Little hierarchy problem for new physics just beyond the LHC

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(Received 4 January 2013; published 1 February 2013)

We discuss two possible extensions to the standard model in which an inert singlet scalar state that only interacts with the Higgs boson is added together with some fermions. In one model, the fermions provide for a seesaw mechanism for the neutrino masses; in the other model, for grand unification of the gauge couplings. Masses and interaction strengths are fixed by the requirement of controlling the finite one-loop corrections to the Higgs boson mass, thus addressing the little hierarchy problem. The inert scalar could provide a viable dark matter candidate. Direct detection of this scalar singlet in nuclear scattering experiments is possible with a cross section within reach of future experiments.

DOI: 10.1103/PhysRevD.87.036001 PACS numbers: 11.30.Qc, 12.60.Fr, 14.80.Bn, 95.35.+d

I. INTRODUCTION

If the absence of new states below the TeV scale [1] is confirmed in the next few years as the integrated luminosity of the LHC increases, it will become unfortunately necessary to move the scale at which to expect new physics outside the reach of the experiments. Such a higher scale is somewhat in agreement with what has already been found at LEP, where the cutoff scale for higher-order operators encoding new physics is constrained to be larger than 5 TeV [2]. Recent fits of supersymmetric models [3] also indicate that the masses of the new particles may be just beyond the LHC's reach at between 5 and 10 TeV.

The presence of new physics above 5 TeV raises the problem of the little hierarchy: For the Higgs boson mass [4] [and the electroweak (EW) vacuum expectation value] to be in the 100 GeV range—that is, roughly between 1 and 2 orders of magnitude smaller than the new physics scale—renormalization effects must at least partially cancel out in order to prevent a shift to the higher energy scale.

One may implement such a cancellation by an appropriated choice of the Higgs boson bare mass, but this would imply a fine-tuning of such a counterterm in which low- and high-energy degrees of freedom are mixed. A more natural choice requires the cancellation to occur at the higher scale, either because of a symmetry (like in the supersymmetric case) or merely because the various terms accidently conspire to cancel against each other. In the latter case, the cancellation is best thought of as the effect of a dynamical mechanism—at work at the high-energy scale and arising from new physics that we do not know. The built-in fine-tuning of such a conspiracy (the same as we would have at the level of the bare-mass counterterm) is of the order of the ratio of the two energy scales, in our case about 10%.

In what follows, we want to address this little hierarchy problem in two possible scenarios of new physics: a representative seesaw model for neutrino masses, and a grand unification model. In both cases, the new states will shift the Higgs boson mass to the new scale by a large one-loop renormalization unless their contribution is compensated by the presence of additional states. The identification of what states (their masses and couplings to the Higgs boson) must be present for such a cancellation to occur provides the heuristic power of the little hierarchy problem.

While many possible new states can be added to prevent large corrections to the Higgs boson mass, the simplest choice consists in including just one inert scalar state [5]; that is, a scalar particle only interacting with the Higgs boson (and gravity), and thus transforming as the singlet representation of the EW gauge group $SU(2) \times U(1)$ (and similarly not charged under the color group) and acquiring no vacuum expectation value. Such a choice minimizes unwanted effects on EW radiative corrections and other physics well described by the standard model (SM).

If, in addition, we impose a Z_2 symmetry under which the inert scalar is odd and all the SM fields are even, the new state will couple to the SM Higgs doublet only through quartic interactions in the scalar potential. By construction, we only look for solutions with a vanishing vacuum expectation value; thus Z_2 is unbroken, and after EW symmetry breaking, the singlet state can, as we shall discuss, potentially be a viable cold dark matter (DM) candidate.

The little hierarchy problem is often discussed in terms of the quadratic divergence arising in the mass term of the Higgs boson in a momentum-dependent regularization (or, equivalently, in a pole in $d = 2$ dimensions in dimensional regularization). In the past, this quadratic divergence has been canceled either by assuming a symmetry (usually, supersymmetry) or by assuming that the Veltman condition [6] is satisfied—namely, that the new sector couples to the SM Higgs boson just so as to make the one-loop quadratic divergences to the SM Higgs boson mass vanish (see Ref. [7] for various applications of this idea). These divergent terms are a different and independent problem from the one discussed here, which only depends on integrating out the heavy modes in the low-energy effective theory. The terms we worry about are finite terms similar to those arising in a supersymmetric theory with soft mass terms, where the quadratic divergencies are canceled while, after integrating out the heavy states, there are finite terms whose contribution shifts the value of the Higgs boson mass.

II. SEESAW MODEL

The first SM extension we consider is a representative seesaw model [8] for the neutrino masses. Three right-handed neutrinos N_i are added. The Lagrangian of the model is given by the kinetic and Yukawa terms of the SM with the addition of the neutrino Yukawa terms:

$$\mathcal{L} = \mathcal{L}_{Y_{\rm SM}} + y^{\nu}_{ij}\bar{N}_i\tilde{H}^{\dagger}L_j + \frac{1}{2}M_{N_i}N_iN_i. \quad (1)$$

We work in the basis in which the right-handed neutrino mass matrix is real and diagonal.

We compute the one-loop finite contributions to the Higgs boson mass using dimensional regularization. The SM particle contributions are negligible. To compute the one-loop contribution arising from the right-handed neutrinos, we rotate the Yukawa couplings y^{ν}_{ij} into the basis in which the neutrino mass matrix, defined as

$$m_{\nu} = -y^{\nu T}\cdot\frac{1}{M_N}\cdot y^{\nu}v_W^2, \quad (2)$$

is diagonal. According to the Casas-Ibarra parametrization [9], we have that

$$\hat{y}^{\nu}_{ij} = (y^{\nu}U)_{ij} = M^{1/2}_{N_i}R^{\dagger}_{ij}\hat{m}^{1/2}_{\nu_j}, \quad (3)$$

where $\hat{m}_{\nu}$ is the light neutrino diagonal mass matrix and R an arbitrary orthogonal complex matrix.

In the traditional seesaw model, the Yukawa couplings are of order 1, and the masses M_{N_i} are very large (of the order of 10^{16} GeV). If the Yukawa couplings are taken to be small, the M_{N_i} can be accordingly lighter.

Taking into account the one-loop contribution, and assuming right-handed neutrino degeneracy as well as R real, the Higgs boson mass receives a shift given by

$$\frac{1}{16\pi^2}\frac{M_N^3}{v_W^2}\sum m_{\nu}\left(\frac{3}{2} - \log\frac{M_N^2}{\mu^2}\right), \quad (4)$$

with μ being the matching scale that in this case we identify with M_N. The sum of the neutrino masses, the term $\sum m_{\nu}$ in Eq. (4), has a lower bound of about 0.055 eV [10], which corresponds to a normal neutrino mass hierarchy with vanishing lightest mass. On the other hand, cosmological constraints set an upper bound on $\sum m_{\nu}$ that, even if model dependent, is always ≤ 0.44 eV [11].

Because of the smallness of the neutrino mass term, as long as the new states have masses up to around 10^4 TeV, the shift in the Higgs boson mass is of the order of its mass, and no hierarchy problem arises. Notice that the one-loop correction of right-handed neutrinos with $M_N \sim 10^4$ TeV gives rise to a correction to the Higgs boson mass of the order of

$$\left(\frac{\sqrt{M_N\sum m_{\nu}}}{v_W}\right)M_N \sim 2.5 \text{ TeV}, \quad (5)$$

for which also two-loop corrections are under control.

On the other hand, if the new fermion masses $M_{N_i} \simeq M_N$ are larger than 10^4 TeV, we do have a little hierarchy problem and must balance their one-loop contribution against some other contribution in order to keep the overall renormalization of the Higgs boson mass of the order of the weak scale.

To provide for such a contribution, we add the simplest state: an inert scalar particle S. The scalar potential is given by

$$V(H,S) = \mu_H^2(H^{\dagger}H) + \mu_S^2S^2 + \lambda_1(H^{\dagger}H)^2 + \lambda_2S^4 + \lambda_3(H^{\dagger}H)SS. \quad (6)$$

Linear and trilinear terms for S are absent due to the Z_2 symmetry mentioned above.

Taking into account the one-loop contribution induced by the scalar state S, the overall shift to μ_H^2, taking $\mu = M_S$ to minimize the logarithmic terms in the matching, becomes

$$\delta\mu_H^2(M_S) = \frac{1}{16\pi^2}\left[-\lambda_3M_S^2 - \frac{M_N^3}{v_w^2}\sum m_{\nu}\left(\log\frac{M_N^2}{M_S^2} - \frac{3}{2}\right)\right]. \quad (7)$$

We want the correction in Eq. (7) to be of the order of the Higgs boson mass itself. For simplicity, we can just impose that $\delta\mu_H^2 = 0$ and obtain

$$\lambda_3 = \frac{3}{2}\left(\frac{M_N^3\sum m_{\nu}}{M_S^2v_W^2}\right)\left[1 - \frac{4}{3}\log\frac{M_N^2}{M_S^2}\right]. \quad (8)$$

In the region $M_S \ll M_N$, $M_N > 10^4$ TeV, a cancellation is possible provided λ_3 is negative. The value of λ_3 is bounded by

$$\lambda_3 \geq -2\sqrt{\lambda_1\lambda_2}, \quad (9)$$

to ensure the stability of the scalar potential at infinity. The value of λ_1 is fixed by the value of the Higgs boson mass to be $\lambda_1 \sim 0.13$. Equation (8) and the above condition are satisfied for $M_S > 5$ TeV.

For M_S around 10 TeV, $\lambda_3 \simeq 0.2$. As the value of M_S comes close to that of M_N—and the logarithmic term becomes smaller—the value of λ_3 becomes positive and smaller; it is of the order of 10^{-7} for $M_S \simeq M_N$.

The order of the the one-loop contribution to the Higgs boson mass is $\sqrt{\lambda_3}M_S$. The two-loop contributions are under control as long as this correction is less than 10 TeV.

III. GRAND UNIFICATION MODEL

The other SM extension we consider is one in which a minimal set of fermions is introduced to provide for gauge coupling unification. The possible sets have been discussed in the context of split supersymmetry models [12]. They are given by $(Q + \bar{Q}) + (D + \bar{D})$, two chiral couples of left-handed fermions with quantum numbers identical to the left-handed quark doublet and right-handed down quark, respectively; or by $(L + \bar{L}) + V + G$, one chiral couple of left-handed lepton-like fermions and a wino-like as well as a gluino-like fermion multiplet. We choose the first option as the minimal and representative set.

The new fermions couple to the Higgs boson SM through the Yukawa Lagrangian

$$M_Q \bar{Q} Q + M_D \bar{D} D + k_1 \bar{Q} D H + k_2 \bar{D} Q H^*, \quad (10)$$

and they give a shift to the Higgs boson mass equal to

$$\frac{|k|^2}{16\pi^2}\left[(3M_Q^2 - M_Q M_D) - 3M_Q M_D \log\frac{M_Q^2}{\mu^2}\right], \quad (11)$$

with $|k|^2 = |k_1|^2 + |k_2|^2$ and $M_Q \sim M_D$. We identify the matching scale μ with M_D.

If the new fermions are lighter than 1 TeV, there is no little hierarchy problem. On the other hand, if they are heavier, the problem exists, and we introduce an inert singlet scalar S to protect the Higgs boson mass. Therefore, we add the terms

$$k_{3_i} \bar{Q} q_i S + k_{4_i} d^c_{L_i} D S + \text{H.c.} - V(H, S), \quad (12)$$

to the Lagrangian in Eq. (10). In Eq. (12), $V(H, S)$ coincides with Eq. (6) with S odd under an additional Z_2 symmetry. We have also imposed for the extra fermions to be odd under Z_2. The total one-loop contribution to μ_H^2 at the scale $\mu = M_S$ is given by

$$\delta\mu_H^2(M_S) = \frac{1}{16\pi^2}\left[-\lambda_3 M_S^2 + |k|^2(3M_Q^2 - M_Q M_D) - 3|k|^2 M_Q M_D \log\frac{M_Q^2}{M_S^2}\right]. \quad (13)$$

Let us consider the case in which all couplings are of order 1. As before, for simplicity, we just impose that $\delta\mu_H^2 = 0$. Taking $M_Q \sim M_S$, and with the singlet S the lightest Z_2-odd particle, this condition is satisfied by writing λ_3 as a function of $|k|^2$:

$$\lambda_3 = |k|^2\left(\frac{M_Q}{M_S}\right)^2\left(2 - 3\log\frac{M_Q^2}{M_S^2}\right) \sim 2|k|^2. \quad (14)$$

Contrary to the previous example of the seesaw model, in this case it is always possible to find an appropriate value of λ_3 so as to control the renormalization of the Higgs boson mass.

IV. DARK MATTER

We may ask whether, in the two models considered, the inert scalar S is a viable DM candidate. It is a gauge singlet and therefore only interacts with the SM particles through the Higgs boson h. The pointlike interaction $\lambda_3/2SShh$ and the scattering mediated by h—in both the s and t channels—contribute to the cross section $SS \to hh$. The Higgs boson h also mediates the scattering processes $SS \to f\bar{f}$, $SS \to W^+W^-$, $SS \to ZZ$.

It has been shown [13] that a single inert singlet that couples with the Higgs boson with a small coupling is a realistic cold DM candidate with a mass $\lesssim v_W$. In our case, the singlet may account for the correct relic density in the opposite regime, where its mass is much larger than v_W and its coupling with the Higgs boson relatively large. In this case, the scattering amplitude is dominated by the pointlike $SS \to hh$ vertex, which gives a contribution to the total cross section equal to

$$\langle\sigma v\rangle \simeq \frac{1}{16\pi}\frac{\lambda_3^2}{M_S^2}. \quad (15)$$

To estimate the viability of S as a DM candidate, we make use of the approximated analytical solution [14]. The relic abundance $n_{\rm DM}$ is written as

$$\frac{n_{\rm DM}}{s} = \sqrt{\frac{180}{\pi g_*}}\frac{1}{M_{pl} T_f \langle\sigma v\rangle}, \quad (16)$$

where M_{pl} is the Planck mass, and T_f is the freeze-out temperature, which for our and similar candidates is given by $m_S/T_f \sim 26$. The constant $g_* = 106.75 + 1$ counts the number of SM degrees of freedom in thermal equilibrium plus the additional degrees of freedom related to the singlets, and s is their total entropy density. Current data fits within the standard cosmological model give a relic abundance with $\Omega_{\rm DM} h^2 = 0.112 \pm 0.006$ [15], which corresponds to

$$\frac{n_{\rm DM}}{s} = \frac{(0.40 \pm 0.02)}{10^9\, M_S/\text{GeV}}. \quad (17)$$

By combining Eq. (17) with Eq. (15), we may write λ_3 as a function of M_S, obtaining

$$|\lambda_3| \simeq 0.44\frac{M_S}{\text{TeV}}. \quad (18)$$

In the first model we considered, the condition in Eq. (18) can only be satisfied for $M_S \ll M_N$ (see Fig. 1). For $M_S \simeq M_N$, the smallness of the neutrino Yukawa couplings forces λ_3 to be very small, thus destroying its potential role as a DM candidate. More dangerously, in the latter case, it could give rise to the overclosure of the Universe. Since its production mechanism could be nonthermal, any conclusion should be drawn only after a detailed analysis that goes beyond the purposes of this work. In any case, we could let S acquire a small vacuum

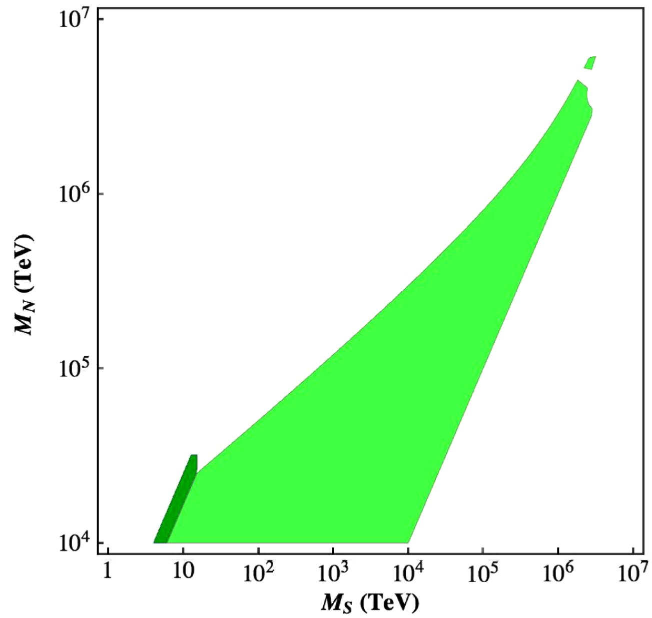

FIG. 1 (color online). Seesaw model: The two regions for which the one-loop contribution vanishes and λ_3 satisfies Eq. (18) (narrow, dark green/grey region) and $|\lambda_3| < 0.44\, M_S/\mathrm{TeV}$ (light green/grey region). The points have been selected by requiring the following: $\lambda_3 > -1.6$ to avoid too large a value for λ_2 according to Eq. (9), $\sqrt{\lambda_3} M_S \lesssim 10$ TeV to control the two-loop corrections, $M_N \geq M_S$ according to our assumption, and $\sum m_\nu$ in the range 0.055–0.44 eV (see the text). For the points in the narrow, dark green region, the model provides a viable DM candidate; whereas for those in the light green region, a detailed analysis of the singlet production mechanism should be performed before ruling out the model, as commented in the text.

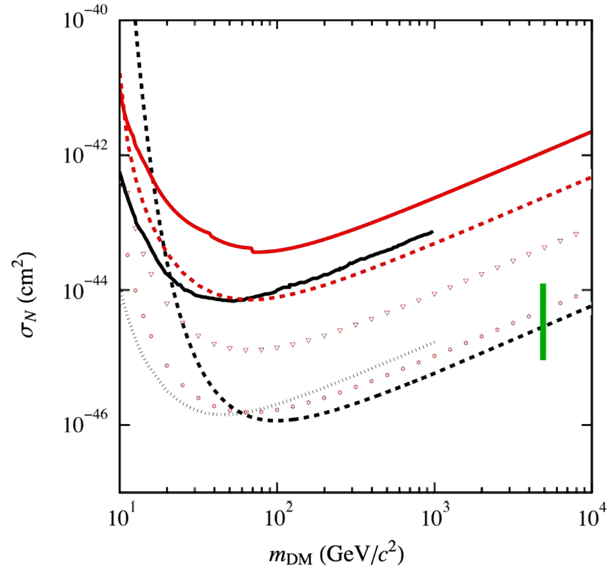

FIG. 2 (color online). Spin-independent cross section per nucleon versus DM candidate masses [18]. The black (red/dark grey) solid line corresponds to the XENON100 (CDMSII) data. Black points and the black dashed line are the projections for upgraded XENON100 and XENON1T, respectively. The red/dark grey dashed line, down triangles, and stars correspond to different projections for SCDMS. The green/light gray vertical line is the prediction of the inert model discussed in this work.

expectation value $\sim v_W^2/M_S$ and not impose the Z_2 symmetry, so that the scalar state would rapidly decay into SM particles through its mixing with the SM Higgs boson.

In the second model we discussed, λ_3 depends only on the ratio M_Q/M_S and is scale independent; thus, the correct relic density may be accommodated for any value of M_S—in particular, for $M_S \geq 10$ TeV.

Let us briefly comment on the possibility of detecting the inert scalar S in nuclear scattering experiments.

The λ_3 quartic term in Eq. (6) gives rise also to the three-field interaction SSh, which gives the effective singlet-nucleon vertex

$$f_N \frac{\lambda_3 m_N}{m_h^2} SS\bar{\psi}_N \psi_N. \tag{19}$$

The (nonrelativistic) cross section for the process is given by [16]

$$\sigma_N = f_N^2 m_N^2 \frac{\lambda_3^2}{4\pi}\left(\frac{m_r}{m_S m_h^2}\right)^2, \tag{20}$$

where m_r is the reduced mass for the system, which is, to a very good approximation in our case, equal to the nucleon mass m_N; the factor f_N contains many uncertainties due to the computation of the nuclear matrix elements, and it can vary from 0.3 to 0.6 [17]. Substituting the values we have found for our model, we obtain, depending on the choice of parameters within the given uncertainties, a cross section σ_N between 10^{-45} and 10^{-44} cm^2, a value within reach of the next generation of experiments (see Fig. 2).

V. CONCLUSIONS

As the scale of new physics is pushed to around the 10 TeV scale or higher, the stability of the Higgs boson mass against finite one-loop corrections induced by the new states gives rise to the little hierarchy problem. Since these new states are beyond the current experimental reach, we can use this problem in a heuristic manner to determine the masses and couplings of the new particles. We have shown that for two representative new physics scenarios—seesaw neutrino mass generation and gauge coupling unification—the addition of an inert scalar state suffices in solving the little hierarchy problem and provides in addition a viable candidate for DM. Such a candidate may well be the only experimentally testable signature of the new physics.

ACKNOWLEDGMENTS

We thank P. Ullio for explaining to us some aspects of DM physics. M. F. thanks SISSA for the hospitality.

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