PHYSICAL REVIEW D **87,** 044007 (2013)

Bondi accretion onto cosmological black holes

Janusz Karkowski and Edward Malec
Instytut Fizyki Mariana Smoluchowskiego, Instytut Fizyki, Uniwersytet Jagielloński, Reymonta 4, 30-059 Kraków, Poland
(Received 6 November 2012; published 5 February 2013)

In this paper we investigate a steady accretion within the Einstein-Straus vacuole, in the presence of the cosmological constant. The dark energy damps the mass accretion rate and—above a certain limit—completely stops the steady accretion onto black holes, which, in particular, is prohibited in the inflation era and after (roughly) 10^{12} years from the big bang (assuming the presently known value of the cosmological constant). Steady accretion would not exist in the late phases of the Penrose's scenario—known as the Weyl curvature hypothesis—of the evolution of the Universe.

DOI: 10.1103/PhysRevD.87.044007 PACS numbers: 04.20.−q, 04.25.D−, 04.40.Nr, 98.35.Mp

I. INTRODUCTION

The classical Bondi accretion model [1] describes, in the test gas and stationarity approximations, the spherical accretion/wind of a barotropic fluid onto a Newtonian gravity center. That description has been extended to general-relativistic spacetimes. The steady accretion in the Schwarzschildean geometry has been investigated by Michel [2] and Shapiro and Teukolsky [3], and in a spherical symmetric spacetime with backreaction by Malec [4]. A remarkable universal behavior has been discovered for self-gravitating transonic flows, under suitable boundary conditions: the ratio of respective mass accretion rates $\dot{M}_{GR}/\dot{M}_N$ appears independent of the fractional mass of the gas and depends only on the asymptotic temperature. It is close to 1 in the regime of low asymptotic temperatures and can grow by 1 order of magnitude at high temperatures [5,6]. Here $\dot{M}_{GR}$ and $\dot{M}_N$ are the general-relativistic and the Newtonian mass accretion rates, respectively. The stability of steady accretion has been established by Mach and his coworkers [7–9].

In this paper we consider the accretion onto a black hole that is immersed in a cosmological universe. We adopt the Einstein-Straus "Swiss cheese" model [10]. A spherically symmetric black hole surrounded with accreting gas sits inside a vacuole that is matched—through the Darmois-Israel junction conditions [11,12] to an external Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime. We calculate how the presence of a cosmological constant Λ would affect the most important system characteristic—the mass accretion rate $\dot{M}$ of a fluid onto the center. We found that the mass accretion rate is damped by the presence of the cosmological constant. This effect is stronger for negative values than for positive values of the cosmological constant and it depends on the ratio of the dark energy density to the fluid density.

In the next two sections we remind readers of the Kottler solutions and draw the general picture of the steady accretion, together with suitable equations. Sections IV and V present three analytic sets of results. These show that the matter distribution can be affected by the cosmological system. More importantly, we prove that if the cosmological constant exceeds a certain limit, then the steady accretion ceases to exist. Section VI presents results of numerical integrations. It appears that the presence of dark energy, either negative (attractive) or positive (repulsive), damps the mass accretion rate. It appears—in accordance with analytic predictions—that the steady accretion does not exist above certain values of the cosmological constant. The last section summarizes the main results and applies them to two cosmological epochs when the dark energy is dominant. It is pointed out that our results do not agree with the Penrose's "Weyl curvature hypothesis."

II. ACCRETION IN AN EINSTEIN-STRAUS VACUOLE

We assume relativistic units with $G = c = 1$. Let a FLRW spacetime be filled with dust and dark energy. Its line element reads

$$ds^2 = -d\tau^2 + a^2(\tau)\left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\right), \qquad (1)$$

where the variables have obvious meaning [13] and $k = 0, \pm 1$. We do not need to analyze the FLRW equations, for a reason explained below.

We cut off, following Einstein and Straus [10], a ball from this FLRW spacetime and insert instead a spherical symmetric inhomogeneity—a black hole surrounded by a spherical cloud of gas—satisfying Einstein's equation. It is known that the boundary of the ball must be comoving if the FLRW geometry contains only dust and the dark energy represented by Λ, assuming the absence of a boundary layer and continuity of pressure [14–16]. Crenon and Lake show that the no-boundary layer assumption is necessary for having a comoving boundary [14].

Inside the excision ball itself we can distinguish an inner accretion region such that the areal radius R does not exceed a limiting radius R_∞ (that in turn should be smaller, or perhaps much smaller, than the areal radius of the boundary of the ball), and the asymptotic vacuum region outside R_∞. In the asymptotic region the metric is given by the Kottler [Schwarzschild—de Sitter (SdS, $\Lambda > 0$) and Schwarzschild—anti—de Sitter (SAdS, $\Lambda < 0$)] spacetime line element [17]

$$ds^2 = -\left(1 - \frac{2m}{R} - \frac{\Lambda}{3}R^2\right)dt^2 + \frac{dR^2}{1 - \frac{2m}{R} - \frac{\Lambda}{3}R^2} + R^2 d\Omega^2. \quad (2)$$

The Darmois-Israel gluing conditions [11,12] demand that along the comoving boundary the first and second fundamental forms are continuous. In the spherically symmetric case the radius R has to be continuous and in addition (i) the excess mass m should be exactly equal to the mass of dust in the excised ball and (ii) the boundary should be comoving with the Hubble velocity H. This guarantees that the stress-energy tensor $T_{\mu\nu}$ does not form the surface layer at the junction surface. Balbinot *et al.* have found an explicit solution that describes the corresponding matching between the Schwarzschild—de Sitter and FLRW (dust + Λ) spacetimes [18]. Therefore we stop discussion on FLRW and SdS (SAdS) solutions at this point and focus on the description of the accretion zone.

We assume that in the accretion region the Einstein equations can be approximated by a set of stationary equations in time intervals that are much smaller than a characteristic time T that is defined in the next section. Let R_∞ be the size of the cloud of gas and a_∞ the asymptotic speed of sound. Outside R_∞ the geometry can be connected smoothly to the SdS or SAdS spacetime geometry by a transient zone of a negligible mass. Let us remark that the assumption of the approximate stationarity can be checked *a posteriori*, after finding appropriate solutions. Approximately stationary accretion equations have been derived in Ref. [4]. A quasistationary solution, of a similar accretion problem in the asymptotically flat spacetime with a spherical black hole, appears to be stable under axially symmetric perturbations [8].

III. EQUATIONS OF STEADY ACCRETION

The metric in the accretion zone is spherically symmetric and has the form

$$ds^2 = -N^2dt^2 + \hat{a}dr^2 + R^2(d\theta^2 + \sin^2(\theta)d\phi^2), \quad (3)$$

where we use comoving coordinates t, r, $0 \le \theta \le \pi$, $0 \le \phi < 2\pi$—time, coordinate radius, and two angle variables, respectively. R denotes the areal radius and N is the lapse. $\hat{a}$ is the radial-radial metric component. The radial velocity of gas is given by $U = \frac{1}{N}\frac{dR}{dt}$.

The energy-momentum tensor reads $T^B_{\mu\nu} = (\rho + p)U_\mu U_\nu + pg_{\mu\nu}$ with the timelike normalized four-velocity U_μ, $U_\mu U^\mu = -1$. A comoving observer would measure local mass density $\rho = T^{\mu\nu}U_\mu U_\nu$. Let n_μ be the unit normal to a coordinate sphere lying in the hypersurface $t = \text{const}$ and let k be the related mean curvature scalar, $k = \frac{R}{2}\nabla_i n^i = \frac{1}{\sqrt{\hat{a}}}\partial_r R$. We assume the perfect gas equation of state $p = (\Gamma - 1)\rho_0\epsilon$, where ϵ is the specific internal energy and Γ is a constant. Assuming that the accretion is isentropic, one can derive the polytropic equation of state $p = K\rho_0^\Gamma$, with a constant K. The internal energy $E = \rho_0\epsilon$ and the rest ρ and baryonic ρ_0 mass densities are related by $\rho = \rho_0 + E = \rho_0 + p/(\Gamma - 1)$.

The matter-related and geometric quantities satisfy Einstein equations and the baryonic mass conservation. (But let us point out that for isentropic flows the conservation of baryonic mass also follows from Einstein equations [4].) One can find the mean curvature k from the Einstein constraint equations $G_{\mu 0} = 8\pi T_{\mu 0}$ [4],

$$k = \sqrt{1 - \frac{2m(R)}{R} - \frac{\Lambda}{3}R^2 + U^2}, \quad (4)$$

where $m(R)$ is the quasilocal mass,

$$m(R) = m - 4\pi\int_R^{R_\infty} drr^2\rho. \quad (5)$$

In the line element (3) we have the comoving time. In the polar gauge foliation one has a new time $t_S(t, r)$ with $\partial_{t_S} = \partial_t - NU\partial_R$. The field ∂_{t_S} is tangent to the cylinder of constant areal radius, $\partial_{t_S}R = 0$.

One can show that

$$\dot{M}_\rho \equiv \partial_{t_S}m(R) = 4\pi NUR^2(\rho + p). \quad (6)$$

The mass contained in an annulus (R, R_∞) changes if the fluxes on the right-hand side, one directed outward and the other inward, do not cancel. The baryonic current density reads $j^\mu \equiv \rho_0 U^\mu$. Its continuity equation reads

$$\nabla_\mu j^\mu = 0. \quad (7)$$

For stationary flows the local baryonic flux

$$\dot{M} = -4\pi UR^2\rho_0 \quad (8)$$

is time-independent at a fixed R.

We say that the accretion process is stationary (or quasistationary) if all physically relevant observables, which are measured at a fixed areal radius R, remain approximately constant during time intervals much smaller than the runaway instability time scale $T = M/\dot{M}$. That means that $\partial_{t_S}X \equiv (\partial_t - NU\partial_R)X = 0$ for $X = \rho_0, \rho, j, U, \ldots, \dot{M}$. For quasistationary flows we have $\partial_R\dot{M}_\rho = 0$ and $\partial_R\dot{M} = 0$ [4]. Therefore both $\dot{M}_\rho$ and $\dot{M}$ are equal, modulo a constant factor; they can be identified. The quantity $\dot{M}$ will be called the mass accretion rate. One obtains from (8) an expression

$$\partial_R U^2 = -\frac{4U^2}{R} - 2U^2\partial_R\ln(\rho_0). \quad (9)$$

The speed of sound is defined as $a = \sqrt{\partial_\rho p}$. It is a useful and straightforward exercise to express the hydrodynamic quantities in terms of a:

$$p = \rho_0 \frac{\Gamma - 1}{\Gamma} \frac{a^2}{\Gamma - 1 - a^2},$$
$$\rho = \rho_0 \frac{\Gamma - 1}{\Gamma - 1 - a^2} - p, \qquad (10)$$
$$\rho_0 = \rho_{0\infty} \left(\frac{a}{a_\infty}\right)^{\frac{2}{\gamma-1}} \left(\frac{1 - \frac{a_\infty^2}{\Gamma - 1}}{1 - \frac{a^2}{\Gamma-1}}\right)^{\frac{1}{\Gamma-1}}.$$

Here (and below) quantities with the suffix ∞ do refer to their asymptotic values (i.e., at R_∞). Notice that p, ρ_0 and a^2 show the same monotonicity behavior: $\partial_R \rho_0 = C_1 \partial_R p = C_2 \partial_R a^2$, where C_1, C_2 are strictly positive functions.

There are two conservation equations that originate from the contracted Bianchi identities $\nabla_\mu T^\mu_\nu = 0$. One of them can be eliminated, if we choose instead the baryonic mass conservation. The case when $\nu = 0$ is the relativistic version of the Euler equation,

$$N \frac{d}{dR} p + (p + \rho) \frac{d}{dR} N = 0. \qquad (11)$$

One can solve Eq. (11), using Eq. (10):

$$N = \tilde{C}(\Gamma - 1 - a^2), \qquad (12)$$

where $\tilde{C}$ is a constant. The whole system of algebraic equations (4)–(12) closes with the imposition of the Einstein equation, $G_{rr} = 8\pi T_{rr}$, which is the only integro-differential equation. It can be put in the following form:

$$\frac{d}{dR}\ln(a^2) = -\frac{\Gamma - 1 - a^2}{a^2 - \frac{U^2}{k^2}} \times \frac{1}{k^2 R}\left(\frac{m(R)}{R} - 2U^2 + 4\pi R^2 p - \frac{\Lambda R^2}{3}\right). \qquad (13)$$

Finally the line element, in (t, R) coordinates, is given by

$$ds^2 = -(N^2 - U^2)dt^2 - 2\frac{N}{k} dt dR + \frac{dR^2}{k^2} + R^2 d\Omega^2. \qquad (14)$$

We shall study transonic accretion flows. For them the principal object of interest is a sonic point, where both the denominator $a^2 - \frac{U^2}{k^2}$ and the numerator $\frac{m(R)}{R} - 2U^2 + 4\pi R^2 p - \frac{\Lambda R^2}{3}$ of Eq. (13) vanish. The corresponding value of the areal radius is called the sonic radius, denoted as R_*. A closer inspection of the sonic point shows that it is a critical point, with branching pairs of solutions describing accretion or wind. In the accretion branch, below the sonic point the infall velocity $|U|/k$ is bigger than a, while outside the sonic sphere the converse is true. This analysis is very much standard, replicating the work done in the Newtonian case by Bondi [1] and in the general-relativistic case by Malec [4].

IV. QUALITATIVE ANALYSIS OF SOLUTIONS: SDS BLACK HOLES

In this section we shall investigate some properties of transonic solutions for positive values of Λ. The speed of sound a (and thus also ρ_0 and p) decreases as a function of R—for accreting solutions—when the cosmological constant is absent. We shall show, in Lemma 2, that for large values of ΛR^2 the converse is possible.

Lemma 1: Assume $\Lambda < \frac{3m}{2R_\infty^3}$. Then the speed of sound, the pressure, and the mass density ρ_0 are decreasing outside the sonic sphere, $\frac{d}{dR}X < 0$ for $X = a^2$, p, and ρ_0.

Proof of Lemma 1: Let us define $C(R) \equiv \frac{m(R)}{R} - 2U^2(R) + 4\pi R^2 p - \frac{\Lambda R^2}{3}$ and let $C_\infty \equiv C(R_\infty)$. One easily obtains from this definition, using Eq. (9) in order to eliminate $\partial_R U^2$, that

$$\frac{d}{dR}C(R) = \frac{-4C}{R} + 4\pi R\left(\rho + 6p - \frac{\Lambda}{2\pi}\right) + \frac{3m}{R^2} + 4U^2 \frac{\frac{d}{dR}\rho_0}{\rho_0} + 4\pi R^2 \frac{d}{dR} p. \qquad (15)$$

The condition $\Lambda < \frac{3m}{R_\infty^3}$ guarantees that $C_\infty > 0$ and—using the argument of continuity—the positivity of the function $C(R)$ in an open interval (R_s, R_∞). We show that $R_s = R_*$.

Notice that while at the sonic point $C(R_*) = 0$, the function $\partial_R C$ must be strictly positive. Indeed, assume the opposite; from continuity, C would have to be negative just above R_*. Then from Eq. (15) it would follow that both terms with derivatives must be negative, that is—see the remark below (10)—$\partial_R a^2 < 0$. But Eq. (13) gives $\partial_R a^2 > 0$, if $C < 0$ and $R > R_*$. This contradiction proves our statement and implies the positivity of C in an interval (R_*, R_b).

Now, assume that C changes sign at R_b, that is, $C(R_b) = 0$ and $\frac{d}{dR}C(R)_{R=R_b} < 0$. But if $C(R_b) = 0$ then—from Eq. (13)—the derivative $\partial_R a^2$ would have to vanish. From (10) it would then follow $\partial_R \rho_0 = \partial_R p = 0$. Thus Eq. (15) yields at R_b

$$\frac{d}{dR}C = 4\pi R_b\left(\rho(R_b) + 6p(R_b) - \frac{\Lambda}{2\pi}\right) + \frac{3m}{R_b^2}, \qquad (16)$$

but this implies $\frac{d}{dR}C > 0$ at R_b, due to the condition $\Lambda < \frac{3m}{R_b^3}$. This contradiction proves that $C(R)$ is strictly positive outside the sonic sphere.

The positivity of C allows one to conclude—from Eq. (13)—that $\partial_R a^2$ is strictly decreasing outside the sonic sphere. This in turn implies the decrease of the pressure p and the baryonic mass density ρ_0 [see the remark under Eq. (10)].

Lemma 2: Assume $\Lambda R_\infty^2 > \frac{18\alpha a_\infty^2}{\Gamma - 1 - a_\infty^2}$ and define $R_0 = (\frac{6m}{\Lambda})^{\frac{1}{3}}$. The necessary condition for the existence of solutions is $\alpha \leq 1$. If $\alpha > 1/3$ then $R_0 \ll R_\infty$ and the speed of sound, the pressure, and the mass densities are increasing in the interval (R_0, R_∞): $\frac{d}{dR}X > 0$ for $X = a^2$, p, and ρ_0.

Proof of Lemma 2: The estimate $R_0 \ll R_\infty$ follows immediately from the boundary condition $a_\infty^2 \gg m/R_\infty$ and the assumed value of Λ. In the asymptotic end (R_0, R_∞) the function $-C(R)$ can be bounded from below as follows,

$$-C(R) = \frac{-m(R)}{R} + 2U^2(R) - 4\pi R^2 p + \frac{\Lambda R^2}{3} \geq \frac{-2m}{R} + \frac{\Lambda R^2}{3}. \quad (17)$$

The second line follows from $U^2 \geq 0$ and from the observation that $p \leq \rho$—thus $4\pi R^2 p \leq 4\pi \int_R^{R_\infty} dr r\rho$—and $4\pi \int_R^{R_\infty} dr r\rho \leq \frac{m}{R}$.

Taking into account the last estimate, we can estimate from below the right-hand side of Eq. (13) by

$$D(R) \equiv \frac{\Gamma - 1 - a_\infty^2}{a^2}\left(-\frac{2m}{R^2} + \frac{\Lambda R}{3}\right). \quad (18)$$

We obtain from Eqs. (13) and (18) the inequality

$$\frac{d}{dR}a^2 \geq (\Gamma - 1 - a_\infty^2)\left(-\frac{2m}{R^2} + \frac{\Lambda R}{3}\right). \quad (19)$$

It is clear that (i) R_0 is the null point of $D(R)$ and (ii) the function $D(R)$ is strictly positive for $R \in (R_0, R_\infty)$. Therefore a^2 (and consequently the remaining gas characteristics p and ρ_0) does increase in this interval, provided that a solution does exist.

The necessary condition for the existence of a solution is that $a(R_0)$ should be strictly positive. Integration of Eq. (19), between R_0 and R_∞, yields

$$a_\infty^2 - a^2(R_0) \geq (\Gamma - 1 - a_\infty^2) \times \left(-\frac{2m}{R_0} - \frac{\Lambda R_0^2}{6} + \frac{2m}{R_\infty} + \frac{\Lambda R_\infty^2}{6}\right). \quad (20)$$

We can replace $\frac{2m}{R_0}$ by $\frac{\Lambda R_0^2}{3}$, using the definition of R_0, and get the following:

$$\begin{aligned} a^2(R_0) &\leq (\Gamma - 1 - a_\infty^2)\frac{\Lambda R_\infty^2}{18}\left(\frac{6R_0^2}{R_\infty^2} - 1\right) + a_\infty^2 - (\Gamma - 1 - a_\infty^2)\frac{2m}{R_\infty} \\ &\leq a_\infty^2 + (\Gamma - 1 - a_\infty^2)\frac{\Lambda R_\infty^2}{18}\left(\frac{6 \times 36^{\frac{1}{3}}(\frac{m}{R_\infty})^{\frac{2}{3}}}{(\Lambda R_\infty^2)^{\frac{2}{3}}} - 1\right). \end{aligned} \quad (21)$$

In the second inequality we dropped the positive term with m/R_∞ and used the definition of R_0. We already assumed that $m/R_\infty \ll a_\infty^2$ which ensures that the first term within the second bracket is small and the right-hand side of the last inequality in (21) is approximated by $E(R) \equiv a_\infty^2 - \frac{\Lambda R_\infty^2}{18}(\Gamma - 1 - a_\infty^2)$. Therefore if the cosmological constant is given by $\Lambda R_\infty^2 = 18\alpha \frac{a_\infty^2}{\Gamma - 1 - a_\infty^2}$, with α larger than 1, then $E(R)$ must be negative and the square of the speed of sound at R_0 must be nonpositive, which means that solutions are absent. On the other hand solutions can exist for $0.01 < \alpha < 1$; in this case $C(R) < 0$ between (R_0, R_∞), which implies that $\frac{d}{dR}X < 0$ for $X = a^2$, p, ρ_0, and ρ.

The two lemmas proven in this section have a transparent physical interpretation. Lemma 1 assumes that the dark energy density $\Lambda/4\pi$ is smaller than one half of the averaged matter density $3m/4\pi R_\infty^3$. The presence of Λ is expected to have some quantitative impact onto accretion mass rate $\dot{M}$, but qualitative features of the flow are not influenced—in particular, the gas characteristics a, $|U|$, ρ_0, and ρ are all decreasing functions.

Lemma 2 assumes the opposite—that the dark energy density $\Lambda/4\pi$ is much larger than the average matter density $3m/4\pi R_\infty^3$. Above a particular value—that depends on the boundary characteristics of the flow, its volume, and the black hole mass—steadily accreting solutions do not exist. This is intuitively understandable—large Λ implies a large Hubble expansion velocity which can obstruct or even prohibit accretion, through the junction condition on the boundary of the vacuole. Therefore one can expect a significantly smaller mass accretion rate $\dot{M}$ and even its absence.

V. QUALITATIVE ANALYSIS OF SOLUTIONS: SADS BLACK HOLES

Lemma 3: Assume $\Lambda < 0$. Then,

(i) For $R > R_*$ the speed of sound, the pressure, and the mass densities ρ_0 and ρ are decreasing, $\frac{d}{dR}X < 0$ for $X = a^2$, p, and ρ_0.

(ii) Solutions are absent if $\Lambda R_\infty^2 \leq -6\frac{\Gamma - 1 - a_\infty^2}{\Gamma - 1 - a_*^2}$.

Proof of Lemma 3:

Part (i). The key observation is that the function C is strictly positive, if $\Lambda < 0$. Indeed, let $R_b > R_*$ be a smallest radius at which $C(R_b)$ vanishes, and assume $C < 0$ in the open interval (R_*, R_b) [notice that $C(R_*)$ vanishes]. But Eq. (15) would imply in such a case that both terms with derivatives become negative in a subinterval (R_*, R_1) of (R_*, R_b), that is—see the remark below (10)—$\partial_R a^2 < 0$. But this in turn would contradict Eq. (13)—if $C < 0$ then $\partial_R a^2 > 0$ outside the sonic sphere. Thus we arrive at the contradiction in this subinterval (R_*, R_1) and the argument of continuity and the definition of R_b imply the positivity of C in the open interval (R_*, R_b).

In the next step we show that C cannot vanish outside the sonic sphere. Indeed, (13) and (15) imply that $\frac{dC}{dR}$ is strictly positive whenever $C = 0$ for $R > R_*$; that in turn yields (by the argument of continuity) the strict positivity of C outside the sonic sphere. Equation (13) allows one to conclude that the speed of sound is monotonically decreasing, and that implies also decrease of the mass density ρ_0 and the pressure.

Part (ii). In order to prove the necessary existence condition, notice that Eq. (13) gives, after integration,

$$|a_\infty^2 - a^2(R)| \geq \int_R^{R_\infty} dr \left|(\Gamma - 1 - a^2)\frac{1}{r}\left(-2U^2 - \frac{\Lambda r^2}{3}\right)\right|. \tag{22}$$

Notice that $a^2 < \Gamma - 1$; thus $|a_\infty^2 - a^2(R)| \leq \Gamma - 1 - a_\infty^2$. The conservation of the mass accretion rate $\dot{M}$ and $\frac{d}{dR}\rho_0 \leq 0$ lead to the estimate $\frac{d}{dR}(U^2R^4) \geq 0$, and this in turn implies inequality

$$U^2 \leq U_\infty^2 \frac{R_\infty^4}{R^4}. \tag{23}$$

Collecting this information, one finally obtains

$$\Gamma - 1 - a_\infty^2 \geq (\Gamma - 1 - a^2(R))\left(-\frac{U_\infty^2}{2} - \frac{\Lambda R_\infty^2}{6}\left(1 - \frac{R^2}{R_\infty^2}\right)\right). \tag{24}$$

Let $R = R_*$ and $\frac{\Lambda R_\infty^2}{6} < -\frac{\Gamma-1-a_\infty^2}{\Gamma-1-a_*^2}$; then—taking into account that $\frac{R^2}{R_\infty^2}$ is negligibly small, and the same is true for the velocity term U^2—we conclude that the right-hand side of Eq. (22) exceeds its left-hand side. This contradiction proves the necessary condition of Lemma 3.

VI. DESCRIPTION OF NUMERICS

Our aim is to find transonic accretion flows. We assume the total mass $m = 1$, $\Gamma = 4/3$, and $R_\infty = 10^6$. The boundary values at R_∞ are $k = N = \sqrt{1 - \frac{2m}{R_\infty} - U_\infty^2 - \frac{\Lambda}{3}R_\infty^2}$ and $a_\infty^2 = 2 \times 10^{-4}$. There are three remaining parameters that are free: the cosmological constant Λ, the mass accretion rate $\dot{M}$, and the asymptotic density $\rho_{0\infty}$—after specifying them, equations are integrated inward, starting from R_∞. These quantities should be such as to ensure the asymptotic conditions: $|U_\infty| \ll \sqrt{2m/R_\infty} \ll a_\infty$.

A numerical run, with the above boundary data, has been regarded as successful, if

(i) the integration started from R_∞ and reached the outermost apparent horizon, that the point R_i such that $\frac{2m(R_i)}{R_i} + \frac{\Lambda R_i^2}{3} = 1$, along the accretion branch (with the increasing—in the direction to the center—infall velocity $|U|$);

(ii) there appears a sonic point at a radius $R_* \geq R_i$.

More specifically, in order to find a single transonic flow, we fix Λ and the mass accretion rate $\dot{M}$, and treat the asymptotic value of the baryonic mass density as a free parameter. For a random choice of $\rho_{0\infty}$ there are three possibilities, one exceptional and two generic:

(a) an almost unlikely event, that the numerical run is successful in the sense defined above and we find the sought accretion flow;

(b) the solution numerically fails—a singularity appears during integration;

(c) there exists a numerical solution without a sonic point—a piece of the solution describes accelerating accreting flow, while the other corresponds to a decelerating accreting gas.

In the case (a) the job is done. If one of the remaining generic alternatives occurs—say (b), with the asymptotic density $\rho_{b)0\infty}$—we change $\rho_{0\infty}$ until we get the situation described in (c) with the asymptotic density $\rho_{c)0\infty}$. Now we use the bisection method in order to find an intermediate value $\rho_{i0\infty}$ between the two values $\rho_{b)0\infty}$ and $\rho_{c)0\infty}$, for which the numerical run is successful and the transonic flow exists.

The numerics itself is standard—we use commonly known algorithms of the 8th order.

The calculations have been performed for a few dozen values—negative and positive—of the cosmological constant Λ. In the sector of negative cosmological constant we take $\Lambda \in (-5 \times 10^{-13}, -10^{-19})$; the absolute value over the bound is larger by 1 order of magnitude than the value found in Lemma 3 as a necessary existence condition. In the positive part of the spectrum we choose $\Lambda \in (10^{-19}, 7.2 \times 10^{-15})$; this is quite close to the value established as the necessary condition in Lemma 2. For each fixed value of Λ we find about 100 solutions corresponding to different values of $\dot{M}$.

The obtained information is summarized in Figs. 1 and 2. The abscissa shows the ratio $x = M_g/m$ of the mass $M_g = 4\pi \int_{R_i}^{R_\infty} drr^2\rho$ of gas to the total mass m. The total mass has been normalized to 1. We obtain, although in a less transparent form, the same result as in Ref. [5]—the maximum

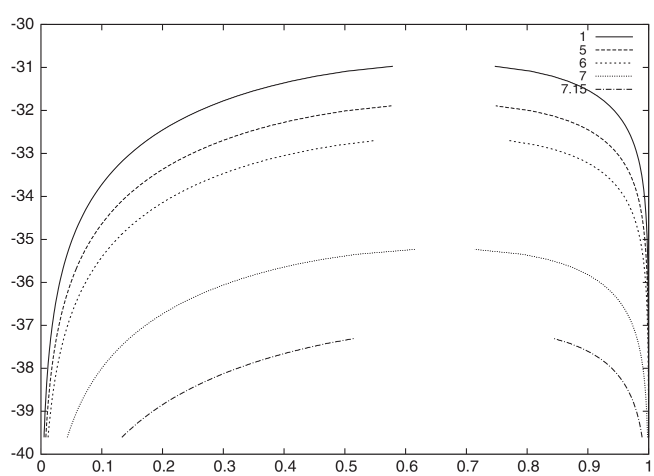

FIG. 1. The ordinate shows the mass accretion rate $\dot{M}$ and the abscissa shows $1 - x$, where x is the relative mass of gas in the system. The various lines correspond to $\Lambda R_\infty^2/10^{-3} = 1, 5, 6, 7, 7.15$ in the order from the top.

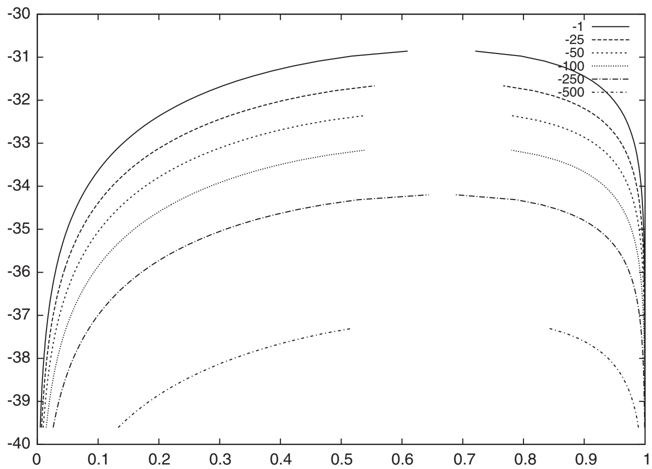

FIG. 2. The ordinate shows the mass accretion rate $\dot{M}$ and the abscissa shows $1 - x$, where x is the relative mass of gas in the system. The various lines correspond to $\Lambda R_\infty^2/10^{-3} = -1, -25, -50, -100, -250, -500$ in the decreasing order from the top.

of the mass accretion rate corresponds to $x \approx 1/3$. For a fixed value of $\dot{M}$ there exist two flows, one with heavier center and lighter gas, and the other with opposite characteristics. Just around the maximum of $\dot{M}$ there are blank points—we find that in this region it is difficult to find a numerical solution. We think that this is a purely numerical artifact. There is a slight shift of this maximum towards smaller values of x with the increase of the absolute value of the cosmological constant.

The only new physically relevant information is that concerning the mass accretion rate $\dot{M}$. It appears that the presence of the cosmological constant manifests only when the dark energy density $\Lambda/(4\pi)$ exceeds the averaged matter density $3m/(4\pi R_\infty^3) \approx 4 \times 10^{-19}$ by at least 3 orders of magnitude. The mass accretion rate $\dot{M}$ is decreasing with the increase of the absolute value of the cosmological constant. It is clear that the rate of the falloff depends on the sign of Λ—it is faster for positive values—but the falloff itself occurs for both positive and negative cosmological constant.

VII. COSMOLOGICAL IMPLICATIONS

Results that are reported in the preceding section demonstrate that while the mass accretion rate depends on the cosmological constant, this effect becomes significant only when the dark energy fraction Ω_Λ is much larger than the material fraction Ω_m. On the other hand, this impact can be dramatic, a sevenfold increase of Λ—starting from its value corresponding to $\Omega_\Lambda/\Omega_m \approx 10^3$—leads to the diminishing of $\dot{M}$ by 7 orders in magnitude, as seen in Fig. 1. This is so for the accretion onto the SdS black hole, but the phenomenon is strong also in the case of SAdS (Fig. 2). When applying these results to our Universe, we have to take into account that $\Omega_\Lambda \gg \Omega_m$ in two epochs—during inflation and after 10^{11} years from the big bang.

Inflation strictly excludes steady accretion onto primordial black holes, due to Lemmas 2 and 3, because during the inflation era Ω_Λ exceeds Ω_m by something like 50 orders of magnitude. This fact is concordant with the well known freezing of structures that are bigger than a particle horizon [19], but the difference is that now it applies to small accretion systems that are located well within the horizon.

In the present Universe the dark energy and material densities are roughly the same, which suggests that the effect of dark energy is negligible. The mass density of largest bound structures—galactic superclusters—exceeds the cosmological mass density by 1 order of magnitude. Notice, however, that the tenfold ageing of the Universe would increase the ratio Ω_Λ/Ω_m by a factor of 100. That means that in the Universe older than 10^{11} years the steady accretion would become less efficient and at the time $t \to 10^{12}$ years its efficiency goes to zero.

Penrose conjectured the so-called Weyl curvature hypothesis [20]. It asserts, in its informal version, that a Friedmann-Lemaître-Robertson-Walker spacetime, where the Ricci curvature is nonzero but the Weyl curvature vanishes, evolves towards a vacuum spacetime filled with black holes and gravitational radiation—with nonvanishing Weyl curvature and negligible Ricci tensor. (A more formal scenario has been worked out by Tod [21].) This picture assumes that initially small inhomogeneities of the FLRW universe accrete matter and transform themselves into black holes, which gradually merge, leaving at the end a net of huge black holes and a lot of gravitational radiation. A cautious interpretation of our findings would be to say that the role of steady accretion in the realization of this scenario, in the presence of dark energy, is insignificant. It might well happen, however, that this property of dark energy of diminishing the steady spherical accretion signals a more general feature—that dark energy damps any accreting process. In such a case there arises another fundamental question: how does dark energy impact this scenario outlined in the Weyl curvature hypothesis?

ACKNOWLEDGMENTS

This research was carried out with the supercomputer Deszno purchased thanks to the financial support of the European Regional Development Fund in the framework of the Polish Innovation Economy Operational Program (Contract No. POIG. 02.01.00-12-023/08).

[1] H. Bondi, Mon. Not. R. Astron. Soc. **112**, 195 (1952).
[2] F. C. Michel, Astrophys. Space Sci. **15**, 153 (1972).
[3] S. Shapiro and S. Teukolsky, *Black Holes, White Dwarfs and Neutron Stars* (Wiley, New York, 1983).
[4] E. Malec, Phys. Rev. D **60**, 104043 (1999).
[5] J. Karkowski, B. Kinasiewicz, P. Mach, E. Malec, and Z. Świerczyński, Phys. Rev. D **73**, 021503(R) (2006).
[6] E. Malec and T. Rembiasz, Phys. Rev. D **82**, 124005 (2010).
[7] P. Mach, Rep. Math. Phys. **64**, 257 (2009).
[8] P. Mach and E. Malec, Phys. Rev. D **78**, 124016 (2008).
[9] P. Mach, Acta Phys. Pol. B **38**, 3935 (2007).
[10] A. Einstein and A. G. Straus, Rev. Mod. Phys. **17**, 120 (1945).
[11] G. Darmois, Mm. Sci. Math. **XXV**, 1 (1927).
[12] W. Israel, Nuovo Cimento B **44**, 1 (1966); **48**, 463 (1967).
[13] S. Weinberg, *Cosmology* (Oxford University Press, Oxford, UK, 2008).
[14] C. Grenon and K. Lake, Phys. Rev. D **84**, 083506 (2011).
[15] N. P. Humphreys, R. Maartens, and D. R. Matravers, Gen. Relativ. Gravit. **44**, 3197 (2012).
[16] D. Giulini and M. Carrera, Rev. Mod. Phys. **82**, 169 (2010).
[17] F. Kottler, Ann. Phys. (Berlin) **361**, 401 (1918); H. Weyl, Phys. Z. **20**, 31 (1919); see also a concise description in V. Perlick, Living Rev. Relativity **7**, 9 (2004).
[18] R. Balbinot, R. Bergamini, and A. Comastri, Phys. Rev. D **38**, 2415 (1988).
[19] V. Mukhanov, *Physical Foundations of Cosmology* (Cambridge University Press, Cambridge, England, 2005).
[20] R. Penrose, *The Road to Reality: A Complete Guide to the Laws of the Universe* (Jonathan Cape, London, 2005).
[21] P. Tod, Classical Quantum Gravity **20**, 521 (2003).