PHYSICAL REVIEW D **87,** 034502 (2013)

Low-lying Λ baryons from the lattice

Georg P. Engel,[1] C. B. Lang,[1] and Andreas Schäfer[2]

(BGR [Bern-Graz-Regensburg Collaboration])

[1]*Institut für Physik, FB Theoretische Physik, Universität Graz, A-8010 Graz, Austria*
[2]*Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany*
(Received 20 September 2012; published 4 February 2013)

In a lattice QCD calculation with two light dynamical chirally improved quarks we determine ground state and some excited state masses in all four Λ baryon channels $\frac{1}{2}^{\pm}$ and $\frac{3}{2}^{\pm}$. We perform an infinite volume extrapolation and confirm the widely discussed Λ(1405). We also analyze the amount of octet-singlet mixing, which is helpful in comparing states with the quark model.

DOI: 10.1103/PhysRevD.87.034502 PACS numbers: 11.15.Ha, 12.38.Gc

One of the central aims of hadron spectroscopy is to understand the spin-flavor-parity structure of the excitation spectra for different quantum numbers. It seems in particular that the nucleon and Λ spectra show significant differences which, if properly understood, should illuminate the influence of quark mass and flavor on hadron structure. The lowest $\Lambda(\frac{1}{2}^-)$ mass lies below the Roper-like $\Lambda(1600,\frac{1}{2}^+)$ and the negative parity nucleon $N^*(1535)$; unlike the nucleon sector it does have standard level ordering lying between the positive parity ground state and the first positive parity excitation. Λ baryons can be flavor singlets or octets or, due to the difference in light and strange quark masses, mixtures of both. Various continuum model studies discuss that mixing. This is the first lattice QCD analysis of the Λ baryons for dynamical quarks, that includes *all four*, namely the $J^P=\frac{1}{2}^{\pm}$ and $\frac{3}{2}^{\pm}$ channels. We obtain ground states compatible with experiments in three of those. We also study the infinite volume limit and give for the first time lattice results on singlet-octet composition for all four sectors, obtaining the mass of the Λ(1405) and confirming its flavor singlet nature.

Lattice studies in the quenched case generally had problems finding the low-lying Λ(1405). Even a study with two dynamical light quarks [1,2] found mass values that were too large. Only recent results [3] obtained with PACS-CS (2 + 1)-flavor dynamical quark configurations [4] show a level ordering which is compatible with experiments.

We study the baryons for a set of seven ensembles with two dynamical chirally improved (CI) quarks [5,6]. The CI fermion action consists of several hundred terms and obeys the Ginsparg-Wilson relation in a truncated approximation. The pion mass ranges from 255 to 596 MeV; the lattice spacing lies between 0.1324 and 0.1398 fm. The bulk of our results was obtained for lattices of size $16^3\times 32$. For two ensembles with light pion masses (255 and 330 MeV) we also used 24×48 lattices. Thus $m_\pi L$ which controls the finite size effects is 4.08 and 5.61 in these two situations. Further details on the action and our simulation, as well as results for mesons, can be found in Refs. [7,8]. The strange quark is introduced as a valence quark and its mass fixed by the Ω mass.

The Dirac and flavor structure of the interpolating fields used in our study is motivated by the quark model [9,10]; see also Ref. [11]. Within the relativistic quark model there have been many determinations of the hadron spectrum, based on confining potentials and different assumptions on the hyperfine interaction (see, e.g., Refs. [12–14]). The singlet, octet and decuplet attribution [11] of the states has been evaluated based on such model calculations, e.g., in Ref. [15] (see also the summary in Ref. [16]).

For the Λ baryons we use sets of up to 24 interpolating fields in each quantum channel. The singlet and octet combinations of Table I are combined with three possible choices of Dirac matrices $(\Gamma_1,\Gamma_2)=(\mathbb{1},C\gamma_5)$, (γ_5,C) and $(i\mathbb{1},C\gamma_t\gamma_5)$ (denoted by χ_1, χ_2 and χ_3 for short) for $J=\frac{1}{2}$ and eight combinations of Gaussian smeared quarks [17,18] with two smearing widths (n,w). The operator numbering is given in Table II. All interpolators are projected to definite parity and all Rarita-Schwinger fields (spin $\frac{3}{2}$ interpolators in Table II) are projected to definite spin $\frac{3}{2}$ using the continuum formulation of the Rarita-Schwinger projector [19]. C denotes the charge conjugation matrix; γ_t and γ_i are the time and the spatial direction Dirac matrices, respectively.

For pointlike quark fields, Fierz identities reduce the actual number of independent operators (see, e.g., Ref. [20]). In particular, there are no nonvanishing pointlike interpolators for $\Delta(\frac{1}{2})$ and singlet $\Lambda(\frac{3}{2})$. We use different quark smearing widths such that the Fierz identities do not apply. Hence χ_1, χ_2 and χ_3 are all independent; for $J=\frac{3}{2}$ all interpolators are nonvanishing.

From the cross-correlation matrix $C_{ik}(t)=\langle O_i(t)O_k(0)^\dagger\rangle$ we obtain the energy levels with the help of the variational method [21,22]. One solves the generalized eigenvalue problem $C(t)\vec{u}_n(t)=\lambda_n(t)C(t_0)\vec{u}_n(t)$ in order to approximately recover the energy eigenstates $|n\rangle$. The eigenvalues $\lambda_n(t)\sim\exp(-E_n t)$ allow us to get

TABLE I. Baryon interpolators: The possible choices for the Dirac matrices $\Gamma_{1,2}^{(i)}$ in the spin $\frac{1}{2}$ channels are discussed in the text. Summation convention applies; for spin $\frac{3}{2}$ observables, the open Lorentz index (after spin projection) is summed after taking the expectation value of correlation functions.

Spin	Flavor	Name	Interpolator
$\frac{1}{2}$	Singlet	$\Lambda_{1/2}^{(1,i)}$	$\epsilon_{abc}\Gamma_1^{(i)}u_a(d_b^T\Gamma_2^{(i)}s_c - s_b^T\Gamma_2^{(i)}d_c)$ + cyclic permutations of u, d, s
$\frac{1}{2}$	Octet	$\Lambda_{1/2}^{(8,i)}$	$\epsilon_{abc}[\Gamma_1^{(i)}s_a(u_b^T\Gamma_2^{(i)}d_c - d_b^T\Gamma_2^{(i)}u_c) + \Gamma_1^{(i)}u_a(s_b^T\Gamma_2^{(i)}d_c) - \Gamma_1^{(i)}d_a(s_b^T\Gamma_2^{(i)}u_c)]$
$\frac{3}{2}$	Singlet	$\Lambda_{3/2}^{(1,i)}$	$\epsilon_{abc}\gamma_5 u_a(d_b^T C\gamma_5\gamma_i s_c - s_b^T C\gamma_5\gamma_i d_c)$ + cyclic permutations of u, d, s
$\frac{3}{2}$	Octet	$\Lambda_{3/2}^{(8,i)}$	$\epsilon_{abc}[\gamma_5 s_a(u_b^T C\gamma_5\gamma_i d_c - d_b^T C\gamma_5\gamma_i u_c) + \gamma_5 u_a(s_b^T C\gamma_5\gamma_i d_c) - \gamma_5 d_a(s_b^T C\gamma_5\gamma_i u_c)]$

the energy values and the eigenvectors serve as fingerprints of the states, indicating their content in terms of the lattice interpolators. The quality of the results depends on the statistics and the provided set of lattice operators. The dependence on t_0 is used to study the systematic error; in the final analysis we use $t_0 = 1$ (with the origin at 0). The statistical error is determined with a single elimination jackknife. For the fits we use single exponential behavior but check the stability with double exponential fits; we take the correlation matrix for the correlated fits from the complete sample [8].

For the extrapolation to the physical pion mass we fit to the leading order chiral behavior, which is linear in m_π^2. Two ensembles (at pion masses 255 MeV and 588 MeV) suffer from a slight mistuning of the strange quark mass. These two ensembles are therefore discarded in the extrapolation to the physical pion mass, whenever the effects are found to be significant. This is the case for the lowest energy levels in each channel (the three lowest ones in $\frac{1}{2}^-$). The quoted systematic errors for these levels include the corresponding deviation and the dependence of the energy levels on the choice of interpolators and fit ranges for the eigenvalues.

In the present study we are restricted to three-quark operators for the baryon. Note that ideally one should take into account also meson-baryon interpolators (see, e.g., the discussion in Ref. [23]). This leads to many more contributing graphs and necessitates also the inclusion of backtracking quark loops. The related computational and algorithmic effort prevented such lattice calculations so far, although such studies are in progress [24]. Due to sea quarks, in principle, three-quark operators have overlap with meson-baryon states as well. The corresponding coupling was however found to be weak in actual simulations [7,25]. We will argue below that in particular in the s-wave channels we find hints of such coupling even for our interpolators.

TABLE II. Baryon interpolators: Quark smearing types (n/w for narrow/wide) and naming convention for the interpolators in the different channels. The three columns for the $J = \frac{1}{2}$ interpolators refer to χ_1–χ_3.

Quark	Numbering of associated interpolators			
smearing types	$\Lambda_{3/2}^{(1,i)}$	$\Lambda_{3/2}^{(8,i)}$	$\Lambda_{1/2}^{(1,i)}$	$\Lambda_{1/2}^{(8,i)}$
(nn)n	1	9	1,9,17	25,33,41
(nn)w	2	10	2,10,18	26,34,42
(nw)n	3	11	3,11,19	27,35,43
(nw)w	4	12	4,12,20	28,36,44
(wn)n	5	13	5,13,21	29,37,45
(wn)w	6	14	6,14,22	30,38,46
(ww)n	7	15	7,15,23	31,39,47
(ww)w	8	16	8,16,24	32,40,48

(a) $J^P = \frac{1}{2}^+$, *finite volume:* In Fig. 1 we show results for the four lowest eigenstates for interpolator set (1, 2, 11, 20, 25, 26, 33, 34, 43). After extrapolation to the physical point our lowest energy level agrees well with the experimental $\Lambda(1116)$. The systematic error estimated from different combinations of interpolators and fit ranges is indicated in the summary Fig. 6. Analyzing the eigenvectors, we find that the ground state is dominated by octet interpolators of Dirac structure χ_1 and χ_3 (in agreement with a relativistic quark model calculation [15]). Our first excitation is dominated by singlet interpolators (first Dirac structure) matching the $\Lambda(1810)$ (singlet in the quark model). The Roper-like $\Lambda(1600)$ (octet in the quark model) seems to be missing.

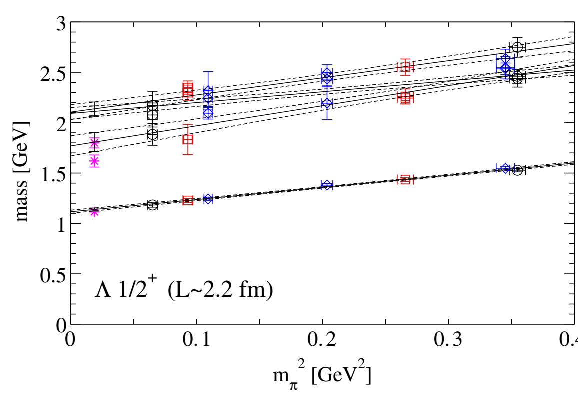

FIG. 1 (color online). Energy levels for Λ spin $\frac{1}{2}^+$ in a finite box of linear size $L \approx 2.2$ fm. Stars denote the experimental values [16]; other symbols denote results from the simulation. The full lines show the extrapolations linear in the m_π^2; the broken curves indicate the statistical error bands.

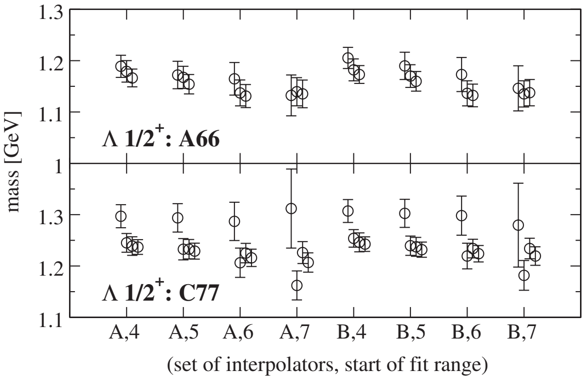

FIG. 2. Systematic error of the Λ $\frac{1}{2}^+$ ground state energy levels. The levels are shown for different choices of interpolators and fit ranges, labeled on the horizontal axis. For example, "A4" denotes the set of interpolator "A" and a fit range for the eigenvalues from $t = 4a$ to the onset of noise. "A" denotes interpolators (2, 3, 10, 18, 26, 27, 34, 42); "B" denotes (3, 11, 18, 27, 34). For each set of interpolators and fit range, results for small to large lattices (spatial size 16, 24 for ensemble A66, and 12, 16, 24 for C77; for notation see Ref. [8]) are shown from left to right. The corresponding infinite volume limits are the rightmost points.

This resembles the situation in the $N(\frac{1}{2}^+)$ channel [7]. The second and third excitations are again dominated by octet interpolators.

Infinite volume extrapolation: We performed a volume analysis for several sets of interpolators and various fit ranges. The results in finite volume and the infinite volume extrapolations for the ground state for specific interpolators are shown in Fig. 2. The extrapolation follows the method of Ref. [26]. A stable choice is the set of interpolators A and $t_{\mathrm{min}} = 5a$. The corresponding systematic error estimated from different interpolators and fit ranges is indicated in the summary Fig. 6. Our final result is 1126(17)(11) MeV (statistical and systematic error), which agrees nicely with the experimental Λ(1116) mass.

(b) $J^P = \frac{1}{2}^-$, *finite volume:* We use different sets of interpolators and fit ranges. We stress that our basis is large compared to that of other studies with three types of Dirac structures for both singlet and octet interpolators. We can extract the lowest four energy levels, shown in Fig. 3, using interpolators (2, 3, 10, 18, 26, 27, 34, 42). We find that the ground state energy level agrees well with the experimental Λ(1405). The dependence of the levels on the tuning of the strange quark mass appears to be sizeable, albeit an accident of our simulation. This is one reason for the large systematic error depicted in Fig. 6.

The chosen set of interpolators is particularly suitable for an analysis of the content of the states. The spatial support of the quark fields is equivalent in all interpolators and hence does not require additional renormalization when comparing the contribution of different interpolators to the eigenstate. In addition, we use several interpolators for each given combination of flavor and Dirac structures, which allows us to identify a possibly higher number of states with a similar structure. Within the basis used, the ground state is dominated by singlet interpolators of all three Dirac structures. There is, however, a considerable mixing with octet interpolators (second Dirac structure) of 15%–20% in ensemble A66, i.e., at $m_\pi \approx 255$ MeV (see Fig. 4). This mixing is expected to increase towards the physical point, which may complicate the functional dependence of the energy levels on the pion mass. The first and second excitation are both dominated by octet interpolators, by the second and first Dirac structure, respectively.

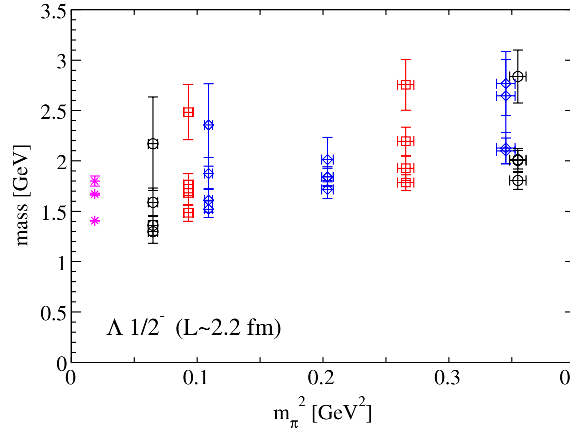

FIG. 3 (color online). Energy levels for Λ spin $\frac{1}{2}^-$ in a finite box of linear size $L \approx 2.2$ fm; for the legend see the caption of Fig. 1. Fits are omitted for clarity.

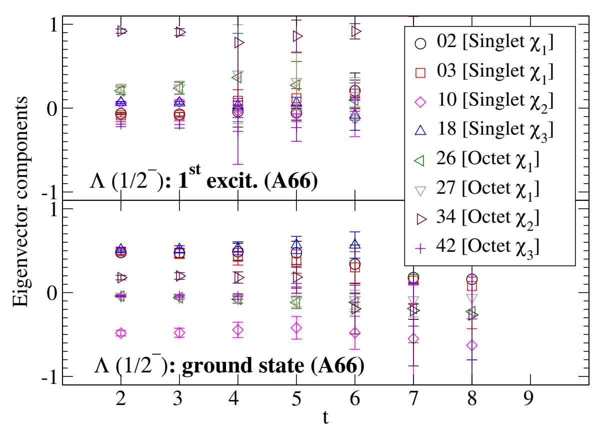

FIG. 4 (color online). Eigenvectors for the Λ spin $\frac{1}{2}^-$ ground state and first excitation for ensemble A66 ($m_\pi \approx 255$ MeV). The ground state is dominated by singlet interpolators of all three Dirac structures. The first (and also the second excitation, not shown) is dominated by octet interpolators. Note that a considerable mixing of singlet and octet is observed (15%–20% for the ground state).

The first and second excited energy levels are both a bit low but compatible with the experimental $\Lambda(1670)$ and $\Lambda(1800)$. In general in the $J^P = \frac{1}{2}^-$ channel one may expect coupling to $\pi\Sigma$ and $\bar{K}N$ in the s wave. In Refs. [27,28] the expected energy levels in finite volumes are discussed based on a continuum hadron exchange model. There (with physical hadron masses), the low-lying scattering state levels in the $\frac{1}{2}^-$ channel are well separated for $m_\pi L \lesssim 3$. For the pion masses of our study, the noninteracting meson-baryon thresholds lie close but (except for one point) above the lowest energy level observed. For example, for the lowest pion mass, the values are $m_\Sigma + m_\pi \approx 1.52$ GeV, $m_N + m_K \approx 1.62$ GeV, above the lowest level. This resembles the situation in the $N(\frac{1}{2}^-)$ channel. Earlier work argued that the coupling of single baryons to baryon-meson channels may be too weak to lead to observable effects [7,25]. However, in our case, in s-wave scattering, we cannot exclude that one or even two of the observed three lowest energy levels correspond to scattering states. Note that the measured ground state energy level is always (except for one point) below the s-wave threshold, thus supporting the $\Lambda(1405)$ identification.

It has been conjectured from chiral unitary theory that the lowest state may have a double pole [23,29] and a identification strategy for lattice simulations is suggested in Ref. [30]. This would require asymmetric boxes or moving frames.

Infinite volume extrapolation: We study the volume dependence of the three lowest states for different sets of interpolators and various fit ranges. We emphasize that the stability of the signals of the excitations is comparable to the ones of the ground state. The volume dependence of all three low states appears fairly flat in our simulation, in a few cases showing even negative finite volume corrections. These features are compatible with significant contributions of an attractive s-wave scattering state. For interpolators (2, 3, 10, 18, 26, 27, 34, 42) and $t_{\text{min}} = 5a$, after infinite volume extrapolation, we show the result of the final extrapolation of the ground state energy level to the physical pion mass in Fig. 6. The final result for the ground state agrees very well with the experimental $\Lambda(1405)$. Both the first and the second excitation appear to be a bit low but are compatible with the experimental $\Lambda(1670)$ and $\Lambda(1800)$ (see Figs. 3 and 6) and might also possibly be s-wave $\pi\Sigma$ and $\bar{K}N$ signals.

(c) $J^P = \frac{3}{2}^+$, *finite volume:* In spin $\frac{3}{2}$ channels, for symmetric quark fields, singlet interpolators vanish exactly due to Fierz identities. We use different quark smearing widths and thus circumvent the Fierz identities constructing singlet interpolators nevertheless. We derive results for the lowest three energy levels of the variational analysis of interpolators (2, 9, 10, 16). Only the ground state can be clearly identified and its extrapolation agrees with the experimental $\Lambda(1890)$. Within the finite basis used, this state is dominated by octet interpolators. The first excitation is dominated by singlet interpolators with non-negligible octet contributions at our lightest pion mass (see Fig. 5). We want to emphasize the importance of singlet interpolators for the low-lying states in this channel, even though those interpolators are vanishing exactly for symmetric pointlike quark fields.

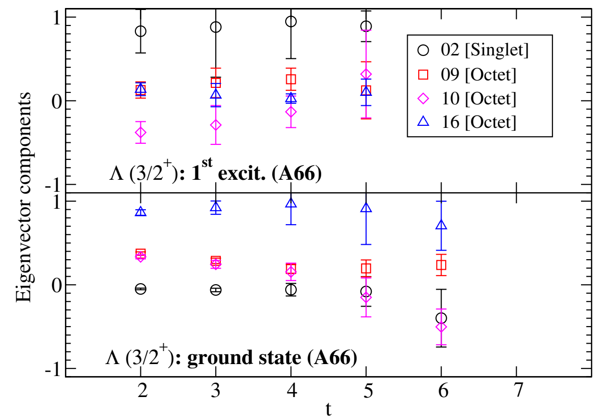

FIG. 5 (color online). Eigenvectors for the Λ spin $\frac{3}{2}^+$ ground state and first excitations for ensemble A66 ($m_\pi \approx 255$ MeV). We emphasize the domination of singlet interpolators for the first excitation. Such interpolators are nonvanishing only for broken Fierz identities, realized by the use of different quark smearing widths.

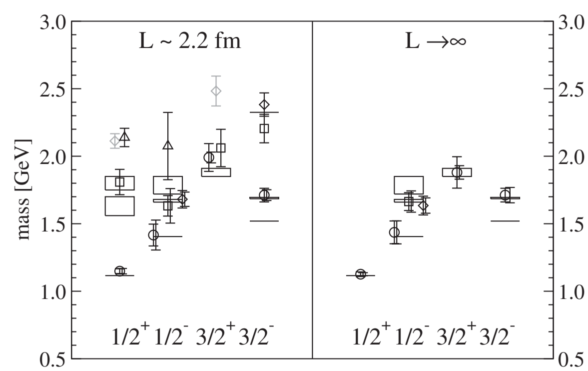

FIG. 6. Energy levels extrapolated to the physical pion mass in finite volume $L \approx 2.2$ fm (lhs) and low-lying levels after infinite volume extrapolation (rhs). The horizontal lines or boxes represent experimentally known states [16], where the box size indicates the statistical uncertainty of 1σ. The statistical uncertainty of our results is indicated by error bars of 1σ; second bars set closely to the right indicate systematic errors estimated from the use of different sets of interpolators, fit ranges of the eigenvalues and the strange quark mass. Gray symbols denote a poor $\chi^2/\text{d.o.f.} > 3$ of the chiral fits.

Infinite volume extrapolation: Within errors we do not observe a clear volume dependence. The final result agrees with the experimental Λ(1890) mass, but with large uncertainty.

(d) $J^P = \frac{3}{2}^-$, *finite volume:* We choose interpolators (2, 7, 9, 10, 15) and find a clear gap between ground state and excitations. The extrapolation of the ground state energy level lies clearly above the experimental ground state Λ(1520) and is compatible with the Λ(1690). The excitations extrapolate close to the Λ(2325).

From the eigenvectors we find that the two lowest states are dominated by octet, and the second excitation by singlet interpolators. The quark model shows for Λ(1520) singlet dominance [like for its companion Λ(1405)]. We conclude that we might miss a signal for the ground state altogether, or, alternatively, there is a strong deviation from the leading chiral behavior towards the physical point. The latter case is intriguing as it might be related to strong coupling to an s-wave $\pi\Sigma(1385)$ state, which is discussed, e.g., in Refs. [31,32].

Infinite volume extrapolation: The volume dependence turns out to be fairly flat, in a few cases even compatible with negative finite volume corrections. The final result in the infinite volume limit at the physical point again misses the experimental Λ(1520) and agrees with the Λ(1690) mass.

Summary: We present a comprehensive study of spin $\frac{1}{2}$ and $\frac{3}{2}$ Λ baryon ground states and excitations, utilizing a large basis of interpolators in the variational analysis including differently smeared quark sources (which allows us to sidestep the Fierz identities), three Dirac structures, and singlet and octet forms. Figure 6 shows the results for ground states and excitations for lattices of linear size $L \approx 2.2$ fm (lhs) and the results for the ground states in the infinite volume limit (rhs). Systematic errors from the choice of interpolators, the fit ranges of the eigenvalues and the tuning of the strange quark mass have been investigated. In both $\frac{1}{2}$ channels and in the $\frac{3}{2}^+$ channel we find ground states extrapolating to the experimental values, in particular we reproduce Λ(1405) and also find two low-lying excitations. In our simulation, Λ(1405) is dominated by singlet contributions, but at $m_\pi \approx 255$ MeV octet interpolators contribute roughly 15%–20%, which may increase towards physical pion masses. The observation of Λ(1405) with the employed basis suggests that this state has a non-negligible singlet three-quark content. The Roper-like (octet) state Λ(1600), on the other hand, may not couple to our three-quark interpolators.

We analyze the volume dependence and find that only the ground state of spin $\frac{1}{2}^+$ shows a clear exponential dependence as expected for bound states. For all other discussed states, the volume dependence is either fairly flat or obscured by the statistical error. For the $\frac{1}{2}^+$, $\frac{1}{2}^-$ and $\frac{3}{2}^+$ channels the remaining small deviations at the physical point can be easily caused by the continuum limit and/or dynamical strange quarks. The discrepancy for the $\frac{3}{2}^-$ ground state might point to some effect which is so far not properly accounted for.

Special thanks to Leonid Y. Glozman and Daniel Mohler for valuable criticism. Discussions with Christof Gattringer, Markus Limmer, Mario Schröck and Valentina Verduci are gratefully acknowledged. The calculations have been performed on the SGI Altix 4700 of the LRZ Munich and on local clusters at the University of Graz. G. P. E. and A. S. acknowledge support by the DFG project SFB/TRR-55.

[1] T. T. Takahashi and M. Oka, Proc. Sci. LAT2009 (2009) 108.
[2] T. T. Takahashi and M. Oka, Phys. Rev. D **81**, 034505 (2010).
[3] B. J. Menadue, W. Kamleh, D. B. Leinweber, and M. S. Mahbub, Phys. Rev. Lett. **108**, 112001 (2012).
[4] S. Aoki, K.-I. Ishikawa, N. Ishizuka, T. Izubuchi, D. Kadoh, K. Kanaya, Y. Kuramashi, Y. Namekawa, M. Okawa, Y. Taniguchi, A. Ukawa, N. Ukita, and T. Yoshie, Phys. Rev. D **79**, 034503 (2009).
[5] C. Gattringer, Phys. Rev. D **63**, 114501 (2001).
[6] C. Gattringer, I. Hip, and C. B. Lang, Nucl. Phys. **B597**, 451 (2001).
[7] G. P. Engel, C. B. Lang, M. Limmer, D. Mohler, and A. Schäfer (BGR [Bern-Graz-Regensburg]), Phys. Rev. D **82**, 034505 (2010).
[8] G. P. Engel, C. B. Lang, M. Limmer, D. Mohler, and A. Schäfer, Phys. Rev. D **85**, 034508 (2012).
[9] N. Isgur and G. Karl, Phys. Rev. D **18**, 4187 (1978).
[10] N. Isgur and G. Karl, Phys. Rev. D **19**, 2653 (1979).
[11] L. Y. Glozman and D. O. Riska, Phys. Rep. **268**, 263 (1996).
[12] S. Capstick and N. Isgur, Phys. Rev. D **34**, 2809 (1986).
[13] L. Y. Glozman, W. Plessas, K. Varga, and R. Wagenbrunn, Phys. Rev. D **58**, 094030 (1998).
[14] U. Loring, B. C. Metsch, and H. R. Petry, Eur. Phys. J. A **10**, 395 (2001).
[15] T. Melde, W. Plessas, and B. Sengl, Phys. Rev. D **77**, 114002 (2008).
[16] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).

[17] S. Güsken, U. Löw, K.-H. Mütter, R. Sommer, A. Patel, and K. Schilling, Phys. Lett. B **227**, 266 (1989).
[18] C. Best, M. Göckeler, R. Horsley, E.-M. Ilgenfritz, H. Perlt, P. Rakow, A. Schäfer, G. Schierholz, A. Schiller, and S. Schramm, Phys. Rev. D **56**, 2743 (1997).
[19] D. Lurié, *Particles and Fields* (Interscience Publishers, New York, 1968).
[20] H.-X. Chen, V. Dmitrasinovic, A. Hosaka, K. Nagata, and S.-L. Zhu, Phys. Rev. D **78**, 054021 (2008).
[21] M. Lüscher and U. Wolff, Nucl. Phys. **B339**, 222 (1990).
[22] C. Michael, Nucl. Phys. **B259**, 58 (1985).
[23] J. A. Oller and U.-G. Meißner, Phys. Lett. B **500**, 263 (2001).
[24] D. Mohler, Proc. Sci. LATTICE2012 (2012) 003.
[25] J. Bulava, R. G. Edwards, E. Engelson, B. Joó, H.-W. Lin, C. Morningstar, D. G. Richards, and S. J. Wallace, Phys. Rev. D **82**, 014507 (2010).
[26] S. Dürr, Z. Fodor, J. Frison, C. Hoelbling, R. Hoffmann, S. Katz, S. Krieg, T. Kurth, L. Lellouch, T. Lippert, K. Szabo, and G. Vulvert, Science **322**, 1224 (2008).
[27] M. Lage, U.-G. Meißner, and A. Rusetsky, Phys. Lett. B **681**, 439 (2009).
[28] M. Döring, J. Haidenbauer, U.-G. Meißner, and A. Rusetsky, Eur. Phys. J. A **47**, 163 (2011).
[29] D. Jido, J. A. Oller, E. Oset, A. Ramos, and U.-G. Meißner, Nucl. Phys. **A725**, 181 (2003).
[30] A. M. Torres, M. Bayar, D. Jido, and E. Oset, Phys. Rev. C **86**, 055201 (2012).
[31] E. E. Kolomeitsev and M. F. M. Lutz, Phys. Lett. B **585**, 243 (2004).
[32] S. Sarkar, E. Oset, and M. J. VicenteVacas, Nucl. Phys. A **750**, 294 (2005).