Two-loop effective potential for the Wess-Zumino model in 2 + 1 dimensions

R. V. Maluf[1,2,*] and A. J. da Silva[1,†]

[1]*Instituto de Física, Universidade de São Paulo, Caixa Postal 66318, 05315-970 São Paulo, São Paulo, Brazil*
[2]*Departamento de Física, Universidade Federal do Ceará, C.P. 6030, 60455-760 Fortaleza, Ceará, Brazil*
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By using superfield techniques, the effective potential of the $\mathcal{N} = 1$ Wess-Zumino model in $2 + 1$ dimensions is computed off-shell up to two loops. It is shown that supersymmetry is not dynamically broken and that dynamical generation of mass does not occur perturbatively. We also investigate the renormalization of the effective potential and determine the renormalization group gamma and beta functions, showing that this model is infrared-free. Comparison with some other results in the literature is provided.

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I. INTRODUCTION

Although supersymmetry (SUSY) is a key concept in the physics of elementary particles and fields, it is not supported (up to now) by experimental evidence. So, any realistic model involving SUSY must include some mechanism of breakdown. Many different mechanisms of breakdown have been considered in the literature. For instance, the minimal supersymmetric standard model with explicit soft SUSY-breaking operators has been suggested as a way of solving the scale of grand unification and the hierarchy problems [1]. The breakdown due to instanton solutions [2] and its connection with R-symmetry breaking [3] and with the Witten index [4,5] have also been intensely investigated throughout the years. Yet, several variations or extensions of the models of O'Raifeartaigh and of Fayet-Iliopoulos [6], which present spontaneous SUSY breaking, have been considered and more recently; theories which exhibit metastable vacua with broken SUSY [7] have also been proposed. Another interesting question is whether a purely perturbative mechanism, i.e., a dynamical symmetry breaking induced by radiative corrections, can be achieved (in this case, a mass scale would be dynamically generated).

In $3 + 1$ spacetime dimensions, this possibility is ruled out by nonrenormalization theorems [8]. On the other hand, in $2 + 1$ dimensions, such restriction (at least for $\mathcal{N} = 1$ SUSY) does not exist [9,10]. The usual way of investigating the vacuum structure in quantum field theory involves the calculation of the effective potential [11]. Recently, the two-loop effective potential for the three-dimensional $\mathcal{N} = 2$ Wess-Zumino (WZ) model was evaluated in Ref. [12]. For the case of $\mathcal{N} = 1$, the effective potential for the WZ model and massless electrodynamics up to one loop were first calculated in Ref. [13] long ago. In both models, that author showed that neither SUSY nor the gauge invariance are broken by radiative corrections up to one-loop order. Nevertheless, in $2 + 1$ dimensions, terms involving logarithms of the classical fields only appear in two or more loops. Since these logarithmic contributions have a crucial role in the dynamical symmetry breakdown, the calculations must be carried up at least to two loops.

In the component field formalism, the two-loop effective potential of the WZ model was evaluated off-shell and on-shell in Refs. [14,15], respectively. In Ref. [14], it is reported a problem with the renormalization of the effective potential: a divergent term which cannot be absorbed by the rescaling of the classical Lagrangian appears. On the other hand, in Ref. [15], difficulties with the renormalization are not found, but it is claimed that SUSY is broken and a dynamical mass generation takes place. In that paper, however, the evaluation of the effective potential did not take into account radiative corrections to the equation of motion of the auxiliary field [16]. These facts lead us to conclude that the renormalization and the vacuum structure to the three-dimensional WZ model are issues not yet satisfactorily answered.

The present work aims to calculate the two-loop effective potential of the WZ model by using the superfield formulation. We claim that the renormalization of the effective potential with dimensional reduction regularization is achieved in the usual way. Moreover, we show that SUSY is not broken, and dynamical generation of mass is not perturbatively consistent. We have also determined the beta function associated with the fourfold self-interaction and verified that it agrees with the result in Ref. [17], which is obtained by direct calculation of the one-particle irreducible Green's functions in components fields. The anomalous dimension of the superfield is also determined.

The paper is organized as follows. In Sec. II, the model is defined, and the tree-level potential is analyzed for different setups of the coupling constants. In Sec. III, the effective potential in one- and two-loop order is calculated, and its renormalization is analyzed for the most general WZ model of a single real scalar superfield. In Sec. IV, the possibility of dynamical symmetry breakdown, for the (sub)model that is classically scale invariant, is studied with the conclusion that the symmetries are preserved.

*rmaluf@fma.if.usp.br
†ajsilva@fma.if.usp.br

The beta function of the coupling constant is also calculated, showing that the model has a Landau pole in the UV limit. In Sec. V, we summarize our conclusions. In Appendix A, the ζ-function method for the calculation of the one-loop contribution is outlined, and in Appendix B, some details of the two-loop calculations are presented.

II. THE MODEL

The most general renormalizable action for the $\mathcal{N}=1$ WZ model, containing a single real scalar superfield in 2 + 1 dimensions, is given by

$$\mathcal{S}[\Phi]=\int d^5z\left\{-\frac{1}{4}D^\alpha\Phi D_\alpha\Phi+W(\Phi)+\mathcal{L}_{CT}\right\}, \quad (1)$$

where $W=a\Phi+\frac{1}{2}m\Phi^2+\frac{\lambda}{3!}\Phi^3+\frac{g}{4!}\Phi^4$ is the superpotential, $\Phi(x,\theta)=\phi(x)+\theta^\alpha\psi_\alpha(x)-F(x)\theta^2$ is a scalar superfield, $d^5z\equiv d^3xd^2\theta$ is the superspace element of volume, and $\mathcal{L}_{CT}$ is the counterterm Lagrangian. Our conventions and notations for the superfield formalism are the same as in Ref. [18]. The mass dimensions of the scalar superfield and the coupling constants are $[\Phi]=1/2$, $[\lambda]=1/2$, $[g]=0$, $[a]=3/2$. When $\lambda=a=0$, the classical action is invariant under the discrete symmetry transformation $\Phi\to-\Phi$, and, if in addition $m=0$, the model is also classically scale-invariant.

The component form of Eq. (1) is easily obtained by doing the θ integration:

$$\begin{aligned}\mathcal{S}=\int d^3x\Big\{&\frac{1}{2}(\phi\Box\phi+\psi^\alpha i\partial_\alpha{}^\beta\psi_\beta+F^2)\\&+m(\psi^2+\phi F)+\lambda\left(\phi\psi^2+\frac{1}{2}\phi^2F\right)\\&+\frac{g}{6}\phi^3F+\frac{g}{2}\phi^2\psi^2+aF+\mathcal{L}_{CT}\Big\}.\end{aligned} \quad (2)$$

The above action is invariant under the supersymmetry transformations

$$\begin{aligned}\delta\phi&=-\epsilon^\alpha\psi_\alpha,\\ \delta\psi_\alpha&=-\epsilon^\beta(C_{\alpha\beta}F+i\partial_{\alpha\beta}\phi),\\ \delta F&=-\epsilon^\alpha i\partial_\alpha{}^\beta\psi_\beta,\end{aligned} \quad (3)$$

where ϵ^α is a constant fermionic parameter.

The tree-level effective potential, as can be read directly from Eq. (2), is given by

$$V^{(0)}(\phi,F)=-\frac{1}{2}F^2-FS(\phi), \quad (4)$$

where $S(\phi)\equiv W'(\phi)=(a+m\phi+\frac{\lambda}{2}\phi^2+\frac{g}{6}\phi^3)$. By eliminating the auxiliary field F through its algebraic equation of motion $0=\partial V^{(0)}/\partial F=-F-S(\phi)$, the classical potential becomes only a function of the physical field ϕ, such that

$$V^{(0)}(\phi)=\frac{1}{2}(S(\phi))^2\geq 0. \quad (5)$$

As is well-known, for any unbroken supersymmetric theory, the vacuum state must correspond to a global minimum of the effective potential with $S(\phi_{\rm min})=0$ and $V(\phi_{\rm min})=0$ [19]. For $g\neq 0$, the model (at tree level) has a SUSY-preserving phase, since $S(\phi_{\rm min})=0$ always has at least one real solution for $\phi_{\rm min}$. In this case, if $a=\lambda=0$, we have a minimum at $\phi_{\rm min}=0$, and if, besides that, we also have $-6m/g>0$, there exist two other solutions, $\phi_{\rm min}=\pm\sqrt{-6m/g}$, that spontaneously break the symmetry $\phi\to-\phi$. Anyway, for $g\neq 0$, SUSY is classically preserved.

Another possibility is $g=0$ and $\lambda\neq 0$, in which case the model is super-renormalizable. If $2a\lambda\leq m^2$, the equation $S=0$ has two real solutions $\phi_{\rm min}=-\frac{m}{\lambda}\pm(\frac{m^2}{\lambda^2}-\frac{2a}{\lambda})^{1/2}$, and SUSY is preserved. If instead $2a\lambda>m^2$, the minimum of $V^{(0)}(\phi)$ occurs for $\phi_{\rm min}=-\frac{m}{\lambda}$ (solution of $dV^{(0)}/d\phi=SS'=0$, for which $S=\frac{m^2}{2\lambda}-a\neq 0$) and implies $V(\phi_{\rm min})=\frac{1}{8\lambda^2}(2\lambda a-m^2)^2>0$, showing a spontaneous breakdown of SUSY at the classical level. When only a and λ are non-null, solutions, $\phi=\pm(2|a/\lambda|)^{1/2}$ exist for $a\lambda<0$ and do not exist for $a\lambda>0$, showing a breakdown of SUSY, with $V(\phi_{\rm min}=0)=a^2/2$.

III. THE EFFECTIVE POTENTIAL

There are several methods by which one can calculate loop corrections to the effective potential in ordinary field theory. We will employ the Jackiw's functional method [20] whose extension to superspace is straightforward. The recipe is shift the quantum superfield Φ by a classical superfield $\phi_{\rm cl}$ and consider the action

$$\hat{\mathcal{S}}[\Phi,\phi_{\rm cl}]\equiv\mathcal{S}[\Phi+\phi_{\rm cl}]-\mathcal{S}[\phi_{\rm cl}]-\int d^5z\Phi\frac{\delta\mathcal{S}}{\delta\Phi}\bigg|_{\Phi=\phi_{\rm cl}}, \quad (6)$$

where $\phi_{\rm cl}(\theta)=\sigma_1-\theta^2\sigma_2$, with $\sigma_1=\langle\phi\rangle$ and $\sigma_2=\langle F\rangle$ being the constant vacuum expectation values of the scalar component fields (the Lorentz invariance of the vacuum requires that $\langle\psi^\alpha\rangle=0$). The action $\hat{\mathcal{S}}$ takes the form

$$\begin{aligned}\hat{\mathcal{S}}[\Phi,\phi_{\rm cl}]=\int d^5z\Big[&\frac{1}{2}\Phi\left(D^2+m+\lambda\phi_{\rm cl}+\frac{g}{2}\phi_{\rm cl}^2\right)\Phi\\&+\frac{1}{3!}(\lambda+g\phi_{\rm cl})\Phi^3+\frac{g}{4!}\Phi^4\Big].\end{aligned} \quad (7)$$

The effective potential can be written in a manifestly supercovariant form as

$$\begin{aligned}V_{\rm eff}(\sigma_1,\sigma_2)=&V^{(0)}(\sigma_1,\sigma_2)-\frac{i}{2\Omega}\ln{\rm Det}[i\Delta_F^{-1}(z,z')]\\&+\frac{i}{\Omega}\langle 0|T\exp i\int d^5z\hat{\mathcal{L}}_{\rm int}(\Phi,\phi_{\rm cl})|0\rangle.\end{aligned} \quad (8)$$

The first term in Eq. (8) is the tree-level potential as given in Eq. (4). The second term is the one-loop correction, where

$$i\Delta_F^{-1}(z,z') = \frac{\delta^2 S[\Phi]}{\delta\Phi_z\delta\Phi_{z'}}\bigg|_{\Phi=\phi_{\rm cl}} = \left(D^2 + m + \lambda\phi_{\rm cl} + \frac{g}{2}\phi_{\rm cl}^2\right)\delta^5(z-z'), \tag{9}$$

and $\Omega \equiv \int d^3x$ is the spacetime volume. The third term encodes the higher-loop corrections: the sum of one-particle-irreducible vacuum superdiagrams with two and more loops computed from the shifted action (7). Let us note that the effective potential is only a function of the constant (x^μ-independent fields σ_1 and σ_2). Actually, the superfield approach adopted here guarantees that after all the D-algebra manipulations, only a single θ integration remains to be done. This allows us to read off the effective potential as it was previously made in Eqs. (2) and (4).

A. One-loop contribution

The one-loop contribution $V^{(1)}$ to the effective potential is enclosed by the functional determinant in Eq. (8). It can be evaluated by the ζ-function method as described in Ref. [13]. Following the calculations outlined in Appendix A, we get

$$V^{(1)} = -\frac{i}{2}\int\frac{d^3k}{(2\pi)^3}\ln\left[\frac{k^2+M^2}{k^2+\mu_1^2}\right] = \frac{1}{12\pi}[(\mu_1^2)^{3/2} - (M^2)^{3/2}], \tag{10}$$

where dimensional reduction with minimal subtraction was used to perform the integrals. The parameter $\mu_1 = S'$ is the fermionic mass, $\mu_2^2 = \sigma_2 S''$ (note that μ_2^2 may assume positive or negative values), and $M^2 = \mu_1^2 - \mu_2^2 = S'^2 - \sigma_2 S''$ is the squared bosonic mass. It must also be noted that the perturbative calculation is valid only for M^2 positive (for $M^2 < 0$, the effective potential becomes complex). The prime denotes derivation with respect to σ_1. Therefore, up to one-loop order, the effective potential is given by

$$V_{\rm eff}(\sigma_1,\sigma_2) = -\frac{1}{2}\sigma_2^2 - \sigma_2 S + \frac{2}{3}\alpha[(S'^2)^{3/2} - (S'^2 - \sigma_2 S'')^{3/2}] + \mathcal{O}(\alpha^2), \tag{11}$$

where we defined $\alpha = \hbar/8\pi = 1/8\pi$ as the parameter that characterizes the strength of the one-loop terms. The $\mathcal{O}(\alpha^2)$ stand for higher-loop orders of approximation.

Let us now investigate the possibility of SUSY breaking and the stability of the effective potential. The stationary points of $V_{\rm eff}$ are determined from the conditions

$$0 = \frac{\partial V_{\rm eff}}{\partial\sigma_2} = -\sigma_2 - S + \alpha S''(S'^2 - \sigma_2 S'')^{1/2} + \mathcal{O}(\alpha^2), \tag{12}$$

$$0 = \frac{\partial V_{\rm eff}}{\partial\sigma_1} = -\sigma_2 S' + \alpha[2S'S''(S'^2)^{1/2} - (2S'S'' - g\sigma_2)(S'^2 - \sigma_2 S'')^{1/2}] + \mathcal{O}(\alpha^2). \tag{13}$$

As we are calculating the effective potential in loops approximations (powers of α), we must, for consistency, solve Eq. (12) perturbatively, as a power series in α (see the discussion below in this section and in Sec. V of Ref. [21]). By substituting the trial form $\sigma_2 = -S + \alpha A(\sigma_1) + \mathcal{O}(\alpha^2)$ in Eq. (12), we get:

$$\sigma_2(\sigma_1) = -S + \alpha S''(S'^2 + S''S)^{1/2} + \mathcal{O}(\alpha^2). \tag{14}$$

If SUSY is preserved, the minimum of the effective potential must be $V_{\rm eff} = 0$, occurring for some real σ_1 and for $\sigma_2 = 0$ (which means that the bosonic and fermionic masses, M and μ_1, remain equal). As can be seen, Eqs. (11) and (13) are identically satisfied for $\sigma_2 = 0$. So, for SUSY to be preserved, Eq. (12) must have a solution of $\sigma_2 = 0$, which means that the equation

$$0 = -S + \alpha S''(S'^2 + S''S)^{1/2} + \mathcal{O}(\alpha^2), \tag{15}$$

must have a real solution $\sigma_1 = \bar\sigma_1$. In this case, the field configuration ($\sigma_1 = \bar\sigma_1$, $\sigma_2 = 0$) is both a stationary point and a zero of $V_{\rm eff}$. If instead this equation does not have a real solution for σ_1, then $\sigma_2 = 0$ is not a solution of Eq. (12), and SUSY is broken.

Suppose that $\bar\sigma_1$ does exist. Inserting the solution (15) back in the effective potential, we get the "physical" effective potential:

$$U_{\rm eff}(\sigma_1) = V_{\rm eff}(\sigma_1,\sigma_2(\sigma_1)) = \frac{1}{2}S^2 + \frac{2}{3}\alpha((S'^2)^{3/2} - (S'^2 + SS'')^{3/2}) + \mathcal{O}(\alpha^2). \tag{16}$$

It still remains to determine if Eq. (14) does have a solution $\bar\sigma_1$ and if $U_{\rm eff}(\sigma_1) \geq 0$ in the region around $\sigma_1 = \bar\sigma_1$, in which we can trust the loop calculation. As already observed, if Eq. (15) does not have a real solution for σ_1, then SUSY is broken. So, let us start by analyzing the solutions of Eq. (15). Moving S to the left side, taking the square, and solving for S, we get $S \mp \alpha S'S'' = \mathcal{O}(\alpha^2)$, for $S' = \pm|S'|$. Up to order α, this equation reads

$$(a \mp \alpha m\lambda) + [m \mp \alpha(gm + \lambda^2)]\sigma_1 + \frac{\lambda}{2}(1 \mp 3\alpha g)\sigma_1^2 + \frac{g}{6}(1 \mp 3\alpha g)\sigma_1^3 = 0. \tag{17}$$

For $g \neq 0$, this equation has at least one real solution for σ_1. In the particular case $a = m = \lambda = 0$, this (triple) solution is $\sigma_1 = 0$. If $a = \lambda = 0$, we have a solution $\sigma_1 = 0$, and if, additionally, $m/g < 0$, two other solutions, the roots of $\sigma_1^2 = -\frac{6m}{g}(1 \mp 2\alpha g)$, which break the symmetry $\Phi \to -\Phi$ but not SUSY.

If $g = 0$, real solutions exist if $m^2 + \alpha^2\lambda^4 > 2a\lambda$ and do not exist otherwise (dropping the term with α in this condition, we get back to the classical condition for SUSY preservation).

Many other particular cases can be studied, but we will fix in the more interesting case, in which only the parameter $g \neq 0$, for which the model is classically scale-invariant. By substituting $S = \frac{g}{6}\sigma_1^3$ in Eq. (16), we get

$$U_{\text{eff}} = \frac{g^2}{72}\sigma_1^6\left[1 - \alpha\frac{g}{12}\left[\left(\frac{5}{3}\right)^{3/2} - 1\right]\right], \qquad (18)$$

which is positive (or null) for $g \ll 1$. So, for this subcase, $(\sigma_1 = 0, \sigma_2 = 0)$ is the minimum of the effective potential, and SUSY is preserved.

For $g = m = 0$ and $a\lambda < 0$, Eq. (16) becomes

$$U_{\text{eff}} = \frac{1}{2}\left(a + \frac{\lambda}{2}\sigma_1^2\right)^2 + \frac{2\alpha}{3} \times\left[(\lambda^2\sigma_1^2)^{3/2} - \left(\frac{3}{2}\lambda^2\sigma_1^2 - |a\lambda|\right)^{3/2}\right]. \qquad (19)$$

We must remember that the calculations can only be trusted for $m_B^2 = \frac{3}{2}\lambda^2\sigma_1^2 - |a\lambda| > 0$; that is, for $\sigma_1^2 > \frac{2}{3}|\frac{a}{\lambda}|$, in which case U_{eff} is positive (its zeros occur for $\sigma_1 = \pm(2|\frac{a}{\lambda}|)^{1/2} \pm \alpha\lambda$). In this case, the discrete symmetry is broken, and SUSY is preserved. For $a\lambda = |a\lambda|$, SUSY is broken, as in the classical case.

These results are in accordance with those of Ref. [22] where, using Wilson renormalization group equations, it is shown that SUSY is preserved for superpotentials with an even highest power of ϕ ($g \neq 0$), but can or cannot be conserved (depending on the relation among the parameters) for the odd highest power of ϕ ($g = 0$ and $\lambda \neq 0$).

An observation is in order. In Ref. [23], the authors observe that the physical effective potential is positive for any value of σ_1, if the auxiliary field σ_2 is eliminated by exactly solving its equation of motion [Eq. (12) in the present paper]. As they say, this positivity must result from effects of higher orders in α, involved in the exact solution of Eq. (12). We did not try to confirm this claim; we instead took the viewpoint that Eq. (12) is valid up to first order in α, and so its solution [our Eq. (14)] must also be trusted up to this same order in α (an interesting discussion about these alternative views is given in Sec. V of Ref. [21]). In the approximation that we are considering, the solution of Eq. (12) can become complex (for values of σ_1 so that $SS'' + S'^2 < 0$) and imply that the effective potential becomes complex in the region in which the classical potential ($U = S^2/2$) is not convex. This is a characteristic of loop calculations and not a particularity of SUSY [24].

B. Two-loop contribution

As is well-known, for symmetry breaking to occur by radiative corrections, we need the induction of terms of the form $h(\sigma_1, \sigma_2)\ln f(\sigma_1, \sigma_2)$. In 2 + 1 dimensions, this only happens in two- (or more) loop approximations. To study this possibility and to make a detailed analysis of the UV counterterms needed to renormalize the effective potential, we will consider the general case in which all the parameters in Eq. (1) are non-null.

Let us start by establishing the supergraph Feynman rules for the shifted theory (7). The Feynman propagator satisfies the Green equation:

$$\hat{\mathcal{O}}_z\Delta_F(z - z') = i\delta^5(z - z'), \qquad (20)$$

where $\hat{\mathcal{O}}_z = D_z^2 + \mu_1 - \mu_2^2\theta^2$ with μ_1 and μ_2^2 defined as before.

To invert the operator $\hat{\mathcal{O}}$, we make use of the projection operators method, developed in Ref. [25]. A basis for the space of scalar operators is formed by the set of six linearly independent operators:

$$P_0 = 1, \quad P_1 = D^2, \quad P_2 = \theta^2, \quad P_3 = \theta^\alpha D_\alpha, \quad P_4 = \theta^2 D^2, \quad P_5 = i\partial_{\alpha\beta}\theta^\alpha D^\beta,$$

satisfying the multiplication table shown in Table I.

After a straightforward algebra, the superpropagator in momentum space is given by

$$\Delta_F(k; \theta - \theta') = i\left(\sum_{i=0}^{5} c_i P_i\right)\delta^2(\theta - \theta'), \qquad (21)$$

where

TABLE I. Multiplication table employed in the inversion of $\hat{\mathcal{O}}$. In addition, we have the trivial relations $P_0P_i = P_iP_0 = P_i$, with $i = 0, \ldots, 5$.

	P_1	P_2	P_3	P_4	P_5
P_1	$\Box$	$-P_0 + P_3 + P_4$	$2P_1 + P_5$	$-P_1 + \Box P_2 - P_5$	$\Box(-2P_0 + P_3)$
P_2	P_4	0	0	0	0
P_3	$-P_5$	$2P_2$	$P_3 - 2P_4$	$2P_4$	$2\Box P_2 + P_5$
P_4	$\Box P_2$	$-P_2$	$2P_4$	$-P_4$	$-2\Box P_2$
P_5	$-\Box P_3$	0	$-2\Box P_2 + P_5$	0	$\Box(P_3 + 2P_4)$

$$c_0 = \frac{\mu_1}{k^2 + M^2},$$
$$c_1 = -\frac{1}{k^2 + M^2},$$
$$c_2 = -\frac{(k^2 - \mu_1^2)\mu_2^2}{(k^2 + \mu_1^2)(k^2 + M^2)},$$
$$c_3 = -\frac{\mu_1\mu_2^2}{(k^2 + \mu_1^2)(k^2 + M^2)},$$
$$c_4 = -\frac{2\mu_1\mu_2^2}{(k^2 + \mu_1^2)(k^2 + M^2)},$$
$$c_5 = -\frac{\mu_2^2}{(k^2 + \mu_1^2)(k^2 + M^2)}.$$

The interaction vertices may be read from Eq. (7), and the symmetry factors can be determined by Wick's theorem in the conventional way.

The two-loop superdiagrams contributing for the effective potential are drawn in Fig. 1. The associated analytical expressions are shown in Appendix B, and the resulting two-loop momentum integrals are evaluated by dimensional reduction using the formulas presented in Ref. [26].

The contribution of the diagram (a), denoted by $V_a^{(2)}$, turns out to be finite, since it is constituted by the product of nonoverlapping one-loop integrals. The diagram (b) instead has divergences proportional to all the terms present in the tree-level potential $V^{(0)}$, which is consistent with the usual renormalizability of the model. In summary, we have the following results:

$$\begin{aligned}
V_a^{(2)} &= -\frac{g}{32\pi^2}\frac{M\mu_1\mu_2^2}{(M+\mu_1)},\\
V_b^{(2)} &= \frac{(\lambda+g\sigma_1)^2}{64\pi^2}\left[\frac{\mu_2^2}{2}I_{\rm div} - 6\mu_1^2\ln\left(\frac{2M+\mu_1}{\mu}\right) + (M^2+5\mu_1^2)\ln\left(\frac{3M}{\mu}\right)\right]\\
&\quad + \frac{(\lambda+g\sigma_1)^2}{64\pi^2}\left[-M^2\ln\left(\frac{M}{\mu}\right) + \frac{M^2}{3}\left\{1+\ln\left(\frac{M+2\mu_1}{27\mu}\right)\right\} - \frac{2}{3}M\mu_1\right.\\
&\quad \left. + \frac{\mu_1^2}{3}\left\{1 - 6\ln\left(\frac{3M}{\mu}\right) - 10\ln\left(\frac{M+2\mu_1}{\mu}\right)\right\} + \frac{2}{3}(M^2+8\mu_1^2)\ln\left(\frac{2M+\mu_1}{\mu}\right)\right]\\
&\quad + \frac{g\sigma_2}{64\pi^2}\left[\left\{I_{\rm div} - 2\ln\left(\frac{3M}{\mu}\right)\right\}\left(\lambda\mu_1 + g\mu_1\sigma_1 - \frac{g}{6}\sigma_2\right)\right],
\end{aligned} \tag{22}$$

where $I_{\rm div} = \frac{1}{\epsilon} + \ln[4\pi e^{(1-\gamma_E)}]$ and μ is an arbitrary mass parameter introduced via dimensional regularization.

The effective potential up to two loops is given by

$$V_{\rm eff} = V^{(0)} + V^{(1)} + V_a^{(2)} + V_b^{(2)} + V_{CT}, \tag{23}$$

in which V_{CT} is the counterterm contribution to the potential

$$V_{CT} = -\left[\frac{1}{2}\delta Z\sigma_2^2 + \delta m\sigma_1\sigma_2 + \frac{\delta\lambda}{2}\sigma_1^2\sigma_2 + g\frac{\delta g}{6}\sigma_1^3\sigma_2 + \delta a\sigma_2\right], \tag{24}$$

as can be read from the classical Lagrangian in Eq. (4); δZ is the wave function renormalization counterterm, and the other counterterms are self-explaining.

The divergent parts of $V^{(2)}$ can be collected in

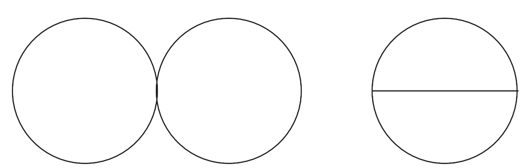
$$V_{\rm div}^{(2)} = \frac{I_{\rm div}}{128\pi^2}\left[-\frac{1}{3}g^2\sigma_2^2 + (2g^2m + 5g\lambda^2)\sigma_1\sigma_2 + 6g^2\lambda\sigma_1^2\sigma_2 + 2g^3\sigma_1^3\sigma_2 + (2gm\lambda + \lambda^3)\sigma_2\right]. \tag{25}$$

As seen from this equation, the renormalization of the effective potential requires all the counterterms in Eq. (24):

$$\begin{aligned}
\delta Z &= -\frac{1}{3}\hat{g}^2 I_{\rm div} + \delta Z_{\rm fin}\\
\delta a &= \frac{1}{2}(2m\hat{g}\,\hat{\lambda} + \lambda\hat{\lambda}^2)I_{\rm div} + \delta a_{\rm fin}\\
\delta m &= \frac{1}{2}(2m\hat{g}^2 + 5g\hat{\lambda}^2)I_{\rm div} + \delta m_{\rm fin}\\
\delta\lambda &= 6\hat{g}^2\lambda I_{\rm div} + \delta\lambda_{\rm fin}\\
\delta g &= 6\hat{g}^2 I_{\rm div} + \delta g_{\rm fin},
\end{aligned} \tag{26}$$

where we defined $\hat{g} = g/8\pi$ and $\hat{\lambda} = \lambda/8\pi$.

Let us compare our results with some others in the literature. In Ref. [12], the effective potential of the $\mathcal{N} = 2$ WZ model in 2 + 1 dimensions was studied in the two-loop approximation. The authors conclude that only a wave



(a) (b)

FIG. 1. Two-loop vacuum bubble supergraphs.

function renormalization is needed. As they say, that result is not unexpected; the $\mathcal{N} = 2$ superspace formulation of supersymmetry in $2 + 1$ dimensions can be gotten from the $\mathcal{N} = 1$ superspace formulation in $3 + 1$ dimensions by dimensional reduction, and so the $3 + 1$-dimensional nonrenormalization theorems are expected to work with $\mathcal{N} = 2$ in $2 + 1$-dimensional supersymmetry. In our results on the other side, no nonrenormalization theorem applies, and the renormalization of all the parameters is necessary. Different from ours, in which three different arguments appear in the generated logarithms, their expression has only a single argument in the generated logarithms. This difference is, maybe, due to their approximation, in which spinorial derivatives $D_\alpha\Phi$ and $D^2\Phi$, besides the usual spatial $\partial\Phi/\partial x^\mu$, are dropped during the calculations. Our results also contradict the result for a similar $\mathcal{N} = 1$ model, reported in Ref. [14], in which a counterterm of the form σ_1^6, not present in the classical Lagrangian, was found to be required.

For the model with $g \neq 0$ and $\lambda \neq 0$, the renormalization also requires that δa and δm be non-null. The submodel with only $g \neq 0$ is renormalizable; that is, it only requires the renormalization of g besides that of Z. We will study this subcase in the next section.

If $g = 0$ and $\lambda \neq 0$ (in which case the model is super-renormalizable), the cancellation of the UV divergences, up to two loops, only requires that $\delta a \neq 0$ ($\delta a = \frac{1}{2}\lambda\hat{\lambda}^2 I_{\rm div}$). As the divergent parts of $\delta\lambda$, δm, and δZ are zero, no running of these constants or an anomalous scaling of the field occur; these results disagree with those in Ref. [22]. This fact is not surprising, considering that the involved approximations in the two methods of calculation are very different. In the two-loop approximation, the only parameter that runs with the scale is a. The renormalization group equation for a is obtained from the relation between the unrenormalized a_0 and the renormalized a, which is given by

$$a_0 = \mu^{-\epsilon/2}\frac{a + \delta a}{(1 + \delta Z)^{1/2}} = \mu^{-\epsilon/2}\left[a + \ln(4\pi e^{1-\gamma}) + \frac{1}{2}\lambda\hat{\lambda}^2\frac{1}{\epsilon} + \cdots\right]. \quad (27)$$

From the equation $0 = \mu(\partial a_0/\partial\mu)$, we get $\mu(\partial a/\partial\mu) = \lambda\hat{\lambda}^2/4$, which, after integration, gives

$$a(\mu) = a(\mu_0) + \frac{\lambda\hat{\lambda}^2}{4}\ln\left(\frac{\mu}{\mu_0}\right). \quad (28)$$

This result means that a change in the parameter μ can be compensated by a simultaneous change in a, leaving the effective potential invariant.

IV. THE UNBROKEN SUSY VACUUM

Let us now investigate in more details the submodel with $g \neq 0$ and $m = \lambda = a = 0$, which is of particular interest for being classically scale-invariant. As discussed in the previous section, the model only requires the δZ and δg counterterms. The total renormalized effective potential $V_{\rm eff}$ takes the form

$$\begin{aligned} V_{\rm eff} = &-\frac{1+\delta Z_{\rm fin}}{2}\sigma_2^2 - g\frac{1+\delta g_{\rm fin}}{6}\sigma_1^3\sigma_2 \\ &+\frac{2\alpha}{3}(\mu_1^3 - M^3) + 2\alpha^2 g\bigg[\frac{1}{3}\mu_1(\mu_1 - M)(\mu_1 - 4M) \\ &-\frac{2}{3}\mu_1(\mu_1^2 - M^2)\ln\left(\frac{2M+\mu_1}{\mu}\right) \\ &-\frac{1}{3}\mu_1(10\mu_1^2 - M^2)\ln\left(\frac{M+2\mu_1}{\mu}\right) \\ &+\left(\mu_1(2\mu_1^2 + M^2) + \frac{g}{6}\sigma_2^2\right)\ln\left(\frac{3M}{\mu}\right)\bigg], \end{aligned} \quad (29)$$

where $\mu_1 = g\sigma_1^2/2$, $\mu_2^2 = g\sigma_1\sigma_2$, and $M = (\mu_1^2 - \mu_2^2)^{1/2}$. The parameters α and α^2 indicate the contributions of one and two loops. Observe that $V_{\rm eff}$ is real only for M real, that is, if $(g\sigma_1^4 - 4\sigma_1\sigma_2) > 0$. The singularity in $\sigma_1 = 0$, for $\sigma_2 \neq 0$, in the last term of $V_{\rm eff}$ is a reminiscence of the IR divergences due to the null mass of the model. So, $\sigma_1 = 0$ is not a convenient spot to impose renormalization conditions. The point $\sigma_1^2 = \mu$, where μ is the mass parameter introduced by the dimensional regularization, is a more natural spot. To see this fact, let us expand the expression of the effective potential in powers of σ_2. The result is

$$\begin{aligned} V_{\rm eff} = &-\frac{g}{6}\sigma_2\sigma_1^3\bigg[1 + \bigg(\delta g_{\rm fin} - 3\hat{g} + 9\hat{g}^2 \\ &+ 12\hat{g}^2\ln\left(\frac{3g}{2}\right)\bigg) + 12\hat{g}^2\ln\left(\frac{\sigma_1^2}{\mu}\right)\bigg] \\ &-\frac{1}{2}\sigma_2^2\bigg[1 + \bigg(\delta Z_{\rm fin} + \hat{g} - \frac{29}{9}\hat{g}^2 - \frac{2}{3}\hat{g}^2\ln\left(\frac{3g}{2}\right)\bigg) \\ &-\frac{2}{3}\hat{g}^2\ln\left(\frac{\sigma_1^2}{\mu}\right)\bigg] + \sigma_2^3\mathcal{F}(\sigma_1, \sigma_2), \end{aligned} \quad (30)$$

where, as before, $\hat{g} = g/8\pi$. We choose $\delta g_{\rm fin}$ and $\delta Z_{\rm fin}$ by imposing that the terms in the parentheses be null. These choices imply that at the point $\sigma_1^2 = \mu$, the coefficients of the two monomials ($\sigma_2\sigma_1^3$ and σ_2^2) are the same as in the classical potential $V_{\rm cl} = -(g/6)\sigma_2\sigma_1^3 - (1/2)\sigma_2^2$. The first condition fixes the renormalized coupling constant, and the second implies that the coefficient of the kinetic term of the effective renormalized Lagrangian at $\sigma_1 = \mu$ is one. In the expanded form, the renormalized potential results in

$$\begin{aligned} V_{\rm eff} = &-\frac{g}{6}\sigma_2\sigma_1^3\left(1 + 12\hat{g}^2\ln\left(\frac{\sigma_1^2}{\mu}\right)\right) \\ &-\frac{1}{2}\sigma_2^2\left(1 - \frac{2}{3}\hat{g}^2\ln\left(\frac{\sigma_1^2}{\mu}\right)\right) + \sigma_2^3\mathcal{F}(\sigma_1, \sigma_2). \end{aligned} \quad (31)$$

In the previous section, we analyzed the effective potential up to one-loop order with minimal subtractions ($\delta Z_{\rm fin} = \delta g_{\rm fin} = 0$). In the present section, we made finite renormalizations, so that in the expansion up to the second power of σ_2, no one loop correction survived; the only corrections to the classical potential come from the two-loop order.

Let us now investigate the possibility of supersymmetry breakdown. It is easy to check that $V_{\rm eff}(\sigma_1, \sigma_2 = 0) = 0$, from which it also follows that $\partial V_{\rm eff}/\partial\sigma_1|_{\sigma_2=0} \equiv 0$. The condition $\partial V_{\rm eff}/\partial\sigma_2|_{\sigma_2=0} = 0$ leads to the following (gap) equation for σ_1:

$$\sigma_1^3\left[1 + 12\hat{g}^2 \ln\left(\frac{\sigma_1^2}{\mu}\right)\right] = 0. \tag{32}$$

This equation has a trivial solution $\sigma_1^{\rm min} = 0$ that ensures that SUSY as well as the discrete symmetry are not broken by the radiative corrections. Looking at the term in the parentheses, a possible nonzero solution $\sigma_1^{\rm min} \neq 0$ would be given by

$$1 + 12\hat{g}^2 \ln\left(\frac{\sigma_1^2}{\mu}\right) = 0. \tag{33}$$

However, by looking at Eq. (31), we see that the two-loop corrections are proportional to $\hat{g}^2 \ln(\sigma_1^2/\mu)$, which, for the validity of the perturbative approach, must be small as compared to the factor (one) coming from the zero-loop potential. So, this minimum lies very far from the range of validity of the two-loop approximation; we conclude that no nontrivial vacuum is induced by radiative corrections, and no SUSY breaking nor mass generation occurs. This result contradicts the claim made in Ref. [15] that the two-loop corrections are able to induce supersymmetry breaking and dynamical generation of mass. On the other hand, a similar conclusion to ours was obtained in Ref. [27] for the O(N) WZ model in the $1/N$ approximation. The same conclusion was also gotten in Ref. [22] through a functional renormalization group analysis. In fact, as discussed in the seminal paper [11], by Coleman and Weinberg, spontaneous symmetry breaking and mass generation, through radiative corrections, can only occur in models with more than one coupling constant and is made possible through an interplay among these constants.

In two loops, the equation $0 = \partial V/\partial\sigma_2$ is a transcendental equation. Yet, a solution as a power series in α can be obtained and inserted back into V to get the physical potential up to order α^2. The solution for σ_2 is of the form $\sigma_2 = C_1\sigma_1^3 + C_2\sigma_1^3 \ln(\sigma_1^2/\mu) + \mathcal{O}(\alpha^3)$, where C_1 and C_2 are functions of α, g, δZ, and δg. The potential results in the form $U_{\rm eff} = c_1\sigma_1^6 + c_2\sigma_1^6 \ln(\sigma_1^2/\mu) + \mathcal{O}(\alpha^3)$ with c_1 and c_2 to be fixed by renormalization conditions. The detailed analysis does not give any new information in relation to our previous and simpler discussion.

Finally, let us determine the renormalization group function β_g for the particular case with $g \neq 0$ and $m = \lambda = a = 0$. Introducing the bare Φ_0 and renormalized superfield Φ and the renormalized coupling constant g through the definitions

$$\Phi_0 = Z_\Phi^{\frac{1}{2}}\Phi = (1 + \delta Z)^{\frac{1}{2}}\Phi, \tag{34}$$

$$g_0 = \mu^\varepsilon g Z_g = \mu^\varepsilon g\left[\frac{1 + \delta g}{Z_\Phi^2}\right], \tag{35}$$

and writing explicitly the counterterms from Eq. (26) as

$$\delta Z = -\frac{g^2}{192\pi^2}\frac{1}{\varepsilon} + \text{finite}, \tag{36}$$

$$\delta g = \frac{3g^2}{32\pi^2}\frac{1}{\varepsilon} + \text{finite}, \tag{37}$$

we obtain the beta function at leading order:

$$\beta_g = \mu\frac{\partial g}{\partial\mu} = \frac{5g^3}{24\pi^2} - \varepsilon g = \frac{5g^3}{24\pi^2} \quad (\text{for } \varepsilon \to 0). \tag{38}$$

This result is in agreement with that obtained in Ref. [17] by calculating the divergent parts of several vertex functions in the component fields formalism. The solution of Eq. (38) is given by

$$\bar{g}^2 = \frac{g^2}{1 - \frac{5}{12\pi^2} g^2 \ln(\frac{\bar{\mu}}{\mu})}. \tag{39}$$

Starting with a $g^2 \ll 1$ at a scale μ, we see that the effective coupling constant $\bar{g}^2$ increases as the scale $\bar{\mu}$ is increased showing a Landau pole at some scale $\bar{\mu}$. So, at short distances, the above results are not reliable: higher-loop corrections become more and more important compared to the second order. If instead we make $\bar{\mu} \to 0$, we get $\bar{g}^2 \to 0$, showing an IR-free limit.

An anomalous scaling of the model is also induced as can be seen by calculating the anomalous dimension of the field:

$$\gamma_\Phi = \frac{1}{2}\mu\frac{d\ln Z_\Phi}{d\mu}. \tag{40}$$

From Eq. (34), we can write Eq. (41) in the form

$$2(1 + \delta Z)\gamma_\Phi = \mu\frac{\partial\delta Z}{\partial g}\frac{\partial g}{\partial\mu}. \tag{41}$$

By replacing Eqs. (36) and (38) into Eq. (41), we get

$$2\left(1 - \frac{g^2}{192\pi^2}\frac{1}{\varepsilon}\right)\gamma_\Phi = \frac{g^2}{96\pi^2} - \frac{5g^4}{24\cdot 96\pi^4}\frac{1}{\varepsilon}, \tag{42}$$

which yields $\gamma_\Phi = \frac{g^2}{192\pi^2}$.

V. CONCLUSIONS

In the present paper, we calculate the effective potential for the $\mathcal{N} = 1$ WZ model in $2 + 1$ dimensions. We employ

Jackiw's functional method combined with the superfield formalism. A detailed analysis of the renormalizability and vacuum structure of the model is presented, up to two loops. One of the main results is that the renormalization of the theory requires, besides the wave function counterterm, also mass and coupling constant counterterms but not any new one. This result differs from that reported in Ref. [14], where the renormalization of the model requires an extra σ_1^6 counterterm. It also differs from that in Ref. [12] for the $\mathcal{N} = 2$ WZ model in $2 + 1$ dimensions, in which only a wave function renormalization was found to be required. For the massless Φ^4 (sub)model, we also determined the β_g function which agrees with the results of Dilkes *et al.* [17], showing a Landau pole in the UV limit. At the same time, we found that the quantum vacuum state preserves supersymmetry and the discrete symmetry $\Phi \to -\Phi$ of the classical theory, contrary to the remark in Ref. [15], but in agreement with the results in Refs. [22,27]. A group renormalization study of the pure $g \neq 0$ model, besides the calculation of the effective potential for the $\mathcal{N} = 2$ model, will be addressed in a forthcoming paper.

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APPENDIX A: THE ζ-FUNCTION METHOD

In this appendix, we compute the one-loop contribution $V^{(1)}$ by the ζ-function method following Ref. [13]. The functional determinant $\mathrm{Det}\hat{\mathcal{O}}$ is understood as the product of the eigenvalues of $\hat{\mathcal{O}}$. Starting with the eigenvalues equation

$$\int d^5 z' \hat{\mathcal{O}}_z(z, z') f_n(z') = \alpha_n f_n(z), \tag{A1}$$

and defining the ζ function associated to $\hat{\mathcal{O}}(z, z') \equiv \hat{\mathcal{O}}_z \delta^5(z - z')$ as

$$\zeta(s) = \sum_n \frac{1}{\alpha_n^s}, \tag{A2}$$

the functional determinant of $\hat{\mathcal{O}}_z$ can be written in the form

$$\mathrm{Det}\hat{\mathcal{O}}_z \equiv \prod_n \alpha_n = \exp[-\zeta'(0)]. \tag{A3}$$

So, the calculation of the determinant requires us to get an analytic representation for $\zeta(s)$. To this end, let us introduce a two-point superspace function $G(z, z'; \tau)$, which obeys the equation

$$\hat{\mathcal{O}}_z G(z, z'; \tau) + \frac{\partial G}{\partial \tau} = 0, \tag{A4}$$

with the initial condition $G(x, \theta; x', \theta'; \tau = 0) = \delta^3(x - x')\delta^2(\theta - \theta')$.

It is straightforward to check that

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty d\tau \tau^{s-1} \int d^3x d^2\theta G(x = x', \theta = \theta'; \tau), \tag{A5}$$

for $G(z, z'; \tau) \equiv \sum_n \exp[-\alpha_n \tau] f_n(z) f_n^*(z')$.

To proceed, we must now determine an explicit solution of $G(z, z'; \tau)$ satisfying Eq. (A4) subject to the initial condition above. To this aim, we will assume that this function is spacetime-translational-invariant so that it can be written as

$$G(x, \theta; x', \theta'; \tau) = \int \frac{d^3k}{(2\pi)^3} g(k, \theta, \theta'; \tau) \exp[-ik(x - x')], \tag{A6}$$

with the following ansatz for $g(k, \theta, \theta'; \tau)$:

$$g(k, \theta, \theta'; \tau) = A(k, \tau) + \theta^\alpha \theta'^\beta k_{\alpha\beta} B(k, \tau) + \theta^\alpha \theta'_\alpha C(k, \tau) + \theta^2 D(k, \tau) + \theta'^2 E(k, \tau) + \theta^2 \theta'^2 H(k, \tau). \tag{A7}$$

To find the coefficients A, B, C, D, E, and H, we have to use the explicit form of $\hat{\mathcal{O}}_z$ read off from Eq. (20) and insert Eq. (A7) into Eq. (A4). This equation splits into six linear ordinary differential equations with the initial conditions:

$$A(k, 0) = 0 \qquad B(k, 0) = 0 \qquad C(k, 0) = 1$$
$$D(k, 0) = -1 \qquad E(k, 0) = -1 \qquad H(k, 0) = 0, \tag{A8}$$

so that the solution of this system is readily found. From these results, we now construct the ζ function as prescribed in Eq. (7).

After integration and using the relation $V^{(1)} = -(i/2\Omega) \ln \mathrm{Det}\hat{\mathcal{O}} = (i/2\Omega)\zeta'(0)$, we are able to get the result described in Eq. (10).

APPENDIX B: TWO-LOOP DIAGRAMS

The analytical expressions for the two-loop vacuum bubbles that contribute to the effective potential displayed in Fig. 1 are ($d^D k \equiv \mu^\varepsilon d^{3-\varepsilon} k$)

$$V_a^{(2)} = -\frac{g}{8} \int \frac{d^D k d^D q}{(2\pi)^{2D}} d^2\theta \Delta_F(k; \theta - \theta_1)|_{\theta=\theta_1} \times \Delta_F(q; \theta - \theta_2)|_{\theta=\theta_2}$$
$$= -\frac{g}{2} \int \frac{d^D k d^D q}{(2\pi)^{2D}} \left[\frac{\mu_1 \mu_2^2}{(k^2 + M^2)(q^2 + \mu_1^2)(q^2 + M^2)} \right], \tag{B1}$$

and

$$V_b^{(2)} = -3i\int\frac{d^Dkd^Dq}{(2\pi)^{2D}}d^2\theta_1 d^2\theta_2 I(\theta_1^2,\theta_2^2)\Delta_F(k;\theta_1-\theta_2)\Delta_F(q;\theta_1-\theta_2)\Delta_F(-k-q;\theta_1-\theta_2), \tag{B2}$$

where

$$I(\theta_1^2,\theta_2^2) = \frac{1}{36}[(\lambda+g\sigma_1)^2 - (\lambda g\sigma_2 + g^2\sigma_1\sigma_2)(\theta_1^2+\theta_2^2) + g^2\sigma_2^2\theta_1^2\theta_2^2]. \tag{B3}$$

After performing the D algebra and carrying out the remaining θ integration, we obtain the following two-loop momentum integrals:

$$\begin{aligned} V_b^{(2)} = &\int\frac{d^Dkd^Dq}{(2\pi)^{2D}}\frac{-\mu_2^2(\lambda+g\sigma_1)^2}{12(k^2+M^2)(q^2+M^2)(k^2+\mu_1^2)(q^2+\mu_1^2)[(k+q)^2+M^2][(k+q)^2+\mu_1^2]} \\ &\times\{k^4(q^2+\mu_1^2)+2k.q[(k^2+\mu_1^2)(q^2+\mu_1^2)-(k^2+q^2-(k+q)^2+\mu_1^2)\mu_2^2] \\ &+\mu_1^2[q^4-15\mu_1^4-4q^2[(k+q)^2+2\mu_1^2]+6\mu_1^2\mu_2^2+2(k+q)^2(-5\mu_1^2+\mu_2^2)] \\ &+k^2q^4-4k^2\mu_1^2[(k+q)^2+2\mu_1^2]+k^2q^2[2(k+q)^2-\mu_1^2-4\mu_2^2]\} \\ &+\int\frac{d^Dkd^Dq}{(2\pi)^{2D}}\frac{-6g\mu_1(\lambda+g\sigma_1)\sigma_2+g^2\sigma_2^2}{12(k^2+M^2)(q^2+M^2)[(k+q)^2+M^2]}. \end{aligned} \tag{B4}$$

The two-loop integrals were performed by dimensional reduction scheme, using the formulas from Ref. [26]. The final results are written in Eq. (22).

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