha i}U_{\beta i}U^*_{\alpha j}U^*_{\beta j}$ that appears in Eqs. (3) and (5) [18].

In summary, we have derived the probabilities and *CP*-violating asymmetries of neutrino-antineutrino oscillations in the standard three-flavor framework.[5] We have particularly illustrated the *CP*-violating effects in neutrino-antineutrino oscillations by considering two phenomenologically allowed limits of the neutrino mass spectrum: (a) the normal hierarchy with $m_1 = 0$ and (b) the inverted hierarchy with $m_3 = 0$. The importance of the Majorana phases in generating the *CP*-violating asymmetries, even when the Dirac phase is absent, has been demonstrated. Our analytical results can easily be generalized to accommodate the light or heavy sterile Majorana neutrinos and antineutrinos.

We reiterate that this work is motivated by a meaningful question that we have asked ourselves: what can we proceed to do to pin down the full picture of flavor mixing and *CP* violation if the massive neutrinos are identified to be the Majorana particles via a convincing measurement of the neutrinoless double beta decay in the future? By then we might be able to find a better way out,[6] or just pay more attention to the feasibility of detecting neutrino-antineutrino oscillations and *CP* violation in them.

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[5]Although it is impossible for the Dirac neutrinos to have neutrino-antineutrino oscillations, it is possible for them to oscillate between their left-handed and right-handed states in a magnetic field and in the presence of matter effects (see Ref. [19] for a review of such spin-flavor precession processes).

[6]Some new techniques for producing and measuring neutrinos and antineutrinos, such as the one using atoms or molecules [20], will probably be developed in the future.

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