

**Lorentz violation in the Higgs sector and the noncommutative standard model**S. Aghababaei,<sup>1,\*</sup> M. Haghghat,<sup>1,†</sup> and A. Kheirandish<sup>1,2,‡</sup><sup>1</sup>*Department of Physics, Isfahan University of Technology, Isfahan 84156-83111, Iran*<sup>2</sup>*Wisconsin IceCube Particle Astrophysics Center (WIPAC), University of Wisconsin, Madison, Wisconsin 53703, USA*

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The noncommutative standard model apparently violates the Lorentz symmetry. We compare the Lorentz-violating terms in the Higgs sector of the noncommutative standard model with their counterparts in the standard model extension. We show that the Lorentz-violating parameters in the Higgs sector can be expressed directly in terms of the noncommutative parameter without any background field. The absence of the background field enhances the obtained bounds on the noncommutative parameter from the standard model extension.

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**I. INTRODUCTION**

Although the Lorentz symmetry is a well-tested symmetry of nature, the possibility that new physics involves a Lorentz symmetry violation has been considered in many works. The main motivation to consider such a violation is in the fact that in the Planck scale, where quantum gravity should be considered, the Lorentz symmetry violation arises naturally. V. A. Kostelecky and S. Samuel [1] showed that the Lorentz symmetry can be broken spontaneously in the context of string theory. Consequently, D. Colladay and V. Alan Kostelecky [2], irrespective of the underlying fundamental theory, introduced a general extension of the minimal standard model that violates both Lorentz invariance and *CPT*. The phenomenological aspects of the so-called standard model extension (SME) have been extensively considered by many authors in terrestrial [3] and astrophysical systems [4], and the bounds on the Lorentz-violating (LV) parameters are collected in Ref. [5]. Meanwhile, in noncommutative (NC) space-time, where in its canonical version, the coordinates are operators and satisfy the relation

$$[\hat{x}_\mu, \hat{x}_\nu] = \theta_{\mu\nu} = \frac{\varepsilon_{\mu\nu}}{\Lambda_{\text{NC}}^2}, \quad (1)$$

the real, constant, and antisymmetric parameter  $\theta_{\mu\nu}$  breaks the Lorentz symmetry intrinsically. Therefore, the standard model in noncommutative space-time may be considered as a subset of the SME. There are two approaches to constructing the gauge theories—and consequently, the standard model—in noncommutative space. In the first one, the gauge group is restricted to  $U(n)$ , and the symmetry group of the standard model is achieved by the reduction of  $U(3) \times U(2) \times U(1)$  to  $SU(3) \times SU(2) \times U(1)$  by an appropriate symmetry breaking [6]. In the second one, the noncommutative gauge theory can be constructed for

a  $SU(n)$  gauge group via a Seiberg-Witten map [7] where the fields themselves, in contrast with the first approach, depend on the parameter of noncommutativity [8]. However, the NC field theories and their phenomenological aspects based on both versions have been examined for many years [9–11]. The relation between the NC field theory with the  $U(1)$  gauge group based on both approaches is compared with the QED part of the SME in Ref. [12]. In both versions, the Lorentz-violating parameters depend on the NC parameter through a magnetic field as a background; and in the absence of the background, all the LV parameters are zero. In this article, we would like to explore the relation between the Higgs part of full SME and the noncommutative standard model (NCSM) based on the second approach to find those explicit relations between NC and LV parameters without any axillary fields.

In Sec. II, we introduce the Higgs sector of the NCSM based on the  $SU(3) \times SU(2) \times U(1)$  gauge group and its counterpart in the Lorentz-violating extension of the standard model. In Sec. III, we derive the LV parameters in terms of the NC parameter in the absence of the background field. The bounds on the noncommutative scale and some concluding remarks are given in Sec. IV.

**II. THE HIGGS PART OF THE SME AND NCSM**

The SME is an extension of the standard model in which all possible Lorentz-violating terms that could arise from the spontaneous symmetry breaking at a fundamental level are included and should preserve the gauge symmetry  $SU(3) \times SU(2) \times U(1)$  with a power-counting renormalizability. Therefore, the Higgs part of the standard model

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \phi)^\dagger D^\mu \phi - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2, \quad (2)$$

where the covariant derivative with the field strength  $B_{\mu\nu}$  for the hypercharge and  $W_{\mu\nu}$  for  $SU(2)$  is defined as

$$D_\mu = \partial_\mu + ig' \frac{Y}{2} B_\mu + ig T_a W_\mu^a, \quad (3)$$

can be extended to [2]

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$$\mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Higgs}}^{\text{CPT-even}} + \mathcal{L}_{\text{Higgs}}^{\text{CPT-odd}}, \quad (4)$$

where

$$\begin{aligned} \mathcal{L}_{\text{Higgs}}^{\text{CPT-even}} = & \left( \frac{1}{2} (k_{\phi\phi})^{\mu\nu} (D_\mu \phi^\dagger) D_\nu \phi + \text{H.c.} \right) \\ & - \frac{1}{2} (k_{\phi B})^{\mu\nu} \phi^\dagger \phi B_{\mu\nu} - \frac{1}{2} (k_{\phi W})^{\mu\nu} \phi^\dagger W_{\mu\nu} \phi \end{aligned} \quad (5)$$

indicates the *CPT*-preserving part of the Lagrangian, and

$$\mathcal{L}_{\text{Higgs}}^{\text{CPT-odd}} = i(k_\phi)^\mu \phi^\dagger D_\mu \phi + \text{H.c.}, \quad (6)$$

which is odd under the *CPT* symmetry.

In NC space-time, the coordinates are operators, and in the canonical version they satisfy Eq. (1). To construct the noncommutative field theory, according to the Weyl-Moyal correspondence, an ordinary function can be used instead of the corresponding noncommutative one by replacing the ordinary product with the star product, as in

$$(f \star g)(x) = \exp\left(\frac{i}{2} \theta^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y^\nu}\right) f(x) g(y)|_{y \rightarrow x}, \quad (7)$$

where up to the first order of  $\theta^{\mu\nu}$ ,

$$f \star g = f \cdot g + \frac{i}{2} \theta^{\mu\nu} \partial_\mu f \cdot \partial_\nu g + \mathcal{O}(\theta^2). \quad (8)$$

Using this correspondence is not enough to construct a gauge theory. Only a  $U(n)$  gauge theory with some restriction on the allowed representations can be simply extended to the noncommutative gauge theory. However, a minimal way to get rid of these restrictions is the construction of a gauge theory with a  $SU(n)$  gauge group via the Seiberg-Witten map which provides the noncommutative fields as local functions of the ordinary fields [8]. Therefore, to construct the NCSM, we should replace the ordinary products and fields in the ordinary standard model with the star product and NC fields, respectively. Consequently, the action for the Higgs part of the NCSM can be easily constructed from Eq. (2) as follows:

$$\begin{aligned} S_{\text{Higgs-NCSM}} = & \int d^4x (\rho_0(\hat{D}_\mu \hat{\Phi})^\dagger \star \rho_0(\hat{D}^\mu \hat{\Phi}) \\ & - \mu^2 \rho_0(\hat{\Phi})^\dagger \star \rho_0(\hat{\Phi}) \\ & - \lambda \rho_0(\hat{\Phi})^\dagger \star \rho_0(\hat{\Phi}) \star \rho_0(\hat{\Phi})^\dagger \star \rho_0(\hat{\Phi})), \end{aligned} \quad (9)$$

where the hat shows the noncommutative field and  $\rho_0$  realizes an appropriate representation for the hybrid Seiberg-Witten map. The covariant derivative is defined as

$$D_\mu = \partial_\mu + iV_\mu, \quad (10)$$

where the gauge potential  $V_\mu$  is defined as

$$V_\mu = g' B_\mu(x) \frac{Y}{2} + g \sum_{a=1}^3 W_{\mu a}(x) T_L^a, \quad (11)$$

in which  $Y$  and  $T_L^a$  are the generators of  $U(1)_Y$  and  $SU(2)_L$ , respectively.

Now, the Higgs action [Eq. (9)] can be expanded to all orders of  $\theta^{\mu\nu}$ . For this purpose, one needs the  $\theta^{\mu\nu}$  dependent of the gauge and Higgs fields to all orders. Up to the first order of the NC parameter, one has

$$\rho_0(\hat{\Phi}) = \phi + \rho_0(\phi^1) + \mathcal{O}(\theta^2) \quad (12)$$

for the Higgs field with

$$\begin{aligned} \rho_0(\phi^1) = & -\frac{1}{2} \theta^{\alpha\beta} (g' B_\alpha + g W_\alpha) \partial_\beta \phi \\ & + i \frac{1}{4} \theta^{\alpha\beta} (g' B_\alpha + g W_\alpha) (g' B_\beta + g W_\beta) \phi, \end{aligned} \quad (13)$$

where  $W_\alpha = W_\alpha^a T_a$ . Meanwhile, the expansion for the mathematical field  $V$  up to the leading order is given by

$$\hat{V}_\mu = V_\mu + i\Gamma_\mu + \mathcal{O}(\theta^2), \quad (14)$$

with

$$\begin{aligned} \Gamma_\mu = & i \frac{1}{4} \theta^{\alpha\beta} \{g' B_\alpha + g W_\alpha, g' \partial_\beta B_\mu + g \partial_\beta W_\mu \\ & + g' B_{\beta\mu} + g W_{\beta\mu}\}, \end{aligned} \quad (15)$$

where  $B_{\mu\nu}$  and  $W_{\mu\nu}$  are the ordinary field strengths for the hypercharge and the  $SU(2)$  gauge fields. However, one should note that the NC field strength has the following expansion:

$$\hat{F}_{\mu\nu} = F_{\mu\nu} + F_{\mu\nu}^1 + \mathcal{O}(\theta^2), \quad (16)$$

with

$$F_{\mu\nu} = g' B_{\mu\nu} + g W_{\mu\nu} \quad (17)$$

and

$$F_{\mu\nu}^1 = \frac{1}{2} \theta^{\alpha\beta} \{F_{\mu\alpha}, F_{\nu\beta}\} - \frac{1}{4} \theta^{\alpha\beta} \{V_\alpha, (\partial_\beta + D_\beta) F_{\mu\nu}\}. \quad (18)$$

Therefore, the Higgs action up to the first order of the NC parameter results in

$$\begin{aligned} S_{\text{Higgs}} = & \int d^4x \left( (D_\mu^{\text{SM}} \phi)^\dagger D^{\text{SM}\mu} \phi - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi) (\phi^\dagger \phi) \right) + \int d^4x \left( (D_\mu^{\text{SM}} \phi)^\dagger \left( D^{\text{SM}\mu} \rho_0(\phi^1) \right. \right. \\ & + \frac{1}{2} \theta^{\alpha\beta} \partial_\alpha V^\mu \partial_\beta \phi + \Gamma^\mu \phi \left. \left. + \left( D_\mu^{\text{SM}} \rho_0(\phi^1) + \frac{1}{2} \theta^{\alpha\beta} \partial_\alpha V_\mu \partial_\beta \phi + \Gamma_\mu \phi \right)^\dagger D^{\text{SM}\mu} \phi \right. \right. \\ & \left. \left. + \frac{1}{4} \mu^2 \theta^{\mu\nu} \phi^\dagger (g' B_{\mu\nu} + g W_{\mu\nu}) \phi - \lambda i \theta^{\alpha\beta} \phi^\dagger \phi (D_\alpha^{\text{SM}} \phi)^\dagger (D_\beta^{\text{SM}} \phi) \right) \right) + \mathcal{O}(\theta^2). \end{aligned} \quad (19)$$

TABLE I. Higgs sector LV coefficients.

Coefficient	NC-expression	Exponent [5,13]	System	$\Lambda_{\text{NC}}$ (TeV)
$k_{\phi\phi}^A$	$-\left(\frac{M_H}{\Lambda_{\text{NC}}}\right)^2$	$3 \times 10^{-16}$	Cosmological	$7 \times 10^6$
$k_{\phi B}$	$-\frac{e}{4 \cos \theta_W} \left(\frac{M_H}{\Lambda_{\text{NC}}}\right)^2$	$0.9 \times 10^{-16}$	Cosmological	$4 \times 10^6$
$k_{\phi W}$	$-\frac{e}{4 \sin \theta_W} \left(\frac{M_H}{\Lambda_{\text{NC}}}\right)^2$	$1.7 \times 10^{-16}$	Cosmological	$4 \times 10^6$

### III. LORENTZ-VIOLATING COEFFICIENTS

Noncommutative coordinates [Eq. (1)] apparently violate the Lorentz symmetry. Therefore, the Higgs action given in Eq. (19) violates the symmetry too. In previous works [12], the NC parameter was related to its correspondence in the SME (in fact, to the QED part of the SME) through a magnetic field as a background. Here we are looking for the direct relation between the parameters of both theories. To this end, we compare Eq. (19) with Eqs. (5) and (6) regarding the absence of background. One can easily find

$$(k_{\phi\phi})^{\mu\nu} = -2i\lambda\phi^\dagger\phi\theta^{\mu\nu} + (K_{\phi\phi})^{\mu\nu}(B, W), \quad (20)$$

where  $K_{\phi\phi}$  stands for the gauge-field-dependent part of  $k_{\phi\phi}$ , which is zero in the absence of background. It should be noted that in general,  $k_{\phi\phi}$  has symmetric and antisymmetric parts, as

$$(k_{\phi\phi})^{\mu\nu} = (k_{\phi\phi}^S + ik_{\phi\phi}^A)^{\mu\nu}. \quad (21)$$

Therefore, in the Higgs part of the NCSM, only the antisymmetric part of  $k_{\phi\phi}$  is nonzero, where after the symmetry breaking one has

$$(k_{\phi\phi}^A)^{\mu\nu} = -2\lambda\left(\frac{\mathbf{v}}{\Lambda_{\text{NC}}}\right)^2 \varepsilon^{\mu\nu} = -\left(\frac{M_H}{\Lambda_{\text{NC}}}\right)^2 \varepsilon^{\mu\nu}. \quad (22)$$

As is expected for the *CPT*-conserved NC field theory,  $(k_{\phi})_{\mu} = 0$  in Eq. (19). Comparing Eq. (19) with Eq. (5) for the LV parameters  $(k_{\phi B})_{\mu\nu}$  and  $(k_{\phi W})_{\mu\nu}$  leads to

$$(k_{\phi B})^{\mu\nu} = -\frac{1}{2}\lambda g' \left(\frac{\mathbf{v}}{\Lambda_{\text{NC}}}\right)^2 \varepsilon^{\mu\nu} = -\frac{e}{4 \cos \theta_W} \left(\frac{M_H}{\Lambda_{\text{NC}}}\right)^2 \varepsilon^{\mu\nu} \quad (23)$$

and

$$(k_{\phi W})^{\mu\nu} = -\frac{1}{2}\lambda g \left(\frac{\mathbf{v}}{\Lambda_{\text{NC}}}\right)^2 \varepsilon^{\mu\nu} = -\frac{e}{4 \sin \theta_W} \left(\frac{M_H}{\Lambda_{\text{NC}}}\right)^2 \varepsilon^{\mu\nu}. \quad (24)$$

The value  $\frac{|k_{\phi B}|}{|k_{\phi W}|} = \tan \theta_W$  is in agreement with the experimental values given in Table I. In fact, in Ref. [13], the experimental bounds on the LV parameters in the Higgs sector are indirectly obtained by evaluating the photon vacuum polarization at one loop and comparing the obtained results with the experimental bounds on the  $k_F$  term. As Eq. (5) shows, the photon-Higgs coupling in the hypercharge term is  $\sim \cos \theta_W$ , while in the  $W^3$  term it is  $\sim \sin \theta_W$ , which leads to  $\left(\frac{|k_{\phi B}|}{|k_{\phi W}|}\right)_{\text{Exp}} = \tan \theta_W$ . One should note that in Eqs. (22)–(24), there are subdominant terms depending on  $\frac{eB}{M_H^2}$  that are very small, even for a magnetic field as large as  $10^{13}$  teslas. Therefore, even in the presence of the magnetic field, only the  $B$ -independent terms are enough to find a bound as large as  $10^6$  TeV for the NC parameter. Such a bound on the  $\Lambda_{\text{NC}}$  is too large, compared with the bounds of GeV in low-energy experiments [9] and a few TeV in high-energy scattering [10].

### IV. CONCLUSION

We examined the Higgs sector of the NCSM based on the  $SU(3) \times SU(2) \times U(1)$  gauge symmetry. As a subset of the standard model extension, we compared the Higgs sectors in the both theories. We found the LV parameters  $k_{\phi B}$ ,  $k_{\phi W}$ , and  $k_{\phi\phi}$  for the Higgs sector as a function of the NC parameter [see Eqs. (22)–(24)]. In the NCSM, only the antisymmetric part of  $k_{\phi\phi}$  survives in the absence of an electromagnetic background field. For all the LV parameters in the NC space, there are also corrections of an order of  $\frac{eB}{M_H^2}$  smaller than the background independent part, which is too small to be considered here. In the previous works to relate the parameters of the both theories, one needs a constant magnetic background and, in the absence of the background, the LV and the NC parameters decoupled from each other [12]. As the obtained results show, here is the first place in which the noncommutativity is

directly expressed in terms of the LV parameters. The experimental bounds on the antisymmetric parts of  $k_{\phi\phi}$ ,  $k_{\phi B}$ , and  $k_{\phi W}$  leads to a bound on the NC parameter as large as  $\Lambda_{\text{NC}} \sim 10^6$  TeV (see Table I). In fact, the main result is twofold:

- (1) Direct relation between the LV and NC parameters without any axillary field.
- (2) A very large bound on the NC parameter compared to the current bound of the order of a few TeV.

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