Gauge symmetry and Virasoro algebra in quantum charged rigid membrane: A first order formalism

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(Received 31 December 2012; published 5 February 2013)

The quantum charged rigid membrane model, which is a higher derivative theory, has been considered to explore its gauge symmetries using a recently developed first order formalism [R. Banerjee *et al.*, J. High Energy Phys. 08 (2011) 085]. Hamiltonian analysis has been performed and the gauge symmetry of the model is identified as reparametrization symmetry. First class constraints are shown to have a truncated Virasoro algebraic structure. An exact correspondence between the higher derivative theory and the first order formalism has been shown from the point of view of equations of motion.

I. INTRODUCTION

Higher derivative field theories are inseparable from modern day theoretical physics. Long ago physicists started considering Lagrangians with higher time derivatives [1–3]. Initially they were introduced to avoid infinities appearing in the scattering amplitudes. But, due to their distinctive properties, higher derivative (HD) theories find their place in various contexts of physics, e.g., electrodynamics [1,2], nonlocal theories [4], relativistic particle model with curvature and torsion [5–7], string theory [8], supersymmetry [9,10], noncommutative theory [11], dark energy physics [12–14], cosmology [15–17], inflation theory [18], brane world scenario [15], and supergravity [19,20]. In quantum gravity, Stelle showed that adding higher derivative terms can ensure renormalizability [21] although it breaks unitarity. But a suitable choice of the coefficients of the higher derivative terms can lead to unitarity too [22]. People constructed $f(R)$ gravity where higher curvature terms were added to Einstein-Hilbert action and opened a vast sector of research. For HD gravity, the list is huge. Interesting features appeared when higher derivative terms were added to study the Higgs mechanism [23]. Also, people working in one of the most exciting fields of recent theoretical physics like AdS/CFT correspondence have considered HD theories [24–27] which indicates the importance and relevance of considering HD theories.

Existence of gauge symmetries in theories with higher derivatives can be an interesting domain to study. For theories with single derivatives only, there exists the well established Dirac's method [28–32]. But HD theories have some extra difficulties while performing canonical analysis and needed careful observation. For performing Hamiltonian analysis of the HD theories, Ostrogradski's method can be useful [33]. The only discomfort is the nontrivial definition of the momenta. For a long period the method was used in various sectors for higher

derivatives theories. This method was presumably first applied in the invariant regularization of gauge theories [34]. Other applications were done in various examples like equivalence theorems for spectrum changing transformations [35], relativistic particle model [5,6], Regge-Teitelboim—type cosmology [17], geodetic brane cosmology [36], and recently for unambiguous quantization of non-Abelian gauge theories [37]. Other than this, an inspired first order formalism exists in the literature where the HD fields are considered as independent fields and usual Hamiltonian analysis can be performed (along with a trivial definition of the momenta) [38,39]. For abstracting the gauge symmetries there exists a powerful method [40–42] but only for first order theories with no higher derivative terms. Recently, we provided a general method for abstracting gauge symmetries with higher derivative theories [38,39] which we referred to as first order formalism. We obtained some peculiar result in gauge symmetries of HD theories. We took the relativistic particle model with curvature [5] and found that there are two independent primary first class constraints but with only one independent gauge symmetry, which is clearly contrary to the accepted result which states that the number of independent gauge symmetries is equal to number of independent primary first class constraints [32,41]. Surprisingly, there appears two gauge symmetries, viz. diffeomorphism and W symmetry, when we considered the mass term to be zero [38]. These results inspired us to consider a thorough analysis of gauge symmetries of models with HD terms (especially with curvature terms). Such a model is Dirac's membrane model for the electron [43,44].

Theories with extrinsic curvatures are frequently studied especially in string theory, although the concept is not new, but recent inclusion of these in some physically interesting models added an extra urgency to revisit the symmetry features of this type of surfaces. Because of extrinsic curvature effects, there appears geometrical frustration when nematic liquid crystals are constrained to a curved surface [45]; whereas, graphene too can be considered as an electronic membrane and its rippling generates spatially

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varying electrochemical potential that is proportional to the square of the local curvature [46]. These extrinsic curvature terms also appear in various brane world scenarios [47–51]. Recently, this concept of extrinsic curvature in membranes also has been incorporated for studying fluid dynamics [52]. Generally, these surfaces come into the picture where we consider the evolution of a surface with a background metric. The lowest dimensional generalization is a point particle evolving in spacetime with a background metric [53]. Applying this idea, in 1962 an extensible relativistic model of the electron was proposed by Dirac [43]. With spherical symmetry, the model was in stable equilibrium due to its surface tension. In this paper we shall investigate the gauge symmetries of an updated version of the Dirac's membrane model for the electron where extrinsic curvature terms of the world volume were included as second order correction terms [44].

The paper is organized in the following manner. In Sec. II we gave a general overview of higher derivative theories and their conversion to first order formalism. Construction of the gauge generator and the master equation for extracting independent gauge symmetries is introduced in this section. Section III comprises mainly a very brief introduction to the model of quantum charged rigid membrane, since literature is available for the model and its variants. Section IV is purely new as our main work is concentrated here. In this section we derive the equation of motion from the variational principle and perform a Hamiltonian analysis of the model. Section V is devoted to find out gauge symmetries. Interestingly, the first class constraints form truncated Virasoro algebra. In Sec. VI we show the equivalence between the higher derivative and the first order formalism via matching the equation of motion. Finally, we conclude with Sec. VII.

II. ABSTRACTION OF GAUGE SYMMETRIES FOR HIGHER DERIVATIVE THEORIES: A FIRST ORDER FORMALISM

A general form for the HD Lagrangian is given by[1]

$$L = L(x, \dot{x}, \ddot{x}, \ldots, x^{(\nu)}), \qquad (1)$$

where $x = x_n (n = 1, 2, \ldots, \nu)$ are the coordinates and an overdot means derivative with respect to time. The νth order derivative of time is denoted by $x^{(\nu)}$.

In the first order formalism, we convert the higher derivative Lagrangian (1) into a first order Lagrangian by defining the variables $q_{n,\alpha} (\alpha = 1, 2, \ldots, \nu - 1)$ as

$$q_{n,1} = x_n, \qquad q_{n,\alpha} = \dot{q}_{n,\alpha-1}, \qquad (\alpha > 1). \qquad (2)$$

Because of redefinition of the variables there emerges the following constraints:

$$q_{n,\alpha} - \dot{q}_{n,\alpha-1} = 0, \qquad (\alpha > 1) \qquad (3)$$

which can be added to the HD Lagrangian via the Lagrange multipliers $\lambda_{n,\beta} (\beta = 2, \ldots, \nu - 1)$. Consequently, we can write down an auxiliary Lagrangian,

$$L'(q_{n,\alpha}, \dot{q}_{n,\alpha}, \lambda_{n,\beta}) = L(q_{n,1}, q_{n,2}, \ldots, q_{n,\nu-1}, \dot{q}_{n,\nu-1})$$
$$+ \sum_{\beta=2}^{\nu-1} (q_{n,\beta} - \dot{q}_{n,\beta-1})\lambda_{n,\beta}. \qquad (4)$$

Considering the Lagrangian multipliers to be independent fields in addition to the fields $q_{n,\alpha}$, we define momenta as

$$p_{n,\alpha} = \frac{\partial L'}{\partial \dot{q}_{n,\alpha}}, \qquad \pi_{n,\beta} = \frac{\partial L'}{\partial \dot{\lambda}_{n,\beta}}. \qquad (5)$$

Having found out the primary constraints of the theory, we can write down the total Hamiltonian as

$$H_T = H_C + u_{n,\beta}\pi_{n,\beta} + v_{n,\beta}\Phi_{n,\beta}, \qquad (6)$$

where $u_{n,\beta}$, $v_{n,\beta}$ are Lagrange multipliers and $\pi_{n,\beta}$, $\Phi_{n,\beta}$ are primary constraints. So we can proceed to have all the secondary constraints by demanding time variation of the constraints as zero. After we have extracted all the constraints, we can move to distinguish the first class and second class constraints. Now, according to Dirac, the first class constraints generate gauge transformation. The second class constraints can be removed by introduction of Dirac brackets. Therefore, our theory is a first order theory with only first class constraints. To find out the gauge symmetries of the model, we define the gauge generator as

$$G = \sum_a \epsilon_a \Phi_a. \qquad (7)$$

Here $\{\Phi_a\}$ is the whole set of primary constraints. All the gauge parameters ϵ_a may not be independent. To identify all the independent gauge transformations, we refer to the method developed in Refs. [41,42] and write down the master equation relating the Lagrange multipliers Λ_{a_1} and the gauge parameters ϵ_a:

$$\delta\Lambda_{a_1} = \frac{d\epsilon_{a_1}}{dt} - \epsilon_a(V_{aa_1} + \Lambda_{b_1}C_{b_1aa_1}), \qquad (8)$$

$$0 = \frac{d\epsilon_{a_2}}{dt} - \epsilon_a(V_{aa_2} + \Lambda_{b_1}C_{b_1aa_2}). \qquad (9)$$

Here the indices $a_1, b_1, \ldots$ refer to the primary first class constraints while the indices $a_2, b_2, \ldots$ correspond to the secondary first class constraints. The coefficients $V_a^{a_1}$ and $C_{b_1a}^{a_1}$ are the structure functions of the involutive algebra, defined as[2]

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[1]For an extended version of this first order formalism, please see Ref. [38].

[2]From now on, we have to use only Dirac brackets since we removed all second class constraints. Poisson brackets are denoted by $\{,\}$, whereas, $\{,\}_D$ refers to Dirac brackets.

$$\{H_{\text{can}}, \Phi_a\}_D = V_{ab}\Phi_b, \qquad \{\Phi_a, \Phi_b\}_D = C_{abc}\Phi_c. \quad (10)$$

Because of the HD nature, a relation between the gauge transformations of the fields can be written as

$$\delta q_{n,\alpha} - \frac{d}{dt}\delta q_{n,\alpha-1} = 0, \qquad (\alpha > 1) \quad (11)$$

which may impose some extra condition on the gauge parameters.

Abstracting all independent gauge transformations, we can write gauge transformation of the basic fields as

$$\delta_{\epsilon_a} q_{n,\alpha} = \{q_{n,\alpha}, G\}_D. \quad (12)$$

This completes our analysis of finding the gauge transformation for HD theories.

III. QUANTUM CHARGED RIGID MEMBRANE

In a background Minkowski spacetime $\eta^{\mu\nu}$,[3] consider the evolving surface Σ. The surface is described by the local coordinate x^μ of the background spacetime. The embedding function $X^\mu(\xi^a) = x^\mu$ is a function of the local coordinates of the world volume m, swept out by the surface. We consider the following effective action underlying the dynamics of the surface Σ [44]:

$$S[X^\mu] = \int_m d^3\xi(-\alpha K + \beta j^a e^\mu{}_a A_\mu), \quad (13)$$

where $K = g^{ab}K_{ab}$ is the extrinsic curvature[4] and α, β are constants related to the rigidity parameter and form factor, respectively. On the other hand, j^a which minimally couples the charged surface and the electromagnetic field A_μ [54], is a constant electric current density distributed over the world volume and is locally conserved on m with $\partial_a j^a = 0$. Variation of the action with respect to the embedding function $X^\mu(\xi^a)$ leads to the equation of motion,

$$\alpha \mathcal{R} = \frac{\beta}{\sqrt{-g}} j^a n^\mu e^\nu{}_a F_{\mu\nu}. \quad (14)$$

The above equation (14) can be thought of as a Lorentz force equation with $\mathcal{R}$ being the Gaussian curvature and $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$ the electromagnetic field tensor. Under a suitable choice of the embedding functions $[X^\mu(\tau, \theta, \varphi) = (t(\tau), r(\tau), \theta, \varphi)]$ Eq. (13) boils down to [44]

$$S = 4\pi \int d\tau L(r, \dot{r}, \ddot{r}, \dot{t}, \ddot{t}), \quad (15)$$

where the Lagrangian L, which is HD in nature, is given by

$$L = -\frac{\alpha r^2}{\dot{t}^2 - \dot{r}^2}(\ddot{r}\,\dot{t} - \dot{r}\,\ddot{t}) - 2\alpha r\dot{t} - \frac{\beta q^2 \dot{t}}{r}. \quad (16)$$

So, Lagrangian (16) will be our sole interest which is reparametrization invariant under the parameter τ.

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[3] Values of the indices are $\mu, \nu = 0, 1, 2, 3$ and $a, b = 0, 1, 2$.
[4] g_{ab} is the world volume metric and $e^\mu{}_a = X^\mu{}_{,a}$ are tangent vectors to the world volume.

Promptly, we can write down the equation of motion for the HD Lagrangian:

$$\frac{d}{d\tau}\left(\frac{\dot{r}}{\dot{t}}\right) = -\frac{\dot{t}^2 - \dot{r}^2}{2r\dot{t}^3}\left(\dot{t}^2 - \frac{\beta(\dot{t}^2 - \dot{r}^2)^2 q^2}{2\alpha r^2}\right). \quad (17)$$

IV. HAMILTONIAN ANALYSIS

Before we start the Hamiltonian analysis we need to convert the HD Lagrangian (16) to a first order Lagrangian, named as the auxiliary Lagrangian, by introduction of the new fields,

$$\dot{r} = R, \qquad \dot{t} = T. \quad (18)$$

So, we write down the auxiliary Lagrangian as[5]

$$L' = -\frac{\alpha r^2}{N^2}(\dot{R}T - R\dot{T}) - 2\alpha rT - \frac{\beta q^2 T}{r}$$
$$+ \lambda_1(R - \dot{r}) + \lambda_2(T - \dot{t}). \quad (19)$$

Inclusion of new fields imposes constraints

$$R - \dot{r} \approx 0, \qquad T - \dot{t} \approx 0, \quad (20)$$

which are taken care of via the multipliers λ_1 and λ_2. Variation of L' with respect to r, R, t, T, λ_1, and λ_2 gives rise to the following equation of motions:

$$-\frac{2\alpha r}{N^2}(\dot{R}T - R\dot{T}) - 2\alpha T + \frac{\beta q^2 T}{r^2} + \dot{\lambda}_1 = 0, \quad (21)$$

$$-\frac{2\alpha r^2}{N^4}R(\dot{R}T - R\dot{T}) + \frac{d}{d\tau}\left(\frac{\alpha r^2}{N^2}T\right) + \frac{\alpha r^2}{N^2}\dot{T} + \lambda_1 = 0, \quad (22)$$

$$\dot{\lambda}_2 = 0, \quad (23)$$

$$\frac{2\alpha r^2}{N^4}T(\dot{R}T - R\dot{T}) - \frac{d}{d\tau}\left(\frac{\alpha r^2}{N^2}R\right) - \frac{\alpha r^2}{N^2}\dot{R} - 2\alpha r$$
$$- \frac{\beta q^2}{r} + \lambda_2 = 0, \quad (24)$$

$$R - \dot{r} = 0, \quad (25)$$

$$T - \dot{t} = 0. \quad (26)$$

Equations (25) and (26) are obvious since they correspond to (20).

Before proceeding to Hamiltonian formulation, we identify the new phase space which is constituted of the variables (r, Π_r), (t, Π_t), (R, Π_R), (T, Π_T), $(\lambda_1, \Pi_{\lambda_1})$, $(\lambda_2, \Pi_{\lambda_2})$. Here $\Pi_{x^\mu} = \frac{\partial L'}{\partial \dot{x}^\mu}$ are the momenta corresponding to x^μ which generically stands for the variables r, R, t, T, λ_1, λ_2. We immediately obtain the primary constraints as listed below:

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[5] Consider $N^2 = T^2 - R^2$, for convenience.

$$\Phi_1 = \Pi_r + \lambda_1 \approx 0, \qquad \Phi_2 = \Pi_t + \lambda_2 \approx 0,$$

$$\Phi_3 = \Pi_R + \frac{\alpha r^2}{N^2} T \approx 0, \qquad \Phi_4 = \Pi_T - \frac{\alpha r^2}{N^2} R \approx 0,$$

$$\Phi_5 = \Pi_{\lambda_1} \approx 0, \qquad \Phi_6 = \Pi_{\lambda_2} \approx 0. \tag{27}$$

The Poisson brackets between the field variables are defined as

$$\{x^\mu, \Pi_{x^\nu}\} = \delta_{\mu\nu}, \qquad \{x^\mu, x^\nu\} = \{\Pi_{x^\mu}, \Pi_{x^\nu}\} = 0. \tag{28}$$

With the aid of (28), the nonzero Poisson brackets between the primary constraints can be written as

$$\{\Phi_1, \Phi_3\} = -\frac{2\alpha r}{N^2} T, \qquad \{\Phi_1, \Phi_4\} = \frac{2\alpha r}{N^2} R,$$

$$\{\Phi_1, \Phi_5\} = 1, \qquad \{\Phi_2, \Phi_6\} = 1. \tag{29}$$

We can take the following combination of the constraints:

$$\Phi_3' = R\Phi_3 + T\Phi_4 \approx 0, \tag{30}$$

$$\Phi_4' = \Phi_4 - \frac{2\alpha r R}{N^2} \Phi_5 \approx 0, \tag{31}$$

so that the new set of primary constraints is Φ_1, Φ_2, Φ_3', Φ_4', Φ_5, Φ_6. The complete algebra of primary constraints is now given by (only the nonzero brackets are listed)

$$\{\Phi_1, \Phi_5\} = \{\Phi_2, \Phi_6\} = 1. \tag{32}$$

We can write the canonical Hamiltonian via Legendre transformation as

$$H_{\text{can}} = 2\alpha r T + \frac{\beta q^2 T}{r} - \lambda_1 R - \lambda_2 T. \tag{33}$$

The total Hamiltonian is

$$H_T = H_{\text{can}} + \Lambda_1 \Phi_1 + \Lambda_2 \Phi_2 + \Lambda_3 \Phi_3'$$
$$+ \Lambda_4 \Phi_4' + \Lambda_5 \Phi_5 + \Lambda_6 \Phi_6. \tag{34}$$

Here $\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \Lambda_5, \Lambda_6$ are the Lagrange multipliers which are arbitrary at this stage. Only those multipliers which are attached to the primary second class constraints will be determined; others corresponding to primary first class constraints will remain undetermined (although they can be determined too via equation of motion). At this level, loosely speaking Φ_3' and Φ_4' are first class constraints (this classification may be changed after we get the full list of constraints). These two may provide us two new secondary constraints and the list can still keep increasing until we get all the constraints. Now, we move towards extracting all constraints of this system. This can be done by demanding that Poisson brackets of the constraints with the total Hamiltonian (time evolution) of the constraints are zero. Preserving Φ_1, Φ_2, Φ_5, Φ_6 in time solves the following multipliers, respectively,

$$\Lambda_5 = 2\alpha T - \frac{\beta q^2 T}{r^2}, \qquad \Lambda_6 = 0,$$

$$\Lambda_1 = R, \qquad \Lambda_2 = T. \tag{35}$$

Time conservation of the primary constraints Φ_3' and Φ_4' leads to the secondary constraints Ψ_1 and Ψ_2 respectively given by

$$\Psi_1 = -2\alpha r T - \frac{\beta q^2 T}{r} + \lambda_1 R + \lambda_2 T \approx 0,$$

$$\Psi_2 = -2\alpha r - \frac{\beta q^2}{r} + \lambda_2 - \frac{2\alpha r}{N^2} R^2 \approx 0. \tag{36}$$

Before proceeding further, we list below all the nonzero Poisson brackets of the secondary constraints Ψ_1, Ψ_2 with other constraints:

$$\{\Phi_1, \Psi_1\} = 2\alpha T - \frac{\beta q^2 T}{r^2}, \qquad \{\Phi_5, \Psi_1\} = -R,$$

$$\{\Phi_6, \Psi_1\} = -T, \qquad \{\Phi_1, \Psi_2\} = \frac{2\alpha}{N^2} T^2 - \frac{\beta q^2}{r^2}, \tag{37}$$

$$\{\Phi_4', \Psi_2\} = -\frac{4\alpha r}{N^4} T R^2, \qquad \{\Phi_6, \Psi_2\} = -1.$$

Now, time preservation of the secondary constraint Ψ_1 gives identically $0 = 0$, and the requirement of $\dot\Psi_2 = 0$ solves the Lagrange multiplier $\Lambda_4 = -\frac{A}{B} R$, with $A = \frac{2\alpha T^2}{N^2} - \frac{\beta q^2}{r^2}$, and $B = -\frac{4\alpha r T R^2}{N^4}$.

From the constraint algebra (32) and (37), one can clearly assert that there is only one first class constraint Φ_3' with seven other second class constraints Φ_1, Φ_2, Φ_4', Φ_5, Φ_6, Ψ_1, Ψ_2. One point worth noting since there is an odd number of second class constraints, it indicates there might be some other first class constraint to make the pair of second class constraints even. Judiciously, we can choose a combination $\Psi_1' = \Psi_1 - \Lambda_1 \Phi_1 - \Lambda_2 \Phi_2 - \Lambda_4 \Phi_4' - \Lambda_5 \Phi_5 - \Lambda_6 \Phi_6$ so that the pair (Φ_3', Ψ_1') becomes first class. This completes our constraint classification.

Having completed the constraint classification, its time to get rid of the unphysical sector $(\lambda_1, \Pi_{\lambda_1})$ and $(\lambda_2, \Pi_{\lambda_2})$ by imposing the primary second class constraints Φ_1, Φ_2, Φ_5, Φ_6 strongly zero. This can be done by replacing all Poisson brackets by Dirac brackets for the rest of the calculations. Surprisingly, Dirac brackets between the basic fields remain the same as their corresponding Poisson brackets. So, now our phase space is spanned by $\{r, \Pi_r, t, \Pi_t, R, \Pi_R, T, \Pi_T\}$. For convenience of future calculations, we rename the constraints as

$$F_1 = \Phi_3' = R\Phi_3 + T\Phi_4 \approx 0. \tag{38}$$

$$F_2 = \Psi_1 - \Lambda_4 \Phi_4 \approx 0. \tag{39}$$

$$S_1 = \Phi_4 \approx 0. \tag{40}$$

$$S_2 = \Psi_2 = -\Pi_t - 2\alpha r - \frac{\beta q^2}{r} - \frac{2\alpha r R^2}{N^2} \approx 0. \tag{41}$$

Here, F_1, F_2 is the first class pair with F_1 as the primary first class constraint. So far we observed that in this theory, there is only one primary first class constraint with one undetermined multiplier which clearly indicates existence of gauge symmetry(s) in the system. In the next section we will extract the gauge symmetries of this quantum charged rigid membrane.

V. GAUGE SYMMETRY AND VIRASORO ALGEBRA

To study gauge symmetry we need to remove all the second class constraints from the system by setting them strongly zero and performing the Dirac bracket defined by

$$\{f, g\}_D = \{f, g\} - \sum_{i,j=1,2} \{f, S_i\} \triangle_{ij}^{-1} \{S_j, g\}, \quad (42)$$

where f and g corresponds to the phase space variables or their functions. To compute $\triangle_{ij}^{-1}$ for the set of of second class constraints, we have $\{S_1, S_2\} = -\frac{4\alpha r t R^2}{N^2}$. So, we can compute the Dirac brackets between the basic fields. The nonzero Dirac brackets are

$$\{r, \Pi_r\}_D = 1, \qquad \{\Pi_r, t\}_D = -\frac{N^2}{2TR},$$

$$\{\Pi_r, t\}_D = \frac{A}{B}, \qquad \{\Pi_r, \Pi_T\}_D = -\frac{2\alpha rR}{N^2} + \frac{Ar}{2R},$$

$$\{t, \Pi_t\}_D = 1 \qquad \{t, \Pi_R\}_D = \frac{r(T^2 + R^2)}{4TR^2},$$

$$\{t, T\}_D = -\frac{1}{B}, \qquad \{t, \Pi_T\}_D = -\frac{r}{2R},$$

$$\{R, \Pi_R\}_D = 1, \qquad \{\Pi_R, T\}_D = -\frac{T}{R},$$

$$\{\Pi_r, \Pi_R\}_D = \frac{2\alpha rT}{N^2} + \frac{A}{B} \frac{\alpha r^2(T^2 + R^2)}{N^4},$$

$$\{\Pi_R, \Pi_T\}_D = \frac{\alpha r^2}{N^2}. \quad (43)$$

The generator of the gauge transformation is given by a linear combination of all first class constraints,

$$G = \epsilon_1 F_1 + \epsilon_2 F_2, \quad (44)$$

where ϵ_1 and ϵ_2 are gauge parameters. We need to find out whether these gauge parameters are independent or not.

The Dirac brackets between the first class constraints are given by

$$\{F_i, F_j\}_D = -\epsilon_{ij}F_2; \qquad i, j = 1, 2. \quad (45)$$

Using a suggestive notation we rename the constraints F_1 and F_2 as

$$L_0 = F_1, \quad (46)$$

$$L_1 = F_2. \quad (47)$$

We can easily identify a sort of truncated Virasoro algebra of the form

$$\{L_m, L_n\}_D = (m - n)L_{m+n} \quad (48)$$

with $m = 0$, $n = 1$ as proposed in Ref. [55] for HD cases.

Now, using Eqs. (10) and (45), we compute the structure constraints as $C_{122} = -1 = -C_{212}$ and $V_{12} = 1$ (other structure constraints are zero). Exploiting the master equations (9) we find the following relation between the gauge parameters:

$$\epsilon_1 = -\Lambda_3\epsilon_2 - \dot{\epsilon}_2, \quad (49)$$

and it is clear that we have only one independent gauge symmetry in this system which is supported by the fact that there is only one undetermined multiplier. We consider ϵ_2 to be independent and compute the gauge transformation of the fields

$$\delta r = -\epsilon_2 R, \quad (50)$$

$$\delta t = -\epsilon_2 T, \quad (51)$$

$$\delta R = \epsilon_1 R, \quad (52)$$

$$\delta T = \epsilon_1 T + \epsilon_2 \frac{A}{B} R. \quad (53)$$

We can identify this gauge symmetry as reparametrization symmetry in the following manner. Consider an infinitesimal transformation of r and t on the world volume as $\tau \to \tau + \sigma$. For some infinitesimal σ, we can write

$$\delta r = -\sigma r, \qquad \delta t = -\sigma t. \quad (54)$$

Clearly, a comparison between (50) and (51) and both equations of (54) shows that the reparametrization parameter is given by $\sigma = \epsilon_2$. Using (54) we compute gauge variation of the Lagrangian (16) which simplifies to

$$\delta L = \frac{d}{d\tau}(\sigma L), \quad (55)$$

and ensure the invariance of the action under (54).

VI. CONSISTENCY CHECK

It would be worth finding out the Hamiltonian equations of motion are given by

$$\dot{r} = R, \quad (56)$$

$$\dot{t} = T, \quad (57)$$

$$\dot{R} = \Lambda_3 R, \quad (58)$$

$$\dot{T} = -\frac{A}{B}R + \frac{\dot{R}}{R}T. \quad (59)$$

Equations (56) and (57) are obvious as they arise as constraints at the Lagrangian level and agree with (25) and (26). Taking the time derivative of (50) and (51), we get

$$\frac{d}{d\tau}\delta r = -\dot{\epsilon}_2 R - \epsilon_2 \dot{R}, \tag{60}$$

$$\frac{d}{d\tau}\delta t = -\dot{\epsilon}_2 T - \epsilon_2 \dot{T}. \tag{61}$$

Using Eq. (49) along with (58) and (59), the above equations (60) and (61) simplify to

$$\frac{d}{d\tau}\delta r = \delta R, \tag{62}$$

$$\frac{d}{d\tau}\delta t = \delta T \tag{63}$$

which is a direct verification for (11). Equation (59) along with the trivial equations of motion (56) and (57) can be cast into the form so that they verify (17). This indeed is an important outcome of this analysis which agrees with the validity of this first order formalism via matching the equation of motion at higher derivative and first order level.

Taking the gauge variation of Eq. (58) and using (52), we get

$$\delta\Lambda_3 = \dot{\epsilon}_1, \tag{64}$$

which in turn verifies the first master equation (8).

VII. DISCUSSION

Studies in higher derivative field theories have been an intense field of research [1,2,6,7]. Symmetry studies have always been interesting for theoreticians. We already have shown some results concerning inequality in the number of independent first class constraints and the number of independent gauge symmetries for a relativistic particle model with curvature [38]. This mismatch inspired us to a further study of some physically interesting models. Dirac's relativistic membrane model for the electron can be a candidate with a future prospect in brane inspired cosmology [47].

In this paper we presented a fresh Hamiltonian analysis purely in a first order formalism where higher time derivatives are considered to be independent fields and the corresponding momenta are defined in the usual way. Gauge symmetries were analyzed in a novel way by constructing the gauge generator and extracting the independent gauge parameter. The number of independent primary first class constraints is exactly in accord with the number of independent gauge symmetries leading to no mismatch. Also, the constraint structure is shown to obey truncated Virasoro algebra. Reparametrization parameters have been identified through a suitable transformation of the fields.

The model continues to be the highlight of recent interests like branes, cosmology, and dark energy [47,48,52,54,56]. Consideration of other variants of the model with more symmetries can be of utmost interest as future projects.

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[1] B. Podolsky, Phys. Rev. **62**, 68 (1942).

[2] B. Podolsky and C. Kikuchi, Phys. Rev. **65**, 228 (1944); **67**, 184 (1945).

[3] D. J. Montgomery, Phys. Rev. **69**, 117 (1946).

[4] A. Pais and G. E. Uhlenbeck, Phys. Rev. **79**, 145 (1950).

[5] R. D. Pisarski, Phys. Rev. D **34**, 670 (1986).

[6] V. V. Nesterenko, J. Phys. A **22**, 1673 (1989).

[7] M. S. Plyushchay, Int. J. Mod. Phys. A **04**, 3851 (1989); Nucl. Phys. **B362**, 54 (1991).

[8] D. A. Eliezer and R. P. Woodard, Nucl. Phys. **B325**, 389 (1989).

[9] J. Iliopoulos and B. Zumino, Nucl. Phys. **B76**, 310 (1974).

[10] F. S. Gama, M. Gomes, J. R. Nascimento, A. Yu. Petrov, and A. J. da Silva, Phys. Rev. D **84**, 045001 (2011).

[11] C. S. Chu, J. Lukierski, and W. J. Zakrzewski, Nucl. Phys. **B632**, 219 (2002).

[12] G. W. Gibbons, arXiv:hep-th/0302199.

[13] S. M. Carroll, M. Hoffman, and M. Trodden, Phys. Rev. D **68**, 023509 (2003).

[14] R. P. Woodard, Lect. Notes Phys. **720**, 403 (2007).

[15] I. P. Neupane, J. High Energy Phys. 09 (2000) 040.

[16] S. Nojiri, S. D. Odintsov, and S. Ogushi, Phys. Rev. D **65**, 023521 (2001).

[17] R. Cordero, A. Molgado, and E. Rojas, Phys. Rev. D **79**, 024024 (2009).

[18] A. Anisimov, E. Babichev, and A. Vikman, J. Cosmol. Astropart. Phys. 06 (2005) 006.

[19] R. Andringa, E. A. Bergshoeff, M. de Roo, O. Hohm, E. Sezgin, and P. K. Townsend, Classical Quantum Gravity **27**, 025010 (2010).

[20] E. A. Bergshoef, O. Hohm, J. Rosseel, E. Sezgin, and P. K. Townsend, Classical Quantum Gravity **28**, 015002 (2011).

[21] K. S. Stelle, Phys. Rev. D **16**, 953 (1977).

[22] S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. (N.Y.) **140**, 372 (1982); **185**, 406(E) (1988).

[23] K. Jansen, J. Kutl, and C. Llu, Phys. Lett. B **309**, 119 (1993); **309**, 127 (1993).

[24] S. Nojiri and S. D. Odintsov, J. High Energy Phys. 07 (2000) 049.

[25] S. Nojiri and S. D. Odinitsov, Int. J. Mod. Phys. A **15**, 413 (2000).

[26] S. Nojiri and S. D. Odintsov, Phys. Lett. B **471**, 155 (1999).

[27] M. Fukuma, S. Matsuura, and T. Sakai, Prog. Theor. Phys. **105**, 1017 (2001).

[28] P. A. M. Dirac, Can. J. Math. **2**, 129 (1950); *Lectures on Quantum Mechanics* (Yeshiva University, unpublished).

[29] A. Hanson, T. Regge, and C. Tietelboim, *Constrained Hamiltonian System* (Accademia Nazionale Dei Lincei, Roma, 1976).

[30] H. J. Rothe and K. D. Rothe, *Classical and Quantum Dynamics of Constrained Hamiltonian Systems*, Lecture Notes in Physics Vol. 81 (World Scientific, New York, 2010).

[31] K. Sundermeyer, *Lecture Notes in Physics 169, Constrained Dynamics* (Springer-Verlag, Berlin, 1982).

[32] M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems* (Princeton University Press, Princeton, New Jersey, 1994).

[33] M. Ostrogradsky, Mem. Acad. St. Petersbourg **6**, 385 (1850).

[34] A. A. Slavnov, Nucl. Phys. B **31**, 301 (1971).

[35] A. A. Slavnov, Phys. Lett. B **258**, 391 (1991).

[36] R. Cordero, M. Cruz, A. Molgado, and E. Rojas, Classical Quantum Gravity **29**, 175010 (2012).

[37] A. A. Slavnov, J. High Energy Phys. 08 (2008) 047; A. Quadri and A. A. Slavnov, J. High Energy Phys. 07 (2010) 087.

[38] R. Banerjee, P. Mukherjee, and B. Paul, J. High Energy Phys. 08 (2011) 085.

[39] P. Mukherjee and B. Paul, Phys. Rev. D **85**, 045028 (2012).

[40] D. M. Gitman and I. V. Tyutin, *Quantization of Fields with Constraints* (Springer-Verlag, Berlin, Heidelberg, 1990), p. 291.

[41] R. Banerjee, H. J. Rothe, and K. D. Rothe, Phys. Lett. B **463**, 248 (1999); **479**, 429 (2000).

[42] R. Banerjee, H. J. Rothe, and K. D. Rothe, J. Phys. A **33**, 2059 (2000).

[43] P. A. M. Dirac, Proc. R. Soc. A **268**, 57 (1962).

[44] R. Cordero, A. Molgado, and E. Rojas, Classical Quantum Gravity **28**, 065010 (2011).

[45] G. Napoli and L. Vergori, Phys. Rev. Lett. **108**, 207803 (2012).

[46] E. Kim and A. H. C. Neto, Europhys. Lett. **84**, 57007 (2008).

[47] A. Davidson and S. Rubin, Classical Quantum Gravity **26**, 235006 (2009).

[48] A. Davidson and S. Rubin, Classical Quantum Gravity **28**, 125005 (2011).

[49] N. T. Yilmaz, Classical Quantum Gravity **27**, 145019 (2010).

[50] V. G. Czinner, Phys. Rev. D **80**, 104017 (2009).

[51] M. Trzetrzelewski, Phys. Lett. B **684**, 256 (2010).

[52] M. B. Roberts, Central Eur. J. Phys. **9**, 1016 (2011).

[53] J. Nambu, Copenhagen Summer Symposium, 1970 (unpublished); T. Gotto, Prog. Theor. Phys. **46**, 1560 (1971).

[54] A. O. Barut and M. Pavsic, Phys. Lett. B **306**, 49 (1993).

[55] P. M. Ho, Phys. Lett. B **558**, 238 (2003).

[56] A. Aurilia, M. Palmer, and E. Spallucci, Phys. Rev. D **40**, 2511 (1989).