

Inhibition of the dynamical Casimir effect with Robin boundary conditionsAndreson L. C. Rego,¹ B. W. Mintz,^{1,2} C. Farina,¹ and Danilo T. Alves³¹*Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, 21945-970 Rio de Janeiro, Brazil*²*Departamento de Física Teórica, Universidade do Estado do Rio de Janeiro, Rua São Francisco Xavier 524, Maracanã, 20550-900 Rio de Janeiro, Brazil*³*Faculdade de Física, Universidade Federal do Pará, 66075-110 Belém, Brazil*

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We consider a real massless scalar field in $3 + 1$ dimensions satisfying a Robin boundary condition at a nonrelativistic moving mirror. Considering vacuum as the initial field state, we compute explicitly the number of particles created per unit frequency and per unit solid angle, exhibiting in this way the angular dependence of the spectral distribution. The well-known cases of Dirichlet and Neumann boundary conditions may be reobtained as particular cases from our results. We show that the particle creation rate can be considerably reduced (with respect to the Dirichlet and Neumann cases) for particular values of the Robin parameter. Our results extend, for $3 + 1$ dimensions, previous results found in the literature for $1 + 1$ dimensions. Further, we also show that this inhibition of the dynamical Casimir effect occurs for different angles of particle emission.

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I. INTRODUCTION

The dynamical Casimir effect (DCE) basically consists of the emission of quanta from a moving body in vacuum due to its interaction with a quantized field [1–5]. Another manifestation of the DCE, which is a direct consequence of the particle creation phenomenon if we invoke the energy conservation law, is a radiation reaction force acting on the moving body. This dissipative force gives rise to an irreversible exchange of energy between the moving body and the quantized field. In other words, the energy dissipated from the moving body is converted into real excitations of the quantized field, i.e., real particles. One can also understand the DCE in the opposite way, namely, from the fluctuation-dissipation theorem [6,7] and from the fact that the static Casimir force acting on a fixed body, though zero, has nonvanishing fluctuations [8]. In this scenario, one expects that a moving plate may be acted by a dissipative force (under certain circumstances) which is proportional to the fluctuations of the Casimir force on the static plate [9–11] (for the case of a moving sphere, see Ref. [12]).

However, the quantized field and the moving body can also exchange energy reversibly, which means that the force exerted on the moving body acquires, in this case, a dispersive part, as it occurs when Robin boundary conditions (BCs) are considered [13,14]. During the first two decades after the pioneering paper by Moore [1], the calculations on the DCE were usually done with scalar fields. The consideration of electromagnetic fields was made for the first time in 1994 [15] (see also Refs. [16,17]), and since then great attention has been devoted to the DCE. Detailed reviews on the DCE can be found in Refs. [18,19]. Several experimental proposals to observe the DCE have been made in the last years. We shall briefly comment on a couple

of them (for more details see, for instance, Dodonov's paper [20]).

The so-called motion induced radiation (MIR) experiment [21] is based on the simulation of a mirror's motion by changing the reflectivity of a semiconductor by irradiating it with appropriate laser pulses, an ingenious idea first introduced by Yablonovitch in 1989 [22] in a paper where the main concern was to propose ways of simulating highly accelerated frames in order to enhance the Unruh radiation. A few years later, this same idea of creating a dense electron-hole plasma in a thin semiconductor by irradiating it with laser pulses was also discussed by Losovik *et al.* [23]. Though there are many promising aspects in the MIR experiment, it is worth mentioning that the MIR experimentalists may have to deal with some difficulties. A first one is related to the limitations in the signal-to-noise ratio present in their experiment caused by thermal effects, when they run the experiment at 4.6 K, as pointed out by Kim *et al.* [24]. A second one is the influence of damping in a parametric amplification process. And in the MIR experiment, the electric permittivity of the semiconductor slab after excitation by the laser pulse acquires a non-negligible imaginary part [25], so that dissipation effects are inevitable. In this case, as shown by Dodonov, damping plays an important role and the emergence of a superchaotic quantum state may occur, leading to a highly super-Poissonian statistics for the distribution function of quanta [26].

Another interesting proposal was made by Kim *et al.* [27]. They suggest that an indirect measurement of the dynamical Casimir photons generated by means of the mechanical motion of a film bulk acoustic resonator is detected with the aid of a superradiance mechanism. More recently, Dezael and Lambrecht [28] proposed that a (dynamical) Casimir-like radiation may arise from an

effective motion of mirrors obtained by the interactions of an optical parametric oscillator with a thin nonlinear crystal slab inside. In 2011, Kawakubo and Yamamoto [29] proposed that photons could be generated by means of a nonstationary plasma mirror, with the photons being detectable by an excitation process of Rydberg atoms through the atom-field interaction. Still, in 2011, Faccio and Carusotto [30] proposed a photon generation mechanism in the near-infrared domain obtained by a train of laser pulses applied perpendicularly to a cavity, made of nonlinear optical fiber, which modulates in time the refractive index of the medium filling the cavity.

Finally, 40 years after its theoretical prediction made by Moore [1], the first experimental observation of the DCE was announced by Wilson and collaborators [31], in an experiment where these authors use a superconducting circuit consisting of a unidimensional coplanar transmission line with a tunable electrical length. The change of the electrical length is performed by modulating the inductance of a superconducting quantum interference device fixed at one end of the transmission line. This modulation is achieved with the aid of a time-dependent magnetic flux through the superconducting quantum interference device. The electromagnetic field along the transmission line is described in terms of a field operator given by a scalar field $\phi(t, x)$ obeying a massless Klein-Gordon equation in $1 + 1$ dimensions and submitted to a Robin BC with a time-dependent Robin parameter $\gamma(t)$ in the following manner [32]:

$$\phi(t, 0) \approx \gamma(t)(\partial_x \phi)(t, 0). \quad (1)$$

This kind of BC was also considered in Refs. [33,34]. In the context of the DCE, the Robin BC appeared for the first time in the papers by Mintz and collaborators [13,35], who investigated a real massless scalar field $\phi(t, x)$ satisfying a Robin BC at a moving plate when observed from an inertial frame in which the plate was instantaneously at rest (we shall refer to this frame as the tangential frame), according to the formula

$$\phi'(t', x') = \gamma(\partial_{x'} \phi')(t', x'), \quad (2)$$

where the prime superscripts are to remind us that the BC is taken in a tangential frame and γ is now a time-independent Robin parameter.

For an oscillating mirror that imposes a Dirichlet BC on a scalar field in $1 + 1$ dimensions, the total particle creation rate is a monotonic function of the mechanical frequency of the mirror [36]. However, it was shown in Ref. [35] that this is not always the case when the field obeys a Robin BC given by Eq. (2). For a given value of γ , there is an interval in which an increase in the oscillation frequency of the mirror leads to a decrease in the particle emission. This can be understood as a kind of “decoupling” between the mirror and some of the field modes.

One important question that remains is whether or not this interesting effect still occurs in $3 + 1$ dimensions. This is not a trivial issue, once the phase space available for the field in $d = 3 + 1$ is much larger than that in $1 + 1$ dimensions. Answering this question is the main purpose of our paper, and we shall do that by considering a real massless scalar field in $3 + 1$ dimensions satisfying a Robin BC at a nonrelativistic moving mirror (in the tangential frame). As far as we know, all papers about DCE with a Robin BC deal with models in $1 + 1$ dimensions, except one [37], which did not discuss the problem in which we are interested here.

An extra motivation to study this model is the connection between the DCE for a real massless scalar field in $3 + 1$ dimensions and the DCE for the electromagnetic field interacting with a perfectly conducting plate. The latter problem can be separated into two problems: a vector potential representing the transverse electric (TE) polarization, which is associated with a Dirichlet BC, and a vector potential representing the transverse magnetic (TM) polarization, which is associated with a Neumann BC. Since the parameter γ allows a continuous interpolation between Dirichlet and Neumann BCs, we expect that in the limit $\gamma \rightarrow 0$ the DCE for the massless scalar field coincides with the TE polarization contribution to the electromagnetic DCE, while for $\gamma \rightarrow \infty$ the TM polarization contribution to the electromagnetic DCE is recovered.

The structure of this paper is as follows. In Sec. II, we calculate the relations between the creation and annihilation field operators in the remote past (“in” operators) and in the far future (“out” operators), in the Heisenberg picture. In Sec. III, we show our results for the particle emission rate for a specific but typical motion of the mirror. Finally, in Sec. IV, we discuss our results and make a few comments on their possible consequences.

II. BOGOLIUBOV TRANSFORMATION AND PARTICLE SPECTRUM

Let us consider—in the Heisenberg picture—a massless scalar field ϕ in $3 + 1$ dimensions written in terms of the time-dependent operators $a^\dagger(t, \mathbf{k})$ and $a(t, \mathbf{k})$, where \mathbf{k} is the wave vector. In the distant past, these operators are relabeled as the creation and annihilation operators $a_{\text{in}}^\dagger(\mathbf{k})$ and $a_{\text{in}}(\mathbf{k})$, whereas in the far future they are relabeled as $a_{\text{out}}^\dagger(\mathbf{k})$ and $a_{\text{out}}(\mathbf{k})$. The time evolution of the operators a^\dagger and a depends on the interaction between the field and an external agent (modeled, in the present paper, by a moving boundary). The out operators can be expressed as a combination of the in operators via Bogoliubov transformations [38]

$$a_{\text{out}}(\mathbf{k}) = \alpha(\mathbf{k})a_{\text{in}}(\mathbf{k}) + \beta(\mathbf{k})a_{\text{in}}^\dagger(\mathbf{k}), \quad (3)$$

where α and β are named Bogoliubov coefficients. Assuming that in the remote past the system is in the

vacuum state $|0\rangle$, the spectral density of the created particles after the movement of the mirror has ceased is given by

$$\frac{dN}{d^3k}(\mathbf{k}) = \frac{1}{(2\pi)^3} \langle 0 | a_{\text{out}}^\dagger(\mathbf{k}) a_{\text{out}}(\mathbf{k}) | 0 \rangle = |\beta(\mathbf{k})|^2. \quad (4)$$

Notice that there will be particle production if, and only if, $\beta(\mathbf{k}) \neq 0$, i.e., if the annihilation operator $a_{\text{out}}(\mathbf{k})$ is “contaminated” by the creation operator $a_{\text{in}}^\dagger(\mathbf{k})$. The relation between the in and out operators can be calculated exactly for the case 1 + 1 dimensions [5] with Dirichlet or Neumann BCs. However, only approximate approaches are currently known for higher dimensional space-times. If the movement of the mirror is nonrelativistic and has a small amplitude, the perturbative method introduced by Ford and Vilenkin [39] applies. In the following, we will use the Ford-Vilenkin approach to find the Bogoliubov transformation for the massless scalar field satisfying the wave equation

$$\frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi = 0 \quad (5)$$

and obeying a Robin BC imposed by a moving mirror. We start by writing the field operator as the field under a static BC plus a perturbation, that is,

$$\phi(t, \mathbf{r}) = \phi_0(t, \mathbf{r}) + \delta\phi(t, \mathbf{r}), \quad (6)$$

where we assume that the undisturbed field ϕ_0 obeys the static boundary condition

$$[\phi_0(t, \mathbf{r}) - \gamma \partial_z \phi_0(t, \mathbf{r})]_{z=0} = 0, \quad (7)$$

where γ is a time-independent parameter.

The perturbation $\delta\phi$ (which is assumed to be small) gives the first-order contribution to the total field $\phi(t, \mathbf{r})$ caused by the movement of the mirror and, therefore, will be responsible for the emergence of the DCE.

Let $z = \delta q(t)$ be the position of the mirror at a given instant. We assume that, in the laboratory reference frame, the mirror starts at rest at $z = 0$, undergoes a given prescribed movement, and finally settles down at $z = 0$ for large times. The perturbation $\delta\phi$ will be small as long as the speed of the mirror is continuous with continuous derivatives and is much smaller than the speed of light, $|\delta\dot{q}(t)| \ll 1$, in natural units. Once we consider a bounded movement for the mirror, we also require that it possesses a small amplitude. More specifically, this means that $|\delta q| \ll 1/\omega_0$, where ω_0 is the dominant mechanical frequency, and this assumption will allow us to neglect terms $\mathcal{O}[(\delta q)^2]$, $\mathcal{O}[(\delta\phi)^2]$, $\mathcal{O}[\delta\phi\delta q]$, and $\mathcal{O}[(\delta\dot{q})^2]$.

We shall follow the same procedure as in Ref. [35]. At the tangential frame at a given instant, the BC is given by

$$\phi'(t', \mathbf{r}') = \gamma' \frac{\partial \phi'}{\partial z'}(t', \mathbf{r}'), \quad (8)$$

where γ' is a t' -independent parameter. This BC can be cast in the laboratory frame with the help of the appropriate

Lorentz transformations. For nonrelativistic velocities, one may expand in first order in $\delta\dot{q}(t)$ to find

$$[\partial_z \phi(t, \mathbf{r}) + \delta\dot{q}(t) \partial_t \phi(t, \mathbf{r}) - \gamma^{-1} \phi(t, \mathbf{r})]_{z=\delta q(t)} = 0, \quad (9)$$

where, in this approximation, we consider that the Robin parameter is not affected by the Lorentz transformation, so that $\gamma' = \gamma$.

We substitute (6) in (9) and expand the result thus obtained in the small parameters, retaining only terms up to linear order in δq and $\delta\phi$; we find

$$\begin{aligned} \partial_z \delta\phi(t, \mathbf{r})|_{z=0} - \gamma^{-1} \delta\phi(t, \mathbf{r})|_{z=0} \\ = \delta q(t) \gamma^{-1} \partial_z \phi_0(t, \mathbf{r})|_{z=0} - \delta q(t) \partial_z^2 \phi_0(t, \mathbf{r})|_{z=0} \\ - \delta\dot{q}(t) \partial_t \phi_0(t, \mathbf{r})|_{z=0}. \end{aligned} \quad (10)$$

Therefore, the perturbation $\delta\phi(t, \mathbf{r})$ obeys a time-dependent BC at a fixed position ($z = 0$), which is associated with the motion of the mirror via $\delta q(t)$.

It is now convenient to express the field in the frequency domain using a Fourier transform, as follows. Let us define $\Phi(\omega, \mathbf{k}_\parallel; z)$ as the Fourier transform of the field $\phi(t, \mathbf{r})$, so that

$$\Phi(\omega, \mathbf{k}_\parallel; z) := \int_{-\infty}^{\infty} dt \int d^2 \mathbf{r}_\parallel e^{i\omega t} e^{-i\mathbf{k}_\parallel \cdot \mathbf{r}_\parallel} \phi(t, \mathbf{r}). \quad (11)$$

Notice that $\Phi(\omega, \mathbf{k}_\parallel; z)$ obeys the Helmholtz equation

$$(\omega^2 - k_\parallel^2 + \partial_z^2) \Phi(\omega, \mathbf{k}_\parallel; z) = 0. \quad (12)$$

Once the massless free scalar field is a solution of the d'Alembertian equation (5), we obtain

$$k_z = [(\omega + i\epsilon)^2 - k_\parallel^2]^{1/2}, \quad \text{with } \epsilon \rightarrow 0^+. \quad (13)$$

With this definition, k_z is a complex function of ω , with a branch cut along the real axis in the range $-k_\parallel < \omega < k_\parallel$ [16], where $k_\parallel = |\mathbf{k}_\parallel|$.

The undisturbed field is the solution of the wave equation (5) subject to the static BC (7):

$$\begin{aligned} \phi_0(t, \mathbf{r}) = \int_{-\infty}^{\infty} d^2 \mathbf{k}_\parallel \int_0^{\infty} dk_z \sqrt{\frac{1}{4\pi^3 |\omega_{\mathbf{k}}|} \left(\frac{1}{1 + \gamma^2 k_z^2} \right)} \\ \times [a(\mathbf{k}) e^{-i\omega_{\mathbf{k}} t} e^{i\mathbf{k}_\parallel \cdot \mathbf{r}_\parallel} + a^\dagger(\mathbf{k}) e^{i\omega_{\mathbf{k}} t} e^{-i\mathbf{k}_\parallel \cdot \mathbf{r}_\parallel}] \\ \times [\sin(k_z z) + \gamma k_z \cos(k_z z)], \end{aligned} \quad (14)$$

where the field normalization is chosen so that the creation and annihilation operators obey the commutation relations

$$[a(\mathbf{k}), a^\dagger(\mathbf{k}')] = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}'). \quad (15)$$

We can now use (11) to write down the Fourier transform of the undisturbed field,

$$\begin{aligned} \Phi_0(\omega, \mathbf{k}_\parallel; z) = A(\omega, k_z; \gamma) [\sin(k_z z) + \gamma k_z \cos(k_z z)] \\ \times [a(\mathbf{k}) \Theta(\omega) - a^\dagger(-\mathbf{k}) \Theta(-\omega)] \Theta(k_z^2), \end{aligned} \quad (16)$$

with the normalization factor

$$A(\omega, k_z; \gamma) = \sqrt{\frac{16\pi^3|\omega|}{k_z^2} \left(\frac{1}{1 + \gamma^2 k_z^2} \right)}. \quad (17)$$

The Fourier transformations of the perturbation $\delta\phi(t, \mathbf{r})$ and the mirror's law of motion $\delta q(t)$ are written as

$$\delta\Phi(\omega, \mathbf{k}_{\parallel}; z) = \int_{-\infty}^{\infty} dt \int d^2\mathbf{r}_{\parallel} e^{i\omega t} e^{-i\mathbf{k}_{\parallel}\cdot\mathbf{r}_{\parallel}} \delta\phi(t, \mathbf{r}) \quad (18)$$

and

$$\delta Q(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} \delta q(t). \quad (19)$$

Notice that $\delta\Phi(\omega, \mathbf{k}_{\parallel}; z)$ obeys the Helmholtz equation

$$(\omega^2 - k_{\parallel}^2 + \partial_z^2)\delta\Phi(\omega, \mathbf{k}_{\parallel}; z) = 0. \quad (20)$$

Recalling that we must take only the solution that propagates outwards from the moving mirror, the solution of the previous equation for $z > 0$ is given by

$$\delta\Phi(\omega, \mathbf{k}_{\parallel}; z) = \frac{1}{ik_z} \partial_z \delta\Phi(\omega, \mathbf{k}_{\parallel}; 0) e^{ik_z z}. \quad (21)$$

We now notice also that $\delta\Phi(\omega, \mathbf{k}_{\parallel}; z)$ obeys the BC given by the Fourier transform of (10), namely,

$$\begin{aligned} & \partial_z \delta\Phi(\omega, \mathbf{k}_{\parallel}; 0) - \gamma^{-1} \delta\Phi(\omega, \mathbf{k}_{\parallel}; 0) \\ &= \gamma^{-1} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \delta Q(\omega - \omega') [\partial_z \Phi_0(\omega', \mathbf{k}_{\parallel}; 0) \\ &+ (\gamma\omega\omega' - \gamma\mathbf{k}_{\parallel}^2)\Phi_0(\omega', \mathbf{k}_{\parallel}; 0)]. \end{aligned} \quad (22)$$

$$\delta\Phi(\omega, \mathbf{k}_{\parallel}; z') = \gamma^{-1} g(\omega, \mathbf{k}_{\parallel}; 0, z') \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \delta Q(\omega - \omega') [\partial_z \Phi_0(\omega', \mathbf{k}_{\parallel}; 0) + (\gamma\omega\omega' - \gamma\mathbf{k}_{\parallel}^2)\Phi_0(\omega', \mathbf{k}_{\parallel}; 0)]. \quad (26)$$

Replacing (26) in (23) one finds

$$\begin{aligned} \Phi(\omega, \mathbf{k}_{\parallel}; z') &= \Phi_0(\omega, \mathbf{k}_{\parallel}; z') + \frac{1}{\gamma} g(\omega, \mathbf{k}_{\parallel}; 0, z') \\ &\times \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \delta Q(\omega - \omega') [\partial_z \Phi_0(\omega', \mathbf{k}_{\parallel}; 0) \\ &+ (\gamma\omega\omega' - \gamma\mathbf{k}_{\parallel}^2)\Phi_0(\omega', \mathbf{k}_{\parallel}; 0)]. \end{aligned} \quad (27)$$

The field $\Phi(\omega, \mathbf{k}_{\parallel}; z')$ can be written in two different forms. The first one involves Φ_{in} , the Fourier transform of the unperturbed field $\phi_0(t, \mathbf{r}_{\parallel}; z')$ in the distant past, that is, the ‘‘in’’ field, with which we associate retarded Green's functions. Then,

$$\begin{aligned} \Phi(\omega, \mathbf{k}_{\parallel}; z') &= \Phi_{\text{in}}(\omega, \mathbf{k}_{\parallel}; z') + \frac{1}{\gamma} g_{\text{ret}}(\omega, \mathbf{k}_{\parallel}; 0, z') \\ &\times \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \delta Q(\omega - \omega') [\partial_z \Phi_0(\omega', \mathbf{k}_{\parallel}; 0) \\ &+ (\gamma\omega\omega' - \gamma\mathbf{k}_{\parallel}^2)\Phi_0(\omega', \mathbf{k}_{\parallel}; 0)], \end{aligned} \quad (28)$$

Once the Fourier transform of the Ford-Vilenkin ansatz (6) is

$$\Phi(\omega, \mathbf{k}_{\parallel}; z) = \Phi_0(\omega, \mathbf{k}_{\parallel}; z) + \delta\Phi(\omega, \mathbf{k}_{\parallel}; z), \quad (23)$$

we need to find an expression for $\Phi(\omega, \mathbf{k}_{\parallel}; z)$ that allows us to relate the Fourier transforms of the field in the remote past and distant future. This can be done straightforwardly by using Green's functions, as in Ref. [35].

Let us start with the one-dimensional version of Green's identity, $\partial_z\{g[\partial_z f] - f[\partial_z g]\} = g[\partial_z^2 f] - f[\partial_z^2 g]$. We now identify the function f in this formula as the perturbation $\delta\Phi$ and the function g as the Green's function of the Helmholtz operator, i.e., a function such that $(\omega^2 - k_{\parallel}^2 + \partial_z^2)g(\omega, \mathbf{k}_{\parallel}; z, z') = \delta(z - z')$. After integration by parts, we can show that

$$\begin{aligned} \delta\Phi(\omega, \mathbf{k}_{\parallel}; z') &= [g(\omega, \mathbf{k}_{\parallel}; 0, z') \partial_z \delta\Phi(\omega, \mathbf{k}_{\parallel}; 0) \\ &- \partial_z g(\omega, \mathbf{k}_{\parallel}; 0, z') \delta\Phi(\omega, \mathbf{k}_{\parallel}; 0)]. \end{aligned} \quad (24)$$

We may freely define the Green's function g so that it obeys the Robin BC at the surface of the static mirror, i.e., $g(\omega, \mathbf{k}_{\parallel}; 0, z') = \gamma \partial_z g(\omega, \mathbf{k}_{\parallel}; 0, z')$. Then, the identity (24) leads to

$$\begin{aligned} \delta\Phi(\omega, \mathbf{k}_{\parallel}; z') &= g(\omega, \mathbf{k}_{\parallel}; 0, z') [\partial_z \delta\Phi(\omega, \mathbf{k}_{\parallel}; 0) \\ &- \gamma^{-1} \delta\Phi(\omega, \mathbf{k}_{\parallel}; 0)] \end{aligned} \quad (25)$$

and, using Eq. (22), we can write the perturbation as

where $\Phi_{\text{in}}(\omega, \mathbf{k}_{\parallel}; z')$ is the Fourier transform of $\phi_{\text{in}}(t, \mathbf{r})$, which is simply the unperturbed field at the remote past,

$$\phi_0(t, \mathbf{r}) \underset{t \rightarrow -\infty}{\sim} \phi_{\text{in}}(t, \mathbf{r}). \quad (29)$$

Analogously, one can use advanced Green's functions to express the field $\Phi_0(\omega, \mathbf{k}_{\parallel}; z')$ using the out field configuration, as follows,

$$\begin{aligned} \Phi(\omega, \mathbf{k}_{\parallel}; z') &= \Phi_{\text{out}}(\omega, \mathbf{k}_{\parallel}; z') + \frac{1}{\gamma} g_{\text{adv}}(\omega, \mathbf{k}_{\parallel}; 0, z') \\ &\times \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \delta Q(\omega - \omega') [\partial_z \Phi_0(\omega', \mathbf{k}_{\parallel}; 0) \\ &+ (\gamma\omega\omega' - \gamma\mathbf{k}_{\parallel}^2)\Phi_0(\omega', \mathbf{k}_{\parallel}; 0)], \end{aligned} \quad (30)$$

where $\Phi_{\text{out}}(\omega, \mathbf{k}_{\parallel}; z')$ is the Fourier transform of

$$\phi_0(t, \mathbf{r}) \underset{t \rightarrow \infty}{\sim} \phi_{\text{out}}(t, \mathbf{r}). \quad (31)$$

From Eqs. (30) and (28), we can relate the in and out field operators through the advanced and retarded Green's functions,

$$\begin{aligned} \Phi_{\text{out}}(\omega, \mathbf{k}_{\parallel}; z') &= \Phi_{\text{in}}(\omega, \mathbf{k}_{\parallel}; z') + \frac{1}{\gamma} [g_{\text{ret}}(\omega, \mathbf{k}_{\parallel}; 0, z') - g_{\text{adv}}(\omega, \mathbf{k}_{\parallel}; 0, z')] \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \delta Q(\omega - \omega') [\partial_z \Phi_0(\omega', \mathbf{k}_{\parallel}; 0) \\ &\quad + (\gamma\omega\omega' - \gamma\mathbf{k}_{\parallel}^2) \Phi_0(\omega', \mathbf{k}_{\parallel}; 0)]. \end{aligned} \quad (32)$$

Using the solution (16) of the Helmholtz equation for the field subject to a Robin BC, one can reexpress (32) as

$$\begin{aligned} \Phi_{\text{out}}(\omega, \mathbf{k}_{\parallel}; z') &= \Phi_{\text{in}}(\omega, \mathbf{k}_{\parallel}; z') + \left(\frac{2i}{1 + \gamma^2 k_z^2} \right) [\sin(k_z z') + \gamma k_z \cos(k_z z')] \\ &\quad \times \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} A(\omega', k_z'; \gamma) \delta Q(\omega - \omega') k_z' (1 - \gamma^2 \mathbf{k}_{\parallel}^2 + \gamma^2 \omega\omega') [a(\mathbf{q})\Theta(k_z') - a^\dagger(-\mathbf{q})\Theta(-k_z')]\Theta(k_z'^2), \end{aligned} \quad (33)$$

where we defined

$$\mathbf{q} := \mathbf{k}_{\parallel} + k_z' \hat{z}, \quad (34)$$

with

$$k_z' = [(\omega' + i\epsilon)^2 - \mathbf{k}_{\parallel}^2]^{1/2}. \quad (35)$$

Once again, using (16) it is straightforward to show that

$$\begin{aligned} a_{\text{out}}(\mathbf{k}) &= a_{\text{in}}(\mathbf{k}) + \frac{2ik_z}{\sqrt{|\omega|(1 + \gamma^2 k_z^2)}} \int_{-\infty}^{\infty} \frac{d\omega''}{2\pi} \\ &\quad \times \sqrt{\frac{|\omega''|}{(1 + \gamma^2 k_z'^2)}} \delta Q(\omega - \omega'') \\ &\quad \times (1 - \gamma^2 \mathbf{k}_{\parallel}^2 + \gamma^2 \omega\omega'') [a_{\text{in}}(\mathbf{q}'')\Theta(k_z'') \\ &\quad - a_{\text{in}}^\dagger(-\mathbf{q}'')\Theta(-k_z'')] \Theta(k_z''^2), \end{aligned} \quad (36)$$

with $\mathbf{q}'' = \mathbf{k}_{\parallel} + k_z'' \hat{z}$. The last expression is a linear relation between the annihilation field operator in the far future and the creation and annihilation operators at the remote past. That is, (36) is the Bogoliubov transformation we were looking for.

As advertised, the vacuum state is annihilated by the in operator $a_{\text{in}}(\mathbf{k})$, but not by the out operator $a_{\text{out}}(\mathbf{k})$ due to the presence of $a_{\text{in}}^\dagger(\mathbf{k})$ on the right-hand side of (36). This clearly indicates a nonzero particle creation in 3 + 1 dimensions due to the DCE for a massless field subject to a Robin BC at a moving mirror.

After some straightforward manipulations, from Eq. (4) and using the Bogoliubov transformation (36), we can finally calculate the spectral density of created particles:

$$\begin{aligned} \frac{dN(\mathbf{k})}{d^3\mathbf{k}} &= \frac{A}{(2\pi)^3} \frac{4k_z^2}{|\omega|(1 + \gamma_0 k_z^2)} \int_{|\mathbf{k}_{\parallel}|}^{\infty} \frac{d\omega'}{2\pi} k_z' \\ &\quad \times \frac{|\delta Q(\omega + \omega')|^2}{1 + \gamma_0 k_z'^2} (1 - \gamma^2 \mathbf{k}_{\parallel}^2 - \gamma^2 \omega\omega')^2, \end{aligned} \quad (37)$$

where A is the area of the moving plate. Taking into account the axial symmetry of the problem, it is interesting to rewrite (37) in terms of the variables

$$k_z = \omega \cos \theta \quad \text{and} \quad |\mathbf{k}_{\parallel}| = \omega \sin \theta, \quad (38)$$

with $\omega = |\mathbf{k}|$ and $\theta \in [0, \pi/2]$. Analogously, we have

$$k_z' = \sqrt{\omega'^2 - \mathbf{k}_{\parallel}^2} = \sqrt{\omega'^2 - \omega^2 \sin^2 \theta}, \quad (39)$$

which leads us to our expression for the particle spectrum per unit area, per unit solid angle,

$$\begin{aligned} \frac{1}{A} \frac{dN}{d\omega d\Omega} &= \frac{1}{(2\pi)^3} \frac{4\omega^3 \cos^2 \theta}{1 + \gamma^2 \omega^2 \cos^2 \theta} \int_{\omega \sin \theta}^{\infty} \frac{d\omega'}{2\pi} \\ &\quad \times \sqrt{\omega'^2 - \omega^2 \sin^2 \theta} \frac{|\delta Q(\omega + \omega')|^2}{1 + \gamma^2 (\omega'^2 - \omega^2 \sin^2 \theta)} \\ &\quad \times (1 - \gamma^2 \omega^2 \sin^2 \theta - \gamma^2 \omega\omega')^2. \end{aligned} \quad (40)$$

The expression (40) is valid for an arbitrary motion of the mirror, as long as it is sufficiently slow ($|\delta\dot{q}| \ll 1$) and has a small amplitude. In the following section, we present our results for a specific but very useful law of motion, that of a mirror which oscillates, practically, at a single frequency.

III. RESULTS AND DISCUSSION

Let us consider that the movement of the mirror is described by

$$\delta q(t) = \epsilon_0 \cos(\omega_0 t) e^{-|t|/\tau}, \quad (41)$$

which is a typical motion considered in investigations of the DCE [16], where ϵ_0 represents its amplitude, ω_0 the dominating mechanical frequency, and τ the effective time interval of the oscillation. The Fourier transform of Eq. (41) is

$$\delta Q(\omega) = \epsilon_0 \tau \left[\frac{1}{1 + (\omega + \omega_0)^2 \tau^2} + \frac{1}{1 + (\omega - \omega_0)^2 \tau^2} \right]. \quad (42)$$

In the limit with $\omega_0 \tau \gg 1$, we obtain

$$|\delta Q(\omega)|^2 \approx \frac{\pi}{2} \epsilon_0^2 \tau [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)], \quad (43)$$

which is reasonable once (42) possesses two narrow peaks around $\omega = \pm\omega_0$ in this limit. Substituting (43) into (40) and defining $\tilde{\mathcal{N}}$, the normalized particle spectrum per unit area, per unit time, per unit solid angle, given by

$$\tilde{\mathcal{N}} = \frac{1}{A} \frac{dN}{d\omega d\Omega} \left(\frac{\epsilon_0^2 \tau}{8\pi^3} \right)^{-1}, \quad (44)$$

we can write

$$\tilde{\mathcal{N}}(\omega, \omega_0, \gamma, \theta) = \omega_0^4 \tilde{\mathcal{F}}(\alpha, \beta, \theta), \quad (45)$$

where

$$\tilde{\mathcal{F}}(\alpha, \beta, \theta) = \frac{\alpha^3 \cos^2 \theta}{1 + \alpha^2 \beta^2 \cos^2 \theta} \sqrt{(\alpha - 1)^2 - \alpha^2 \sin^2 \theta} \frac{[1 - \alpha^2 \beta^2 \sin^2 \theta - \alpha \beta^2 (1 - \alpha)]^2}{1 + \beta^2 [(1 - \alpha)^2 - \alpha^2 \sin^2 \theta]} \Theta[1 - \alpha(1 + \sin \theta)], \quad (46)$$

with $\alpha = \omega/\omega_0$ and $\beta = \omega_0 \gamma$. Notice that the Heaviside function present in Eq. (45) indicates the following interesting features of the particle spectrum: particles associated with a certain frequency ω can be created in all angles if $0 \leq \omega \leq \omega_0/2$; for particles associated with a frequency $\omega > \omega_0/2$, there is no particle emission for angles larger than $\theta_0(\omega)$, with

$$\theta_0(\omega) = \arcsin\left(\frac{\omega_0 - \omega}{\omega}\right). \quad (47)$$

Moreover, observe that expression (45) coincides with the results found in Ref. [16] in two particular cases. In the limit $\gamma \rightarrow 0$, Eq. (45) reproduces the particle spectrum for TE photons, while for $\gamma \rightarrow \infty$ it corresponds to the spectrum of TM photons.

The normalized particle spectrum per unit area, per unit time, here labeled as \mathcal{N} , can be obtained by integrating (45) in the solid angle, with an angular range $\theta \in [0, \pi/2]$:

$$\mathcal{N}(\omega, \omega_0, \gamma) = \omega_0^4 \mathcal{F}(\alpha, \beta) \quad (48)$$

with

$$\mathcal{F}(\alpha, \beta) = 2\pi \int_0^{\pi/2} d\theta \sin \theta \tilde{\mathcal{F}}(\alpha, \beta, \theta). \quad (49)$$

The results for $\omega_0 = 1$ and different values of γ in $\mathcal{N}(\omega, \omega_0, \gamma)$ are shown in Fig. 1. The spectrum is

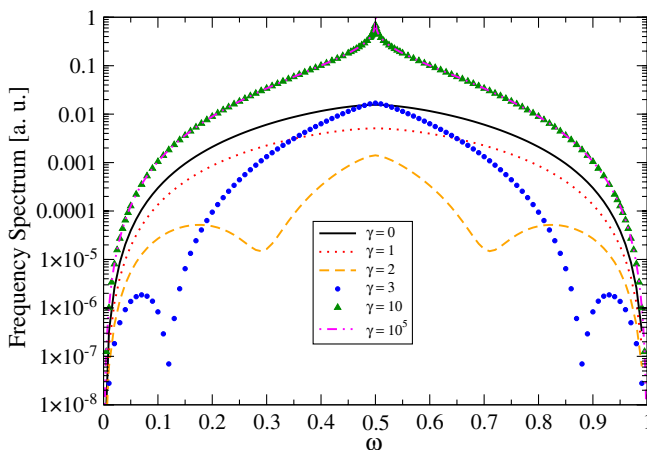


FIG. 1 (color online). The normalized spectrum \mathcal{N} in arbitrary units, as a function of ω , for different values of γ , with $\omega_0 = 1$. The particle emission at a given frequency ω is initially reduced as γ increases from zero until a given angle and then it starts increasing, reaching a fixed plateau for $\gamma \rightarrow \infty$.

symmetric around $\omega_0/2$ for every value of γ . This is in agreement with the expectation that the particles are created in pairs with frequencies ω_1 and ω_2 so that $\omega_1 + \omega_2 = \omega_0$, the driving frequency of the mirror [16].

One can also analyze the angular spectrum of emitted particles. Our results for $\tilde{\mathcal{N}}$, which follow from Eq. (45) for a particular frequency $\omega = \omega_0/2$ and $\omega_0 = 1$, are shown in Fig. 2. A few comments are in order. First, for small values of the Robin parameter, $\gamma \lesssim 1$, the emission is mainly forward, with large angle emission being suppressed. In the second place, for $\gamma \simeq 2$, the emission is strongly inhibited for every angle, but it also acquires a maximum around $\theta \simeq 1$ rad. If the γ is further increased, the emission rate rises again as a whole, but the particle production is highest around $\theta \simeq 1$ rad and not around the normal direction $\theta = 0$. Another interesting and subtle point becomes evident when we compare the angular spectrum in the case of Robin boundary conditions with $\gamma \gg 1$ and that of the Neumann BCs. We see from Fig. 2 that there is no emission for $\theta \rightarrow \pi/2$ no matter how large the values of γ are. This is basically due to the factor $\cos^2 \theta$ in (45). However, as shown in Ref. [16], there is a *finite* emission rate at $\theta \rightarrow \pi/2$ for Neumann BCs. The apparent paradox can be solved by noticing that the limits $\theta \rightarrow \pi/2$

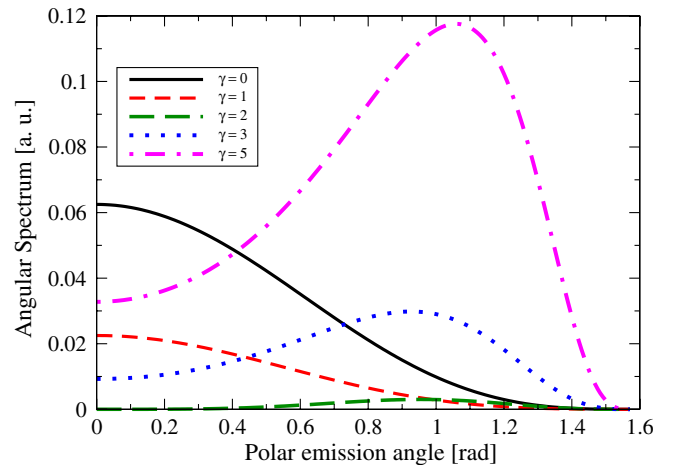


FIG. 2 (color online). Angular spectrum $\tilde{\mathcal{N}}$ of emitted particles with frequency $\omega = \omega_0/2$, with $\omega_0 = 1$, as a function of the emission angle (measured with respect to the normal to the mirror) for various values of γ . Solid (black) line: $\gamma = 0$; dashed (red) line: $\gamma = 1$; long dashed (green) line: $\gamma = 2$; dotted (blue) line: $\gamma = 3$; dashed-dotted (magenta) line: $\gamma = 5$.

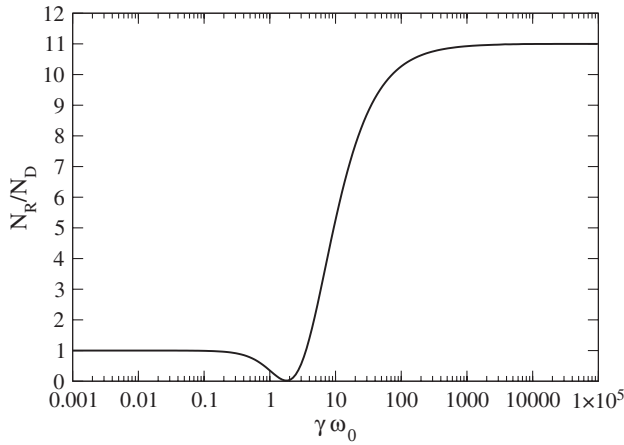


FIG. 3. Total emission rate for a Robin BC normalized by the Dirichlet rate. Notice that the Neumann limit ($\gamma \rightarrow \infty$) is correctly recovered. The strong decoupling between the mirror and the quantum vacuum field at $\gamma\omega_0 \simeq 2$ is typical of the Robin BC [13,35].

and $\gamma \rightarrow \infty$ in Eq. (40) do not commute. Fortunately, this subtlety affects only grazing angles, and the limit $\gamma \rightarrow \infty$ can be identified with the Neumann BC whenever $\theta = \pi/2$ is not the only angle under consideration.

Finally, let us turn our attention to the total number of particles emitted, N , which is given by

$$N(\omega_0, \gamma) = \omega_0^5 F(\beta), \quad (50)$$

where

$$F(\beta) = \int_0^\infty d\alpha \mathcal{F}(\alpha, \beta). \quad (51)$$

The ratio between the emission rate $N_R = N(\omega_0, \gamma)$ and the emission rate $N_D = N(\omega_0, 0)$ (Dirichlet BC) is given by

$$N_R/N_D = F(\beta)/F(0), \quad (52)$$

which is shown in Fig. 3. As first demonstrated in Ref. [16], the total emission rate for Neumann BC is 11 times larger than that for Dirichlet BCs in 3 + 1 dimensions. Our results not only confirm the factor 11 but also show that the Dirichlet and Neumann cases are connected by a non-monotonic curve. Indeed, the Robin BC with $\gamma\omega_0 \simeq 2$ provides a very strong inhibition of the particle production. This property of the DCE with the Robin BC has already been noticed in 1 + 1 dimensions [35], but it was not obvious *a priori* that this property would be preserved at higher dimensions.

IV. FINAL REMARKS

We have investigated the DCE for a real massless scalar field in 3 + 1 dimensions satisfying a Robin BC at a non-relativistic moving plate (in the tangential frame). We used

the perturbative method of Ford and Vilenkin [39], valid for nonrelativistic motions of small amplitudes, to evaluate the Bogoliubov transformations between creation and annihilation operators in the remote past, a_{in} , and distant future, a_{out} . This allowed us to determine the spectral and angular distributions of the created particles caused by the moving mirror. Equation (40) exhibits our expression for the particle spectrum per unit area, valid for a general (nonrelativistic) law of motion for the moving plate. Assuming the oscillating law of motion given by Eq. (41)—a typical motion considered in investigations of the DCE—we obtained an explicit expression for the spectrum per unit area, given by Eq. (45). In the limit $\gamma \rightarrow 0$ (Dirichlet case) we recovered the particle spectrum for TE photons, whereas for $\gamma \rightarrow \infty$ (Neumann case) we reobtained the spectrum of emitted TM photons.

Our results also show that, although in the limits $\gamma \rightarrow 0$ and $\gamma \rightarrow \infty$ the total number of created particles is a monotonic function of the mechanical frequency of the plate, ω_0 , the same is not true for intermediate values of γ . In fact, for any fixed positive γ , the total number of created particles is not a monotonic function of ω_0 . More than that, the strong inhibition of the DCE that occurs in 1 + 1 dimensions, when the Robin BC is present, also occurs in 3 + 1 dimensions. Indeed, the total number of created particles shown in Fig. 3 is dramatically reduced for a quasiharmonic motion with a frequency ω_0 such that $\gamma\omega_0 \simeq 2$, where γ is the parameter that characterizes the Robin condition. Naively thinking, this surprising effective decoupling between the plate and the quantized field, predicted previously in 1 + 1 dimensions [13,35], was not expected in 3 + 1, since in the latter case only the field modes that propagate perpendicularly to the plate are expected to behave as the field modes in 1 + 1 dimensions.

The suppression just discussed shows how the dynamical Casimir effect may be strongly dependent on the boundary conditions employed. This fact is very important, since any extra information about the created particles, either in their total number or in the angular dependence of the corresponding spectral distribution, can be extremely useful to better identify the dynamical Casimir photons in a given experiment. Since the Robin parameter γ can be interpreted as the plasma wavelength of a given material, the results presented here may be of some help for future experiments, at least as a source of concern and caution when dimensioning these experiments. For the moment, realistic values for the product $\gamma\omega_0$ are still very far from a strong suppression, but this may not be the case in future experiments. It would be interesting if the peculiar signature of the DCE with Robin conditions—the strong inhibition in the particle emission—could eventually be captured in experiments. Considering the connections between the Robin BC and the theoretical model underlying the first experimental observation of

the DCE [31], we believe that a thorough study of the implications of the Robin boundary conditions on the DCE is crucial.

As a final comment, since the dissipative force acting on a moving mirror is closely related to the particle creation, we expect that this dissipative force will also suffer a similar inhibition in $3 + 1$ dimensions, as it occurs in the $1 + 1$ -dimensional case [13]. This calculation will be left for a future work.

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