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Rescattering contributions to rare B-meson decays

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Several B and B_s decays have been observed that have been cited as evidence for exchange (E), penguin annihilation (PA), and annihilation (A) processes, such as $\bar{b}d \to \bar{u}u$, $\bar{b}s \to \bar{u}u$, and $\bar{b}u \to W^* \to \bar{c}s$, respectively. These amplitudes are normally thought to be suppressed, as they involve the spectator quark in the weak interaction and thus should be proportional to the B-meson decay constant f_B. However, as pointed out a number of years ago, they can also be generated by rescattering from processes whose amplitudes do not involve f_B, such as color-favored tree amplitudes. In this paper we investigate a number of processes such as $B^0 \to K^+K^-$, $B_s \to \pi^+\pi^-$, and $B^+ \to D_s^+\phi$, and identify promising states from which they can be generated by rescattering. We find that E-and PA-type processes are characterized, respectively, by amplitudes ranging from 5% to 10% and from 15% to 20% with respect to the largest amplitude from which they can rescatter. Based on this regularity, using approximate flavor SU(3) symmetry in some cases and time-reversal invariance in others, we predict the branching fractions for a large number of as-yet-unseen B and B_s decays in an extensive range from order 10^{-9} to 10^{-4}.

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I. INTRODUCTION

The decays of B mesons to two-body final states provide rich data for determining parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which is thought to describe the observed violations of CP symmetry. These processes also yield valuable tests of the SU(3) flavor symmetry obeyed by final-state u, d, and s quarks. Following early SU(3) analyses of B decays [1–3], a hierarchy of invariant amplitudes was established, based on a convenient graphical language [4]. Dominant amplitudes were found to be color-favored tree (T) followed by color-suppressed tree (C) and penguin (P). These three amplitudes involve only the decaying $\bar{b}$ quark in the initial B meson and hence are approximately independent of the light "spectator" quark. Amplitudes considerably suppressed in comparison with them, all of which require participation of the spectator quark, are exchange (E), annihilation (A), and penguin annihilation (PA). All six amplitudes are illustrated in Fig. 1.

As pointed out a number of years ago [5,6], effects of the amplitudes E, A, and PA can also be generated by rescattering from processes whose amplitudes [color-favored tree (T), color-suppressed tree (C), or penguin (P)] do not involve f_B. Since then, both electron-positron and hadron collisions have yielded a wealth of information on many suppressed processes, such as new limits on the branching fraction for $B^0 \to K^+K^-$ [7,8] and observation of the decays $B_s \to \pi^+\pi^-$ [7,9] and $B^+ \to D_s^+\phi$ [10]. In the present paper we study such processes systematically, identifying promising intermediate states contributing to rescattering. We find that the suppressed processes have typical E amplitudes ranging from 5% to 10% of the largest amplitude contributing to rescattering, while PA amplitudes are somewhat larger. Based on this regularity, and using relations based on U-spin or on time reversal, we predict the branching fractions for a large number of as-yet-unseen B and B_s decays.

Calculations of E-, A-, and PA-type amplitudes in QCD factorization are quite challenging. In B decays with one charmed meson in the final state these amplitudes involve

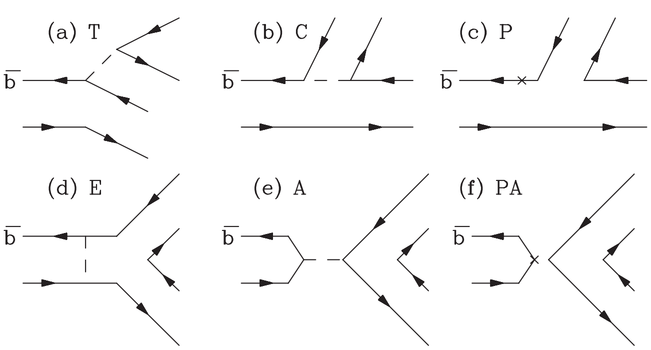

FIG. 1. Graphical representation of invariant amplitudes describing B-meson decays. (a) Color-favored tree (T); (b) color-suppressed tree (C); (c) penguin (P); (d) exchange (E); (e) annihilation (A); (f) penguin annihilation (PA). Dashed lines indicate W exchanges; $\times$ denotes a penguin $\bar{b} \to \bar{d}$ or $\bar{b} \to \bar{s}$ insertion.

unknown matrix elements of nonlocal four-quark operators [11], while $E/A/PA$ amplitudes for charmless decays depend on divergent integrals [12]. References [13–15] and a few references quoted therein have presented model-dependent attempts to calculate E, A, and PA amplitudes within QCD.

In Sec. II we outline our strategy for evaluating rescattering contributions to suppressed E, A, and PA amplitudes. In Sec. III we use current data to obtain ranges of ratios characterizing the suppression of these amplitudes relative to relevant T, C, and P amplitudes. We then apply these ratios in Sec. IV to predict branching ratios for a number of B and B_s decays. Section V highlights predictions based on flavor SU(3) and time-reversal invariance, while Sec. VI concludes.

II. *E*, *A*, AND *PA* AMPLITUDES FROM RESCATTERING

The manner in which a suppressed amplitude is generated by rescattering can be illustrated by some examples. Figure 2(a) depicts the contribution to an exchange (E) amplitude for $B^0 \to K^+K^-$ from the $\rho^+\rho^-$ intermediate state, where the initial amplitude is of the tree (T) form. Figure 2(b) describes a penguin annihilation (PA) amplitude for $B_s \to \pi^+\pi^-$ obtaining a contribution from a K^+K^- intermediate state, where the initial amplitude is of the penguin (P) form. Figure 3 shows the contribution of a $D^{*0}K^{*+}$ intermediate state [initial amplitude of the color-suppressed tree (C) form] to an annihilation (A) amplitude

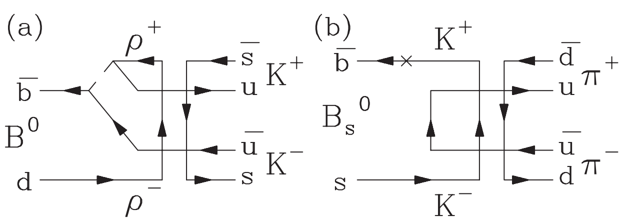

FIG. 2. Rescattering contributions. (a) To $B^0 \to K^+K^-$; initial tree (T) amplitude, $\rho^+\rho^-$ intermediate state contributing to exchange (E) amplitude. (b) To $B_s \to \pi^+\pi^-$; initial penguin (P) amplitude, K^+K^- intermediate state contributing to penguin annihilation (PA) amplitude.

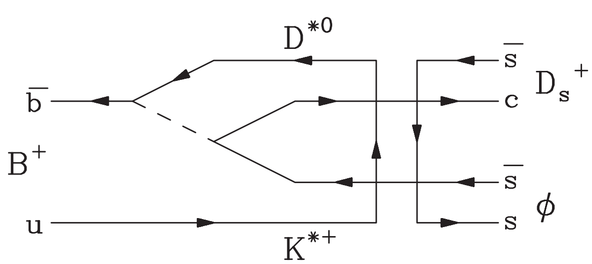

FIG. 3. Rescattering contributions to $B^+ \to D_s^+\phi$ from a $D^{*0}K^{*+}$ intermediate state whose amplitude is of the color-suppressed tree (C) form.

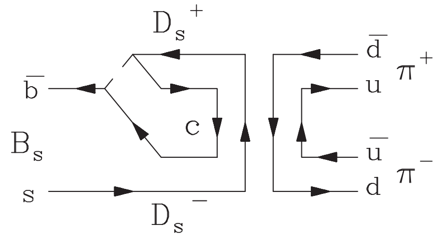

FIG. 4. Rescattering contributions to $B_s \to \pi^+\pi^-$ from a $D_s^+D_s^-$ intermediate state whose amplitude is of the color-favored tree (T) form.

for $B^+ \to D_s^+\phi$. Finally, Fig. 4 shows the contribution of a $D_s^+D_s^-$ intermediate state (from T) to a penguin annihilation (PA) amplitude in $B_s \to \pi^+\pi^-$.

In B decays, whose average multiplicity is quite large, a given final state can be generated by rescattering from any number of intermediate states, many of which have not yet been observed. Even if they were seen, it would not be clear with what relative phases their contributions should be added together. We do expect (quasi-)two-body intermediate states to dominate, because rescattering from three-body or higher-multiplicity states to two-body final states is expected to be greatly suppressed. For instance, while momenta are fixed for decays to two particles, they fill the plane of the Dalitz plot for three-body decays. We assume that rescattering is dominated by light-quark exchange. Rescattering via heavy charm-quark exchange, depicted in Fig. 4, is highly suppressed and will be mentioned briefly at the end of Sec. III.

To circumvent the shortcoming due to having several contributing states, we identify the (quasi-)two-body intermediate state with the largest branching fraction, whose T, C, or P amplitude we compare with the E, A, or PA amplitude of the suppressed process. For several such processes, we find that the ratio $|E/T|$ lies within a narrow range of values between 0.05 and 0.10 while $|PA/P|$ is between 0.15 and 0.20. Finding no experimental evidence for a nonzero $|A/T|$, we will assume that this ratio takes values in the same range as $|E/T|$. The values of these three ratios are then used to predict branching fractions for a large number of the suppressed processes originally identified in Refs. [5,6].

For simplicity we limit our consideration to intermediate states with two pseudoscalar mesons (PP), one pseudoscalar and one vector (PV), and two vector (VV) mesons. The states contributing to PP and PV final states are summarized in Table I.

TABLE I. PP, PV, and VV intermediate states contributing to $B \to PP$, PV decays. Other states are forbidden to contribute by parity conservation in the strong interactions.

Final state	Contributing intermediate state(s)
PP	PP, $(VV)_{L=0,2}$
PV	PV, $(VV)_{L=1}$

TABLE II. E/A-type decays of nonstrange B mesons to two pseudoscalars, and T-type decays to intermediate states contributing to these decays by rescattering. Measured $E/A/PA$ decays (first line or two lines in each subtable for a given CKM factor), along with possible contributing rescattering decays (subsequent lines). The branching ratios for all measured decays are given, along with the value of R found for each rescattering decay, assuming that it is dominant. The ratio of amplitudes probed in each $E/A/PA$ decay is given at the top of the "ratio" column. For entries that are flagged with letters [such as (a)] further details are given in the text.

CKM	Decay	Type	Intermediate state	Branching ratio	Ratio R
$V_{cb}^*V_{ud}$	$B^0 \to D_s^- K^+$	E		$(2.31 \pm 0.24) \times 10^{-5}$ [16]	$\lvert E/T \rvert$
			$D^-\pi^+$	$(2.68 \pm 0.13) \times 10^{-3}$	0.09 ± 0.01
			$D^{*-}\rho^+$	$0.96 \cdot (6.8 \pm 0.9) \times 10^{-3}$ (a)	0.06 ± 0.01
$V_{cb}^*V_{cd}$	$B^0 \to D^0\bar{D}^0$	E		$<4.3 \times 10^{-5}$	$\lvert E/T \rvert$
	$B^0 \to D_s^+ D_s^-$	E		$<3.6 \times 10^{-5}$	$\lvert E/T \rvert$
			D^+D^-	$(2.11 \pm 0.31) \times 10^{-4}$	<0.4
			$D^{*+}D^{*-}$	$(7.0 \pm 0.8) \times 10^{-4}$ (b)	<0.2
$V_{ub}^*V_{ud}$	$B^0 \to K^+K^-$	E		$<2 \times 10^{-7}$ [8]	$\lvert E/T \rvert$
			$\pi^+\pi^-$	$(5.15 \pm 0.22) \times 10^{-6}$	<0.2
			$\rho^+\rho^-$	$(2.42 \pm 0.31) \times 10^{-5}$ (c)	<0.1
$V_{ub}^*V_{cs}$	$B^+ \to D^+K^0$	A		$<2.9 \times 10^{-6}$	$\lvert A/T \rvert$
			$D_s^+\pi^0$ (d)	$(1.6 \pm 0.5) \times 10^{-5}$	<0.3
$V_{ub}^*V_{cd}$	$B^0 \to D_s^+K^-$	E		$\cdots$	$\cdots$
			$D^+\pi^-$	$(7.8 \pm 1.4) \times 10^{-7}$	$\cdots$
$V_{ub}^*V_{cd}$	$B^+ \to D_s^+\bar{K}^0$	A		$<8 \times 10^{-4}$	$\cdots$
			$D^+\pi^0$ (d)	$\cdots$	$\cdots$

III. RELATIVE MAGNITUDE OF SUPPRESSED AMPLITUDES

We begin by reviewing the status of the suppressed decays discussed in Refs. [5,6]. Table II lists the PP decays of nonstrange B mesons with examples of contributing intermediate states. Table III lists the corresponding final and intermediate states for B_s decays. We note four isospin relations between B_s decay amplitudes to charged and neutral mesons,

$$
\begin{aligned}
&A(B_s\to D^+D^-) = -A(B_s\to D^0\bar{D}^0) \ (\Delta I=0),\\
&A(B_s\to \pi^+\pi^-) = -\sqrt{2}A(B_s\to \pi^0\pi^0) \ (\Delta I=0),\\
&A(B_s\to D^+\pi^-) = -\sqrt{2}A(B_s\to D^0\pi^0) \ (\Delta I=1/2),\\
&A(B_s\to D^-\pi^+) = -\sqrt{2}A(B_s\to \bar{D}^0\pi^0) \ (\Delta I=1/2).
\end{aligned} \tag{1}
$$

One can also list a number of nonstrange B decays through E and A amplitudes to PV final states. (No such B_s decays have been reported yet.) These are given in

TABLE III. E/PA-type decays of B_s mesons to two pseudoscalars, and T-type decays to intermediate states contributing to these decays by rescattering. Information organized as in Table II.

CKM	Decay	Type	Intermediate state	Branching ratio	Ratio R
$V_{cb}^*V_{cs}$	$B_s \to D^+D^-$	E		$\cdots$	$\lvert E/T \rvert$
	$B_s \to D^0\bar{D}^0$	E		$\cdots$	$\lvert E/T \rvert$
			$D_s^+D_s^-$	$(5.3 \pm 0.9) \times 10^{-3}$	$\cdots$
			$D_s^{*+}D_s^{*-}$	$(1.60 \pm 0.29) \times 10^{-2}$ (e)	$\cdots$
$V_{cb}^*V_{cs}$	$B_s \to \pi^+\pi^-$	PA		$(0.73 \pm 0.14) \times 10^{-6}$ [16]	$\lvert PA/P \rvert$
	$B_s \to \pi^0\pi^0$	$PA/\sqrt{2}$		$\cdots$	$\lvert PA/P \rvert$
			K^+K^-	$(2.45 \pm 0.18) \times 10^{-5}$ [16]	0.17 ± 0.02
			$K^{*0}\bar{K}^{*0}$	$(1.7 \pm 0.5) \times 10^{-5}$ (f)	0.21 ± 0.04
$V_{cb}^*V_{us}$	$B_s \to D^-\pi^+$	E		$\cdots$	$\lvert E/T \rvert$
	$B_s \to \bar{D}^0\pi^0$	$E/\sqrt{2}$		$\cdots$	$\lvert E/T \rvert$
			$D_s^-K^+$	(g)	$\cdots$
$V_{ub}^*V_{cs}$	$B_s \to D^+\pi^-$	E		$\cdots$	$\lvert E/T \rvert$
	$B_s \to D^0\pi^0$	$E/\sqrt{2}$		$\cdots$	$\lvert E/T \rvert$
			$D_s^+K^-$	(g)	$\cdots$
$V_{cb}^*V_{cs}$	$B_s \to \pi^+\pi^-$	PA		$(0.73 \pm 0.14) \times 10^{-6}$ [16]	$\lvert PA/T \rvert$
	$B_s \to \pi^0\pi^0$	$PA/\sqrt{2}$		$\cdots$	$\lvert PA/T \rvert$
			$D_s^+D_s^-$	$(5.3 \pm 0.9) \times 10^{-3}$	0.012 ± 0.002
			$D_s^{*+}D_s^{*-}$	$(1.60 \pm 0.29) \times 10^{-2}$ (e)	0.007 ± 0.002

Table IV with examples of contributing nonsuppressed intermediate states. All branching ratios quoted in Tables II, III, and IV are taken from the Particle Data Group [17], unless otherwise indicated. Finally, one can consider suppressed $B \to VV$ decays by replacing both pseudoscalars in Tables II and III by vector mesons.

As noted in the Introduction, rescattering can occur via many intermediate states. We can identify at most a few of them, but there will always be one with the largest branching fraction. We can use that one to calculate a "typical" ratio of the suppressed amplitude to the largest unsuppressed one. We then have to assume that the effect of many intermediate states (whether constructive, incoherent, or destructive) is roughly the same for all cases. With this in mind, we calculate the amplitude ratio for all measured $E/A/PA$-type decays, assuming a single intermediate state, that with the largest branching fraction.

Amplitudes are evaluated as square roots of branching fractions, with phase-space differences ignored. We consider only suppressed amplitudes and amplitudes for intermediate states that share the same CKM factor. Thus, for instance, in the amplitude E for $B^0 \to K^+K^-$ involving $V^*_{ub}V_{ud}$ we ignore a rescattering contribution from $B^0 \to D^+D^-$ involving $V^*_{cb}V_{cd}$. For the amplitude ratio (i.e., the rescattering suppression factor), for which CKM factors associated with the decays in the numerator and denominator cancel, we use the symbol $R \equiv |[E/A/PA]/[T/C/P]|$.

For $B \to PP$ $E/A/PA$-type decays, we give the PP and VV intermediate states; for $B \to PV/VP$ $E/A/PA$ decays, we give the PV, VP, and VV intermediate states. Now, for the VV intermediate states, Ref. [17] gives the branching ratio for the decay to all three helicity states. However, only two (one) of these—those with positive (negative) parity—contribute to rescattering to PP (PV/VP) final states. Thus, the effective rescattering branching ratio is probably smaller than that given in the tables, and the value of R is larger. For many $B \to VV$ decays, the polarization fractions have been measured. This allows us to modify the total branching ratios appropriately, which we do where possible.

In Tables II, III, and IV we list all measured $E/A/PA$-type decays, along with the value of R obtained from individual decays into intermediate rescattering states. Some of the quoted branching ratios of the latter processes require some details, which we give now.

(a) The helicity amplitudes for $B^0 \to D^{*-}\rho^+$ were measured in Ref. [18], with the result $|H_0| = 0.941$, $|H_\parallel| = 0.27$, $|H_\perp| = 0.21$. The fraction of

TABLE IV. E/A-type decays of nonstrange B mesons to PV final states, and T- or C-type decays to intermediate states contributing to these decays by rescattering. Information organized as in Table II.

CKM	Decay	Type	Intermediate state	Branching ratio	Ratio R
$V^*_{cb}V_{ud}$	$B^0 \to D_s^{*-}K^+$	E		$(2.19 \pm 0.30) \times 10^{-5}$	$\lvert E/T\rvert$
	$B^0 \to D_s^- K^{*+}$	E		$(3.5 \pm 1.0) \times 10^{-5}$	$\lvert E/T\rvert$
			$D^{*-}\pi^+$	$(2.76 \pm 0.13) \times 10^{-3}$	0.09 ± 0.01
			$D^-\rho^+$	$(7.8 \pm 1.3) \times 10^{-3}$	0.05 ± 0.01
			$D^{*-}\rho^+$	$(2.7 \pm 0.4) \times 10^{-4}$ (h)	
$V^*_{cb}V_{cd}$	$B^0 \to D^{*0}\bar{D}^0, D^0\bar{D}^{*0}$	E		$<2.9 \times 10^{-4}$	$\lvert E/T\rvert$
	$B^0 \to D_s^\pm D_s^{*\mp}$	E		$<1.3 \times 10^{-4}$	$\lvert E/T\rvert$
			$D^{*+}D^-$	$(6.1 \pm 1.5) \times 10^{-4}$	<0.5
$V^*_{ub}V_{ud}$	$B^0 \to K^{*\pm}K^\mp$	E		$\cdots$	$\lvert E/T\rvert$
			$\rho^\pm\pi^\mp$	$(2.30 \pm 0.23) \times 10^{-5}$	$\cdots$
$V^*_{ub}V_{cs}$	$B^+ \to D^+K^{*0}$	A		$<1.8 \times 10^{-6}$ [10]	$\lvert A/T(C)\rvert$
	$B^+ \to D^{*+}K^0$	A		$<9.0 \times 10^{-6}$	$\lvert A/T(C)\rvert$
			$D_s^{*+}\pi^0(T)$	$<2.6 \times 10^{-4}$	$\cdots$
			$D_s^+\rho^0(T)$	$<3.0 \times 10^{-4}$	$\cdots$
			$D^0K^{*+}(C)$	$\sim 1 \times 10^{-5}$ (i,j)	<0.4
$V^*_{ub}V_{cs}$	$B^+ \to D_s^+\phi$	A		$(1.87^{+1.30}_{-0.82}) \times 10^{-6}$ [10]	$\lvert A/T(C)\rvert$
			$D^0K^{*+}(C)$	$\sim 1 \times 10^{-5}$ (i,j)	$\sim 0.4 \pm 0.1$
			$D_s^+\omega(T)$	$<4 \times 10^{-4}$ (k)	(l)
$V^*_{ub}V_{cd}$	$B^0 \to K^{*-}D_s^+$	E		$\cdots$	$\lvert E/T\rvert$
	$B^0 \to K^-D_s^{*+}$	E		$\cdots$	$\lvert E/T\rvert$
			$D^+\rho^-$	$\cdots$	$\cdots$
			$D^{*+}\pi^-$	$\cdots$	$\cdots$
$V^*_{ub}V_{cd}$	$B^+ \to \bar{K}^{*0}D_s^+$	A		$<4.4 \times 10^{-6}$ [10]	$\lvert A/T\rvert$
	$B^+ \to \bar{K}^0D_s^{*+}$	A		$\cdots$	$\lvert A/T\rvert$
			$D^+\rho^0$	$\cdots$	$\cdots$
			$D^{*+}\pi^0$	$\cdots$	$\cdots$

decays with positive parity is thus $f_+ = (|H_0|^2 + |H_\parallel|^2)/(|H_0|^2 + |H_\parallel|^2 + |H_\perp|^2) = 0.96$. This indicates that the rescattering of $B^0 \to D^{*-}\rho^+$ contributes significantly to $B^0 \to D_s^- K^+$. On the other hand, the fraction of decays with negative parity is 0.04, so that there is little rescattering contribution to $B^0 \to D_s^- K^{*+}$.

(b) The fraction of $B^0 \to D^{*+}D^{*-}$ decays with positive parity is $f_+ = 0.850 \pm 0.025$ [17]. We quote $f_+\mathcal{B}(B^0 \to D^{*+}D^{*-}) = (0.850 \pm 0.025)(8.2 \pm 0.9) \times 10^{-3} = (7.0 \pm 0.8) \times 10^{-3}$.

(c) $B^0 \to \rho^+\rho^-$ is dominated by longitudinal polarizations, $f_L = |H_0|^2/(|H_0|^2 + |H_\parallel|^2 + |H_\perp|^2) = 0.977^{+0.028}_{-0.024}$ [17].

(d) Decay amplitude is given by $T/\sqrt{2}$.

(e) We are assuming that the fraction of $B_s \to D_s^{*+}D_s^{*-}$ decays with positive parity is the same as in $B^0 \to D^{*+}D^{*-}$. This assumption is supported by a calculation based on the heavy-quark expansion and factorization [19]. We quote $f_+\mathcal{B}(B_s \to D_s^{*+}D_s^{*-}) = (0.850 \pm 0.025)(1.88 \pm 0.34) \times 10^{-2} = (1.60 \pm 0.29) \times 10^{-2}$.

(f) The helicity amplitudes for $B_s \to K^{*0}\bar{K}^{*0}$ were measured in Ref. [20], leading to $f_+ = 0.62 \pm 0.12$. We quote $f_+\mathcal{B}(B_s \to K^{*0}\bar{K}^{*0}) = (0.62 \pm 0.12)(2.8 \pm 0.7) \times 10^{-5} = (1.7 \pm 0.5) \times 10^{-5}$, $f_-\mathcal{B}(B_s \to K^{*0}\bar{K}^{*0}) = (1.1 \pm 0.4) \times 10^{-5}$.

(g) Using untagged B_s decays only the charge-averaged branching ratio has been measured, $\mathcal{B}(B_s \to D_s^\pm K^\mp) = (2.9 \pm 0.6) \times 10^{-4}$ [17].

(h) We quote $f_-\mathcal{B}(B^0 \to D^{*-}\rho^+) = 0.04 \cdot (6.8 \pm 0.9) \times 10^{-3} = (2.7 \pm 0.4) \times 10^{-4}$.

(i) The decays $B^+ \to D^{(*)0}K^{(*)+}$ have not been measured, but the decays $B^+ \to \bar{D}^{(*)0}K^{(*)+}$ have: $B(B^+ \to \bar{D}^0 K^{*+}) = (5.3 \pm 0.4) \times 10^{-4}$, $B(B^+ \to \bar{D}^{*0}K^+) = (4.20 \pm 0.34) \times 10^{-4}$, $B(B^+ \to \bar{D}^{*0}K^{*+}) = (8.1 \pm 1.4) \times 10^{-4}$. *BABAR* [21] has found that $r_B \equiv |A(B^+ \to D^0 K^{*+})|/|A(B^+ \to \bar{D}^0 K^{*+})| = 0.31 \pm 0.07$. This gives $B(B^+ \to D^0 K^{*+}) = (0.31 \pm 0.07)^2 \cdot (5.3 \pm 0.4) \times 10^{-4} = (5.1 \pm 2.3) \times 10^{-5}$.

(j) The isospin triangle relation, $A(B^0 \to D^0 K^{*0}) = A(B^+ \to D^0 K^{*+}) + A(B^+ \to D^+ K^{*0})$, shown in Ref. [22] implies that r_B is smaller by at least one σ than its above-mentioned central value. With the experimental limits [17] $B(B^0 \to D^0 K^{*0}) < 1.1 \times 10^{-5}$ and $B(B^+ \to D^+ K^{*0}) < 1.8 \times 10^{-6}$ [10], we have (in units of 10^{-3}) $|A(B^0 \to D^0 K^{*0})| < 3.3$ and $|A(B^+ \to D^+ K^{*0})| < 1.3$. But taking $B(B^+ \to D^0 K^{*+}) = (5.1 \pm 2.3) \times 10^{-5}$ yields $|A(B^+ \to D^0 K^{*+})| = 7.1 \pm 1.6$; for the central value of this last branching ratio, the triangle does not close. It closes only if the branching ratio for $B^+ \to D^0 K^{*+}$ is at least 1.5σ below its central value. This, and independent supporting evidence discussed in the next point below, suggest that a likely value of $\mathcal{B}(B^+ \to D^0 K^{*+})$ is around 1×10^{-5}, corresponding to $r_B \simeq 0.15$. This value is consistent with a value $r_B = 0.115 \pm 0.045$ obtained in Ref. [23] by a global fit to CKM parameters.

(k) A potential rescattering state contributing to $B^+ \to D_s^+\phi$ is $D_s^+\omega$, for which one has a rather old upper bound $\mathcal{B}(B^+ \to D_s^+\omega) < 4 \times 10^{-4}$ [24]. An order of magnitude stronger upper bound, $\mathcal{B}(B^+ \to D_s^+\omega) \lesssim 1.2 \times 10^{-5}$, is obtained if one assumes $\mathcal{B}(B^+ \to D_s^+\omega) \simeq \mathcal{B}(B^+ \to D_s^+\rho^0)$ using an isospin relation, $\mathcal{B}(B^+ \to D_s^+\rho^0) = \mathcal{B}(B^0 \to D_s^+\rho^-)/2 < 1.2 \times 10^{-5}$ [17]. We note that while $B^+ \to D^0 K^{*+}$ involves a color-suppressed amplitude C, $B^+ \to D_s^+\omega$ involves a color-favored tree amplitude $T/\sqrt{2}$, which is usually expected to be larger than C. Recalling our discussion of $B^+ \to D^0 K^{*+}$ in point (j) we are led to conclude that both $\mathcal{B}(B^+ \to D^0 K^{*+})$ and $\mathcal{B}(B^+ \to D_s^+\omega)$ are most likely around 1×10^{-5}. Improved measurements of $\mathcal{B}(B^+ \to D_s^+\omega)$ and $\mathcal{B}(B^+ \to D_s^+\rho^0)$ (a potential dominant rescattering contributor to $B^+ \to D^+ K^{*0}$ and $B^+ \to D^{*+}K^0$), using the *BABAR*, Belle, and LHCb high statistics data, would be of great importance.

(l) The rescattering contribution of $B^+ \to D_s^+\omega$ to $B^+ \to D_s^+\phi$ from ω-ϕ mixing is Okubo-Zweig-Iizuka suppressed [25]. It is given by $\mathcal{B}(B^+ \to D_s^+\phi)_{\omega\text{-}\phi} = \mathcal{B}(B^+ \to D_s^+\omega)\delta^2$. Here δ is the ω-ϕ mixing angle, $\delta = -3.34°$ or $\delta(m = m_\phi) = -4.64°$ in mass-independent or mass-dependent analyses [26]. Assuming $\mathcal{B}(B^+ \to D_s^+\omega) \sim 1 \times 10^{-5}$ as argued above and taking a mass-dependent δ, one finds $\mathcal{B}(B^+ \to D_s^+\phi)_{\omega\text{-}\phi} \sim 0.7 \times 10^{-7}$. This is only a tiny fraction of the measured value of $\mathcal{B}(B^+ \to D_s^+\phi)$.

The information on ratios R given in the last columns of Tables II, III, and IV can be summarized as follows:

(i) The ratio $|E/T|$, obtained from $\mathcal{B}(B^0 \to D_s^- K^+)$, $\mathcal{B}(B^0 \to D^-\pi^+)$, and all their VP analogues, lies in the narrow range $|E/T| = 0.05$–0.1. This range describes well contributions of rescattering in $B^0 \to D^-\pi^+$, $D^{*-}\rho^+ \to D_s^- K^+$ and $B^0 \to D^{*-}\pi^+$, $D^-\rho^+ \to D_s^{*-}K^+$, $D_s^- K^{*+}$. The different angular momenta involved in these decays do not seem to affect much the value of R. A number of other decay modes involving $(d\bar{d}) \to (s\bar{s})$ rescattering are expected to have values of R in the same range.

(ii) The ratio $|A/T|$ cannot be extracted from $\mathcal{B}(B^+ \to D_s^+\phi)$ and $\mathcal{B}(B^+ \to D_s^+\omega)$ because rescattering from $D_s^+\omega$ to $D_s^+\phi$ is Okubo-Zweig-Iizuka suppressed. This seems like a singular case, in which we are unable to identify a dominant intermediate state contributing to rescattering. A less likely interpretation for the branching ratio of $B^+ \to D_s^+\phi$ is that physics beyond the CKM framework is at work.

TABLE V. Predictions for branching ratios of B and B_s decays to two pseudoscalar mesons. $E/A/PA$ decays appear in the first line or two lines in each subtable, while corresponding rescattering decay with the largest branching ratio is given in the last line of each subtable. Entries in the last subtable refer to CP-averaged branching ratios.

CKM factor	Decay	Measured branching ratio	Predicted branching ratio
$V_{cb}^* V_{cd}$	$B^0 \to D^0 \bar{D}^0$	$<4.3 \times 10^{-5}$	$(3.4 \pm 2.0) \times 10^{-6}$
	$B^0 \to D_s^+ D_s^-$	$<3.6 \times 10^{-5}$	$(3.4 \pm 2.0) \times 10^{-6}$
	$B^0 \to D^{*+} D^{*-}$	$(7.0 \pm 0.8) \times 10^{-4}$	
$V_{ub}^* V_{ud}$	$B^0 \to K^+ K^-$	$<2 \times 10^{-7}$	$(1.2 \pm 0.7) \times 10^{-7}$
	$B^0 \to \rho^+ \rho^-$	$(2.42 \pm 0.31) \times 10^{-5}$	
$V_{ub}^* V_{cd}$	$B^0 \to D_s^+ K^-$	$\cdots$	$(3.8 \pm 2.3) \times 10^{-9}$
	$B^0 \to D^+ \pi^-$	$(7.8 \pm 1.4) \times 10^{-7}$	
$V_{ub}^* V_{cs}$	$B^+ \to D^+ K^0$	$<2.9 \times 10^{-6}$	$(1.6 \pm 1.0) \times 10^{-7}$
	$B^+ \to D_s^+ \pi^0$	$(1.6 \pm 0.5) \times 10^{-5}$	
$V_{cb}^* V_{cs}$	$B_s \to D^+ D^-$	$\cdots$	$(7.8 \pm 4.7) \times 10^{-5}$
	$B_s \to D^0 \bar{D}^0$	$\cdots$	$(7.8 \pm 4.7) \times 10^{-5}$
	$B_s \to D_s^{*+} D_s^{*-}$	$(1.60 \pm 0.29) \times 10^{-2}$	
$V_{cb}^* V_{us}$, $V_{ub}^* V_{cs}$	$B_s \to D^{\pm} \pi^{\mp}$	$\cdots$	$(1.4 \pm 0.9) \times 10^{-6}$
	$B_s \to D^0 \pi^0$, $\bar{D}^0 \pi^0$	$\cdots$	$(0.7 \pm 0.4) \times 10^{-6}$
	$B_s \to D_s^{\pm} K^{\mp}$	$(2.9 \pm 0.6) \times 10^{-4}$	

(iii) The value of $|PA/P|$, obtained from $\mathcal{B}(B_s \to \pi^+ \pi^-)$ and $\mathcal{B}(B_s \to K^+ K^-)$, is near 0.2, about twice the value of $|E/T|$. In the last subtable in Table III we also obtain a value for a ratio $|PA/T|$, where T is a color-favored tree amplitude determined by $\mathcal{B}(B_s \to D_s^{(*)+} D_s^{(*)-})$. This very small ratio of order 0.01, corresponding to $D_s^{(*)+} D_s^{(*)-} \to \pi^+ \pi^-$ rescattering, is suppressed by requiring two quark-antiquark rescatterings as shown in Fig. 4. Some portion of the suppression may be because of the exchange of the heavy charm quark in rescattering.

IV. PREDICTIONS BASED ON RANGES OF R

With the above-mentioned ranges of R we can now predict the branching ratios for other $E/A/PA$ decays. We will use the value $|E/T| = 0.07 \pm 0.02$ and will *assume* the same range for $|A/T|$ in cases where one may identify a potentially dominant T-type decay contributing by rescattering to an A-type decay. Finally, the value $|PA/P| = 0.17 \pm 0.02$ extracted from $\mathcal{B}(B_s \to \pi^+ \pi^-)$ and $\mathcal{B}(B_s \to K^+ K^-)$ will be used to predict branching ratios for B_s decays into other pairs of unflavored mesons. The central values and uncertainties in the three ratios are chosen to describe ranges for these parameters. Thus the errors in predicted branching ratios, obtained by adding in quadrature these uncertainties and experimental errors in branching ratios, are not statistical. Rather, under our assumptions, they give reasonable ranges for a large number of branching ratios of decay modes that have not yet been observed.

Using the above values for the ratios $|E/T|$, $|A/T|$, and $|PA/P|$, we obtain predictions for B and B_s decay

TABLE VI. Predictions for branching ratios of B and B_s decays to vector and pseudoscalar mesons organized as in Table V. Entries in all but the second subtable refer to CP-averaged branching ratios.

CKM factor	Decay	Measured branching ratio	Predicted branching ratio
$V_{cb}^* V_{cs}$	$B_s \to D^{*\pm} D^{\mp}$	$\cdots$	$(6.1 \pm 3.6) \times 10^{-5}$
	$B_s \to D^{*0} \bar{D}^0$, $D^0 \bar{D}^{*0}$	$\cdots$	$(6.1 \pm 3.6) \times 10^{-5}$
	$B_s \to D_s^{*\pm} D_s^{\mp}$	$(1.24 \pm 0.21) \times 10^{-2}$	
$V_{cb}^* V_{cs}$	$B_s \to \rho^+ \pi^-$	$\cdots$	$(3.1 \pm 1.4) \times 10^{-7}$
	$B_s \to \rho^- \pi^+$	$\cdots$	$(3.1 \pm 1.4) \times 10^{-7}$
	$B_s \to K^{*0} \bar{K}^{*0}$	$(1.1 \pm 0.4) \times 10^{-5}$ (f)	
$V_{cb}^* V_{cd}$	$B^0 \to D_s^{*\pm} D_s^{\mp}$	$<1.3 \times 10^{-4}$	$(3.0 \pm 1.9) \times 10^{-6}$
	$B^0 \to D^{*0} \bar{D}^0$, $D^0 \bar{D}^{*0}$	$<2.9 \times 10^{-4}$	$(3.0 \pm 1.9) \times 10^{-6}$
	$B^0 \to D^{*\pm} D^{\mp}$	$(6.1 \pm 1.5) \times 10^{-4}$	
$V_{ub}^* V_{ud}$	$B^0 \to K^{*\pm} K^{\mp}$	$\cdots$	$(1.1 \pm 0.7) \times 10^{-7}$
	$B^0 \to \rho^{\pm} \pi^{\mp}$	$(2.30 \pm 0.23) \times 10^{-5}$	

branching ratios. Results for $B, B_s \to PP$ and $B, B_s \to VP$ are presented in Tables V and VI, respectively. Predictions appear in the first one or two lines in each subtable, while the last line in each subtable quotes the corresponding largest measured branching ratio for a process of type T or P. Entries in the last subtable of Table V and in all but the second subtable in Table VI refer to CP-averaged branching ratios that are measured using untagged B^0 and B_s decays. Our prediction for $\mathcal{B}(B^+ \to D^+ K^0)$ in Table V can test our assumption $|A/T| = 0.07 \pm 0.02$.

V. PREDICTIONS BASED ON FLAVOR SU(3) OR TIME REVERSAL

In Eqs. (1) we have presented four isospin relations in pairs of E-and PA-type B_s decay amplitudes, leading to relations between corresponding decay branching ratios. Other relations among E-and A-type B and B_s decay amplitudes follow in the limit of flavor SU(3) symmetry. Two subgroups of SU(3), U-spin and V-spin of which (d, s) and (u, s) are fundamental doublet representations, are useful in deriving these relations. We will focus our attention on relations for decays into two pseudoscalar mesons, discussing in certain cases also relations for $B, B_s \to VP$ and $B, B_s \to VV$.

In the V-spin symmetry limit, applying $u \leftrightarrow s$ reflection, one has

$$A(B^0 \to D_s^+ D_s^-) = A(B^0 \to D^0 \bar{D}^0), \tag{2}$$

as assumed in Tables II and V. Thus, in the V-spin symmetry limit the two corresponding branching ratios are predicted to be equal.

Using approximate symmetry of strong interactions under U-spin reflection, $d \leftrightarrow s$, and considering the U-spin structure of the effective weak Hamiltonian and of initial and final states, we find

$$\begin{aligned} A(B_s \to D^- \pi^+) &= \lambda A(B^0 \to D_s^- K^+), \\ -\lambda A(B_s \to D^+ D^-) &= A(B^0 \to D_s^+ D_s^-), \\ -\lambda A(B_s \to D^+ \pi^-) &= A(B^0 \to D_s^+ K^-), \end{aligned} \tag{3}$$

and

$$A(B^+ \to D_s^+ \bar{K}^0) = -\lambda A(B^+ \to D^+ K^0). \tag{4}$$

Here $\lambda \equiv V_{us}/V_{ud} \approx -V_{cd}/V_{cs} = 0.231$ [17]. Given the value of $\mathcal{B}(B^0 \to D_s^- K^+)$ in Table II, the first of Eqs. (3) leads to predicting $\mathcal{B}(B_s \to D^- \pi^+)$ in the U-spin symmetry limit,

$$\mathcal{B}(B_s \to D^- \pi^+) \simeq (1.23 \pm 0.13) \times 10^{-6}. \tag{5}$$

This value is in agreement with the prediction for the CP-averaged branching ratio quoted in Table V, which involves a larger uncertainty. In a similar manner one has U-spin relations for corresponding $B, B_s \to VP$ decays, such as

$$\begin{aligned} A(B_s \to D^{*-} \pi^+) &= \lambda A(B^0 \to D_s^{*-} K^+), \\ A(B_s \to D^- \rho^+) &= \lambda A(B^0 \to D_s^- K^{*+}). \end{aligned} \tag{6}$$

Taking branching ratios quoted in Table IV, we obtain

$$\begin{aligned} \mathcal{B}(B_s \to D^{*-} \pi^+) &= (1.2 \pm 0.2) \times 10^{-6}, \\ \mathcal{B}(B_s \to D^- \rho^+) &= (1.9 \pm 0.5) \times 10^{-6}. \end{aligned} \tag{7}$$

These predictions add to those already given in Table VI.

We will now show that the predictions obtained in Sec. IV, assuming a dominant rescattering contribution in E-type decays, are consistent with U-spin relations such as Eqs. (3) and (6). We will use the fact that final states on the left-hand side of these equations are rescattering states contributing to corresponding amplitudes on the right-hand side, while final states on the right-hand side contribute as rescattering states to amplitudes on the left-hand side.

Let us focus, for instance, on the first U-spin relation in Eqs. (3) between two E-type amplitudes. We will show now that this relation may be derived using our assumption of dominant rescattering states for which two respective T-type amplitudes are related to each other by U-spin. We are assuming that $B^0 \to D_s^- K^+$ is dominated by a positive-parity $D^{*-} \rho^+$ rescattering state,

$$\begin{aligned} |A(B^0 \to D_s^- K^+)| = {} & |A(B^0 \to [D^{*-} \rho^+]_+)| \\ & \times |A([D^{*-} \rho^+]_+ \to D_s^- K^+)|, \end{aligned} \tag{8}$$

where

$$\begin{aligned} & |A(B^0 \to [D^{*-} \rho^+]_+)| \\ & \quad \equiv \sqrt{|A(B^0 \to [D^{*-} \rho^+]_0)|^2 + |A(B^0 \to [D^{*-} \rho^+]_\parallel)|^2}. \end{aligned} \tag{9}$$

Similarly one obtains

$$\begin{aligned} |A(B_s \to D^- \pi^+)| = {} & |A(B_s \to [D_s^{*-} K^{*+}]_+)| \\ & \times |A([D_s^{*-} K^{*+}]_+ \to D^- \pi^+)|, \end{aligned} \tag{10}$$

where dominance of $B_s \to [D_s^{*-} K^{*+}]_+$ over $B_s \to D_s^- K^+$ is implied by $|A(B^0 \to [D^{*-} \rho^+]_+)| > |A(B^0 \to D^- \pi^+)|$ and U-spin symmetry.

Assuming that the rescattering amplitude is invariant under U-spin, $A([D_s^{*-} K^{*+}]_+ \to D^- \pi^+) = A([D^{*-} \rho^+]_+ \to D_s^- K^+)$, we obtain the first of Eqs. (3) as required,

$$\frac{|A(B_s \to D^- \pi^+)|}{|A(B^0 \to D_s^- K^+)|} = \frac{|A(B_s \to [D_s^{*-} K^{*+}]_+)|}{|A(B^0 \to [D^{*-} \rho^+]_+)|} = \lambda. \tag{11}$$

The second equality, giving the ratio of two positive parity T-type amplitudes, follows from the behavior under U-spin reflection of the effective weak Hamiltonian and of initial and final states.

At this point we wish to comment on the definition of the magnitude of the effective rescattering amplitude for positive parity, $|A([D^{*-} \rho^+]_+ \to D_s^- K^+)|$ in (8), which we have defined in Table II as the ratio $|E/T| = 0.06 \pm 0.01$ in $B^0 \to D_s^- K^+$. Equation (8) may be expanded,

$$\begin{aligned} A(B^0 \to D_s^- K^+) = {} & A(B^0 \to [D^{*-}\rho^+]_0) \\ & \times A([D^{*-}\rho^+]_0 \to D_s^- K^+) \\ & + A(B^0 \to [D^{*-}\rho^+]_{\|}) \\ & \times A([D^{*-}\rho^+]_{\|} \to D_s^- K^+), \end{aligned} \tag{12}$$

where $[D^{*-}\rho^+]_{0,\|}$ are longitudinal and parallel polarization states, and $A([D^{*-}\rho^+]_{0,\|} \to D_s^- K^+)$ are corresponding strong interaction rescattering amplitudes. The $B^0 \to D_s^- K^+$ decay rate is obtained by squaring the above sum and integrating over the angular dependence of the two pairs of final pseudoscalars, $\bar{D}^0\pi^-$ (or $D^-\pi^0$) and $\pi^+\pi^0$. The interference term drops out by integration implying (we omit phase-space factors),

$$\begin{aligned} |A(B^0 \to D_s^- K^+)|^2 = {} & |A(B^0 \to [D^{*-}\rho^+]_0)|^2 \\ & \times |A([D^{*-}\rho^+]_0 \to D_s^- K^+)|^2 \\ & + |A(B^0 \to [D^{*-}\rho^+]_{\|})|^2 \\ & \times |A([D^{*-}\rho^+]_{\|} \to D_s^- K^+)|^2. \end{aligned} \tag{13}$$

Comparing this expression for $|A(B^0 \to D_s^- K^+)|^2$ with that given in (8) and (9), we find

$$\begin{aligned} & |A([D^{*-}\rho^+]_+ \to D_s^- K^+)|^2 \\ & \quad = g_0 |A([D^{*-}\rho^+]_0 \to D_s^- K^+)|^2 \\ & \qquad + g_{\|} |A([D^{*-}\rho^+]_{\|} \to D_s^- K^+)|^2, \end{aligned} \tag{14}$$

where $g_0 = 0.924$ and $g_{\|} = 0.076$ are longitudinal and parallel fractions of $B^0 \to D^{*-}\rho^+$ decays relative to decays with positive parity. [See comment (a) above.] That is, the effective rescattering probability for positive parity is given by a weighted average of the two rescattering probabilities for longitudinal and parallel helicity states.

To conclude this section, let us show that using merely time-reversal invariance and assuming a dominant intermediate state for rescattering permits predicting a ratio of E- and T-type amplitudes for one pair of processes in terms of a similar (sometimes given) ratio of another pair of processes. Applying relations similar to (8) and (10) to VV amplitudes for a given helicity h, one has

$$\begin{aligned} A(B^0 \to [D_s^{*-} K^{*+}]_h) = {} & A(B^0 \to [D^{*-}\rho^+]_h) \\ & \times A([D^{*-}\rho^+]_h \to [D_s^{*-} K^{*+}]_h), \\ A(B_s \to [D^{*-}\rho^+]_h) = {} & A(B_s \to [D_s^{*-} K^{*+}]_h) \\ & \times A([D_s^{*-} K^{*+}]_h \to [D^{*-}\rho^+]_h). \end{aligned} \tag{15}$$

Using time-reversal invariance (neglecting the small B_s-B^0 mass difference),

$$A([D_s^{*-} K^{*+}]_h \to [D^{*-}\rho^+)]_h) = A([D^{*-}\rho^+]_h \to [D_s^{*-} K^{*+}]_h), \tag{16}$$

one obtains

$$\frac{A(B_s \to [D^{*-}\rho^+]_h)}{A(B_s \to [D_s^{*-} K^{*+}]_h)} = \frac{A(B^0 \to [D_s^{*-} K^{*+}]_h)}{A(B^0 \to [D^{*-}\rho^+]_h)}. \tag{17}$$

Thus a similar relation holds also for ratios of square roots of total branching ratios,

$$\sqrt{\frac{\mathcal{B}(B_s \to D^{*-}\rho^+)}{\mathcal{B}(B_s \to D_s^{*-} K^{*+})}} = \sqrt{\frac{\mathcal{B}(B^0 \to D_s^{*-} K^{*+})}{\mathcal{B}(B^0 \to D^{*-}\rho^+)}} = 0.07^{+0.02}_{-0.01}. \tag{18}$$

Here we have used $\mathcal{B}(B^0 \to D_s^{*-} K^{*+}) = (3.2^{+1.5}_{-1.3}) \times 10^{-5}$ [17] and the value of $\mathcal{B}(B^0 \to D^{*-}\rho^+)$ quoted in Table II for the sum of positive and negative parity states. The two ratios of amplitudes in (18), corresponding to values of $|E/T|$ not discussed earlier in our study, lie precisely in the range of $|E/T|$ assumed for all our other predictions.

The relations (11) and (18) have been derived for PP and VV final states belonging to a class of the pair $(D^-\pi^+, D_s^- K^+)$ appearing in the first of Eqs. (3). Similar amplitude relations can be derived for PP, VP, and VV final states belonging to classes of states appearing in the other two equations. For instance, the rescattering relations,

$$\begin{aligned} A(B^0 \to D_s^{*+} D_s^-) = {} & A(B^0 \to D^{*+} D^-) \\ & \times A(D^{*+} D^- \to D_s^{*+} D_s^-), \\ A(B_s \to D^{*+} D^-) = {} & A(B_s \to D_s^{*+} D_s^-) \\ & \times A(D_s^{*+} D_s^- \to D^{*+} D^-), \end{aligned} \tag{19}$$

and time-reversal invariance,

$$A(D^{*+} D^- \to D_s^{*+} D_s^-) = A(D_s^{*+} D_s^- \to D^{*+} D^-), \tag{20}$$

imply

$$\frac{A(B_s \to D^{*+} D^-)}{A(B_s \to D_s^{*+} D_s^-)} = \frac{A(B^0 \to D_s^{*+} D_s^-)}{A(B^0 \to D^{*+} D^-)}. \tag{21}$$

Similarly, the relations

$$\begin{aligned} A(B^0 \to D_s^+ K^{*-}) = {} & A(B^0 \to D^+ \rho^-) \\ & \times A(D^+ \rho^- \to D_s^+ K^{*-}), \\ A(B_s \to D^+ \rho^-) = {} & A(B_s \to D_s^+ K^{*-}) \\ & \times A(D_s^+ K^{*-} \to D^+ \rho^-), \end{aligned} \tag{22}$$

and invariance of rescattering under time reversal lead to

$$\frac{A(B_s \to D^+ \rho^-)}{A(B_s \to D_s^+ K^{*-})} = \frac{A(B^0 \to D_s^+ K^{*-})}{A(B^0 \to D^+ \rho^-)}. \tag{23}$$

While experimental information exists on T-type amplitudes in the two denominators in (21) (see Table VI), the four numerators in this equation and in (23) representing E-type amplitudes have not yet been measured. We expect the magnitudes of all four $|E/T|$ ratios to lie in the range 0.07 ± 0.02.

VI. SUMMARY AND CONCLUSIONS

We have shown that some observed B decays that have been cited as evidence for exchange and annihilation processes can be generated by rescattering from decays whose amplitudes do not involve the spectator quark and hence are not suppressed by powers of f_B/m_B. We have studied a number of processes such as $B^0 \to K^+K^-$, $B_s \to \pi^+\pi^-$, and $B^+ \to D_s^+\phi$, and have identified promising states from which they can be generated by rescattering. We have found that such decays have typical amplitude ratios ranging from 5% to 20% with respect to the largest amplitude from which they can rescatter.

A narrower range between 5% and 10% associated with exchange amplitudes leads to estimated branching fractions in a vast range from $\mathcal{O}(10^{-9})$ to $\mathcal{O}(10^{-4})$ for a large number of as-yet-unseen B and B_s decay processes. These include B^0 decays to K^+K^-, $K^{*\pm}K^{*\mp}$, $D_s^{(*)+}D_s^{(*)-}$, $D^{(*)0}\bar{D}^{(*)0}$, $D^{(*)+}D^{(*)-}$ and B_s decays to $D^{(*)\pm}D^{(*)\mp}$, $D^{(*)0}\bar{D}^{(*)0}$, $D^{\pm}\pi^{\mp}$, $D^0(\bar{D}^0)\pi^0$. Typical values of order a few times 10^{-7} have also been presented for $\mathcal{B}(B^+ \to D^+K^0)$ and $\mathcal{B}(B_s \to \rho^+\pi^-)$, $\mathcal{B}(B_s \to \rho^-\pi^+)$, providing tests for the suppression of annihilation and penguin annihilation amplitudes. Other predictions for $\mathcal{B}(B_s \to D^{*+}\pi^-)$ and $\mathcal{B}(B_s \to D^-\rho^+)$ in the range of $(1\text{–}2) \times 10^{-6}$ have been obtained in the limit of U-spin symmetry. Finally, a class of processes has been identified in which time-reversal invariance of strong interactions leads to further relations between ratios of exchange amplitudes and unsuppressed amplitudes. We emphasize that the above predictions do not stem from first principles, but reflect reasonable ranges within the standard model for branching ratios of many decay modes that have not yet been observed.

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Note added.—: After the completion of this work we were made aware of an unpublished measurement of $\mathcal{B}(B_s \to D^+D^-)$ by the LHCb Collaboration [27], $\mathcal{B}(B_s \to D^+D^-)/\mathcal{B}(B^0 \to D^+D^-) = 1.00 \pm 0.18 \pm 0.09$. Using the values $\mathcal{B}(B^0 \to D^+D^-)$ and $f_+\mathcal{B}(B_s \to D_s^{*+}D_s^{*-})$ in Tables II and III we calculate for $B_s \to D^+D^-$ a ratio $|E/T| = 0.11 \pm 0.02$, on the high side of the range that we have assumed.

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