PHYSICAL REVIEW D **87,** 053013 (2013)

Bilarge neutrino mixing and Abelian flavor symmetry

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(Received 15 January 2013; published 18 March 2013)

We explore two bilarge neutrino mixing *Anzätze* within the context of Abelian flavor symmetry theories: ($\mathtt{BL}_1$) $\sin\theta_{12} \sim \lambda$, $\sin\theta_{13} \sim \lambda$, $\sin\theta_{23} \sim \lambda$, and ($\mathtt{BL}_2$) $\sin\theta_{12} \sim \lambda$, $\sin\theta_{13} \sim \lambda$, $\sin\theta_{23} \sim 1-\lambda$. The first pattern is proposed by two of us and is favored if the atmospheric mixing angle θ_{23} lies in the first octant, while the second one is preferred for the second octant of θ_{23}. In order to reproduce the second texture, we find that the flavor symmetry should be $U(1)\times Z_m$, while for the first pattern the flavor symmetry should be extended to $U(1)\times Z_m\times Z_n$ with m and n of different parity. Explicit models for both mixing patterns are constructed based on the flavor symmetries $U(1)\times Z_3\times Z_4$ and $U(1)\times Z_2$. The models are extended to the quark sector within the framework of $SU(5)$ grand unified theory in order to give a successful description of quark and lepton masses and mixing simultaneously. Phenomenological implications are discussed.

DOI: 10.1103/PhysRevD.87.053013 PACS numbers: 14.60.Pq, 11.30.Hv, 12.10.−g, 12.60.Jv

I. INTRODUCTION

Our knowledge of the neutrino oscillation parameters has enormously improved in recent years. In particular, the Daya Bay [1], RENO [2], and Double Chooz [3] Collaborations have established that the reactor mixing angle $\theta_{13} > 0$ at about five confidence level, confirming the early hints for a nonzero θ_{13} [4,5]. Recent global analyses [6,7] of neutrino oscillation parameters, including the data released at the Neutrino 2012 conference, find that θ_{13} is nonzero at about 10σ and nonmaximal atmospheric mixing angle θ_{23} is preferred. However, it still is not clear which octant θ_{23} lies in. The global fit of Ref. [6] prefers θ_{23} in the second octant with the best fit value $\sin^2\theta_{23} = 0.613(0.600)$ for normal (inverted) neutrino mass hierarchy, although this hint is quite marginal and first octant values of θ_{23} are well inside the 1σ range for normal hierarchy and at 1.2σ for inverted spectrum. While the independent phenomenological analyses of atmospheric neutrino data in Ref. [7] obtain a preference for θ_{23} in the first octant for both mass hierarchies and exclude maximal mixing at the 2σ level, the best fit value is found to be $\sin^2\theta_{23} = 0.386(0.392)$ for normal (inverted) neutrino spectrum. Alternative recent global fits claim both the first and second θ_{23} octants are possible [8]. As for the mass-squared difference, the best fit values of $\Delta m^2_{\mathrm{sol}}$ and $\Delta m^2_{\mathrm{atm}}$ are $7.62\times 10^{-5}\ \mathrm{eV}^2$ and $2.55(2.43)\times 10^{-3}\ \mathrm{eV}^2$, respectively, which lead to $\Delta m^2_{\mathrm{sol}}/\Delta m^2_{\mathrm{atm}} \simeq 0.030(0.031)$. Here the values shown in parentheses correspond to the inverted neutrino mass hierarchy. Note that the three groups give almost the same 3σ ranges for the lepton mixing parameters.

From the theoretical or model-building point of view, one implication of this significant experimental progress is that it excludes the tribimaximal mixing *Ansatz* for neutrino mixing [9], unless the underlying theory is capable of providing sufficiently large corrections. So far many suggestions have been advanced to explain the new data, in particular, the largish θ_{13} [10–15]. Instead of seeking for new mass-independent lepton mixing matrices to replace the tribimaximal pattern [11–13], which may be derived from certain discrete flavor symmetries, Ref. [14] proposed a novel Wolfenstein-like *Ansatz* for the neutrino mixing matrix. In this scheme, all three lepton mixing angles are assumed to be of the same order to first approximation

$$\sin\theta_{12} \sim \lambda, \qquad \sin\theta_{13} \sim \lambda, \qquad \sin\theta_{23} \sim \lambda, \tag{1}$$

where $\lambda \simeq 0.23$ is the Cabibbo angle, and the symbol "$\sim$" implies that the above relations contain unknown factors of order one; the freedom in these factors can be used to obtain an adequate description of the neutrino mixing. Inspecting the global data fitting [6–8], we see that $\sin\theta_{12} \simeq 2.5\lambda$ and $\sin\theta_{13} \simeq \lambda/\sqrt{2}$, which is proposed in the so-called tribimaximal-Cabibbo mixing [15] and also appeared in the context of quark-lepton complementarity [16]. Such bilarge mixing pattern [14] would clearly provide a good leading order approximation for the current neutrino mixing pattern if the atmospheric neutrino mixing angle θ_{23} turns out to lie in the first octant. However, the second octant of θ_{23} cannot be ruled out and is supported by the analyses in Refs. [6,8]. In this case, the texture

$$\sin\theta_{12} \sim \lambda, \qquad \sin\theta_{13} \sim \lambda, \qquad \sin\theta_{23} \sim 1-\lambda \tag{2}$$

could be taken as a viable model-building standard. We shall refer to two mixing patterns as $\mathtt{BL}_1$ and $\mathtt{BL}_2$ textures, respectively. The difference between $\mathtt{BL}_1$ and $\mathtt{BL}_2$ mixing lies in the order of magnitude of the atmospheric mixing angle θ_{23}; the $\mathtt{BL}_1$ mixing pattern would be favored if future experiments establish that θ_{23} belongs to the first octant and the deviation from maximal mixing is somewhat large; otherwise, $\mathtt{BL}_2$ mixing is preferred. It is well known that the observed hierarchies of masses and flavor mixing in the quarks and charged leptons sectors can be conveniently characterized by the Cabibbo angle. As a result, the $\mathtt{BL}_1$ and $\mathtt{BL}_2$ parametrization may have deep implications for the theoretical formulation of the ultimate unified theory of flavor. A lot of work in the literature has demonstrated that the smallness and hierarchy of the quark masses and mixing angles can be naturally generated in theories which, at low energy, are described effectively by an Abelian horizontal symmetry, which is explicitly broken by a small parameter [17–19]. It certainly follows a natural path to try and apply these ideas on Abelian family symmetries developed for the quarks to the lepton sector. In this work, we shall investigate whether and how the $\mathtt{BL}_1$ and $\mathtt{BL}_2$ textures can be reproduced naturally from the Abelian horizontal flavor symmetry. For generality we assume that the light neutrino masses arise from lepton-number-violating effective Weinberg-like operators.

The paper is organized as follows. In Sec. II, we present the effective low energy theory for the Abelian $U(1)$ flavor symmetry and its extension to $U(1)\times Z_m\times Z_n$. We find that in order to produce the $\mathtt{BL}_1$ texture without fine-tuning, the family symmetry should be $U(1)\times Z_m\times Z_n$ with m and n of opposite parity. Models for the $\mathtt{BL}_1$ and $\mathtt{BL}_2$ schemes are constructed in Secs. III and IV, respectively. These models are extended to include quarks within the $SU(5)$ grand unified theory (GUT); the observed patterns of both quark and lepton masses and flavor mixings are reproduced, and the general phenomenological predictions of the models are discussed. Finally, our conclusions are summarized in Sec. V.

II. THEORETICAL FRAMEWORK

Our theoretical framework is defined as follows. For definiteness we consider a low energy effective theory with the same particle content as the supersymmetric Standard Model (SM). In addition to supersymmetry and the SM gauge symmetry, we introduce a horizontal $U(1)$ symmetry and a SM singlet chiral superfield Θ which is charged under the $U(1)$ family symmetry; without loss of generality, we normalize its charge to -1. The effective Yukawa couplings of the quarks and leptons are generated from nonrenormalizable superpotential terms of the form

$$\begin{aligned} W &= (y_u)_{ij}Q_iU^c_jH_u\left(\frac{\Theta}{\Lambda}\right)^{F(Q_i)+F(U^c_j)} \\ &\quad + (y_d)_{ij}Q_iD^c_jH_d\left(\frac{\Theta}{\Lambda}\right)^{F(Q_i)+F(D^c_j)} \\ &\quad + (y_e)_{ij}L_iE^c_jH_d\left(\frac{\Theta}{\Lambda}\right)^{F(L_i)+F(E^c_j)} \\ &\quad + (y_\nu)_{ij}\frac{1}{\Lambda}L_iL_jH_uH_u\left(\frac{\Theta}{\Lambda}\right)^{F(L_i)+F(L_j)}, \end{aligned} \tag{3}$$

where $H_{u,d}$ are Higgs doublets, Q_i and L_i are the left-handed quark and lepton doublets, respectively, U^c_j, D^c_j, and E^c_j are the right-handed up-type quark, down-type quark, and charged lepton superfields, respectively, and i, j are generation indices. The parameter Λ is the cutoff scale of the $U(1)$ symmetry, and $F(\psi)$ denotes the $U(1)$ charge of the field ψ. Note that $F(H_u)$ and $F(H_d)$ do not appear in the exponents since one can always set the horizontal charges of the Higgs doublet H_u and H_u to zero by redefinition of the $U(1)$ charges. The last term of Eq. (3) is the high-dimensional version of the effective lepton-number-violating Weinberg operator.

For the Froggatt-Nielsen flavon field Θ, the supersymmetric action contains a Fayet-Iliopoulos term and the associated D term in the scalar potential provides a large vacuum expectation value (VEV) for the scalar component of Θ. The D term in the potential is given by

$$V_D = \frac{1}{2}(M^2_{\rm FI} - g_\Theta|\Theta|^2)^2, \tag{4}$$

where $M^2_{\rm FI}$ is the Fayet-Iliopoulos term. The vanishing of V_D requires

$$|\langle\Theta\rangle| = M_{\rm FI}/\sqrt{g_\Theta}. \tag{5}$$

We note that this flavor symmetry breaking mechanism is also frequently exploited in discrete flavor symmetry model building [20]. Once the horizontal symmetry is broken by the VEV $\langle\Theta\rangle$, one obtains the quark and lepton mass matrices whose elements are suppressed by powers of the small parameter $\langle\Theta\rangle/\Lambda$, which for simplicity is usually assumed to be characterized by the Cabibbo angle, i.e., $\lambda = \langle\Theta\rangle/\Lambda$. Then we have

$$\begin{aligned} (M_u)_{ij} &= (y_u)_{ij}\lambda^{F(Q_i)+F(U^c_j)}v_u, \\ (M_d)_{ij} &= (y_d)_{ij}\lambda^{F(Q_i)+F(D^c_j)}v_d, \\ (M_e)_{ij} &= (y_e)_{ij}\lambda^{F(L_i)+F(E^c_j)}v_u, \\ (M_\nu)_{ij} &= (y_\nu)_{ij}\lambda^{F(L_i)+F(L_j)}\frac{v^2_u}{\Lambda}, \end{aligned} \tag{6}$$

where $v_{u,d} = \langle H_{u,d}\rangle$ is the electroweak scale VEV of the Higgs doublet $H_{u,d}$. The factors $(y_u)_{ij}$, $(y_d)_{ij}$, $(y_e)_{ij}$, and $(y_\nu)_{ij}$ are not constrained by the flavor symmetry and are usually assumed to be of order one; the freedom in these factors is used in order to obtain a quantitative description of the fermion masses and flavor mixings. Since the holomorphicity of the superpotential forbids nonrenormalizable terms with a negative power of the superfield Θ, one has $(M_u)_{ij} = 0$ if $F(Q_i) + F(U^c_j) < 0$. Similarly $(M_d)_{ij} = 0$ if $F(Q_i) + F(D^c_j) < 0$, $(M_e)_{ij} = 0$ if $F(L_i) + F(E^c_j) < 0$, and $(M_\nu)_{ij} = 0$ if $F(L_i) + F(L_j) < 0$.

In our framework, the light neutrino masses are generated by the high-dimensional effective Weinberg operators shown in the last term of Eq. (3); consequently, the light neutrinos are Majorana particles and its mass matrix M_ν is symmetric with $(M_\nu)_{ij} = (M_\nu)_{ji}$.[1] Furthermore, if all the horizontal charges are positive, the hierarchial structure of the mass matrices shown in Eq. (6) allows a simple order of magnitude estimate for the various mass ratios and mixing angles:

$$\begin{aligned} \frac{m_{u_i}}{m_{u_j}} &\sim \lambda^{F(Q_i)-F(Q_j)+F(U_i^c)-F(U_j^c)}, \\ \frac{m_{d_i}}{m_{d_j}} &\sim \lambda^{F(Q_i)-F(Q_j)+F(D_i^c)-F(D_j^c)}, \\ V_{ij} &\sim \lambda^{F(Q_i)-F(Q_j)}, \\ \frac{m_i}{m_j} &\sim \lambda^{2[F(L_i)-F(L_j)]}, \\ \frac{m_{\ell_i}}{m_{\ell_j}} &\sim \lambda^{F(L_i)-F(L_j)+F(E_i^c)-F(E_j^c)}, \\ \sin\theta_{ij} &\sim \lambda^{F(L_i)-F(L_j)}, \end{aligned} \tag{7}$$

where m_i is the light neutrino mass, and V_{ij} denotes the element of the quark Cabibbo-Kobayashi-Maskawa quark-mixing matrix (CKM) mixing matrix. We note that the sign "$\sim$" implies that there is an unknown order one coefficient in each relation so that the actual value of the mass ratios and mixing angles may slightly depart from the naive "power counting" estimate. Moreover, if some fields carry negative F charges, then holomorphy plays an important role and the estimates (7) could be violated as well. For the BL_1 mixing pattern, both $\sin\theta_{12}$ and $\sin\theta_{23}$ are of order λ, then we should require

$$F(L_1) = F(L_2) + 1, \qquad F(L_2) = F(L_3) + 1. \tag{8}$$

This implies $F(L_1) = F(L_3) + 2$; as a result, we have $\sin\theta_{13} \sim \lambda^2$. Therefore, we conclude that the BL_1 mixing pattern cannot be naturally produced from a pure $U(1)$ flavor symmetry. Turning to the BL_2 mixing pattern given by $\sin\theta_{23} \sim 1$, $\sin\theta_{12} \sim \lambda$, and $\sin\theta_{13} \sim \lambda$, one should choose

$$F(L_2) = F(L_3) = F(L_1) - 1. \tag{9}$$

Then we have the $(2i)$ and $(3i)$ $(i = 1, 2, 3)$ entries of the charged lepton mass matrix of the same order; hence, the diagonalization of the charged lepton mass matrix leads to large 2–3 mixing. In addition, we obtain

$$M_\nu \sim \lambda^{2F(L_3)} \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} \frac{v_u^2}{\Lambda}. \tag{10}$$

Clearly the (2–3) sector of the light neutrino mass matrix has a democratic structure; thus, large mixing in this 2–3 sector is naturally obtained. However, barring the presence of special cancellations, the masses of the second and third light neutrinos are typically expected to be of the same order in this case. As a result, the three light neutrinos are quasidegenerate, and strong parameter fine-tuning is required in order to account for the hierarchy between the measured mass squared differences $\Delta m^2_{\mathrm{sol}}$ and $\Delta m^2_{\mathrm{atm}}$.

In order to avoid this kind of fine-tuning in obtaining an acceptable pattern of neutrino oscillation parameters, we must go beyond the pure $U(1)$ flavor symmetry case considered above. Let us now move to the extended flavor symmetry $U(1) \times Z_m \times Z_n \subset U(1) \times U(1)' \times U(1)''$. This kind of Abelian symmetry is somewhat complex and not yet fully discussed, as far as we know, since most of the previous work concentrated on $U(1)$ or $U(1) \times Z_m \subset U(1) \times U(1)'$ flavor symmetry. We now consider [18,22] three SM singlet superfields Θ_1, Θ_2, and Θ_3 with the horizontal charges

$$\Theta_1: (-1, 0, 0), \quad \Theta_2: (0, -1, 0), \quad \Theta_3: (0, 0, -1). \tag{11}$$

In exactly the same way as the single $U(1)$ case, the three flavons Θ_1, Θ_2, and Θ_3 could get nonvanishing VEVs determined by corresponding the D terms. In general, the VEVs $\langle\Theta_1\rangle$, $\langle\Theta_2\rangle$, and $\langle\Theta_3\rangle$ are different [18,22]. For simplicity, we take in what follows: $\langle\Theta_1\rangle/\Lambda \sim \lambda$, $\langle\Theta_2\rangle/\Lambda \sim \lambda$, and $\langle\Theta_3\rangle/\Lambda \sim \lambda$. The effective Yukawa couplings are given by extending Eq. (3) with new flavons Θ_1, Θ_2, and Θ_3 as follows:

$$\begin{aligned} W = {}& (y_u)_{ij} Q_i U_j^c H_u \left(\frac{\Theta_1}{\Lambda}\right)^{F(Q_i)+F(U_j^c)} \left(\frac{\Theta_2}{\Lambda}\right)^{[Z_m(Q_i)+Z_m(U_j^c)]} \left(\frac{\Theta_3}{\Lambda}\right)^{[Z_n(Q_i)+Z_n(U_j^c)]} \\ &+ (y_d)_{ij} Q_i D_j^c H_d \left(\frac{\Theta_1}{\Lambda}\right)^{F(Q_i)+F(D_j^c)} \left(\frac{\Theta_2}{\Lambda}\right)^{[Z_m(Q_i)+Z_m(D_j^c)]} \left(\frac{\Theta_3}{\Lambda}\right)^{[Z_n(Q_i)+Z_n(D_j^c)]} \\ &+ (y_e)_{ij} L_i E_j^c H_d \left(\frac{\Theta_1}{\Lambda}\right)^{F(L_i)+F(E_j^c)} \left(\frac{\Theta_2}{\Lambda}\right)^{[Z_m(L_i)+Z_m(E_j^c)]} \left(\frac{\Theta_3}{\Lambda}\right)^{[Z_n(L_i)+Z_n(E_j^c)]} \\ &+ (y_\nu)_{ij} \frac{1}{\Lambda} L_i L_j H_u H_u \left(\frac{\Theta_1}{\Lambda}\right)^{F(L_i)+F(L_j)} \left(\frac{\Theta_2}{\Lambda}\right)^{[Z_m(L_i)+Z_m(L_j)]} \left(\frac{\Theta_3}{\Lambda}\right)^{[Z_n(L_i)+Z_n(L_j)]}, \end{aligned} \tag{12}$$

[1]Note that if we introduce three right-handed neutrino superfields N_i^c to generate light neutrino mass via type I seesaw mechanism, the structure of the light neutrino mass matrix M_ν is independent of the N_i^c charge assignment [21,22], unless there are holomorphic zeros in neutrino Dirac mass matrix M_D or in Majorana mass matrix M_N for the heavy fields N^c.

where $Z_{m,n}(\psi)$ is the $Z_{m,n}$ charge of the field ψ, and the brackets $[\ldots]$ around the exponents denote that we are modding out by m (n) according to the Z_m (Z_n) addition rule, namely,

$$[Z_m(Q_i) + Z_m(U^c_j)] = \begin{cases} r & \text{if } r < m \\ r - m & \text{if } r \geq m \end{cases}, \quad (13)$$

where $r = Z_m(Q_i) + Z_m(U^c_j)$. We note that the charge assignments of the Higgs doublets H_u and H_d have been set to (0, 0, 0) by redefining the flavor symmetry charges of the fields. Thus, the fermion mass matrix can be expressed in term of the horizontal charges as

$$\begin{aligned}
(M_u)_{ij} &= (y_u)_{ij}\lambda^{F(Q_i)+F(U^c_j)+[Z_m(Q_i)+Z_m(U^c_j)]+[Z_n(Q_i)+Z_n(U^c_j)]}v_u,\\
(M_d)_{ij} &= (y_d)_{ij}\lambda^{F(Q_i)+F(D^c_j)+[Z_m(Q_i)+Z_m(D^c_j)]+[Z_n(Q_i)+Z_n(D^c_j)]}v_d,\\
(M_e)_{ij} &= (y_e)_{ij}\lambda^{F(L_i)+F(E^c_j)+[Z_m(L_i)+Z_m(E^c_j)]+[Z_n(L_i)+Z_n(E^c_j)]}v_d,\\
(M_\nu)_{ij} &= (y_\nu)_{ij}\lambda^{F(L_i)+F(L_j)+[Z_m(L_i)+Z_m(L_j)]+[Z_n(L_i)+Z_n(L_j)]}\frac{v_u^2}{\Lambda}.
\end{aligned} \quad (14)$$

Consider the quark sector; the flavor mixing angles there are given by

$$V^u_{ij} \sim \lambda^{(F(Q_i)+F(U^c_j))-(F(Q_j)+F(U^c_j))+[Z_{m(n)}(Q_i)+Z_{m(n)}(U^c_j)]-[Z_{m(n)}(Q_j)+Z_{m(n)}(U^c_j)]}, \quad (15)$$

$$V^d_{ij} \sim \lambda^{(F(Q_i)+F(D^c_j))-(F(Q_j)+F(D^c_j))+[Z_{m(n)}(Q_i)+Z_{m(n)}(D^c_j)]-[Z_{m(n)}(Q_j)+Z_{m(n)}(D^c_j)]}. \quad (16)$$

For $m = n = 0$, the CKM matrix elements describing the charged current weak interaction of quarks behave approximately as $V^{u,d}_{ij} \sim \lambda^{F(Q_i)-F(Q_j)}$, and therefore the CKM mixing $V_{\rm CKM} = V^{u\dagger} \cdot V^d$ is expected to scale as $V_{{\rm CKM}_{ij}} \sim \lambda^{F(Q_i)-F(Q_j)}$. In order to compare with the pure $U(1)$ horizontal symmetry case, we can define an effective flavor charge in the general case $m \neq n \neq 0$ as

$$F_{\rm eff}(\psi) = F(\psi) + Z_m(\psi) + Z_n(\psi). \quad (17)$$

Then it is clear that

$$V_{{\rm CKM}_{ij}} \sim \lambda^{F_{\rm eff}(Q_i)-F_{\rm eff}(Q_j)\pm\alpha m\pm\beta n}, \quad (18)$$

where α, $\beta = 0, 1$, and we have used Eq. (13) and the fact that

$$\begin{aligned}
&[Z_m(Q_i) + Z_m(U^c_j)] - [Z_m(Q_j) + Z_m(U^c_j)]\\
&\quad = Z_m(Q_i) - Z_m(Q_j) \pm \alpha m,
\end{aligned} \quad (19)$$

$$\begin{aligned}
&[Z_m(Q_i) + Z_m(D^c_j)] - [Z_m(Q_j) + Z_m(D^c_j)]\\
&\quad = Z_m(Q_i) - Z_m(Q_j) \pm \alpha m,
\end{aligned} \quad (20)$$

where $\alpha = 0, 1$. The condition for the value $\pm\beta n$ follows similarly. Likewise for the lepton sector, one obtains

$$V^l_{ij} \sim \lambda^{F_{\rm eff}(L_i)-F_{\rm eff}(L_j)\pm\alpha m\pm\beta n}. \quad (21)$$

Therefore, the masses and mixing angles can be enhanced or suppressed by $\lambda^{\pm m \pm n}$ relative to the scaling predictions obtained when the family symmetry is the continuous flavor symmetry $U(1) \times U(1)' \times U(1)''$ because of the discrete nature of $Z_m \times Z_n$. Note that in the case where the light neutrino masses are generated by the type I seesaw mechanism and all fermion charges are positive, the neutrino masses and mixing angles still do not depend on the details of the right-handed neutrino sector, except for the possible enhancement or suppression associated to the $Z_m \times Z_n$ flavor symmetry.

Furthermore, when the flavor symmetry is reduced to $U(1) \times Z_m$ by taking $n = 0$, all the above results remain valid. It is remarkable that we can employ the $U(1) \times Z_m$ flavor symmetry to maintain the $\mathtt{BL}_2$ mixing while achieving very different neutrino masses without fine-tuning. We shall restrict our attention to the case of a Z_2 symmetry which is the minimal nontrivial Z_m group (see, for example, the explicit model construction given in Sec. IV below). In this case, just the Z_m symmetry can reproduce a hierarchy in neutrino masses of order λ^2 consistent with the observed ratio of solar-to-atmospheric splittings.

In contrast, note that since the reactor neutrino mixing is necessarily of order $\sin\theta_{13} \sim \lambda^{2\pm\alpha m}$, the $U(1) \times Z_m$ flavor symmetry cannot produce the $\mathtt{BL}_1$ mixing pattern. Indeed for such $\mathtt{BL}_1$ texture, one has $\sin\theta_{12} \sim \lambda$ and $\sin\theta_{23} \sim \lambda$, which is in conflict with the required linear behavior of the reactor mixing angle $\sin\theta_{13} \sim \lambda$. Note parenthetically that the Z_1 group consists of only the identity element, so the group $U(1) \times Z_1$ is isomorphic to $U(1)$, and the Z_1 charge of field is 0; hence, the flavor symmetry $U(1) \times Z_1$ produces a wrong scaling behavior $\sin\theta_{13} \sim \lambda^2$.

We now turn to the realistic case of the $U(1) \times Z_m \times Z_n$ flavor symmetry. If both solar and atmospheric neutrino mixing angles are of order λ then the reactor angle would be constrained to be of order $\sin\theta_{13} \sim \lambda^{2\pm\alpha m\pm\beta n}$. As a result, one can have $\sin\theta_{13} \sim \lambda$ if the parity of m and n is opposite. This is an interesting observation of the present work. In Sec. III, a concrete model for the $\mathtt{BL}_1$ mixing pattern is presented based on the flavor symmetry $U(1) \times Z_3 \times Z_4$.

Since an Abelian flavor symmetry cannot predict the exact value of the $\mathcal{O}(1)$ coefficients in front of each invariant operator, we must content ourselves with explaining the orders of magnitude of fermion masses and flavor mixing parameters. To identify the phenomenologically acceptable mass matrices, we will estimate the various mass ratios and mixing angles as approximate powers of the small parameter λ. The hierarchies in the quark mixing angles are clearly displayed in Wolfenstein's truncated form [23] of the parametrization of the CKM matrix [24]:

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \tag{22}$$

where the quantities A, ρ, and η are experimentally determined to be of order one. Therefore the order of magnitude of the three mixing angles is given in terms of the λ as

$$|V_{us}| \sim \lambda, \qquad |V_{cb}| \sim \lambda^2, \qquad |V_{ub}| \sim \lambda^3 - \lambda^4. \tag{23}$$

The charged fermion mass ratios at the GUT scale should satisfy [25]

$$\begin{aligned} &\frac{m_u}{m_c} \sim \lambda^4, \qquad \frac{m_c}{m_t} \sim \lambda^3 - \lambda^4, \qquad \frac{m_d}{m_s} \sim \lambda^2, \\ &\frac{m_s}{m_b} \sim \lambda^2, \qquad \frac{m_e}{m_\mu} \sim \lambda^2 - \lambda^3, \qquad \frac{m_\mu}{m_\tau} \sim \lambda^2, \end{aligned} \tag{24}$$

as well as

$$\frac{m_b}{m_\tau} \sim 1, \qquad \frac{m_b}{m_t} \sim \lambda^3 \tag{25}$$

for the intrafamily hierarchy. The first identity is the well-known $b - \tau$ unification relation. For the neutrinos, we required that the lepton mixing is of $\mathtt{BL}_1$ or $\mathtt{BL}_2$ type depending on the octant of θ_{23}. For the quark sector, all the explicit models are properly constructed to meet the requirement $m_t/v_u \sim 1$ and $m_b/v_d \sim \lambda^3$.

III. MODEL FOR $\mathtt{BL}_1$ MIXING

As has been shown in the previous section, one can reproduce the $\mathtt{BL}_1$ texture within the framework of $U(1) \times Z_m \times Z_n$ family symmetry, where m and n should have different parity. For concreteness, we shall use $m = 3$ and $n = 4$ for our model. For such symmetry choice the possible model realization of the $\mathtt{BL}_1$ texture is not unique. As a concrete example, here the horizontal charges of the lepton fields are taken to be

$$\begin{aligned} &L_1\colon (4,1,3), \qquad L_2\colon (3,2,2), \qquad L_3\colon (1,1,1), \\ &E_1^c\colon (3,2,2), \qquad E_2^c\colon (1,2,2), \qquad E_3^c\colon (0,0,0). \end{aligned} \tag{26}$$

One immediately obtains the charged lepton mass matrix

$$M_e \sim \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^8 \\ \lambda^7 & \lambda^5 & \lambda^7 \\ \lambda^7 & \lambda^5 & \lambda^3 \end{pmatrix} v_d, \tag{27}$$

which yields the mass ratios

$$\frac{m_e}{m_\mu} \sim \lambda^3, \qquad \frac{m_\mu}{m_\tau} \sim \lambda^2, \tag{28}$$

that are consistent with the experimental requirements. For the charged assignments in Eq. (26), the light neutrino mass matrix is given by

$$M_\nu \sim \begin{pmatrix} \lambda^{12} & \lambda^8 & \lambda^7 \\ \lambda^8 & \lambda^7 & \lambda^7 \\ \lambda^7 & \lambda^7 & \lambda^6 \end{pmatrix} \frac{v_u^2}{\Lambda}. \tag{29}$$

It predicts the light neutrino mass eigenvalues as follows:

$$m_1 \sim \lambda^8 \frac{v_u^2}{\Lambda}, \qquad m_2 \sim \lambda^7 \frac{v_u^2}{\Lambda}, \qquad m_3 \sim \lambda^6 \frac{v_u^2}{\Lambda}. \tag{30}$$

The neutrino mass spectrum is normal hierarchy, this is confirmed by subsequent numerical analysis. It is remarkable that this model gives rise to $m_2/m_3 \sim \lambda$ and $\Delta m^2_{\mathrm{sol}}/\Delta m^2_{\mathrm{atm}} \sim \lambda^2$, which is in excellent agreement with the experimental data. In conventional $U(1)$ or $U(1) \times Z_m$ flavor symmetries, if any ratio between neutrino masses is an odd power of the small breaking parameter, generally the mixing angle between the two neutrinos will vanish [22]. The crucial point is that the element $(M_\nu)_{22}$, which would have been $\mathcal{O}(\lambda^{14})$ under the continuous $U(1) \times U(1)' \times U(1)''$ symmetry, is enhanced to $\mathcal{O}(\lambda^7)$ due to the discrete symmetry $Z_3 \times Z_4$. Diagonalizing the mass matrices in Eqs. (27) and (29) by the standard perturbative techniques described in Refs. [18,22,26], we get the three lepton flavor mixing angles

$$\sin\theta_{12} \sim \lambda, \qquad \sin\theta_{13} \sim \lambda, \qquad \sin\theta_{23} \sim \lambda. \tag{31}$$

Hence, the $\mathtt{BL}_1$ pattern is produced automatically. Note that the solar neutrino mixing $\sin\theta_{12}$ arises from order λ contributions from the diagonalization of both M_e and M_ν, while at leading order the reactor and the atmospheric neutrino mixing angles receive contribution only from the neutrino mass matrix M_ν. The off-diagonal elements $(M_\nu)_{13}$ and $(M_\nu)_{23}$ are enhanced by Z_4 and Z_3, respectively; hence, we have $\sin\theta_{13} \sim \lambda$ and $\sin\theta_{23} \sim \lambda$ instead of the naive expectations $\sin\theta_{13} \sim \lambda^5$ and $\sin\theta_{23} \sim \lambda^4$ characteristic of the continuous flavor symmetry case.

In the following, we shall extend the model to encompass also quark sector. Since GUT relates quarks and leptons, the transformation properties of quark fields can be determined from those of leptons. In order to give a successful description of the observed fermion mass hierarchies and mixings simultaneously under the same flavor symmetry acting on quarks and leptons, we work in the framework of $SU(5)$ for definiteness. Another motivation of considering $SU(5)$ unification is the anomaly cancellation. If the $U(1)$ flavor symmetry is gauged then a general assignment of flavor charges to the fields will be anomalous. One can imagine the anomaly to be canceled via the Green-Schwarz mechanism [27]; however, one must check whether the correct relations are satisfied [28].

A convenient way to ensure that the flavor charges are amenable to cancellation is to have the flavor symmetry to commute with the $SU(5)$ group [29].

Here we propose a model with the quark and lepton matter assignments manifestly compatible with potential unification within $SU(5)$. A complete study of a realistic grand unified model addressing the well-known problems such as the doublet-triplet splitting, the proton lifetime, and gauge coupling unification is beyond the scope of the present paper and will be studied elsewhere.

In the conventional $SU(5)$ grand unified theory, the fields D_i^c and L_i of the same generation are assigned to a $\bar{\mathbf{5}}$ multiplet; the fields Q_i, U_i^c, and E_i^c are unified in the $\mathbf{10}$ representation. Since the flavor symmetry is required to commute with the gauge symmetry, this means that the fields in each gauge multiplet transform in the same way under the flavor symmetry. Consequently, the quantum numbers of the quark fields under the flavor symmetry $U(1)\times Z_3\times Z_4$ are as follows:

$$\begin{array}{lll} Q_{L1}\colon (3,2,2), & Q_{L2}\colon (1,2,2), & Q_{L3}\colon (0,0,0),\\ U_1^c\colon (3,2,2), & U_2^c\colon (1,2,2), & U_3^c\colon (0,0,0),\\ D_1^c\colon (4,1,3), & D_2^c\colon (3,2,2), & D_3^c\colon (1,1,1). \end{array} \tag{32}$$

We note that although there are many possible assignments to produce the BL_1 texture in the neutrino sector, only a few of them can satisfy the quark sector phenomenological constraints within $SU(5)$. It is well known that the minimal $SU(5)$ grand unified theory predicts that the down-type quark mass matrix is the transpose of the charged lepton mass matrix; therefore, the down-type quarks and charged lepton masses are closely related: $m_e = m_d$, $m_\mu = m_s$, and $m_\tau = m_b$, which are in gross disagreement with the measured fermion masses and must be corrected [30]. This can be done through the contribution of renormalizable [30] or nonrenormalizable [31] operators to the Yukawa matrices. Following Ref. [32], we introduce an additional $U(1)\times Z_3\times Z_4$ singlet superfield Σ transforming as a $\mathbf{75}$ of $SU(5)$, which has nonrenormalizable couplings to fermions of the form $\bar{\mathbf{5}}\mathbf{10}H_{\bar{\mathbf{5}}}\Sigma/\Lambda$. The Yukawa couplings of the down-type quark and charged leptons then arise from the two $SU(5)\times U(1)\times Z_3\times Z_4$ invariant superpotential terms,[2]

$$W_d = \left(\mathbf{10}_i(C_1)_{ij}\bar{\mathbf{5}}_j H_{\bar{\mathbf{5}}} + \frac{\Sigma}{\Lambda}\mathbf{10}_i(C_2)_{ij}\bar{\mathbf{5}}_j H_{\bar{\mathbf{5}}}\right)\left(\frac{\Theta_1}{\Lambda}\right)^{F(\mathbf{10}_i)+F(\bar{\mathbf{5}}_j)} \times \left(\frac{\Theta_2}{\Lambda}\right)^{[Z_3(\mathbf{10}_i)+Z_3(\bar{\mathbf{5}}_j)]}\left(\frac{\Theta_3}{\Lambda}\right)^{[Z_4(\mathbf{10}_i)+Z_4(\bar{\mathbf{5}}_j)]}, \tag{33}$$

which, after the scalar components of Σ acquires a VEV, lead to

$$\begin{aligned}(\mathbf{Y}_d)_{ij} &= ((C_1)_{ij} + \kappa(C_2)_{ij})\lambda^{F(Q_i)+F(D_j^c)+[Z_3(Q_i)+Z_3(D_j^c)]+[Z_4(Q_i)+Z_4(D_j^c)]},\\ (\mathbf{Y}_e)_{ij} &= ((C_1)_{ij} - 3\kappa(C_2)_{ij})\lambda^{F(Q_j)+F(D_i^c)[Z_3(Q_j)+Z_3(D_i^c)]+[Z_4(Q_j)+Z_4(D_i^c)]},\end{aligned} \tag{34}$$

where $\kappa = \langle\Sigma\rangle/\Lambda$, which breaks the transposition relation between $\mathbf{Y}_d$ and $\mathbf{Y}_e$ and can explain the difference between down-type quarks and charged lepton masses. In our numerical fits, we take $\kappa = 0.3$ for illustration and find that realistic values for down-type quarks and charged lepton masses can be reproduced. The superpotential for the up-type quark mass is

$$W_u = \mathbf{10}_i(C_3)_{ij}\mathbf{10}_j H_5\left(\frac{\Theta_1}{\Lambda}\right)^{F(\mathbf{10}_i)+F(\mathbf{10}_j)}\left(\frac{\Theta_2}{\Lambda}\right)^{[Z_3(\mathbf{10}_i)+Z_4(\mathbf{10}_j)]} \times\left(\frac{\Theta_3}{\Lambda}\right)^{[Z_4(\mathbf{10}_i)+Z_4(\mathbf{10}_j)]}, \tag{35}$$

where one has $(C_3)_{ij} = (C_3)_{ji}$ due to the constraint of the $SU(5)$ gauge symmetry. Then one can express the effective Yukawa couplings for the up-type quark in terms of the flavor symmetry charges as

$$(\mathbf{Y}_u)_{ij} = (C_3)_{ij}\lambda^{F(Q_i)+F(Q_j)+[Z_3(Q_i)+Z_3(Q_j)]+[Z_4(Q_i)+Z_4(Q_j)]}. \tag{36}$$

[2] The $\mathbf{75}$ could in principle also give a contribution in the up sector. However, following Ref. [32] we neglect such a term since it is not needed to reproduce the up-type quark masses.

With the assignments dictated by Eq. (32), one has the following patterns for the up- and down-type quark mass matrices,

$$M_u \sim \begin{pmatrix}\lambda^7 & \lambda^5 & \lambda^7\\ \lambda^5 & \lambda^3 & \lambda^5\\ \lambda^7 & \lambda^5 & 1\end{pmatrix} v_u, \qquad M_d \sim \begin{pmatrix}\lambda^8 & \lambda^7 & \lambda^7\\ \lambda^6 & \lambda^5 & \lambda^5\\ \lambda^8 & \lambda^7 & \lambda^3\end{pmatrix} v_d, \tag{37}$$

which yield

$$|V_{us}| \sim \lambda^2, \qquad |V_{cb}| \sim \lambda^2, \qquad |V_{ub}| \sim \lambda^4. \tag{38}$$

We note that both the up and down quark sector contribute λ^2 to the mixing element $|V_{us}|$; therefore, an accidental enhancement of $\mathcal{O}(\lambda^{-1})$ among the undetermined order one coefficients $(C_1)_{ij}$, $(C_2)_{ij}$, and $(C_3)_{ij}$ is required in order to describe the correct Cabibbo angle. The remaining CKM mixing angles $|V_{cb}|$ and $|V_{ub}|$ arise solely from the diagonalization of the down-type quark mass matrix M_d. In addition, the pattern given by Eq. (37) leads to the following quark mass scalings:

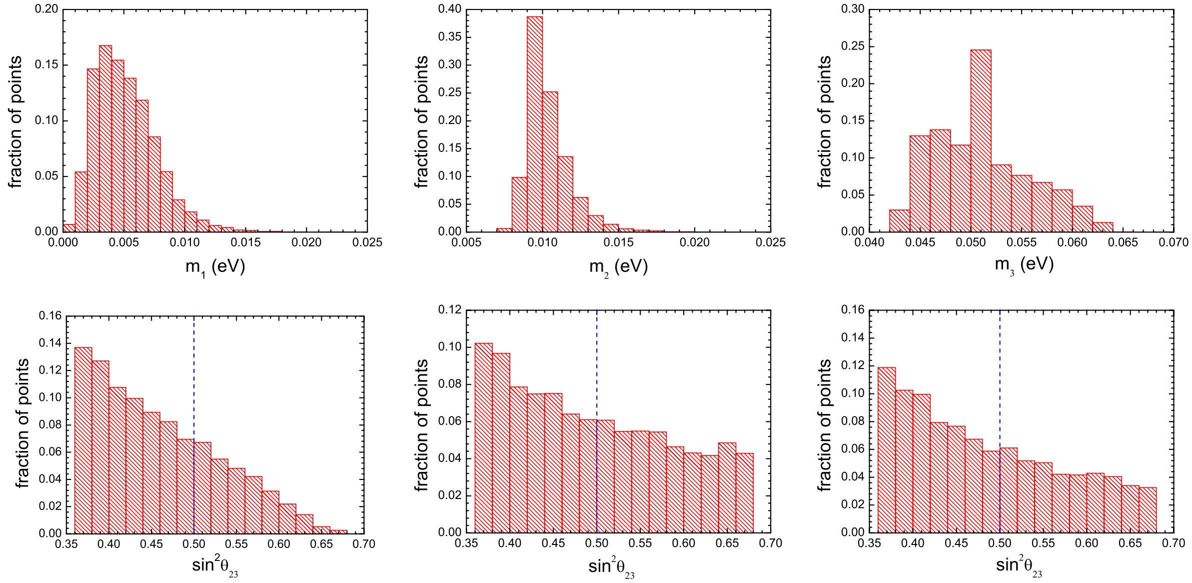

FIG. 1 (color online). Histograms for the distribution of light neutrino masses and atmospheric neutrino mixing parameter in the BL_1 model. In the second row, the left, middle, and right panels are obtained using different seed procedures for the order one Yukawa coefficients; namely, flat, exponential, and Gaussian, respectively, from left to right.

$$\begin{aligned} &m_u \sim \lambda^7 v_u, \quad && m_c \sim \lambda^3 v_u, \quad && m_t \sim v_u, \\ &m_d \sim \lambda^8 v_d, && m_s \sim \lambda^5 v_d, && m_b \sim \lambda^3 v_d, \end{aligned} \tag{39}$$

which describe the experimental data satisfactorily. Note that the second term in Eq. (33) accounts for the mass difference between the down-type quarks and charged leptons, allowing for an acceptable charged fermion mass pattern.

In order to see in a quantitative way how well the model describes the observed values of the fermion masses and mixings, we perform a numerical analysis within three independent different seeding methods; namely, flat, Gaussian, and exponential distributions. The modulus of the undetermined order one coefficients are taken to be random numbers with flat, Gaussian, and exponential distributions; in turn, the corresponding phases are varied between 0 and 2π. The probability density function $f(x)$ of the three distributions is well known,

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & x < a \quad \text{or} \quad x > b \end{cases}. \tag{40}$$

For flat distribution, we take $a = 1/3$ and $b = 3$ for illustration in the present work. In the case of Gaussian distribution,

$$f(x) = \frac{a}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \tag{41}$$

We set the mean $\mu = 1$ and the standard deviation $\sigma = 1.5$ in our numerical calculation. The probability density function for the exponential distribution is

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}. \tag{42}$$

Its statistic mean is $1/\lambda$, and λ is taken to be 1 as a typical value for numerical simulation. To the extent that our results are independent of the choice of seeding method, they are robust and not simply an artifact of the choice of the seed function.

The coefficients $(C_1)_{ij}$, $(C_2)_{ij}$, $(C_3)_{ij}$, and $(y_\nu)_{ij}$ are treated as random complex numbers with arbitrary phases and absolute value in the interval of $[1/3, 3]$. Then we calculate the quark and lepton masses as well as the CKM and lepton mixing matrix entries which are required to lie in the experimentally allowed ranges. The numerical results are found to be nicely consistent with the above theoretical estimates and qualitative discussions. Since the flavor parameters of the quark sector are precisely measured, here we focus on the neutrino sector. As an example, the predicted distributions for the light neutrino masses and atmospheric mixing parameter are shown in Fig. 1. The light neutrino masses follow the normal hierarchy pattern and, for all the points produced, though all nonvanishing, they are rather tiny, with most of the expected m_1 values below 0.015 eV. As to the mixing angles, no specific values of θ_{12} and θ_{13} are favored within 3σ, and hence they are not shown in the figure.[3] In contrast, however, the atmospheric neutrino mixing angle θ_{23} obeys $\sin^2\theta_{23} < 1/2$, which means that nonmaximal θ_{23} values are preferred,

[3]Similarly, we can hardly see any specific preferred pattern for the charge parity (CP) violating phases δ, φ_1, and φ_2; hence, as before, these are not shown.

as indicated by current neutrino oscillation global analyses post-Neutrino 2012 [6–8], with a preference for the first octant. This has been one of our motivations for introducing BL_1 mixing pattern, which leads to $\sin\theta_{23}$ of order λ at leading order.

The rare process, neutrinoless double beta decay ($0\nu 2\beta$), constitutes an important probe for the Majorana nature of neutrino and lepton number violation [33]; a sizable number of new experiments are currently running, under construction, or in the planing phase. The histogram for the distribution of the effective $0\nu 2\beta$-decay mass $|m_{ee}|$ and its correlation with the lightest neutrino mass m_1 are given in Fig. 2. We also show the future sensitivity on the lightest neutrino mass of 0.2 eV from the KATRIN experiment [34]. The horizontal lines represent the sensitivities of the future $0\nu 2\beta$-decay experiments CUORE [35] and MAJORANA [36]/GERDA III [37], which are approximately 18 meV and 12 meV, respectively. Clearly the expected effective mass $|m_{ee}|$ is predicted to be far below the sensitivities of the planned $0\nu 2\beta$ experiments. The reason for this is the strong destructive interference amongst the three light neutrinos, as seen in the right panel. As a result, if $0\nu 2\beta$ decay will be detected in the near future, our construction would be ruled out.

To keep our discussion as generic as possible, we describe the light neutrino masses by the effective higher-dimensional Weinberg operators as shown in Eqs. (3) and (12), which could come from the so-called type I seesaw mechanism by integrating out the right-handed neutrinos. It is interesting to note that $U(1)$ flavor symmetry models have particularly simple factorization properties [21,22]: our various predictions for the light neutrino parameters given above are independent of the $U(1)$ charge assignments of the right-handed neutrinos. For example, suppose we introduce three right-handed neutrinos transforming under the flavor symmetry $U(1)\times Z_3\times Z_4$ as follows:

$$N_1^c: (n_1, 0, 1), \qquad N_2^c: (n_2, 0, 3), \qquad N_3^c: (n_3, 2, 2), \tag{43}$$

where n_i, which are positive integers denoting the $U(1)$ charges of the heavy Majorana neutrinos. Then one can straightforwardly read out the Dirac neutrino mass matrix M_D and the Majorana mass matrix M_N of the right-handed neutrinos,

$$\begin{aligned} M_D &\sim \begin{pmatrix} \lambda^{5+n_1} & \lambda^{7+n_2} & \lambda^{5+n_3} \\ \lambda^{8+n_1} & \lambda^{6+n_2} & \lambda^{4+n_3} \\ \lambda^{4+n_1} & \lambda^{2+n_2} & \lambda^{4+n_3} \end{pmatrix} v_u, \\ M_N &\sim \begin{pmatrix} \lambda^{2+2n_1} & \lambda^{n_1+n_2} & \lambda^{5+n_1+n_3} \\ \lambda^{n_1+n_2} & \lambda^{2+2n_2} & \lambda^{3+n_2+n_3} \\ \lambda^{5+n_1+n_3} & \lambda^{3+n_2+n_3} & \lambda^{1+2n_3} \end{pmatrix} \Lambda. \end{aligned} \tag{44}$$

The resulting effective light neutrino mass matrix is given by the seesaw formula

$$M_\nu = -M_D M_N^{-1} M_D^T \sim \begin{pmatrix} \lambda^9 & \lambda^8 & \lambda^7 \\ \lambda^8 & \lambda^7 & \lambda^7 \\ \lambda^7 & \lambda^7 & \lambda^6 \end{pmatrix} \frac{v_u^2}{\Lambda}. \tag{45}$$

This is the same as obtained in the above effective approach given in Eq. (29) except that the smallest element $(M_\nu)_{11}$ is of order λ^9 instead of λ^{12}; both of them are too small to affect the predictions for the neutrino oscillation parameters. We get the same light neutrino masses in Eq. (30) and the same neutrino mixing angles in Eq. (31) as in the above effective Weinberg operator neutrino mass generation. We would like to emphasize again that the predictions for the neutrino masses and mixing parameters are independent of the charges n_i, which drop out in the seesaw formula for the light neutrino mass matrix. However, different values of the charges n_i obviously give rise to different Dirac neutrino Yukawa

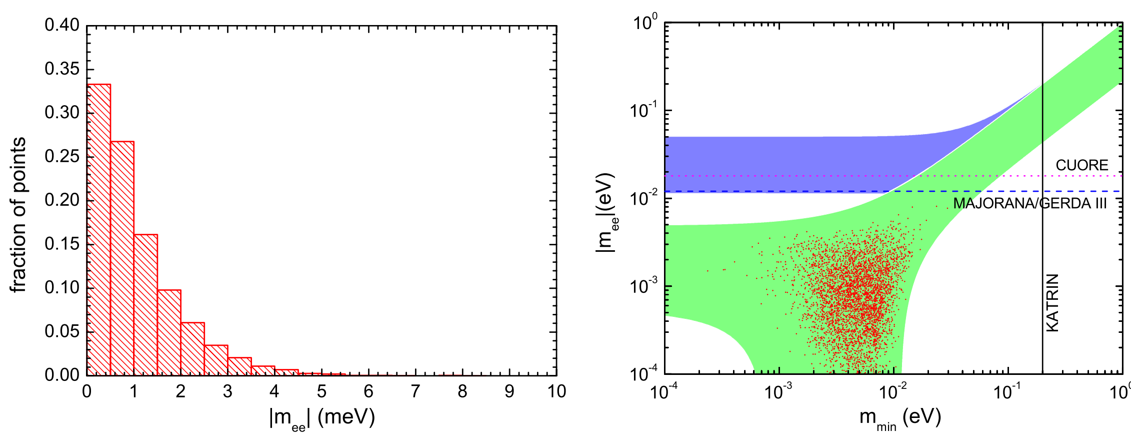

FIG. 2 (color online). Histogram of the effective mass $|m_{ee}|$ (left panel) and the scatter plot of $|m_{ee}|$ versus the lightest neutrino mass m_1 (right panel) for the BL_1 model. The colored bands represent the regions for the 3σ ranges of the oscillation parameters in the normal and inverted neutrino mass spectrum, respectively. The future sensitivity of 0.2 eV of the KATRIN experiment is shown by the vertical solid line, while the future expected bounds on $|m_{ee}|$ from the CUORE and MAJORANA/GERDA III experiments are represented by horizontal lines.

coupling $Y_\nu \equiv M_D/v_u$. As a result, the predictions for charged lepton flavor violation (LFV) processes such as $\mu \to e\gamma$, $\tau \to \mu\gamma$, and $\mu \to 3e$ are quite different [38]. Recalling that the branching ratio of the LFV process is generally proportional to Y_ν^4, the stringent bound on LFV, in particular from $\mu \to e\gamma$, can be easily satisfied for only slightly large n_i [38] while keeping the predictions for neutrino parameters intact.

IV. MODEL FOR BL$_2$ MIXING

As explained in Sec. II, the order one atmospheric neutrino mixing $\sin\theta_{23} \sim 1$ generically implies that the corresponding masses of ν_2 and ν_3 are of the same order of magnitude within pure $U(1)$ family symmetry schemes. As a result, the neutrino mass spectrum is quasidegenerate and strong fine-tuning is required in order to account for the measured mass-squared differences $\Delta m^2_{\rm sol}$ and $\Delta m^2_{\rm atm}$. Furthermore, the renormalization group evolution effects could drastically enhance the neutrino mixing angles due to the degeneracy so that the BL$_2$ texture would be spoiled at the electroweak scale. This can be avoided by extending the flavor symmetry to $U(1) \times Z_m$. Now the whole flavor symmetry is chosen to be $U(1) \times Z_2$; the lepton fields carry the following $U(1) \times Z_2$ charges:

$$\begin{aligned} &L_1\colon (3,0), \qquad L_2\colon (3,1), \qquad L_3\colon (2,0),\\ &E_1^c\colon (4,0), \qquad E_2^c\colon (2,1), \qquad E_3^c\colon (0,1). \end{aligned} \tag{46}$$

Then the light neutrino mass matrix is given, apart from the order one coefficients, as

$$M_\nu \sim \begin{pmatrix} \lambda^6 & \lambda^7 & \lambda^5 \\ \lambda^7 & \lambda^6 & \lambda^6 \\ \lambda^5 & \lambda^6 & \lambda^4 \end{pmatrix} \frac{v_u^2}{\Lambda}, \tag{47}$$

which yields

$$m_1 \sim \lambda^6 v_u^2/\Lambda, \qquad m_2 \sim \lambda^6 v_u^2/\Lambda, \qquad m_3 \sim \lambda^4 v_u^2/\Lambda. \tag{48}$$

One sees that the first two light neutrinos are quasidegenerate in this model, and their masses are suppressed by $\mathcal{O}(\lambda^2)$ with respect to the third one. This prediction is consistent with the observation that the solar neutrino mass difference $\Delta m^2_{\rm sol}$ is much smaller than the atmospheric neutrino mass difference $\Delta m^2_{\rm atm}$. Moreover, the neutrino mass spectrum is predicted to be of the normal hierarchy type here, the same as in the previous BL$_1$ model (this is also confirmed our numerical analysis). The next generation of higher precision neutrino oscillation experiments is designed to be able to measure neutrino mass hierarchy and the CP phase [39]. Should the latter be determined to be of the inverted type by future experiments, both of our models would be ruled out. On the other hand, the charged lepton mass matrix takes the following form:

$$M_e \sim \begin{pmatrix} \lambda^7 & \lambda^6 & \lambda^4 \\ \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^6 & \lambda^5 & \lambda^3 \end{pmatrix} v_d, \tag{49}$$

which has a "lopsided" structure; a large 2–3 mixing arises from the diagonalization of M_e. Obviously it also gives the correct order of magnitude for the charged lepton mass ratios. Combining the contribution from both the neutrino and the charged lepton mass matrices diagonalization, the leptonic mixing angles are given by

$$\sin\theta_{12} \sim \lambda, \qquad \sin\theta_{13} \sim \lambda \qquad \sin\theta_{23} \sim 1. \tag{50}$$

This is exactly the desired BL$_2$ mixing pattern, Eq. (2). Here we would like to point out that since the Super-Kamiokande data indicted large atmospheric neutrino mixing, perhaps even maximal [40], there have been several attempts to account for the large atmospheric neutrino mixing $\sin\theta_{23} \sim 1$ in terms of Abelian flavor symmetries [41]. However, it was usually assumed that the reactor angle θ_{13} was rather small, at most of order λ^2 at that time [42]. In contrast in our construction, the consistency between large $\sin\theta_{23}$ and sizeable $\sin\theta_{13}$ mixing angles emerges naturally.

In what follows, we extend the model to include quarks within the $SU(5)$ unified framework. The fields Q_i and U_i^c together with E_i^c within the same generation fill out the **10** representation, while D_i^c and the left-handed lepton doublet L_i make up the $\bar{\mathbf{5}}$ representation. As a result, we can determine the transformation properties of the quark fields under the $U(1) \times Z_2$ flavor symmetry as follows:

$$\begin{aligned} &Q_1\colon (4,0), \qquad Q_2\colon (2,1), \qquad Q_3\colon (0,1),\\ &U_1^c\colon (4,0), \qquad U_2^c\colon (2,1), \qquad U_3^c\colon (0,1),\\ &D_1^c\colon (3,0), \qquad D_2^c\colon (3,1), \qquad D_3^c\colon (2,0). \end{aligned} \tag{51}$$

The up and down quark mass matrices can be determined in a straightforward way as follows:

$$M_u \sim \begin{pmatrix} \lambda^8 & \lambda^7 & \lambda^5 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix} v_u, \qquad M_d \sim \begin{pmatrix} \lambda^7 & \lambda^8 & \lambda^6 \\ \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^4 & \lambda^3 & \lambda^3 \end{pmatrix} v_d, \tag{52}$$

which lead to

$$\begin{aligned} &|V_{us}| \sim \lambda, \qquad |V_{cb}| \sim \lambda^2, \qquad |V_{ub}| \sim \lambda^3, \qquad \frac{m_u}{m_c} \sim \lambda^4,\\ &\frac{m_c}{m_t} \sim \lambda^4, \qquad \frac{m_d}{m_s} \sim \lambda^2, \qquad \frac{m_s}{m_b} \sim \lambda^2, \qquad \frac{m_b}{m_t} \sim \lambda^3, \end{aligned} \tag{53}$$

which are in excellent agreement with observed quark mass hierarchies and CKM mixing angles. As in Sec. III, we perform a numerical simulation of the expected neutrino oscillation parameters. In Fig. 3 we display the

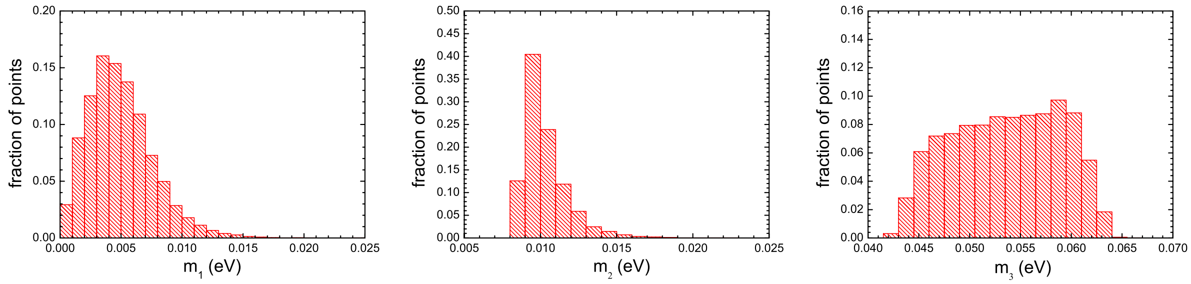

FIG. 3 (color online). Light neutrino masses in the $\mathtt{BL}_2$ model.

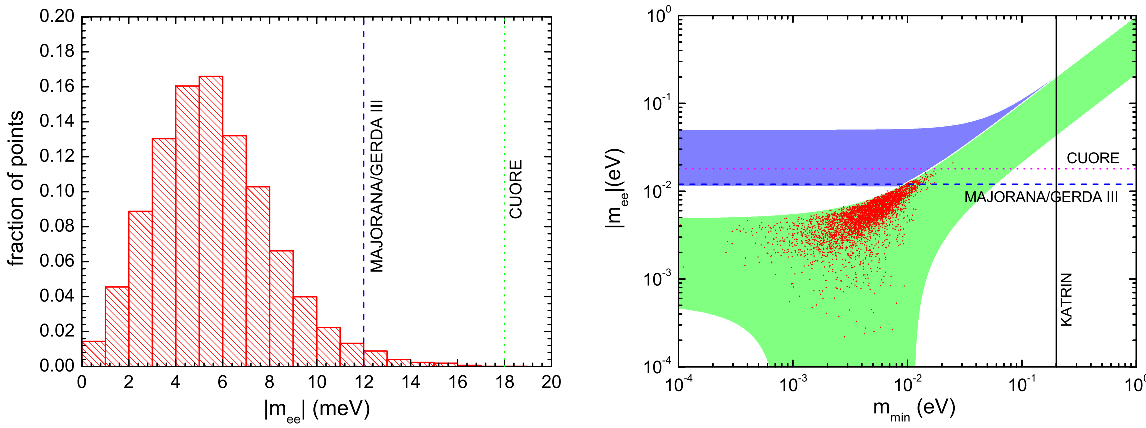

FIG. 4 (color online). Histogram of the effective mass $|m_{ee}|$ (left panel) and the $|m_{ee}|$ versus the lightest neutrino mass m_1 correlation (right panel) predicted in the $\mathtt{BL}_2$ model.

resulting histograms for the neutrino mass eigenvalues.[4] As expected, on the basis of the qualitative estimate in Eq. (48), the light neutrino mass spectrum is normal hierarchy, the degenerate spectrum being strongly disfavored, and almost all the generated points lie in the region of the lightest neutrino mass m_1 smaller than 0.015 eV. The neutrinoless double beta decay predictions are shown in Fig. 4. One sees that in contrast with the $\mathtt{BL}_1$ case, although the effective mass $|m_{ee}|$ is also quite small, with $|m_{ee}|$ around 5 meV preferred, there is a small portion of the parameter space of the model where the predictions for $|m_{ee}|$ approach the future experimental sensitivities. However, the points above the sensitivity limits on next generation experiments are statistically rather low. Therefore, if the signal of $0\nu 2\beta$ decay would be observed by upcoming experiments, the present $\mathtt{BL}_2$ model would also be ruled out, although not completely. We expect that the future $0\nu 2\beta$-decay experiments with sensitivity much higher than MAJORANA/GERDA III should be able to provide a better test of the model.

Now we turn to the seesaw realization of this model; the assignments for the right-handed neutrinos are not unique. As an example, we can introduce three right-handed neutrinos transforming as

$$N_1^c\colon (n_1, 0), \qquad N_2^c\colon (n_2, 1), \qquad N_3^c\colon (n_3, 0). \tag{54}$$

Then we obtain the Dirac neutrino mass matrix M_D as well as the right-handed neutrino mass matrix M_N,

$$\begin{aligned} M_D &\sim \begin{pmatrix} \lambda^{3+n_1} & \lambda^{4+n_2} & \lambda^{3+n_3} \\ \lambda^{4+n_1} & \lambda^{3+n_2} & \lambda^{4+n_3} \\ \lambda^{2+n_1} & \lambda^{3+n_2} & \lambda^{2+n_3} \end{pmatrix} v_u, \\ M_N &\sim \begin{pmatrix} \lambda^{2n_1} & \lambda^{1+n_1+n_2} & \lambda^{n_1+n_3} \\ \lambda^{1+n_1+n_2} & \lambda^{2n_2} & \lambda^{1+n_2+n_3} \\ \lambda^{n_1+n_3} & \lambda^{1+n_2+n_3} & \lambda^{2n_3} \end{pmatrix} \Lambda. \end{aligned} \tag{55}$$

The effective light neutrino mass matrix is given by the seesaw relation

$$M_\nu = -M_D M_M^{-1} M_D^T \sim \begin{pmatrix} \lambda^6 & \lambda^7 & \lambda^5 \\ \lambda^7 & \lambda^6 & \lambda^6 \\ \lambda^5 & \lambda^6 & \lambda^4 \end{pmatrix} \frac{v_u^2}{\Lambda}. \tag{56}$$

This is exactly Eq. (47); consequently, the predictions for neutrino parameters in Eqs. (48) and (50) remain (note that dependence on the right-handed neutrino charges n_i drops out). However, different values of the charges n_i result in

[4]Insofar as the neutrino mixing angles θ_{ij} and CP phases δ, φ_1, and φ_2 are concerned, we do not obtain any special predicted pattern; hence, the results are not displayed.

different LFV predictions, and the model would be less constrained for slightly large n_i assignments [38].

V. CONCLUSIONS

The recent neutrino oscillation experimental highlights (i) rather large value of reactor mixing angle θ_{13} and (ii) indication of significant deviation of the atmospheric neutrino mixing angle θ_{23} from maximality may change our theoretical approach for constructing neutrino mass models. In this paper, we study the Wolfenstein-like mixing schemes: $\mathtt{BL}_1$ mixing in which $\sin\theta_{12} \sim \lambda$, $\sin\theta_{13} \sim \lambda$, $\sin\theta_{23} \sim \lambda$, and $\mathtt{BL}_2$ mixing in which $\sin\theta_{12} \sim \lambda$, $\sin\theta_{13} \sim \lambda$, $\sin\theta_{23} \sim 1$. The largish θ_{13} can be naturally accommodated in both of them; the two mixing patterns differ in the order of magnitude of $\sin\theta_{23}$; the $\mathtt{BL}_1$ texture is favored for θ_{23} in the first octant, while $\mathtt{BL}_2$ is preferred for the second octant θ_{23}. In order to produce the $\mathtt{BL}_1$ mixing without invoking unnatural cancellation, the Abelian flavor symmetry should be $U(1) \times Z_m \times Z_n$ with the parity of m and n being opposite. A concrete model based on $U(1) \times Z_3 \times Z_4$ family symmetry is constructed, where the light neutrino mass hierarchy $m_2/m_3 \sim \lambda$ is realized due to the discrete nature of $Z_3 \times Z_4$. The ratio $\Delta m^2_{\mathrm{sol}}/\Delta m^2_{\mathrm{atm}}$ is expected to be of order λ^2 in this model, which is in good agreement with experimental data in contrast with conventional $U(1)$ or $U(1) \times Z_m$ flavor symmetry constructions. Furthermore, the model is embedded into the $SU(5)$ grand unified theory to describe the quark masses and mixing simultaneously. As for the $\mathtt{BL}_2$ mixing, it can be reproduced within the framework of pure $U(1)$ flavor symmetry. However, the light neutrino mass spectrum is expected to be quasidegenerate; hence, fine-tuning of the neutrino mass parameters is needed in order to achieve the observed mass-squared differences. To improve upon this situation, the family symmetry is enlarged to $U(1) \times Z_2$, which gives rise to both large atmospheric neutrino mixing $\sin\theta_{23} \sim 1$ and hierarchical neutrino masses. The model is extended to $SU(5)$ grand unified theory as well.

We show that both models can give a successful description of the observed quark and lepton masses and mixing angles, and the numerical results are nicely in agreement with the theoretical estimates and the qualitative discussions. The light neutrinos are normal mass hierarchy in both models; quasidegenerate spectrum is strongly disfavored. If the next generation high precision neutrino oscillation experiments determine that the neutrino mass spectrum is inverted hierarchy, both our constructions will be ruled out. The present framework cannot predict the CP violating phases δ, φ_1, and φ_2. The $0\nu2\beta$-decay effective mass $|m_{ee}|$ is predicted to be rather small in both constructions; a substantial part of the data are below the sensitivity of future experiments except for a region of the $\mathtt{BL}_2$ model indicated in Fig. 4. Therefore, future $0\nu2\beta$-decay experiments such as CUORE, MAJORANA, and GERDA III will provide another important test of the present models.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Grant No. 10905053, Chinese Academy KJCX2-YW-N29, DFG Grant No. WI 2639/4-1, and the 973 project with Grant No. 2009CB825200; by the Spanish MINECO under Grants No. FPA2011-22975 and MULTIDARK No. CSD2009-00064 (Consolider-Ingenio 2010 Programme), by Prometeo/2009/091 (Generalitat Valenciana), and by the EU ITN UNILHC PITN-GA-2009-237920. S. M. is supported by a Juan de la Cierva grant.

[1] F. P. An *et al.* (Daya Bay Collaboration), Phys. Rev. Lett. **108**, 171803 (2012).

[2] J. K. Ahn *et al.* (RENO Collaboration), Phys. Rev. Lett. **108**, 191802 (2012).

[3] Y. Abe *et al.* (Double Chooz Collaboration), Phys. Rev. Lett. **108**, 131801 (2012).

[4] K. Abe *et al.* (T2K Collaboration), Phys. Rev. Lett. **107**, 041801 (2011).

[5] P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. Lett. **107**, 181802 (2011).

[6] D. V. Forero, M. Tortola, and J. W. F. Valle, Phys. Rev. D **86**, 073012 (2012).

[7] G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, and A. M. Rotunno, Phys. Rev. D **86**, 013012 (2012).

[8] M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado, and T. Schwetz, J. High Energy Phys. 12 (2012) 123.

[9] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. B **530**, 167 (2002); P. F. Harrison and W. G. Scott, Phys. Lett. B **535**, 163 (2002); Z.-z. Xing, Phys. Lett. B **533**, 85 (2002); X. G. He and A. Zee, Phys. Lett. B **560**, 87 (2003).

[10] Y. Shimizu, M. Tanimoto, and A. Watanabe, Prog. Theor. Phys. **126**, 81 (2011); J.-M. Chen, B. Wang, and X.-Q. Li, Phys. Rev. D **84**, 073002 (2011); Z.-z. Xing, Chinese Phys. C **36**, 101 (2012); E. Ma and D. Wegman, Phys. Rev. Lett. **107**, 061803 (2011); Y.-j. Zheng and B.-Q. Ma, Eur. Phys. J. Plus **127**, 7 (2012); X.-G. He and A. Zee, Phys. Rev. D **84**, 053004 (2011); S. Zhou, Phys. Lett. B **704**, 291 (2011); N. Haba and R. Takahashi, Phys. Lett. B **702**, 388 (2011); D. Meloni, J. High Energy Phys. 10 (2011) 010; S. Morisi, K. M. Patel, and E. Peinado, Phys. Rev. D **84**, 053002 (2011); W. Chao and Y.-j. Zheng, J. High Energy Phys. 02 (2013) 044; H. Zhang and S. Zhou, Phys. Lett. B **704**, 296 (2011); X. Chu, M. Dhen, and

T. Hambye, J. High Energy Phys. 11 (**2011**) 106; P. S. Bhupal Dev, R. N. Mohapatra, and M. Severson, Phys. Rev. D **84**, 053005 (2011); S. Antusch and V. Maurer, Phys. Rev. D **84**, 117301 (2011); W. Rodejohann, H. Zhang, and S. Zhou, Nucl. Phys. **B855**, 592 (2012); Y. H. Ahn, H.-Y. Cheng, and S. Oh, Phys. Rev. D **84**, 113007 (2011); S. F. King and C. Luhn, J. High Energy Phys. 09 (2011) 042; Q.-H. Cao, S. Khalil, E. Ma, and H. Okada, Phys. Rev. D **84**, 071302 (2011); D. Marzocca, S. T. Petcov, A. Romanino, and M. Spinrath, J. High Energy Phys. 11 (2011) 009; S.-F. Ge, D. A. Dicus, and W. W. Repko, Phys. Rev. Lett. **108**, 041801 (2012); R. Shahbuddin, J. High Energy Phys. 07 (2012) 103; 08 (2012) 129(E); F. Bazzocchi, arXiv:1108.2497; T. Araki and C.-Q. Geng, J. High Energy Phys. 09 (2011) 139; S. Antusch, S. F. King, C. Luhn, and M. Spinrath, Nucl. Phys. **B856**, 328 (2012); H. Fritzsch, Z.-z. Xing, and S. Zhou, J. High Energy Phys. 09 (2011) 083; R. N. Mohapatra and M. K. Parida, Phys. Rev. D **84**, 095021 (2011); A. Rashed and A. Datta, Phys. Rev. D **85**, 035019 (2012); P. O. Ludl, S. Morisi, and E. Peinado, Nucl. Phys. **B857**, 411 (2012); N. Okada and Q. Shafi, arXiv:1109.4963; A. Aranda, C. Bonilla, and A. D. Rojas, Phys. Rev. D **85**, 036004 (2012); G.-J. Ding, L. L. Everett, and A. J. Stuart, Nucl. Phys. **B857**, 219 (2012); D. Meloni, J. High Energy Phys. 02 (2012) 090; S. Dev, S. Gupta, R. R. Gautam, and L. Singh, Phys. Lett. B **706**, 168 (2011); K. N. Deepthi, S. Gollu, and R. Mohanta, Eur. Phys. J. C **72**, 1888 (2012); A. Rashed, Phys. Rev. D **85**, 035019 (2012); W. Buchmuller, V. Domcke, and K. Schmitz, J. High Energy Phys. 03 (2012) 008; I. d. M. Varzielas, J. High Energy Phys. 01 (2012) 097; R. de Adelhart Toorop, F. Feruglio, and C. Hagedorn, Nucl. Phys. **B858**, 437 (2012); S. F. King and C. Luhn, J. High Energy Phys. 03 (2012) 036; S. Gupta, A. S. Joshipura, and K. M. Patel, Phys. Rev. D **85**, 031903 (2012); H. Ishimori and T. Kobayashi, Phys. Rev. D **85**, 125004 (2012); S. Dev, R. R. Gautam, and L. Singh, Phys. Lett. B **708**, 284 (2012); P. S. Bhupal Dev, B. Dutta, R. N. Mohapatra, and M. Severson, Phys. Rev. D **86**, 035002 (2012); X. Zhang and B.-Q. Ma, Phys. Lett. B **710**, 630 (2012); I. K. Cooper, S. F. King, and C. Luhn, J. High Energy Phys. 06 (2012) 130; Z.-Z. Xing, Chinese Phys. C **36**, 281 (2012); Y.-L. Wu, Phys. Lett. B **714**, 286 (2012); Y. BenTov and A. Zee, Phys. Lett. B **714**, 80 (2012); H.-J. He and X.-J. Xu, Phys. Rev. D **86**, 111301 (2012); D. Meloni, J. High Energy Phys. 05 (2012) 124; Y. H. Ahn and S. K. Kang, Phys. Rev. D **86**, 093003 (2012); I. d. M. Varzielas and G. G. Ross, J. High Energy Phys. 12 (2012) 041; D. Hernandez and A. Y. Smirnov, Phys. Rev. D **86**, 053014 (2012); C. Hagedorn and D. Meloni, Nucl. Phys. **B862**, 691 (2012); M. Fukugita, Y. Shimizu, M. Tanimoto, and T. T. Yanagida, Phys. Lett. B **716**, 294 (2012); S. Zhou, arXiv:1205.0761; C. Hagedorn, S. F. King, and C. Luhn, Phys. Lett. B **717**, 207 (2012); G. Altarelli, F. Feruglio, L. Merlo, and E. Stamou, J. High Energy Phys. 08 (2012) 021; G. Altarelli, F. Feruglio, and L. Merlo, arXiv:1205.5133; A. Meroni, S. T. Petcov, and M. Spinrath, Phys. Rev. D **86**, 113003 (2012); M. J. Baker, J. Bordes, H. M. Chan, and S. T. Tsou, arXiv:1206.0199; X. Zhang and B.-Q. Ma, Phys. Rev. D **86**, 093002 (2012); E. Ma, A. Natale, and A. Rashed, Int. J. Mod. Phys. A **27**, 1250134 (2012); N. Haba and R. Takahashi, Europhys. Lett. **100**, 31001 (2012); A. G. Dias, A. C. B. Machado, and C. C. Nishi, Phys. Rev. D **86**, 093005 (2012); P. M. Ferreira, W. Grimus, L. Lavoura, and P. O. Ludl, J. High Energy Phys. 09 (2012) 128; G. Altarelli, F. Feruglio, I. Masina, and L. Merlo, J. High Energy Phys. 11 (2012) 139; Y. BenTov, X.-G. He, and A. Zee, J. High Energy Phys. 12 (2012) 093; W. Rodejohann and H. Zhang, Phys. Rev. D **86**, 093008 (2012); R. N. Mohapatra and C. C. Nishi, Phys. Rev. D **86**, 073007 (2012).

[11] R. d. A. Toorop, F. Feruglio, and C. Hagedorn, Phys. Lett. B **703**, 447 (2011).

[12] G.-J. Ding, Nucl. Phys. **B862**, 1 (2012).

[13] S. F. King, C. Luhn, and A. J. Stuart, Nucl. Phys. **B867**, 203 (2013).

[14] S. M. Boucenna, S. Morisi, M. Tortola, and J. W. F. Valle, Phys. Rev. D **86**, 051301 (2012).

[15] S. F. King, Phys. Lett. B **718**, 136 (2012); S. Antusch, C. Gross, V. Maurer, and C. Sluka, Nucl. Phys. **B866**, 255 (2013).

[16] H. Minakata and A. Y. Smirnov, Phys. Rev. D **70**, 073009 (2004); N. Li and B.-Q. Ma, Eur. Phys. J. C **42**, 17 (2005); N. Qin and B. Q. Ma, Phys. Lett. B **702**, 143 (2011); Y. H. Ahn, H.-Y. Cheng, and S. Oh, Phys. Rev. D **83**, 076012 (2011).

[17] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. **B147**, 277 (1979).

[18] M. Leurer, Y. Nir, and N. Seiberg, Nucl. Phys. **B398**, 319 (1993); **B420**, 468 (1994).

[19] E. Dudas, C. Grojean, S. Pokorski, and C. A. Savoy, Nucl. Phys. **B481**, 85 (1996); K. Choi, E. J. Chun, and H. D. Kim, Phys. Lett. B **394**, 89 (1997); N. Irges, S. Lavignac, and P. Ramond, Phys. Rev. D **58**, 035003 (1998); J. K. Elwood, N. Irges, and P. Ramond, Phys. Rev. Lett. **81**, 5064 (1998); J. R. Ellis, G. K. Leontaris, and J. Rizos, J. High Energy Phys. 05 (2000) 001; H. K. Dreiner and M. Thormeier, Phys. Rev. D **69**, 053002 (2004); H. K. Dreiner, H. Murayama, and M. Thormeier, Nucl. Phys. **B729**, 278 (2005); M.-C. Chen, D. R. T. Jones, A. Rajaraman, and H.-B. Yu, Phys. Rev. D **78**, 015019 (2008); L. F. Duque, D. A. Gutierrez, E. Nardi, and J. Norena, Phys. Rev. D **78**, 035003 (2008).

[20] G. Altarelli, F. Feruglio, and L. Merlo, J. High Energy Phys. 05 (2009) 020.

[21] A. Rasin and J. P. Silva, Phys. Rev. D **49**, R20 (1994).

[22] Y. Grossman and Y. Nir, Nucl. Phys. **B448**, 30 (1995); Y. Grossman, Y. Nir, and Y. Shadmi, J. High Energy Phys. 10 (1998) 007; Y. Nir and Y. Shadmi, J. High Energy Phys. 05 (1999) 023.

[23] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[24] J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980); W. Rodejohann and J. W. F. Valle, Phys. Rev. D **84**, 073011 (2011).

[25] H. Fusaoka and Y. Koide, Phys. Rev. D **57**, 3986 (1998); G. Ross and M. Serna, Phys. Lett. B **664**, 97 (2008); Z.-z. Xing, H. Zhang, and S. Zhou, Phys. Rev. D **77**, 113016 (2008).

[26] L. J. Hall and A. Rasin, Phys. Lett. B **315**, 164 (1993).

[27] M. B. Green and J. H. Schwarz, Phys. Lett. B **149**, 117 (1984).

[28] L. E. Ibanez and G. G. Ross, Phys. Lett. B **332**, 100 (1994); P. Binetruy and P. Ramond, Phys. Lett. B **350**, 49 (1995);

V. Jain and R. Shrock, Phys. Lett. B **352**, 83 (1995); E. Dudas, S. Pokorski, and C. A. Savoy, Phys. Lett. B **356**, 45 (1995); P. Binetruy, S. Lavignac, and P. Ramond, Nucl. Phys. **B477**, 353 (1996); P. H. Chankowski, K. Kowalska, S. Lavignac, and S. Pokorski, Phys. Rev. D **71**, 055004 (2005).

[29] A. E. Nelson and D. Wright, Phys. Rev. D **56**, 1598 (1997).

[30] H. Georgi and C. Jarlskog, Phys. Lett. B **86**, 297 (1979).

[31] J. R. Ellis and M. K. Gaillard, Phys. Lett. B **88**, 315 (1979).

[32] G. Altarelli, F. Feruglio, and I. Masina, J. High Energy Phys. 11 (2000) 040.

[33] J. Schechter and J. W. F. Valle, Phys. Rev. D **25**, 2951 (1982).

[34] A. Osipowicz *et al.* (KATRIN Collaboration), arXiv:hep-ex/0109033; see also, http://www-ik.fzk.de/ katrin/index.html.

[35] C. Arnaboldi *et al.* (CUORE Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A **518**, 775 (2004); R. Ardito, C. Arnaboldi, D. R. Artusa, F. T. Avignone III, M. Balata, I. Bandac, M. Barucci, J. W. Beeman *et al.*, arXiv:hep-ex/0501010.

[36] S. R. Elliott (MAJORANA Collaboration), J. Phys. Conf. Ser. **173**, 012007 (2009); V. E. Guiseppe *et al.* (MAJORANA Collaboration), IEEE Nucl. Sci. Symp. Conf. Rec. **2008**, 1793 (2008).

[37] I. Abt, M. F. Altmann, A. Bakalyarov, I. Barabanov, C. Bauer, E. Bellotti, S. T. Belyaev, L. B. Bezrukov *et al.*, arXiv:hep-ex/0404039; S. Schonert (GERDA Collaboration), J. Phys. Conf. Ser. **203**, 012014 (2010).

[38] M. Cannoni, J. Ellis, M. E. Gomez, and S. Lola, arXiv:1301.6002.

[39] L. Zhan, Y. Wang, J. Cao, and L. Wen, Phys. Rev. D **79**, 073007 (2009); X. Qian, D. A. Dwyer, R. D. McKeown, P. Vogel, W. Wang, and C. Zhang, Phys. Rev. D **87**, 033005 (2013).

[40] Y. Fukuda *et al.* (Super-Kamiokande Collaboration), Phys. Rev. Lett. **81**, 1562 (1998).

[41] P. Binetruy, S. Lavignac, S. T. Petcov, and P. Ramond, Nucl. Phys. **B496**, 3 (1997); S. Lola and G. G. Ross, Nucl. Phys. **B553**, 81 (1999); M. Tanimoto, Phys. Lett. B **456**, 220 (1999); M. S. Berger and K. Siyeon, Phys. Rev. D **62**, 033004 (2000); **64**, 053006 (2001); J. Sato and K. Tobe, Phys. Rev. D **63**, 116010 (2001).

[42] M. Maltoni, T. Schwetz, M. A. Tortola, and J. W. F. Valle, New J. Phys. **6**, 122 (2004).