Branching ratios of B_c meson decaying to vector and axial-vector mesons

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We investigate the weak decays of B_c mesons in Cabibbo-Kobayashi-Maskawa-favored and -suppressed modes. We present a detailed analysis of the B_c meson decaying to a vector meson (V) and an axial-vector meson (A) in the final state. We also give the form factors involving the $B_c \to A$ transition in the Isgur-Scora-Grinstein-Wise II framework and, consequently, predict the branching ratios of $B_c \to VA$ and AA decays.

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I. INTRODUCTION

The B_c meson was first discovered by the CDF collaboration at Fermilab [1] in 1998. At present, a more precise measurement of its mass and lifetime is available from the Particle Data Group [2], i.e., $M_{B_c} = 6.277 \pm 0.006$ GeV and $\tau_{B_c} = (0.453 \pm 0.042) \times 10^{-12}$ s. It is believed that LHC-b is expected to produce 5×10^{10} events per year [3–6], which is around 10% of the total B meson data. This will provide a rich amount of information regarding the B_c meson.

The B_c meson is a unique Standard Model particle, being a quark-antiquark bound state ($b\bar{c}$) consisting of two heavy quarks of different flavors, and is therefore flavor asymmetric. The study of the B_c meson is of special interest as compared to the flavor-neutral heavy quarkonium ($b\bar{b}$, $c\bar{c}$) states; while the former only decays *via* weak interactions, the latter predominantly decays via strong interactions and/or electromagnetic interactions. The decay processes of the B_c meson can be divided into three categories: (i) decay of the b quark with a c quark being a spectator, (ii) decay of the c quark with a b quark being a spectator, and (iii) the relatively suppressed annihilation of b and $\bar{c}$, which is ignored in the present work. One can find several theoretical works based on a variety of quark models [7–18] for the semileptonic and nonleptonic decays of B_c emitting s-wave mesons, pseudoscalar (P) mesons, and vector (V) mesons. Relatively less attention has been paid to the p-wave meson-emitting weak decays of the B_c meson [19–25]. In the recent past, several relativistic and nonrelativistic quark models [13,15,19–22] have been used by employing the factorization approach to calculate the branching ratios (BRs) of the B_c meson decaying to a p-wave charmonium ($c\bar{c}$) in the final state. Most recently, the Salpeter method [24] and the Improved Bethe-Salpeter Approach [25] were used to probe nonleptonic decays of the B_c meson. On the experimental side, more measurements regarding the B_c meson will be available soon at the Large Hadron Collider (LHC), LHC-b, and Super-B experiments. The high-precision instrumentation at these experiments may provide precise measurement of BRs of the order of (10^{-6}), which makes the study of B_c meson decays more interesting. The developing theoretical and experimental aspects of B_c meson physics motivate us to investigate weak hadronic decays of B_c mesons emitting vector and axial-vector (A) mesons in the final state. We employ the improved Isgur-Scora-Grinstein-Wise quark model (known as the ISGW II Model) [26,27] to obtain $B_c \to A$ transition form factors. Using the factorization approach, we calculate the decay amplitudes and predict the branching ratios of $B_c \to VA/AA$ decays. For the $B_c \to V$ transition form factors we rely on our previous work [18], which was based on flavor-dependence effects in the Bauer-Stech-Wirbel (BSW) model framework [28].

The presentation of the paper is as follows. We discuss the mass spectrum and the methodology in Secs. II and III, respectively. Decay constants are discussed in Sec. IV. We present the $B_c \to A$ transition form factors in the ISGW II Model and give a brief account of $B_c \to V$ transition form factors in Sec. V. Consequently, the branching ratios are estimated. Results and discussions are presented in Sec. VI, and last Sec. VII contains a summary and conclusions.

II. MASS SPECTRUM

Two types of axial-vector mesons exist, $^3P_1$ ($J^{PC} = 1^{++}$) and $^1P_1$ ($J^{PC} = 1^{+-}$), with respect to the quark model $q\bar{q}$ assignments. These states can exhibit two kinds of mixing behavior: mixing between $^3P_1$ or $^1P_1$ states themselves, and mixing among $^3P_1$ or $^1P_1$ states. The following non-strange and uncharmed mesons states have been observed experimentally [2]:

(a) $^3P_1$ multiplet consists of an isovector $a_1(1.230)$ and four isoscalars: $f_1(1.285)$, $f_1(1.420)$, $f_1'(1.512)$, and $\chi_{c1}(3.511)$.

(b) $^1P_1$ multiplet consists of an isovector $b_1(1.229)$ and three isoscalars: $h_1(1.170)$, $h_1'(1.380)$, and $h_{c1}(3.526)$, where the spin and parity of the

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$h_{c1}(3.526)$ and the C-parity of $h'_1(1.380)$ have yet to be confirmed.[1]

In the present work, we use the following mixing scheme for the isoscalar (1^{++}) mesons:

$$\begin{aligned} f_1(1.285) &= \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})\cos\phi_A + (s\bar{s})\sin\phi_A, \\ f'_1(1.512) &= \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})\sin\phi_A - (s\bar{s})\cos\phi_A, \\ \chi_{c1}(3.511) &= (c\bar{c}). \end{aligned} \tag{1}$$

Likewise, mixing for isoscalar (1^{+-}) mesons is given by

$$\begin{aligned} h_1(1.170) &= \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})\cos\phi_{A'} + (s\bar{s})\sin\phi_{A'}, \\ h'_1(1.380) &= \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})\sin\phi_{A'} - (s\bar{s})\cos\phi_{A'}, \\ h_{c1}(3.526) &= (c\bar{c}), \end{aligned} \tag{2}$$

with

$$\phi_{A(A')} = \theta(\text{ideal}) - \theta_{A(A')}(\text{physical}).$$

It has been observed that $f_1(1.285) \to 4\pi/\eta\pi\pi$, $f'_1(1.512) \to K\bar{K}\pi$, $h_1(1.170) \to \rho\pi$, and $h'_1 \to K\bar{K}^*/\bar{K}K^*$, predominantly, which seems to favor the ideal mixing for both 1^{++} and 1^{+-} nonets, i.e.,

$$\phi_A = \phi_{A'} = 0°. \tag{3}$$

The hidden-flavor diagonal $^3P_1$ and $^1P_1$ states have opposite C-parity, and therefore cannot mix. However, there is no restriction on such mixing in strange and charmed states, which are most likely a mixture of $^3P_1$ and $^1P_1$ states. States involving strange partners of $A(J^{PC} = 1^{++})$ and $A'(J^{PC} = 1^{+-})$ states, i.e., K_{1A} and $K_{1A'}$ mesons mix to generate the physical states in the following manner:

$$\begin{aligned} K_1(1.270) &= K_{1A}\sin\theta_K + K_{1A'}\cos\theta_K, \\ \underline{K}_1(1.400) &= K_{1A}\cos\theta_K - K_{1A'}\sin\theta_K. \end{aligned} \tag{4}$$

Numerous analyses based on phenomenological studies indicate that the mixing angle θ_K of strange axial-vector meson states lies in the vicinity of $\sim 35°$ and $\sim 55°$; see Ref. [29] for details. Experimental information based on $\tau \to K_1(1.270)/K_1(1.400) + \nu_\tau$ data yields $\theta_K = \pm 37°$ and $\theta_K = \pm 58°$ [30]. However, the negative mixing-angle solutions are favored by $D \to K_1(1.270)\pi/K_1(1.400)\pi$ decays and experimental measurements of the ratio of $K_1\gamma$ production in B decays [31]. Following the discussions given in Ref. [29], which states that a mixing angle $\theta_K \sim 35°$ is preferred over $\sim 55°$, we use $\theta_K = -37°$ in our numerical calculations. This is based on the observation that the choice of angle for f-f' and h-h' mixing schemes (which are close to ideal mixing) are intimately related to the choice of the mixing angle θ_K.

[1]Here quantities in the brackets indicate their respective masses (in GeV).

In general, mixing of charmed and strange charmed states is given by

$$\begin{aligned} D_1(2.427) &= D_{1A}\sin\theta_{D_1} + D_{1A'}\cos\theta_{D_1}, \\ \underline{D}_1(2.422) &= D_{1A}\cos\theta_{D_1} - D_{1A'}\sin\theta_{D_1}, \end{aligned} \tag{5}$$

and

$$\begin{aligned} D_{s1}(2.460) &= D_{s1A}\sin\theta_{D_{s1}} + D_{s1A'}\cos\theta_{D_{s1}}, \\ \underline{D}_{s1}(2.535) &= D_{s1A}\cos\theta_{D_{s1}} - D_{s1A'}\sin\theta_{D_{s1}}. \end{aligned} \tag{6}$$

As pointed out in Ref. [31], for heavy mesons the heavy-quark spin S_Q and the total angular momentum of the light antiquark can be used separately as good quantum numbers. In the heavy-quark limit, the physical mass eigenstates $P_1^{3/2}$ and $P_1^{1/2}$ with $J^P = 1^+$ can be expressed as a combination of the $^3P_1$ and $^1P_1$ states as

$$\begin{aligned} |P_1^{1/2}\rangle &= -\sqrt{\frac{1}{3}}|^1P_1\rangle + \sqrt{\frac{2}{3}}|^3P_1\rangle, \\ |P_1^{3/2}\rangle &= \sqrt{\frac{2}{3}}|^1P_1\rangle + \sqrt{\frac{1}{3}}|^3P_1\rangle. \end{aligned} \tag{7}$$

Thus, the states $D_1(2.427)$ and $\underline{D}_1(2.422)$ can be identified as $P_1^{1/2}$ and $P_1^{3/2}$, respectively. However, beyond the heavy-quark limit, there is a mixing between $P_1^{1/2}$ and $P_1^{3/2}$, given by

$$\begin{aligned} D_1(2.427) &= D_1^{1/2}\cos\theta_2 + D_1^{3/2}\sin\theta_2, \\ \underline{D}_1(2.422) &= -D_1^{1/2}\sin\theta_2 + D_1^{3/2}\cos\theta_2. \end{aligned} \tag{8}$$

Similarly, for strange charmed axial-vector mesons,

$$\begin{aligned} D_{s1}(2.460) &= D_{s1}^{1/2}\cos\theta_3 + D_{s1}^{3/2}\sin\theta_3, \\ \underline{D}_{s1}(2.535) &= -D_{s1}^{1/2}\sin\theta_3 + D_{s1}^{3/2}\cos\theta_3. \end{aligned} \tag{9}$$

A detailed analysis by Belle [32] yields the mixing angle $\theta_2 = (-5.7 \pm 2.4)°$, while the quark potential model [33,34] yields $\theta_3 \approx 7°$.

For ω and ϕ vector meson states, we consider ideal mixing, i.e., $\omega = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ and $\phi = \frac{1}{\sqrt{2}}(s\bar{s})$ [2].

III. METHODOLOGY

A. Weak Hamiltonian

The QCD-modified weak Hamiltonian [35] generating the B_c decay involving the $b \to c$ transition in Cabibbo-Kobayashi-Maskawa (CKM)-enhanced modes ($\Delta b = 1$, $\Delta C = 1$, $\Delta S = 0$; $\Delta b = 1$, $\Delta C = 0$, $\Delta S = -1$) is given by

$$H_w^{\Delta b=1}=\frac{G_F}{\sqrt{2}}\{V_{cb}V_{ud}^*[c_1(\mu)(\bar{c}b)(\bar{d}u)+c_2(\mu)(\bar{c}u)(\bar{d}b)] + V_{cb}V_{cs}^*[c_1(\mu)(\bar{c}b)(\bar{s}c)+c_2(\mu)(\bar{c}c)(\bar{s}b)] + V_{cb}V_{us}^*[c_1(\mu)(\bar{c}b)(\bar{s}u)+c_2(\mu)(\bar{c}u)(\bar{s}b)] + V_{cb}V_{cd}^*[c_1(\mu)(\bar{c}b)(\bar{d}c)+c_2(\mu)(\bar{c}c)(\bar{d}b)]\}, \quad (10)$$

and CKM-suppressed ($\Delta b = 1, \Delta C = 1, \Delta S = -1$; $\Delta b = 1, \Delta C = 1, \Delta S = 1$; $\Delta b = 1, \Delta C = -1, \Delta S = -1$; $\Delta b = 1, \Delta C = -1, \Delta S = 0$) $b \to u$ transitions are given by

$$H_w^{\Delta b=1} = \frac{G_F}{\sqrt{2}}\{V_{ub}V_{cs}^*[c_1(\mu)(\bar{u}b)(\bar{s}c) + c_2(\mu)(\bar{s}b)(\bar{u}c)] + V_{ub}V_{ud}^*[c_1(\mu)(\bar{u}b)(\bar{d}u) + c_2(\mu)(\bar{d}b)(\bar{u}u)] + V_{ub}V_{us}^*[c_1(\mu)(\bar{u}b)(\bar{s}u) + c_2(\mu)(\bar{s}b)(\bar{u}u)] + V_{ub}V_{cd}^*[c_1(\mu)(\bar{u}b)(\bar{d}c) + c_2(\mu)(\bar{u}c)(\bar{d}b)]\}, \quad (11)$$

where $\bar{q}q \equiv \bar{q}\gamma_\mu(1-\gamma_5)q$, G_F is the Fermi constant and V_{ij} are the CKM matrix elements; c_1 and c_2 are the standard perturbative QCD coefficients, usually taken as $\mu \approx m_b^2$. In addition to the bottom-changing decays, the bottom-conserving decay channel is also available for the B_c meson, where the charm quark decays to an s or a d quark. However, in the case of $B_c \to VA/AA$ decays, these modes are kinematically forbidden.

B. Decay amplitudes

In the generalized factorization hypothesis the decay amplitudes can be expressed as a product of the matrix elements of weak currents (up to the weak-scale factor of $\frac{G_F}{\sqrt{2}} \times$ CKM elements $\times$ QCD factor), given by

$$\langle MA|H_w|B_c\rangle \sim \langle M|J^\mu|0\rangle\langle A|J_\mu|B_c\rangle + \langle A|J^\mu|0\rangle\langle M|J_\mu|B_c\rangle,$$
$$\langle MA'|H_w|B_c\rangle \sim \langle M|J^\mu|0\rangle\langle A'|J_\mu|B_c\rangle + \langle A'|J^\mu|0\rangle\langle M|J_\mu|B_c\rangle, \quad (12)$$

where $M = V$ or A. Using Lorentz invariance, the hadronic transition matrix elements [26–28] for the relevant weak current between meson states can be expressed as

$$\langle V(k_V)|A_\mu|0\rangle = \varepsilon_\mu^* f_V m_V,$$
$$\langle V(k_V,\varepsilon)|V_\mu|B_c(k_{B_c})\rangle = -i\frac{2}{m_{B_c}+m_V} \times \epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}k_{B_c}^\alpha k_V^\beta V^{B_cV}(q^2),$$
$$\langle V(k_V,\varepsilon)|A_\mu|B_c(k_{B_c})\rangle = (m_{B_c}+m_V)\varepsilon_\mu^* A_1^{B_cV}(q^2) - (\varepsilon^*\cdot k_{B_c}) \times (k_{B_c}+k)_\mu \frac{A_2^{B_cV}(q^2)}{m_{B_c}+m_V} - 2m_V\frac{\varepsilon^*\cdot k_{B_c}}{q^2}q^\mu \times [A_3^{B_cV}(q^2) - A_0^{B_cV}(q^2)], \quad (13)$$

where $q = (k_{B_c} - k_V)_\mu$, $V_3^{B_cV}(0) = V_0^{B_cV}(0)$, and

$$A_3^{B_cV}(q^2) = \frac{m_{B_c}+m_V}{2m_V}A_1^{B_cV}(q^2) - \frac{m_{B_c}-m_V}{2m_V}A_2^{B_cV}(q^2). \quad (14)$$

Similarly, for axial-vector meson states,

$$\langle A(k_A,\varepsilon)|A_\mu|0\rangle = \varepsilon_\mu^* m_A f_A,$$
$$\langle A'(k_{A'},\varepsilon)|A_\mu|0\rangle = \varepsilon_\mu^* m_{A'} f_{A'}, \quad (15)$$

$$\langle A(k_A,\varepsilon)|V_\mu|B_c(k_{B_c})\rangle = l\varepsilon_\mu^* + c_+(\varepsilon^*\cdot k_{B_c})(k_{B_c}+k_A)_\mu + c_-(\varepsilon^*\cdot k_{B_c})(k_{B_c}-k_A)_\mu,$$
$$\langle A(k_A,\varepsilon)|A_\mu|B_c(k_{B_c})\rangle = iq'\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}(k_{B_c}+k_A)^\alpha \times (k_{B_c}-k_A)^\beta,$$
$$\langle A'(k_{A'},\varepsilon)|V_\mu|B_c(k_{B_c})\rangle = r\varepsilon_\mu^* + s_+(\varepsilon^*\cdot k_{B_c})(k_{B_c}+k_{A'})_\mu + s_-(\varepsilon^*\cdot k_{B_c})(k_{B_c}-k_{A'})_\mu,$$
$$\langle A'(k_A,\varepsilon)|A_\mu|B_c(k_{B_c})\rangle = iv\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}(k_{B_c}+k_{A'})^\alpha \times (k_{B_c}-k_{A'})^\beta, \quad (16)$$

where $q_\mu = (k_{B_c} - k_A)_\mu$.

It may be noted that the $B_c \to A/A'$ transition form factors in the ISGW II framework are related to the BSW-type form factor notations [28], i.e., A, $V_{0,1,2}$ are as follows:

$$A(q^2) = -q'(q^2)(m_{B_c}+m_A),$$
$$V_1(q^2) = l(q^2)/(m_{B_c}+m_A),$$
$$V_2(q^2) = -c_+(q^2)(m_{B_c}+m_A),$$
$$V_0(q^2) = \frac{1}{(2m_A)}[(m_{B_c}+m_A)V_1(q^2) - (m_{B_c}-m_A)V_2(q^2) - q^2c_-(q^2)]. \quad (17)$$

Sandwiching the weak Hamiltonian (10) and (11) between the initial and final states, the decay amplitudes for various $B_c \to MA$ decay modes ($M = V$ or A) can be obtained for the following three categories [28]:

(1) Class I transitions consist those decays which are caused by color-favored diagrams. The decay amplitudes are proportional to a_1, where $a_1(\mu) = c_1(\mu) + \frac{1}{N_c}c_2(\mu)$, and N_c is the number of colors.

(2) Class II transitions consist of those decays which are caused by color-suppressed diagrams. The decay amplitude in this class is proportional to a_2, i.e., for the color-suppressed modes $a_2(\mu) = c_2(\mu) + \frac{1}{N_c}c_1(\mu)$.

(3) Class III transitions consist of those decays which are caused by the interference of color-singlet and color-neutral currents, and consist of both color-favored and color-suppressed diagrams, i.e., the amplitudes a_1 and a_2 interfere.

For numerical calculations, we follow the convention of taking $N_c = 3$ to fix the QCD coefficients a_1 and a_2, where we use [35]

$$c_1(\mu) = 1.12, \qquad c_2(\mu) = -0.26 \quad \text{at } \mu \approx m_b^2.$$

A detailed analysis regarding N_c counting and the role of color-octet current operators is available in Ref. [34]. It may be noted that N_c, the number of color degrees of freedom, may be treated as a phenomenological parameter in weak meson decays, which account for nonfactorizable contributions. It implies that the effective expansion parameter is something like $1/(4\pi)N_c$, $1/N_c^2 \ldots$, or nonleading $1/N_c$ terms are suppressed for the some reason [35]. In order to study the variation in decay rates and branching ratios, we effectively vary the parameter N_c from 3 to 10. The obtained results are thus presented as an average with uncertainties between branching ratios at $N_c = 3$ to $N_c = 10$, taking into account the constructive interference observed for B meson decays involving both the color-favored and color-suppressed diagrams [35]. We take the ratio a_2/a_1 to be positive in the present calculations.

C. Decay widths

Like vector mesons (V), axial-vector mesons (A) also carry spin degrees of freedom; therefore, the decay rate [31] of $B_c \to VA$ is composed of three independent helicity amplitudes H_0, H_{+1}, and H_{-1}, which is given by

$$\Gamma(B_c \to MA) = \frac{p_c}{8\pi m_{B_c}^2}(|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2), \quad (18)$$

where p_c is the magnitude of the three-momentum of a final-state particle in the rest frame of the B_c meson, and $M = V$ or A. The helicity amplitudes H_0, H_{+1}, and H_{-1} are defined in terms of the coefficients a, b, and c as follows:

$$H_{\pm 1} = a \pm c(x^2 - 1)^{1/2}, \qquad H_0 = -ax - b(x^2 - 1), \quad (19)$$

where

$$x = \frac{m_{B_c}^2 - m_M^2 - m_A^2}{2m_A m_M}, \quad (20)$$

such that

$$A(B \to MA) \equiv \varepsilon^*_{M\mu}\varepsilon^*_{A\nu}[a g^{\mu\nu} + b k^\mu_{B_c} k^\nu_{B_c} + ic\epsilon^{\mu\nu\alpha\beta}k_{B_c\alpha}k_{M\beta}]. \quad (21)$$

The coefficients a, b, and c describe the s-, d-, and p-wave contributions, respectively. m_M and m_A denotes masses of respective mesons.

IV. DECAY CONSTANTS

The decay constants for axial-vector mesons are defined by the matrix elements given in the previous section. It may be pointed out that the axial-vector meson states are represented by a 3×3 matrix and they transform under the charge conjugation [30] as

$$M_a^b(^3P_1) \to M_b^a(^3P_1), \qquad M_a^b(^1P_1) \to -M_b^a(^1P_1), \quad (a = 1, 2, 3). \quad (22)$$

Since the weak axial-vector current transfers as $(A_\mu)_a^b \to (A_\mu)_b^a$ under charge conjugation, the decay constant of the $^1P_1$ meson should vanish in the SU(3) flavor limit [30]. Experimental information based on τ decays gives a decay constant of $f_{K_1}(1270) = 0.175 \pm 0.019\,\text{GeV}$ [20,31], while the decay constant for $\underline{K}_1(1.400)$ can be obtained from the relation $f_{\underline{K}_1}(1.400)/f_{K_1}(1.270) = \cot\theta_K$, i.e., $f_{\underline{K}_1}(1.400) = (-0.232 \pm 0.025)$ GeV for $\theta_K = -37°$ (the value used in the present work [31]). In the case of non-strange axial-vector mesons, Nardulli and Pham [36] used the mixing angle for strange axial-vector mesons and SU(3) symmetry to determine $f_{a_1} = 0.223$ GeV for $\theta_K = -58°$. Since a_1 and f_1 lie in the same nonet we assume $f_{f_1} \approx f_{a_1}$ under SU(3) symmetry. Due to charge conjugation invariance, the decay constants for $^1P_1$ non-strange neutral mesons $b_1^0(1.235)$, $h_1(1.170)$, and $h_1'(1.380)$ vanish. Also, owing to G-parity conservation in the isospin limit, the decay constant $f_{b_1} = 0$.

For the decay constants of charmed and strange-charmed states, we use $f_{D_{1A}} = -0.127$ GeV, $f_{D'_{1A}} = 0.045$ GeV, $f_{D_{s1A}} = -0.121$ GeV, $f_{D'_{s1A}} = 0.038$ GeV, and $f_{\chi_{c1}} \approx -0.160$ GeV [34,37].

On the other hand, the decay constants for vector mesons are relatively trivial: we use $f_\rho = 0.221$ GeV, $f_{K^*} = 0.220$ GeV, $f_{D^*} = 0.245$ GeV, $f_{D_s^*} = 0.273$ GeV, $f_\phi = 0.195$ GeV, $f_\omega = 0.229$ GeV, and $f_{J/\psi} = 0.411$ GeV [2,15,31,37] in numerical calculations.

V. FORM FACTORS

In this section, we give a short description of how to calculate $B_c \to A$ and $B_c \to V$ transition form factors.

A. $B_c \to A/A'$ transition form factors

We use the ISGW II Model [27] to calculate $B \to A/A'$ transition form factors. The ISGW model is a nonrelativistic constituent quark model [26], which obtains an exponential q^2 dependence of the form factors. It employ variational solutions of the Schrödinger equation based on the harmonic oscillator wave functions, using the Coulomb and linear potentials. In general, the form factors evaluated are considered reliable at $q^2 = q_m^2$, with the maximum momentum transfer being $(m_B - m_X)^2$. The reason for this is that the form factor's q^2 dependence in the ISGW model is proportional to $e^{-(q_m^2 - q^2)}$, and hence the form factor decreases exponentially as a function of $(q_m^2 - q^2)$. This has been improved in the ISGW II model [27] in which the form factor has a more realistic behavior at large $(q_m^2 - q^2)$, which is expressed in terms of a certain polynomial term. In addition to this, the ISGW II model incorporates a number of improvements, such as the heavy-quark symmetry constraints, the heavy-quark-symmetry-breaking color-magnetic interaction, relativistic corrections, etc.

The form factors have the following simplified expressions in the ISGW II model for $B_c \to A/A'$ transitions caused by a $b \to c$ quark transition [26,27]:

$$\begin{aligned} l &= \tilde{m}_{B_c}\beta_{B_c}\left[\frac{1}{\mu_-} + \frac{m_c\tilde{m}_A(\tilde{\omega}-1)}{\beta_{B_c}^2}\right.\\ &\quad\left.\times\left(\frac{5+\tilde{\omega}}{6m_q} - \frac{m_c\beta_{B_c}^2}{2\mu_-\beta_{B_cA}^2}\right)\right]F_5^{(l)},\\ c_+ + c_- &= -\frac{\tilde{m}_A}{2m_{B_c}\beta_{B_c}}\left(1 - \frac{m_c^2\beta_{B_c}^2}{2m_A\mu_-\beta_{B_cA}^2}\right)F^{(c_++c_-)},\\ c_+ - c_- &= -\frac{\tilde{m}_A}{2m_{B_c}\beta_{B_c}}\left(\frac{\tilde{\omega}+2}{3} - \frac{m_c^2\beta_{B_c}^2}{2m_A\mu_-\beta_{B_cA}^2}\right)F^{(c_+-c_-)},\\ q' &= \frac{m_c}{2\tilde{m}_A\beta_{B_c}}\left(\frac{5+\tilde{\omega}}{6m_q}\right)F_5^{(q)}, \end{aligned} \tag{23}$$

$$\begin{aligned} r &= \frac{\tilde{m}_{B_c}\beta_{B_c}}{\sqrt{2}}\left[\frac{1}{\mu_+} + \frac{\tilde{m}_A}{3\beta_{B_c}^2}(\tilde{\omega}-1)^2\right]F_5^{(r)},\\ s_+ + s_- &= \frac{m_c}{\sqrt{2}\tilde{m}_{B_c}\beta_{B_c}}\left(\frac{m_c\beta_{B_c}^2}{2\mu_+\beta_{B_cA}^2}\right)F^{(s_++s_-)},\\ s_+ - s_- &= \frac{1}{\sqrt{2}\beta_{B_c}}\left(\frac{4-\tilde{\omega}}{3} - \frac{m_c^2\beta_{B_c}^2}{2\tilde{m}_A\mu_+\beta_{B_cA}^2}\right)F^{(s_+-s_-)},\\ v &= \left[\frac{\tilde{m}_{B_c}\beta_{B_c}}{4\sqrt{2}m_c\tilde{m}_A} + \frac{(\tilde{\omega}-1)m_c}{6\sqrt{2}\tilde{m}_A\beta_{B_c}}\right]F_5^{(q)}, \end{aligned} \tag{24}$$

where

$$\mu_\pm = \left(\frac{1}{m_c} \pm \frac{1}{m_b}\right)^{-1}, \tag{25}$$

the $t(\equiv q^2)$ dependence is given by

$$\tilde{\omega} - 1 = \frac{t_m - t}{2\bar{m}_{B_c}\bar{m}_A}, \tag{26}$$

and

$$\begin{aligned} F_5^{(l)} &= F_5^{(r)} = F_5\left(\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{1/2}\left(\frac{\bar{m}_A}{\tilde{m}_A}\right)^{1/2},\\ F_5^{(c_++c_-)} &= F_5^{(s_++s_-)} = F_5\left(\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{-3/2}\left(\frac{\bar{m}_A}{\tilde{m}_A}\right)^{1/2},\\ F_5^{(c_+-c_-)} &= F_5^{(s_+-s_-)} = F_5\left(\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{-1/2}\left(\frac{\bar{m}_A}{\tilde{m}_A}\right)^{-1/2},\\ F_5^{(q')} &= F_5^{(v)} = F_5\left(\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{-1/2}\left(\frac{\bar{m}_A}{\tilde{m}_A}\right)^{-1/2}. \end{aligned} \tag{27}$$

The function F_5 is given by

$$F_5 = \left(\frac{\tilde{m}_A}{\tilde{m}_{B_c}}\right)^{1/2}\left(\frac{\beta_{B_c}\beta_A}{\beta_{B_cA}}\right)^{5/2}\left[1 + \frac{1}{18}\chi^2(t_m - t)\right]^{-3}, \tag{28}$$

with

$$\begin{aligned} \chi^2 &= \frac{3}{4m_bm_c} + \frac{3m_c^2}{2\bar{m}_{B_c}\bar{m}_A\beta_{B_cA}^2}\\ &\quad + \frac{1}{\bar{m}_{B_c}\bar{m}_A}\left(\frac{16}{33-2n_f}\right)\ln\left[\frac{\alpha_S(\mu_{QM})}{\alpha_S(m_c)}\right], \end{aligned} \tag{29}$$

and

$$\beta_{B_cA}^2 = \frac{1}{2}(\beta_{B_c}^2 + \beta_A^2). \tag{30}$$

$\tilde{m}$ is the sum of the meson's constituent quarks masses, $\bar{m}$ is the hyperfine-averaged physical masses, n_f is the number of active flavors (which is taken to be five in the present case), $t_m = (m_{B_c} - m_A)^2$ is the maximum momentum transfer, and $\mu_{\rm QM}$ is the quark-model scale. The values of the parameter β for different s-wave and p-wave mesons [26,27] are given in Table I. We use the following quark masses to calculate the form factors for $B_c \to A/A'$ transitions, which are given in Tables II and III:

$$m_u = m_d = 0.31 \pm 0.04, \qquad m_s = 0.49 \pm 0.04,$$
$$m_c = 1.7 \pm 0.04, \quad \text{and} \quad m_b = 5.0 \pm 0.04.$$

It may be pointed out that the form factors are sensitive to the choice of quark masses. The variation in quark masses, particularly in light-quark sector, may lead to uncertainties in the form factors; therefore, we allowed a certain range based on the literature [38]. These uncertainties in the form factors are shown in Tables II and III.

B. $B_c \to V$ transition form factors

For $B_c \to V$ transition form factors we use our previous work [18]—which was based on the BSW framework [28]—in which one of the authors investigated the possible flavor dependence in $B_c \to P/V$ form factors and consequently in $B_c \to PP/PV$ decay widths. It may be noted that in the BSW model [28] the form factors depend upon the average transverse quark momentum inside a meson ω, which is fixed in the model to 0.40 GeV. However, it has been pointed out that ω—being a dimensional quantity—may show flavor dependence. Therefore, it may not be justified to take the same ω for all the mesons. Following

TABLE I. The values of the parameter β for s-wave and p-wave mesons in the ISGW II quark model.

Quark content	$u\bar{d}$	$u\bar{s}$	$s\bar{s}$	$c\bar{u}$	$c\bar{s}$	$u\bar{b}$	$s\bar{b}$	$c\bar{c}$	$b\bar{c}$
β_s (GeV)	0.41	0.44	0.53	0.45	0.56	0.43	0.54	0.88	0.92
β_p (GeV)	0.28	0.30	0.33	0.33	0.38	0.35	0.41	0.52	0.60

TABLE II. $B_c \to A$ transition form factors calculated at q^2_{max} in ISGW II quark model are given in ISGW-model-type (upper) and BSW-model-type notations (lower), respectively.

Modes	Transition	l	c_+	c_-	q'
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = -1$	$B_c \to D_1$	$-3.529^{+0.504}_{-0.430}$	-0.048 ± 0.001	-0.006 ± 0.00	-0.074 ± 0.002
	$B_c \to D_{s1}$	-2.860 ± 0.258	-0.061 ± 0.001	-0.006 ± 0.001	-0.095 ± 0.002
$\Delta b = 1,\ \Delta C = 1,\ \Delta S = 0$	$B_c \to \chi_{c1}(c\bar{c})$	-1.182 ± 0.038	-0.103 ± 0.003	-0.006 ± 0.001	-0.130 ± 0.003
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = -1$	$B_c \to a_1$	-0.243 ± 0.008	-0.036 ± 0.001	0.015 ± 0.001	-0.074 ± 0.002
	$B_c \to f_1$	-0.242 ± 0.008	-0.036 ± 0.001	0.015 ± 0.001	-0.074 ± 0.002
	$B_c^- \to f_1'$	-0.363 ± 0.010	-0.049 ± 0.002	0.018 ± 0.001	-0.095 ± 0.002

Modes	Transition	A	V_1	V_2	V_0
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = -1$	$B_c \to D_1$	0.646 ± 0.15	$-0.406^{+0.63}_{-0.50}$	$0.421^{+0.004}_{-0.003}$	$-1.081^{+0.112}_{-0.097}$
	$B_c^- \to D_{s1}$	0.829 ± 0.020	-0.326 ± 0.029	0.535 ± 0.011	-1.00 ± 0.044
$\Delta b = 1,\ \Delta C = 1,\ \Delta S = 0$	$B_c \to \chi_{c1}(c\bar{c})$	1.273 ± 0.030	-0.120 ± 0.005	1.008 ± 0.032	-0.572 ± 0.007
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = -1$	$B_c \to a_1$	0.553 ± 0.013	-0.032 ± 0.001	0.270 ± 0.010	-0.495 ± 0.009
	$B_c \to f_1$	0.558 ± 0.013	-0.032 ± 0.001	0.272 ± 0.009	-0.476 ± 0.009
	$B_c^- \to f_1'$	0.733 ± 0.017	-0.047 ± 0.001	0.378 ± 0.013	-0.626 ± 0.011

the analysis described in Ref. [18], we estimate ω for different mesons from $|\psi(0)|^2$, i.e., the square of the wave function at the origin obtained from the hyperfine-splitting term for the meson masses, which in turn fixes the quark masses (in GeV) to be $m_u = m_d = 0.31 \pm 0.04$, $m_s = 0.49 \pm 0.04$, $m_c = 1.7 \pm 0.04$, and $m_b = 5.0 \pm 0.04$ for $\alpha_s(m_b) = 0.19$, $\alpha_s(m_c) = 0.25$, and $\alpha_s = 0.48$ (for light flavors u, d, and s). In addition, a variation in α_s may lead to an uncertainty in quark masses [38] and consequently in the form factors. For further details we refer the interested reader to Ref. [18]. We find that all of the form factors get significantly enhanced due to the flavor dependence of ω. The obtained form factors along with the corresponding uncertainties due to the variation in the quark masses are shown in Table IV.

It may also be noted that consistency with the heavy-quark symmetry (HQS) requires certain form factors—such as F_1, A_0, A_2, and V—to have a dipole q^2 dependence [28]. Therefore, we use the following q^2 dependence for the different form factors:

$$A_0(q^2) = \frac{A_0(0)}{(1 - \frac{q^2}{m_P^2})^2}, \qquad A_1(q^2) = \frac{A_1(0)}{(1 - \frac{q^2}{m_A^2})},$$

$$A_2(q^2) = \frac{A_2(0)}{(1 - \frac{q^2}{m_A^2})^2} \quad \text{and} \quad V(q^2) = \frac{V(0)}{(1 - \frac{q^2}{m_V^2})^2},$$

with appropriate pole masses m_i.

TABLE III. $B_c \to A'$ transition form factors calculated at q^2_{max} in ISGW II quark model are given in ISGW-model-type (upper) and BSW-model-type notations (lower), respectively.

Modes	Transition	r	s_+	s_-	v
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = -1$	$B \to \underline{D}_1$	$2.825^{+0.355}_{-0.306}$	0.083 ± 0.000	$-0.055^{+0.005}_{-0.003}$	$0.057^{+0.009}_{-0.006}$
	$B_c \to \underline{D}_{s1}$	2.464 ± 0.193	0.102 ± 0.001	-0.060 ± 0.002	0.046 ± 0.003
$\Delta b = 1,\ \Delta C = 1,\ \Delta S = 0$	$B_c \to h_{c1}(c\bar{c})$	1.674 ± 0.044	0.143 ± 0.004	-0.039 ± 0.001	0.019 ± 0.001
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = -1$	$B_c \to b_1$	0.344 ± 0.007	0.053 ± 0.001	-0.028 ± 0.001	0.010 ± 0.000
	$B_c \to h_1$	0.337 ± 0.007	0.054 ± 0.001	-0.029 ± 0.000	0.011 ± 0.001
	$B_c^- \to h_1'$	0.512 ± 0.010	0.074 ± 0.002	-0.037 ± 0.001	0.014 ± 0.001

Modes	Transition	A	V_1	V_2	V_0
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = -1$	$B_c \to \underline{D}_1$	$-0.498^{+0.067}_{-0.055}$	$0.324^{+0.042}_{-0.034}$	-0.722 ± 0.005	$0.987^{+0.067}_{-0.53}$
	$B_c^- \to \underline{D}_{s1}$	-0.407 ± 0.033	0.280 ± 0.022	-0.898 ± 0.009	0.995 ± 0.025
$\Delta b = 1,\ \Delta C = 1,\ \Delta S = 0$	$B_c \to h_{c1}(c\bar{c})$	-0.183 ± 0.004	0.171 ± 0.003	-1.401 ± 0.032	0.742 ± 0.008
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = -1$	$B_c \to b_1$	-0.079 ± 0.002	0.046 ± 0.001	-0.401 ± 0.008	0.677 ± 0.010
	$B_c \to h_1$	-0.080 ± 0.002	0.045 ± 0.001	-0.402 ± 0.008	0.702 ± 0.010
	$B_c^- \to h_1'$	-0.105 ± 0.002	0.067 ± 0.001	-0.566 ± 0.012	0.866 ± 0.012

TABLE IV. $B_c \to V$ transition form factors at $q^2 = 0$ using flavor-dependent ω in the BSW model [34].

Modes	Transition	V	A_1	A_2	A_0
$\Delta b = 1, \Delta C = 0, \Delta S = -1$	$B_c \to D^*$	0.161 ± 0.014	0.094 ± 0.009	0.108 ± 0.012	0.079 ± 0.008
	$B_c \to D_s^*$	0.284 ± 0.09	0.171 ± 0.008	0.193 ± 0.010	0.150 ± 0.008
$\Delta b = 1, \Delta C = 1, \Delta S = 0$	$B_c \to J/\psi(c\bar{c})$	0.919 ± 0.002	0.624 ± 0.008	0.741 ± 0.020	0.564 ± 0.001
$\Delta b = 1, \Delta C = 0, \Delta S = -1$	$B_c \to \rho$	0.369 ± 0.023	0.577 ± 0.042	0.624 ± 0.046	0.410 ± 0.028
	$B_c \to \omega$	0.272 ± 0.020	0.424 ± 0.036	0.460 ± 0.040	0.296 ± 0.024
	$B_c \to \phi$	0.150 ± 0.017	0.217 ± 0.026	0.245 ± 0.029	0.144 ± 0.017

VI. RESULTS AND DISCUSSION

Using the decay constants and form factors described in Secs. IV and V, respectively, we predict the branching ratios of $B_c \to VA$ and $B_c \to AA$ decays in CKM-favored and CKM-suppressed modes.

A. $B_c \to VA$ decays

The branching ratios for B_c decaying to a vector and an axial-vector meson in the final state for CKM-favored and CKM-suppressed modes are given in column 2 of Tables V, VI, VII, VIII, IX, and X. We also give the helicity amplitudes of the corresponding decay channels in columns 3, 4, and 5 of the respective Tables V, VI, VII, VII, IX, and X. We made the following observations.

1. For CKM-favored modes

(1) The branching ratios for dominant decays in the Cabibbo-enhanced ($\Delta b = 1$, $\Delta C = 1$, $\Delta S = 0$) mode are: $\mathrm{Br}(B_c^- \to J/\psi a_1^-) = 4.14 \pm 0.26 \pm 0.05 \times 10^{-3}$, $\mathrm{Br}(B_c^- \to \rho^- \chi_{c1}) = 1.47 \pm 0.15 \pm 0.01 \times 10^{-3}$, and $\mathrm{Br}(B_c^- \to \rho^- h_{c1}) = 1.24 \pm 0.08 \pm 0.01 \times 10^{-3}$. The next-order branching ratio is $\mathrm{Br}(B_c^- \to D^{*0} D_1^-) = 2.92 \pm 0.84^{+0.52}_{-0.28} \times 10^{-5}$. We wish to remark here that the first quoted uncertainty in the branching ratios is due to the effective variation of the parameter N_c, and the second uncertainty is caused by the variation of the quark masses in the form factors. The same has been followed throughout the presentation of results, including Tables V, VI, VII, VIII, IX, and X. The branching ratios of the remaining decays are of order of magnitude (10^{-6}–10^{-7}), except for the $B_c^- \to J/\psi b_1^-$ decay, which is $\mathcal{O}(10^{-8})$.

(2) The dominant decay channels in the Cabibbo-favored ($\Delta b = 1$, $\Delta C = 0$, $\Delta S = -1$) mode are those which consist of one $\bar{c}c$-meson in the final state, i.e., $\mathrm{Br}(B_c^- \to J/\psi D_{s1}^-) = 2.35 \pm 0.25 \pm 0.01 \times 10^{-3}$, $\mathrm{Br}(B_c^- \to D_s^{*-} \chi_{c1}) = 1.08 \pm 0.08 \pm 0.01 \times 10^{-3}$, $\mathrm{Br}(B_c^- \to D_s^{*-} h_{c1}) = 8.11 \pm 0.48 \pm 0.12 \times 10^{-4}$, and $\mathrm{Br}(B_c^- \to J/\psi \underline{D}_{s1}^-) = 6.33 \pm 0.49 \pm 0.06 \times 10^{-4}$. The rest of the decay modes remain suppressed, with branching ratios of $\mathcal{O}(10^{-7} \sim 10^{-11})$.

(3) It may be noted that the branching ratios for $B_c \to VA$ decays are higher for axial-vectors $A(^3P_1)$ in the final state as compared to the $A(^1P1)$ with the same quark content, except for strange axial-meson-emitting decays, which are roughly of the same order.

(4) We find that the longitudinal helicity amplitudes are higher in magnitude for all of the decay modes.

2. For CKM-suppressed modes

(1) It is interesting to note that the branching ratios for the CKM-suppressed mode ($\Delta b = 1$, $\Delta C = 1$,

TABLE V. Branching ratios and helicity amplitudes of $B_c \to VA$ decays for the CKM-favored ($\Delta b = 1$, $\Delta C = 0$, $\Delta S = -1$) mode.

		Helicity amplitudes		
Decays	Branching ratios	$\lvert H_0 \rvert$	$\lvert H_+ \rvert$	$\lvert H_- \rvert$
$B_c^- J/\psi a_1^-$	$4.14 \pm 0.26 \pm 0.05 \times 10^{-3}$	$1.66 \pm 0.05 \pm 0.01 \times 10^{-1}$	$3.68 \pm 0.11 \pm 0.08 \times 10^{-2}$	$1.04 \pm 0.03 \pm 0.01 \times 10^{-1}$
$B_c^- J/\psi b_1^-$	$2.90 \pm 0.18 \pm 0.04 \times 10^{-8}$	$4.38 \pm 0.13 \pm 0.02 \times 10^{-4}$	$9.77 \pm 0.33 \pm 0.23 \times 10^{-5}$	$2.74 \pm 0.07 \pm 0.03 \times 10^{-4}$
$B_c^- \to \rho^- \chi_{c1}$	$1.47 \pm 0.15 \pm 0.01 \times 10^{-3}$	$1.16 \pm 0.04 \pm 0.01 \times 10^{-1}$	$2.70 \pm 0.08 \pm 0.01 \times 10^{-2}$	$3.83 \pm 0.12 \pm 0.31 \times 10^{-3}$
$B_c^- \to \rho^- h_{c1}$	$1.24 \pm 0.08 \pm 0.01 \times 10^{-3}$	$1.12 \pm 0.04 \pm 0.01 \times 10^{-1}$	$1.05 \pm 0.03 \pm 0.03 \times 10^{-2}$	$5.10 \pm 0.15 \pm 0.17 \times 10^{-3}$
$B_c^- \to D^{*0} D_1^-$	$2.92 \pm 0.84^{+0.52}_{-0.28} \times 10^{-5}$	$1.52 \pm 0.22^{+0.11}_{-0.9} \times 10^{-2}$	$5.64 \pm 0.83^{+0.61}_{-0.37} \times 10^{-3}$	$2.67 \pm 0.30^{+0.40}_{-0.26} \times 10^{-3}$
$B_c^- \to D^{*0} \underline{D}_1^-$	$2.43 \pm 0.70^{+0.44}_{-0.23} \times 10^{-6}$	$4.38 \pm 0.64^{+0.47}_{-0.28} \times 10^{-3}$	$1.52 \pm 0.22 \pm 0.02 \times 10^{-3}$	$0.91 \pm 0.14^{+2.04}_{-1.60} \times 10^{-3}$
$B_c^- \to D^{*-} D_1^0$	$1.19 \pm 0.34 \pm 0.16 \times 10^{-6}$	$1.96 \pm 0.29 \pm 0.16 \times 10^{-3}$	$1.35 \pm 0.20 \pm 0.28 \times 10^{-4}$	$2.67 \pm 0.39 \pm 0.22 \times 10^{-3}$
$B_c^- \to D^{*-} \underline{D}_1^0$	$3.71 \pm 1.06 \pm 0.51 \times 10^{-7}$	$1.09 \pm 0.16 \pm 0.93 \times 10^{-3}$	$7.43 \pm 1.09 \pm 1.55 \times 10^{-5}$	$1.49 \pm 0.22 \pm 0.12 \times 10^{-3}$

TABLE VI. Branching ratios and helicity amplitudes of $B_c \to VA$ decays for the CKM-favored ($\Delta b = 1$, $\Delta C = 0$, $\Delta S = -1$) mode.

		Helicity amplitudes		
Decays	Branching ratios	$\vert H_0\vert$	$\vert H_+\vert$	$\vert H_-\vert$
$B_c^- J/\psi D_{s1}^-$	$2.35 \pm 0.25 \pm 0.01 \times 10^{-3}$	$1.26 \pm 0.18 \pm 0.01 \times 10^{-1}$	$7.06 \pm 0.49 \pm 0.02 \times 10^{-2}$	$1.04 \pm 0.05 \pm 0.00 \times 10^{-1}$
$B_c^- J/\psi \underline{D}_{s1}^-$	$6.33 \pm 0.49 \pm 0.06 \times 10^{-4}$	$6.18 \pm 0.25 \pm 0.02 \times 10^{-2}$	$3.73 \pm 0.19 \pm 0.05 \times 10^{-2}$	$6.57 \pm 0.22 \pm 0.02 \times 10^{-2}$
$B_c^- \to D_s^{*-} \chi_{c1}$	$1.08 \pm 0.08 \pm 0.01 \times 10^{-3}$	$9.85 \pm 0.20 \pm 0.02 \times 10^{-2}$	$8.05 \pm 0.25 \pm 0.03 \times 10^{-2}$	$3.10 \pm 1.40 \pm 0.92 \times 10^{-2}$
$B_c^- \to D_s^{*-} h_{c1}$	$8.11 \pm 0.48 \pm 0.12 \times 10^{-4}$	$1.02 \pm 0.03 \pm 0.05 \times 10^{-1}$	$3.97 \pm 0.11 \pm 0.09 \times 10^{-2}$	$2.61 \pm 0.08 \pm 0.07 \times 10^{-2}$
$B_c^- \to K^{*-} \bar{D}_1^0$	$5.29 \pm 0.31^{+1.12}_{-0.57} \times 10^{-7}$	$2.04 \pm 0.06^{+0.20}_{-0.12} \times 10^{-3}$	$3.10 \pm 0.09^{+0.35}_{-0.21} \times 10^{-4}$	$1.04 \pm 0.03^{+0.22}_{-0.14} \times 10^{-4}$
$B_c^- \to K^{*-} \underline{\bar{D}}_1^0$	$6.08 \pm 0.37^{+1.35}_{-0.67} \times 10^{-8}$	$6.92 \pm 0.20^{+0.74}_{-0.42} \times 10^{-4}$	$8.57 \pm 0.26 \pm 0.40 \times 10^{-5}$	$5.12 \pm 0.15^{+1.26}_{-0.92} \times 10^{-5}$
$B_c^- \to \bar{D}^{*0} K_1^-$	$3.92 \pm 0.23 \pm 0.73 \times 10^{-8}$	$4.57 \pm 0.13 \pm 0.45 \times 10^{-4}$	$1.82 \pm 0.05 \pm 0.39 \times 10^{-5}$	$3.12 \pm 0.09 \pm 0.29 \times 10^{-4}$
$B_c^- \to \bar{D}^{*0} \underline{K}_1^-$	$6.29 \pm 0.36 \pm 1.17 \times 10^{-8}$	$5.66 \pm 0.14 \pm 0.55 \times 10^{-4}$	$2.72 \pm 0.08 \pm 0.51 \times 10^{-5}$	$4.19 \pm 0.12 \pm 0.39 \times 10^{-4}$
$B_c^- \to D_s^{*-} a_1^0$	$1.43 \pm 0.41 \pm 0.08 \times 10^{-9}$	$0.89 \pm 0.13 \pm 0.31 \times 10^{-5}$	$5.36 \pm 0.78 \pm 0.54 \times 10^{-6}$	$5.58 \pm 0.82 \pm 0.21 \times 10^{-5}$
$B_c^- \to D_s^{*-} f_1$	$1.44 \pm 0.41 \pm 0.08 \times 10^{-9}$	$0.89 \pm 0.13 \pm 0.31 \times 10^{-5}$	$5.55 \pm 0.81 \pm 0.54 \times 10^{-5}$	$5.79 \pm 0.85 \pm 0.21 \times 10^{-5}$
$B_c^- \to \phi D_{s1}^-$	$6.61 \pm 1.90 \pm 0.30 \times 10^{-9}$	$2.27 \pm 0.33 \pm 0.06 \times 10^{-4}$	$3.73 \pm 0.55 \pm 0.10 \times 10^{-5}$	$0.95 \pm 0.14 \pm 0.07 \times 10^{-5}$
$B_c^- \to \phi \underline{D}_{s1}^-$	$4.79 \pm 1.38 \pm 0.20 \times 10^{-10}$	$6.08 \pm 0.90 \pm 0.16 \times 10^{-5}$	$1.32 \pm 0.19 \pm 0.15 \times 10^{-5}$	$3.68 \pm 0.54 \pm 0.47 \times 10^{-6}$
$B_c^- \to \rho^0 D_{s1}^-$	$6.65 \pm 1.94 \pm 0.30 \times 10^{-9}$	$2.27 \pm 0.33 \pm 0.06 \times 10^{-4}$	$2.79 \pm 0.41 \pm 0.10 \times 10^{-5}$	$7.02 \pm 1.03 \pm 0.72 \times 10^{-6}$
$B_c^- \to \rho^0 \underline{D}_{s1}^-$	$5.00 \pm 1.44 \pm 0.20 \times 10^{-10}$	$6.22 \pm 0.91 \pm 0.15 \times 10^{-5}$	$1.00 \pm 0.15 \pm 0.02 \times 10^{-5}$	$2.79 \pm 0.41 \pm 0.47 \times 10^{-6}$
$B_c^- \to \omega D_{s1}^-$	$4.15 \pm 1.19 \pm 0.18 \times 10^{-10}$	$5.67 \pm 0.83 \pm 0.15 \times 10^{-5}$	$7.08 \pm 1.04 \pm 0.25 \times 10^{-6}$	$1.78 \pm 0.26 \pm 0.18 \times 10^{-6}$
$B_c^- \to \omega \underline{D}_{s1}^-$	$3.12 \pm 0.90 \pm 0.12 \times 10^{-11}$	$1.55 \pm 0.23 \pm 0.04 \times 10^{-5}$	$2.53 \pm 0.37 \pm 0.04 \times 10^{-6}$	$7.08 \pm 1.04 \pm 1.10 \times 10^{-7}$

$\Delta S = -1$) are of the order (10^{-4}–10^{-5}). The dominant decays are: $\mathrm{Br}(B_c^- \to J/\psi \underline{K}_1^-) = 2.36 \pm 0.14 \pm 0.03 \times 10^{-4}$, $\mathrm{Br}(B_c^- \to J/\psi K_1^-) = 1.49 \pm 0.09 \pm 0.02 \times 10^{-4}$, $\mathrm{Br}(B_c^- \to K^{*-} \chi_{c1}) = 7.07 \pm 0.43 \pm 0.0.04 \times 10^{-5}$, and $\mathrm{Br}(B_c^- \to K^{*-} h_{c1}) = 6.18 \pm 0.37 \pm 0.06 \times 10^{-5}$. The next-order decay has $\mathrm{Br}(B_c^- \to D^{*0} D_{s1}^-) = 2.21 \pm 0.63 \pm 0.12 \times 10^{-6}$. Branching ratios of the other decay modes are of $\mathcal{O}(10^{-7}$–$10^{-8})$.

(2) Only four decay channels have branching ratios of $\mathcal{O}(10^{-4}$–$10^{-5})$ in the CKM-suppressed ($\Delta b = 1$, $\Delta C = 1$, $\Delta S = 1$) mode, i.e., $\mathrm{Br}(B_c^- \to J/\psi D_1^-) = 1.39 \pm 0.14 \pm 0.03 \times 10^{-4}$, $\mathrm{Br}(B_c^- \to D^{*-} \chi_{c1}) = 4.95 \pm 0.33 \pm 0.03 \times 10^{-5}$, $\mathrm{Br}(B_c^- \to D^{*-} h_{c1}) = 3.88 \pm 0.23 \pm 0.04 \times 10^{-5}$, $\mathrm{Br}(B_c^- \to J/\psi \underline{D}_1^-) = 3.45 \pm 0.28 \pm 0.03 \times 10^{-5}$, and $\mathrm{Br}(B_c^- \to \rho^- \bar{D}_1^0) = 1.01 \pm 0.05^{+0.20}_{-0.11} \times 10^{-5}$. However, the branching ratios for $B_c^- \to \rho^- \underline{\bar{D}}_1^0$ and $B_c^- \to \bar{D}^{*0} a_1^-$ decays are of $\mathcal{O}(10^{-6})$; these may also be of experimental interest in the near future.

(3) In the ($\Delta b = 1$, $\Delta C = -1$, $\Delta S = -1$) mode, the branching ratios for the ($B_c^- \to D_s^{*-} \bar{D}_1^0$), ($B_c^- \to D_s^{*-} \underline{\bar{D}}_1^0$), and ($B_c^- \to D_{s1}^- \bar{D}^{*0}$) decays are $2.01 \pm 0.13^{+0.44}_{-0.24} \times 10^{-5}$, $1.71 \pm 0.12^{+0.37}_{-0.19} \times 10^{-6}$, and $1.42 \pm 0.22 \pm 0.10 \times 10^{-6}$, respectively. However, the branching ratios for the ($\Delta b = 1$, $\Delta C = -1$, $\Delta S = 0$) mode remain highly suppressed.

(4) Branching ratios for decays involving $A(^3P_1)$ mesons in the final state are higher than their $A(^1P_1)$ partners for the same flavor content. However, for

TABLE VII. Branching ratios and helicity amplitudes of $B_c \to VA$ decays for the CKM-suppressed ($\Delta b = 1$, $\Delta C = 0$, $\Delta S = -1$) mode.

		Helicity amplitudes		
Decays	Branching ratios	$\vert H_0\vert$	$\vert H_+\vert$	$\vert H_-\vert$
$B_c^- J/\psi K_1^-$	$1.49 \pm 0.09 \pm 0.02 \times 10^{-4}$	$3.12 \pm 0.09 \pm 0.02 \times 10^{-2}$	$7.22 \pm 0.23 \pm 0.17 \times 10^{-3}$	$2.03 \pm 0.06 \pm 0.02 \times 10^{-2}$
$B_c^- J/\psi \underline{K}_1^-$	$2.36 \pm 0.14 \pm 0.03 \times 10^{-4}$	$3.81 \pm 0.07 \pm 0.02 \times 10^{-2}$	$1.00 \pm 0.04 \pm 0.02 \times 10^{-2}$	$2.72 \pm 0.08 \pm 0.02 \times 10^{-2}$
$B_c^- \to K^{*-} \chi_{c1}$	$7.07 \pm 0.43 \pm 0.04 \times 10^{-5}$	$2.59 \pm 0.08 \pm 0.01 \times 10^{-2}$	$7.10 \pm 0.21 \pm 0.00 \times 10^{-3}$	$1.01 \pm 0.03 \pm 0.06 \times 10^{-3}$
$B_c^- \to K^{*-} h_{c1}$	$6.18 \pm 0.37 \pm 0.06 \times 10^{-5}$	$2.50 \pm 0.07 \pm 0.01 \times 10^{-2}$	$2.79 \pm 0.08 \pm 0.08 \times 10^{-3}$	$1.37 \pm 0.06 \pm 0.04 \times 10^{-3}$
$B_c^- \to D^{*0} D_{s1}^-$	$2.21 \pm 0.63 \pm 0.12 \times 10^{-6}$	$4.25 \pm 0.62 \pm 0.13 \times 10^{-3}$	$1.50 \pm 0.22 \pm 0.06 \times 10^{-3}$	$4.25 \pm 0.62 \pm 0.45 \times 10^{-4}$
$B_c^- \to D^{*0} \underline{D}_{s1}^-$	$1.27 \pm 0.37 \pm 0.05 \times 10^{-7}$	$0.96 \pm 0.14 \pm 0.03 \times 10^{-3}$	$5.12 \pm 0.76 \pm 0.07 \times 10^{-4}$	$1.34 \pm 0.20 \pm 0.26 \times 10^{-4}$
$B_c^- \to D_s^{*-} D_1^0$	$1.98 \pm 0.57 \pm 0.12 \times 10^{-7}$	$8.42 \pm 1.23 \pm 0.30 \times 10^{-4}$	$1.03 \pm 0.15 \pm 0.10 \times 10^{-4}$	$1.07 \pm 0.16 \pm 0.04 \times 10^{-3}$
$B_c^- \to D_s^{*-} \underline{D}_1^0$	$6.17 \pm 1.77 \pm 0.38 \times 10^{-8}$	$4.70 \pm 0.69 \pm 0.17 \times 10^{-4}$	$5.73 \pm 0.84 \pm 0.58 \times 10^{-5}$	$5.98 \pm 0.88 \pm 0.22 \times 10^{-4}$

TABLE VIII. Branching ratios and helicity amplitudes of $B_c \to VA$ decays for the CKM-suppressed ($\Delta b = 1$, $\Delta C = 1$, $\Delta S = -1$) mode.

		Helicity amplitudes		
Decays	Branching ratios	$\lvert H_0\rvert$	$\lvert H_+\rvert$	$\lvert H_-\rvert$
$B_c^- J/\psi D_1^-$	$1.39 \pm 0.14 \pm 0.03 \times 10^{-4}$	$3.00 \pm 0.18 \pm 0.04 \times 10^{-2}$	$1.64 \pm 0.10 \pm 0.02 \times 10^{-2}$	$2.77 \pm 0.11 \pm 0.01 \times 10^{-2}$
$B_c^- J/\psi \underline{D}_1^-$	$3.45 \pm 0.28 \pm 0.03 \times 10^{-5}$	$1.42 \pm 0.06 \pm 0.01 \times 10^{-2}$	$7.64 \pm 0.38 \pm 0.10 \times 10^{-3}$	$1.49 \pm 0.05 \pm 0.00 \times 10^{-2}$
$B_c^- \to D^{*-}\chi_{c1}$	$4.95 \pm 0.33 \pm 0.03 \times 10^{-5}$	$2.13 \pm 0.07 \pm 0.01 \times 10^{-2}$	$1.60 \pm 0.05 \pm 0.00 \times 10^{-2}$	$3.01 \pm 1.65 \pm 0.13 \times 10^{-4}$
$B_c^- \to D^{*-}h_{c1}$	$3.88 \pm 0.23 \pm 0.04 \times 10^{-5}$	$2.19 \pm 0.07 \pm 0.00 \times 10^{-2}$	$7.68 \pm 0.22 \pm 0.19 \times 10^{-3}$	$4.87 \pm 0.15 \pm 0.13 \times 10^{-3}$
$B_c^- \to \rho^- \bar{D}_1^0$	$1.01 \pm 0.05^{+0.20}_{-0.11} \times 10^{-5}$	$8.92 \pm 0.27^{+2.02}_{-1.02} \times 10^{-3}$	$1.17 \pm 0.04^{+0.13}_{-0.08} \times 10^{-3}$	$3.89 \pm 0.11^{+0.80}_{-0.50} \times 10^{-4}$
$B_c^- \to \rho^- \bar{\underline{D}}_1^0$	$1.19 \pm 0.07^{+0.29}_{-0.14} \times 10^{-6}$	$3.06 \pm 0.10^{+0.35}_{-0.18} \times 10^{-3}$	$3.21 \pm 0.09 \pm 0.06 \times 10^{-4}$	$1.93 \pm 0.06^{+0.50}_{-0.30} \times 10^{-4}$
$B_c^- \to \bar{D}^{*0}a_1^-$	$1.09 \pm 0.07 \pm 0.20 \times 10^{-6}$	$2.43 \pm 0.07 \pm 0.22 \times 10^{-3}$	$9.34 \pm 0.28 \pm 2.03 \times 10^{-5}$	$1.60 \pm 0.05 \pm 0.15 \times 10^{-3}$
$B_c^- \to \bar{D}^{*0}b_1^-$	$7.38 \pm 0.44 \pm 1.40 \times 10^{-12}$	$6.42 \pm 0.18 \pm 0.63 \times 10^{-6}$	$2.47 \pm 0.07 \pm 0.49 \times 10^{-7}$	$4.22 \pm 0.12 \pm 0.40 \times 10^{-6}$
$B_c^- \to \rho^0 D_1^-$	$7.71 \pm 2.23^{+1.30}_{-0.66} \times 10^{-8}$	$7.73 \pm 1.13^{+0.75}_{-0.40} \times 10^{-4}$	$1.01 \pm 0.15^{+0.10}_{-0.06} \times 10^{-4}$	$4.58 \pm 0.67^{+0.60}_{-0.41} \times 10^{-5}$
$B_c^- \to \rho^0 \underline{D}_1^-$	$8.78 \pm 2.53^{+1.66}_{-0.80} \times 10^{-9}$	$2.65 \pm 0.39^{+0.30}_{-0.15} \times 10^{-4}$	$2.79 \pm 0.41 \pm 0.05 \times 10^{-5}$	$1.67 \pm 0.25^{+0.37}_{-0.26} \times 10^{-5}$
$B_c^- \to \phi D_1^-$	$7.84 \pm 2.26^{+1.35}_{-0.70} \times 10^{-8}$	$7.77 \pm 1.14^{+0.70}_{-0.40} \times 10^{-4}$	$1.36 \pm 0.20^{+0.16}_{-0.09} \times 10^{-4}$	$6.57 \pm 4.58^{+0.67}_{-0.56} \times 10^{-5}$
$B_c^- \to \phi \underline{D}_1^-$	$8.78 \pm 2.53^{+1.50}_{-0.78} \times 10^{-9}$	$2.60 \pm 0.38^{+0.28}_{-0.14} \times 10^{-4}$	$3.74 \pm 0.55 \pm 0.05 \times 10^{-5}$	$2.24 \pm 0.33^{+0.45}_{-0.36} \times 10^{-5}$
$B_c^- \to D^{*-}a_1^0$	$8.16 \pm 2.34 \pm 1.14 \times 10^{-9}$	$2.09 \pm 0.31 \pm 0.19 \times 10^{-4}$	$8.03 \pm 1.18 \pm 1.50 \times 10^{-6}$	$1.37 \pm 0.20 \pm 0.01 \times 10^{-4}$
$B_c^- \to D^{*-}f_1$	$8.23 \pm 2.36 \pm 1.15 \times 10^{-9}$	$2.07 \pm 0.30 \pm 0.18 \times 10^{-4}$	$8.28 \pm 1.21 \pm 1.55 \times 10^{-6}$	$1.42 \pm 0.21 \pm 0.11 \times 10^{-4}$
$B_c^- \to \omega D_1^-$	$4.86 \pm 1.40^{+0.80}_{-0.30} \times 10^{-9}$	$1.94 \pm 0.29^{+0.20}_{-0.10} \times 10^{-4}$	$2.56 \pm 0.38^{+0.51}_{-0.15} \times 10^{-5}$	$8.34 \pm 1.03^{+1.50}_{-1.00} \times 10^{-6}$
$B_c^- \to \omega \underline{D}_1^-$	$5.68 \pm 1.64^{+1.05}_{-0.50} \times 10^{-10}$	$6.62 \pm 0.98^{+0.70}_{-0.36} \times 10^{-5}$	$7.08 \pm 1.04 \pm 0.12 \times 10^{-7}$	$4.23 \pm 0.62^{+0.95}_{-0.67} \times 10^{-6}$

TABLE IX. Branching ratios and helicity amplitudes of $B_c \to VA$ decays for the CKM-suppressed ($\Delta b = 1$, $\Delta C = -1$, $\Delta S = 0$) mode.

		Helicity amplitudes		
Decays	Branching ratios	$\lvert H_0\rvert$	$\lvert H_+\rvert$	$\lvert H_-\rvert$
$B_c^- \to D^{*-}\bar{D}_1^0$	$8.13 \pm 0.51^{+0.96}_{-1.48} \times 10^{-7}$	$2.57 \pm 0.08^{+0.26}_{-0.16} \times 10^{-3}$	$9.39 \pm 0.29^{+0.60}_{-0.67} \times 10^{-4}$	$3.98 \pm 0.18^{+0.50}_{-0.43} \times 10^{-4}$
$B_c^- \to D^{*-}\bar{\underline{D}}_1^0$	$7.01 \pm 0.48^{+1.51}_{-0.78} \times 10^{-8}$	$7.49 \pm 0.26^{+0.81}_{-0.47} \times 10^{-4}$	$2.55 \pm 0.08 \pm 0.04 \times 10^{-4}$	$1.80 \pm 0.08^{+0.43}_{-0.27} \times 10^{-4}$
$B_c^- \to \bar{D}^{*0}D_1^-$	$7.09 \pm 0.99^{+0.50}_{-0.33} \times 10^{-8}$	$6.38 \pm 0.55^{+0.12}_{-0.01} \times 10^{-4}$	$1.37 \pm 0.17^{+0.08}_{-0.02} \times 10^{-4}$	$4.88 \pm 0.19 \pm 0.35 \times 10^{-4}$
$B_c^- \to \bar{D}^{*0}\underline{D}_1^-$	$1.57 \pm 0.17^{+0.02}_{-0.01} \times 10^{-7}$	$2.77 \pm 0.23^{+0.11}_{-0.09} \times 10^{-4}$	$4.34 \pm 0.49^{+0.33}_{-0.24} \times 10^{-5}$	$2.67 \pm 0.10 \pm 0.19 \times 10^{-4}$

TABLE X. Branching ratios and helicity amplitudes of $B_c \to VA$ decays for the CKM-suppressed ($\Delta b = 1$, $\Delta C = -1$, $\Delta S = -1$) mode.

		Helicity amplitudes		
Decays	Branching ratios	$\lvert H_0\rvert$	$\lvert H_+\rvert$	$\lvert H_-\rvert$
$B_c^- \to D_s^{*-}\bar{D}_1^0$	$2.01 \pm 0.13^{+0.44}_{-024.} \times 10^{-5}$	$1.28 \pm 0.04^{+0.14}_{-0.08} \times 10^{-2}$	$4.94 \pm 0.15^{+0.55}_{-0.40} \times 10^{-3}$	$2.25 \pm 0.12^{+0.33}_{-0.24} \times 10^{-3}$
$B_c^- \to D_s^{*-}\bar{\underline{D}}_1^0$	$1.71 \pm 0.12^{+0.37}_{-0.19} \times 10^{-6}$	$3.68 \pm 0.13^{+0.40}_{-0.23} \times 10^{-3}$	$1.34 \pm 0.04 \pm 0.02 \times 10^{-3}$	$9.85 \pm 0.25^{+0.21}_{-0.14} \times 10^{-4}$
$B_c^- \to \bar{D}^{*0}D_{s1}^-$	$1.42 \pm 0.22 \pm 0.10 \times 10^{-6}$	$2.99 \pm 0.28 \pm 0.09 \times 10^{-3}$	$6.76 \pm 0.89 \pm 0.02 \times 10^{-4}$	$2.00 \pm 0.08 \pm 0.15 \times 10^{-3}$
$B_c^- \to \bar{D}^{*0}\underline{D}_{s1}^-$	$3.02 \pm 0.31 \pm 0.39 \times 10^{-7}$	$1.19 \pm 0.08 \pm 0.18 \times 10^{-3}$	$2.67 \pm 0.31 \pm 0.17 \times 10^{-4}$	$1.19 \pm 0.02 \pm 0.04 \times 10^{-3}$

decays involving K_1 and $\underline{K}1$ the branching ratios are of the same order.

(5) The longitudinal helicity amplitudes for the CKM-suppressed decays show the same trend as that observed in CKM-favored modes.

B. $B_c \to AA$ decays

The calculated branching ratios for B_c decaying to two axial-vector mesons in the final state for CKM-favored and CKM-suppressed modes are given in column 2 of Tables XI, XII, XIII, XIV, XV, and XVI. The

TABLE XI. Branching ratios and helicity amplitudes of $B_c \to AA$ decays for the CKM-favored ($\Delta b = 1$, $\Delta C = 1$, $\Delta S = 0$) mode.

Decays	Branching ratios	Helicity amplitudes $\vert H_0\vert$	$\vert H_+\vert$	$\vert H_-\vert$
$B_c^- \to \chi_{c1} a_1^-$	$1.81 \pm 0.11 \pm 0.01 \times 10^{-4}$	$2.15 \pm 0.06 \pm 0.17 \times 10^{-4}$	$6.20 \pm 0.19 \pm 0.52 \times 10^{-3}$	$4.36 \pm 0.13 \pm 0.02 \times 10^{-2}$
$B_c^- \to b_1^- \chi_{c1}$	$1.26 \pm 0.07 \pm 0.01 \times 10^{-9}$	$5.61 \pm 0.17 \pm 0.45 \times 10^{-7}$	$1.64 \pm 0.05 \pm 0.13 \times 10^{-5}$	$1.15 \pm 0.03 \pm 0.00 \times 10^{-4}$
$B_c^- \to h_{c1} a_1^-$	$1.36 \pm 0.08 \pm 0.10 \times 10^{-4}$	$3.32 \pm 0.10 \pm 0.19 \times 10^{-2}$	$8.57 \pm 0.25 \pm 0.29 \times 10^{-3}$	$1.72 \pm 0.05 \pm 0.05 \times 10^{-2}$
$B_c^- \to h_{c1} b_1^-$	$9.52 \pm 0.58 \pm 0.67 \times 10^{-10}$	$8.77 \pm 0.26 \pm 0.50 \times 10^{-5}$	$2.27 \pm 0.07 \pm 0.08 \times 10^{-5}$	$4.55 \pm 0.14 \pm 0.13 \times 10^{-5}$
$B_c^- \to D_1^- D_1^0$	$3.10 \pm 0.90^{+0.81}_{-0.49} \times 10^{-6}$	$4.18 \pm 0.61^{+0.72}_{-0.51} \times 10^{-3}$	$1.33 \pm 0.19^{+0.23}_{-0.16} \times 10^{-3}$	$3.50 \pm 0.50^{+0.41}_{-0.23} \times 10^{-3}$
$B_c^- \to \underline{D}_1^0 D_1^-$	$1.00 \pm 0.30^{+0.25}_{-0.15} \times 10^{-6}$	$2.33 \pm 0.34^{+0.41}_{-0.28} \times 10^{-3}$	$7.42 \pm 1.09 \pm 0.10 \times 10^{-4}$	$1.95 \pm 0.28^{+0.21}_{-0.13} \times 10^{-3}$
$B_c^- \to \underline{D}_1^- D_1^0$	$3.66 \pm 1.01^{+0.60}_{-0.30} \times 10^{-7}$	$1.60 \pm 0.23^{+0.14}_{-0.09} \times 10^{-3}$	$5.53 \pm 0.91^{+1.26}_{-0.95} \times 10^{-4}$	$0.94 \pm 0.14^{+0.02}_{-0.00} \times 10^{-3}$
$B_c^- \to \underline{D}_1^- \underline{D}_1^0$	$1.14 \pm 0.33^{+0.18}_{-0.09} \times 10^{-7}$	$0.89 \pm 0.13^{+0.08}_{-0.05} \times 10^{-3}$	$3.08 \pm 0.45^{+0.71}_{-0.53} \times 10^{-4}$	$5.20 \pm 0.76^{+0.10}_{-0.03} \times 10^{-4}$

TABLE XII. Branching ratios and helicity amplitudes of $B_c \to AA$ decays for the CKM-favored ($\Delta b = 1$, $\Delta C = 0$, $\Delta S = -1$) mode.

Decays	Branching ratios	Helicity amplitudes $\vert H_0\vert$	$\vert H_+\vert$	$\vert H_-\vert$
$B_c^- \to h_{c1} D_{s1}^-$	$3.15 \pm 0.19 \pm 0.18 \times 10^{-5}$	$5.59 \pm 0.17 \pm 0.79 \times 10^{-3}$	$1.40 \pm 0.46 \pm 0.03 \times 10^{-2}$	$1.96 \pm 0.06 \pm 0.04 \times 10^{-2}$
$B_c^- \to \chi_{c1} D_{s1}^-$	$1.22 \pm 0.15 \pm 0.05 \times 10^{-4}$	$1.66 \pm 0.15 \pm 0.15 \times 10^{-2}$	$2.91 \pm 0.71 \pm 1.11 \times 10^{-3}$	$4.51 \pm 0.27 \pm 0.05 \times 10^{-2}$
$B_c^- \to \underline{D}_{s1}^- \chi_{c1}$	$2.88 \pm 0.27 \pm 0.06 \times 10^{-5}$	$8.28 \pm 0.54 \pm 0.58 \times 10^{-3}$	$2.98 \pm 0.72 \pm 0.10 \times 10^{-3}$	$2.31 \pm 0.11 \pm 0.01 \times 10^{-2}$
$B_c^- \to h_{c1} \underline{D}_{s1}^-$	$1.17 \pm 0.07 \pm 0.06 \times 10^{-5}$	$4.89 \pm 0.15 \pm 0.44 \times 10^{-3}$	$8.93 \pm 0.27 \pm 0.21 \times 10^{-3}$	$1.21 \pm 0.04 \pm 0.02 \times 10^{-2}$
$B_c^- \to \underline{K}_1^- \bar{D}_1^0$	$1.90 \pm 0.12^{+0.70}_{-0.42} \times 10^{-7}$	$1.16 \pm 0.03^{+0.24}_{-0.15} \times 10^{-3}$	$1.73 \pm 0.05^{+0.33}_{-0.23} \times 10^{-4}$	$5.10 \pm 0.15^{+0.59}_{-0.33} \times 10^{-4}$
$B_c^- \to \underline{\bar{D}}_1^0 K_1^-$	$1.30 \pm 0.08^{+0.23}_{-0.10} \times 10^{-8}$	$3.08 \pm 0.09^{+0.26}_{-0.14} \times 10^{-4}$	$6.18 \pm 0.19^{+1.50}_{-1.12} \times 10^{-4}$	$1.04 \pm 0.04 \pm 0.02 \times 10^{-4}$
$B_c^- \to \bar{D}_1^0 K_1^-$	$1.22 \pm 0.07^{+0.48}_{-0.27} \times 10^{-7}$	$9.35 \pm 0.28^{+1.75}_{-1.24} \times 10^{-4}$	$1.25 \pm 0.04^{+0.23}_{-0.17} \times 10^{-4}$	$3.70 \pm 0.11^{+0.42}_{-0.24} \times 10^{-4}$
$B_c^- \to \underline{\bar{D}}_1^0 \underline{K}_1^-$	$2.00 \pm 0.10^{+0.34}_{-0.15} \times 10^{-8}$	$3.78 \pm 0.12^{+0.32}_{-0.17} \times 10^{-4}$	$8.39 \pm 0.25^{+0.20}_{-0.15} \times 10^{-5}$	$1.41 \pm 0.05 \pm 0.03 \times 10^{-4}$
$B_c^- \to \underline{D}_{s1}^- a_1^0$	$2.84 \pm 0.82 \pm 0.09 \times 10^{-10}$	$4.57 \pm 0.68 \pm 0.10 \times 10^{-5}$	$4.51 \pm 0.66 \pm 0.75 \times 10^{-6}$	$1.62 \pm 0.24 \pm 0.02 \times 10^{-5}$
$B_c^- \to \underline{D}_{s1}^- f_1$	$2.75 \pm 0.79 \pm 0.90 \times 10^{-10}$	$4.49 \pm 0.66 \pm 0.10 \times 10^{-5}$	$4.71 \pm 0.70 \pm 0.75 \times 10^{-6}$	$1.65 \pm 0.25 \pm 0.03 \times 10^{-5}$
$B_c^- \to D_{s1}^- a_1^0$	$1.02 \pm 0.29 \pm 0.16 \times 10^{-9}$	$7.88 \pm 1.16 \pm 0.90 \times 10^{-5}$	$1.15 \pm 0.17 \pm 0.12 \times 10^{-5}$	$4.54 \pm 0.67 \pm 0.16 \times 10^{-5}$
$B_c^- \to D_{s1}^- f_1$	$1.02 \pm 0.29 \pm 0.16 \times 10^{-9}$	$7.83 \pm 1.16 \pm 0.90 \times 10^{-5}$	$1.20 \pm 0.18 \pm 0.13 \times 10^{-5}$	$4.71 \pm 0.69 \pm 0.16 \times 10^{-5}$

corresponding helicity amplitudes of the decay channels are presented in columns 3, 4, and 5 of Tables XI, XII, XIII, XIV, XV, and XVI. The uncertainties in the obtained results caused by N_c variation and quark mass variation in the form factors, respectively, are also given in Tables XI, XII, XIII, XIV, XV, and XVI. We made the following observations.

1. For CKM-favored modes

(1) The branching ratios for $B_c \to AA$ decays are smaller than those for $B_c \to VA$ decays by an order of magnitude in corresponding CKM modes.

(2) The dominant decays in the Cabibbo-enhanced ($\Delta b = 1$, $\Delta C = 1$, $\Delta S = 0$) mode have branching ratios $\mathrm{Br}(B_c^- \to \chi_{c1} a_1^-) = 1.81 \pm 0.11 \pm 0.01 \times 10^{-4}$ and $\mathrm{Br}(B_c^- \to h_{c1} a_1^-) = 1.36 \pm 0.08 \pm 0.10 \times 10^{-4}$. The branching ratios of the $B_c^- \to D_1^- D_1^0$ and $B_c^- \to D_1^- \underline{D}_1^0$ decays are of the order of 10^{-6}. The order of magnitude for the branching ratios of the remaining decays range from (10^{-7})–(10^{-10}) in this mode.

(3) In e-favored ($\Delta b = 1$, $\Delta C = 0$, $\Delta S = -1$) mode, the dominant decay channels are $\mathrm{Br}(B_c^- \to D_{s1}^- \chi_{c1}) = 1.22 \pm 0.15 \pm 0.05 \times 10^{-4}$, $\mathrm{Br}(B_c^- \to h_{c1} D_{s1}^-) = 3.15 \pm 0.19 \pm 0.18 \times 10^{-5}$, $\mathrm{Br}(B_c^- \to \underline{D}_{s1}^- \chi_{c1}) = 2.88 \pm 0.27 \pm 0.06 \times 10^{-5}$, and $\mathrm{Br}(B_c^- \to h_{c1} \underline{D}_{s1}^-) = 1.77 \pm 0.07 \pm 0.06 \times 10^{-5}$. The remaining decay modes are suppressed, with branching ratios of $\mathcal{O}(10^{-7}$–$10^{-11})$.

(4) In the present analysis, we observe that the magnitudes of the longitudinal helicity amplitudes are higher for all of the decay modes except for decays involving a $c\bar{c}$ meson in the final state. In such decays the transverse helicity amplitude H_- has a larger magnitude.

TABLE XIII. Branching ratios and helicity amplitudes of $B_c \to AA$ decays for the CKM-suppressed ($\Delta b = 1$, $\Delta C = 1$, $\Delta S = -1$) mode.

		Helicity amplitudes		
Decays	Branching ratios	$\|H_0\|$	$\|H_+\|$	$\|H_-\|$
$B_c^- \to \underline{K}_1^- \chi_{c1}$	$1.18 \pm 0.07 \pm 0.01 \times 10^{-5}$	$3.27 \pm 0.10 \pm 0.03 \times 10^{-4}$	$1.61 \pm 0.05 \pm 0.14 \times 10^{-3}$	$1.14 \pm 0.03 \pm 0.00 \times 10^{-2}$
$B_c^- \to \chi_{c1} K_1^-$	$6.76 \pm 0.40 \pm 0.04 \times 10^{-6}$	$9.26 \pm 0.28 \pm 1.70 \times 10^{-5}$	$1.21 \pm 0.04 \pm 0.10 \times 10^{-3}$	$8.48 \pm 0.26 \pm 0.01 \times 10^{-3}$
$B_c^- \to h_{c1} K_1^-$	$4.99 \pm 0.30 \pm 0.34 \times 10^{-6}$	$6.31 \pm 0.19 \pm 0.36 \times 10^{-3}$	$1.73 \pm 0.05 \pm 0.05 \times 10^{-3}$	$3.42 \pm 0.10 \pm 0.10 \times 10^{-3}$
$B_c^- \to h_{c1} \underline{K}_1^-$	$6.63 \pm 0.40 \pm 0.45 \times 10^{-6}$	$6.91 \pm 0.21 \pm 0.44 \times 10^{-3}$	$2.36 \pm 0.07 \pm 0.08 \times 10^{-3}$	$4.59 \pm 0.13 \pm 0.13 \times 10^{-3}$
$B_c^- \to D_{s1}^- D_1^0$	$2.95 \pm 1.05^{+0.20}_{-0.16} \times 10^{-7}$	$7.67 \pm 1.14^{+0.96}_{-0.84} \times 10^{-4}$	$2.75 \pm 0.41^{+0.30}_{-0.27} \times 10^{-4}$	$0.91 \pm 0.13^{+0.04}_{-0.04} \times 10^{-3}$
$B_c^- \to \underline{D}_1^0 D_{s1}^-$	$4.57 \pm 1.32 \pm 0.60 \times 10^{-8}$	$4.28 \pm 0.63 \pm 0.50 \times 10^{-4}$	$1.52 \pm 0.22 \pm 0.16 \times 10^{-4}$	$5.11 \pm 0.75 \pm 0.19 \times 10^{-4}$
$B_c^- \to \underline{D}_{s1}^- D_1^0$	$2.85 \pm 0.82 \pm 0.11 \times 10^{-8}$	$4.46 \pm 0.66 \pm 0.15 \times 10^{-4}$	$7.80 \pm 1.45 \pm 1.60 \times 10^{-5}$	$3.07 \pm 0.45 \pm 1.04 \times 10^{-4}$
$B_c^- \to \underline{D}_{s1}^- \underline{D}_1^0$	$8.92 \pm 2.58 \pm 0.30 \times 10^{-9}$	$2.49 \pm 0.37 \pm 0.08 \times 10^{-4}$	$4.34 \pm 0.64 \pm 0.90 \times 10^{-5}$	$1.71 \pm 0.25 \pm 0.02 \times 10^{-4}$

TABLE XIV. Branching ratios and helicity amplitudes of $B_c \to AA$ decays for the CKM-suppressed ($\Delta b = 1$, $\Delta C = 0$, $\Delta S = 0$) mode.

		Helicity amplitudes		
Decays	Branching ratios	$\|H_0\|$	$\|H_+\|$	$\|H_-\|$
$B_c^- \to \underline{D}_1^- \chi_{c1}$	$1.60 \pm 0.15 \pm 0.04 \times 10^{-6}$	$1.78 \pm 0.13 \pm 0.20 \times 10^{-3}$	$4.08 \pm 2.66 \pm 0.95 \times 10^{-5}$	$5.28 \pm 0.22 \pm 0.02 \times 10^{-3}$
$B_c^- \to \chi_{c1} D_1^-$	$7.48 \pm 0.92 \pm 0.50 \times 10^{-6}$	$4.43 \pm 0.66 \pm 0.46 \times 10^{-3}$	$5.87 \pm 0.67 \pm 0.36 \times 10^{-4}$	$1.52 \pm 0.49 \pm 0.02 \times 10^{-2}$
$B_c^- \to h_{c1} D_1^-$	$1.79 \pm 0.11 \pm 0.01 \times 10^{-6}$	$1.12 \pm 0.03 \pm 0.20 \times 10^{-3}$	$3.26 \pm 0.09 \pm 0.08 \times 10^{-3}$	$4.61 \pm 0.10 \pm 0.11 \times 10^{-3}$
$B_c^- \to h_{c1} \underline{D}_1^-$	$5.59 \pm 0.33 \pm 0.32 \times 10^{-7}$	$6.04 \pm 0.19 \pm 1.10 \times 10^{-4}$	$1.81 \pm 0.05 \pm 0.04 \times 10^{-3}$	$2.60 \pm 0.08 \pm 0.06 \times 10^{-3}$
$B_c^- \to \bar{D}_1^0 a_1^-$	$3.41 \pm 0.20^{+1.30}_{-0.77}. \times 10^{-6}$	$4.96 \pm 0.14^{+0.94}_{-0.66} \times 10^{-3}$	$6.35 \pm 0.20^{+1.22}_{-0.84} \times 10^{-4}$	$1.89 \pm 0.06^{+0.22}_{-0.12} \times 10^{-3}$
$B_c^- \to b_1^- \bar{D}_1^0$	$2.38 \pm 0.14^{+0.92}_{-0.54} \times 10^{-11}$	$1.35 \pm 0.04^{+0.25}_{-0.17} \times 10^{-5}$	$1.68 \pm 0.05^{+0.32}_{-0.22} \times 10^{-6}$	$4.95 \pm 0.10^{+0.58}_{-0.33} \times 10^{-6}$
$B_c^- \to \underline{\bar{D}}_1^0 a_1^-$	$3.38 \pm 0.21^{+0.58}_{-0.26} \times 10^{-7}$	$1.57 \pm 0.05^{+0.13}_{-0.06} \times 10^{-3}$	$3.12 \pm 0.09^{+0.76}_{-0.56} \times 10^{-4}$	$5.22 \pm 0.15^{+0.10}_{-0.08} \times 10^{-4}$
$B_c^- \to \underline{\bar{D}}_1^0 b_1^-$	$2.47 \pm 0.04^{+0.41}_{-0.18} \times 10^{-12}$	$4.15 \pm 0.13^{+0.36}_{-0.18} \times 10^{-6}$	$8.25 \pm 0.25^{+1.95}_{-1.46} \times 10^{-7}$	$1.38 \pm 0.04 \pm 0.03 \times 10^{-6}$
$B_c^- \to D_1^- a_1^0$	$2.61 \pm 0.75^{+0.80}_{-0.45} \times 10^{-8}$	$4.29 \pm 0.64^{+0.75}_{-0.51} \times 10^{-4}$	$5.51 \pm 0.81^{+0.95}_{-0.66} \times 10^{-5}$	$1.62 \pm 0.22^{+0.17}_{-0.10} \times 10^{-4}$
$B_c^- \to D_1^- f_1$	$2.60 \pm 0.75^{+0.80}_{-0.45} \times 10^{-8}$	$4.27 \pm 0.63^{+0.75}_{-0.51} \times 10^{-4}$	$5.74 \pm 0.84^{+0.95}_{-0.66} \times 10^{-5}$	$1.70 \pm 0.25^{+0.17}_{-0.10} \times 10^{-4}$
$B_c^- \to \underline{D}_1^- a_1^0$	$2.60 \pm 0.75^{+0.37}_{-0.16} \times 10^{-9}$	$1.36 \pm 0.20^{+0.10}_{-0.05} \times 10^{-4}$	$2.70 \pm 0.39^{+0.60}_{-0.43} \times 10^{-5}$	$4.53 \pm 0.67 \pm 0.08 \times 10^{-5}$
$B_c^- \to \underline{D}_1^- f_1$	$2.50 \pm 0.72^{+0.37}_{-0.16} \times 10^{-9}$	$1.33 \pm 0.19^{+0.10}_{-0.05} \times 10^{-4}$	$2.79 \pm 0.41^{+0.60}_{-0.43} \times 10^{-5}$	$4.63 \pm 0.68 \pm 0.08 \times 10^{-5}$

TABLE XV. Branching ratios and helicity amplitudes of $B_c \to AA$ decays for the CKM-suppressed ($\Delta b = 1$, $\Delta C = -1$, $\Delta S = 0$) mode.

		Helicity amplitudes		
Decays	Branching ratios	$\|H_0\|$	$\|H_+\|$	$\|H_-\|$
$B_c^- \to \bar{D}_1^0 D_1^-$	$1.00 \pm 0.30^{+0.35}_{-0.21} \times 10^{-7}$	$7.80 \pm 0.33^{+1.46}_{-1.06} \times 10^{-4}$	$2.48 \pm 0.10^{+0.55}_{-0.33} \times 10^{-4}$	$6.51 \pm 0.28^{+0.74}_{-0.46} \times 10^{-4}$
$B_c^- \to \underline{D}_1^- \bar{D}_1^0$	$3.00 \pm 0.23^{+1.00}_{-0.60} \times 10^{-8}$	$4.20 \pm 0.16^{+0.76}_{-0.55} \times 10^{-4}$	$1.34 \pm 0.05^{+0.26}_{-0.19} \times 10^{-4}$	$3.41 \pm 0.12^{+0.38}_{-0.23} \times 10^{-4}$
$B_c^- \to \underline{\bar{D}}_1^0 D_1^-$	$1.42 \pm 0.14^{+0.29}_{-0.16} \times 10^{-8}$	$2.12 \pm 0.15^{+0.34}_{-0.22} \times 10^{-4}$	$1.08 \pm 0.06^{+0.26}_{-0.20} \times 10^{-4}$	$1.95 \pm 0.10^{+0.08}_{-0.01} \times 10^{-4}$
$B_c^- \to \underline{\bar{D}}_1^0 \underline{D}_1^-$	$3.91 \pm 0.35^{+0.74}_{-0.38} \times 10^{-9}$	$1.66 \pm 0.07^{+0.16}_{-0.10} \times 10^{-4}$	$5.75 \pm 0.25^{+1.46}_{-1.10} \times 10^{-5}$	$9.69 \pm 0.41^{+0.20}_{-0.10} \times 10^{-5}$

2. For CKM-suppressed modes

(1) In the CKM-suppressed mode ($\Delta b = 1$, $\Delta C = 1$, $\Delta S = -1$) the highest order of magnitude for the branching ratios of dominant decays is $\sim(10^{-5}$–$10^{-6})$, i.e., $\mathrm{Br}(B_c^- \to \chi_{c1}\underline{K}_1^-) = 1.18 \pm 0.07 \pm 0.01 \times 10^{-5}$, $\mathrm{Br}(B_c^- \to \chi_{c1}K_1^-) = 6.76 \pm 0.40 \pm 0.04 \times 10^{-6}$, $\mathrm{Br}(B_c^- \to h_{c1}\underline{K}_1^-) = 6.63 \pm 0.40 \pm 0.45 \times 10^{-6}$, and $\mathrm{Br}(B_c^- \to h_{c1}K_1^-) = 4.99 \pm 0.30 \pm 0.34 \times 10^{-6}$. The next-order decays have branching ratios of the order $\mathcal{O}(10^{-7}$–$10^{-8})$.

TABLE XVI. Branching ratios and helicity amplitudes of $B_c \to AA$ decays for the CKM-suppressed ($\Delta b = 1$, $\Delta C = -1$, $\Delta S = -1$) mode.

		Helicity amplitudes		
Decays	Branching ratios	$\|H_0\|$	$\|H_+\|$	$\|H_-\|$
$B_c^- \to D_{s1}^- \bar{D}_1^0$	$1.76 \pm 0.15^{+0.55}_{-0.34} \times 10^{-6}$	$3.13 \pm 0.13^{+0.57}_{-0.42} \times 10^{-3}$	$9.96 \pm 0.12^{+1.85}_{-1.36} \times 10^{-4}$	$2.75 \pm 0.12^{+0.28}_{-0.19} \times 10^{-3}$
$B_c^- \to \underline{\bar{D}}_1^0 D_{s1}^-$	$2.47 \pm 0.24^{+0.45}_{-0.26} \times 10^{-7}$	$1.28 \pm 0.05^{+0.13}_{-0.09} \times 10^{-3}$	$4.43 \pm 0.20^{+1.00}_{-0.83} \times 10^{-4}$	$8.55 \pm 0.50^{+0.21}_{-0.01} \times 10^{-4}$
$B_c^- \to \underline{D}_{s1}^- \bar{D}_1^0$	$6.39 \pm 0.50^{+2.00}_{-1.20} \times 10^{-7}$	$1.92 \pm 0.08^{+0.33}_{-0.25} \times 10^{-3}$	$6.29 \pm 0.22^{+1.19}_{-0.87} \times 10^{-4}$	$1.65 \pm 0.06^{+0.18}_{-0.11} \times 10^{-3}$
$B_c^- \to \underline{D}_{s1}^- \underline{\bar{D}}_1^0$	$8.28 \pm 0.74^{+1.46}_{-0.77} \times 10^{-8}$	$7.70 \pm 0.35^{+0.70}_{-0.45} \times 10^{-4}$	$2.59 \pm 0.10^{+0.65}_{-0.51} \times 10^{-4}$	$4.75 \pm 0.22^{+0.08}_{-0.04} \times 10^{-4}$

TABLE XVII. Comparison of branching ratios with available theoretical works.

Decays	This work	Ref. [25]	Ref. [23]	Ref. [13]	Ref. [15]
$B_c^- \to \chi_{c1}\rho^-$	$1.47 \pm 0.03 \pm 0.01 \times 10^{-3}$	2.19×10^{-4}	1.4×10^{-4}	2.86×10^{-4}	9.64×10^{-5}
$B_c^- \to h_{c1}\rho^-$	$1.24 \pm 0.08 \pm 0.01 \times 10^{-3}$	2.19×10^{-3}	9.73×10^{-4}	2.44×10^{-3}	1.24×10^{-3}
$B_c^- \to \chi_{c1}K^{*-}$	$7.07 \pm 0.43 \pm 0.04 \times 10^{-5}$	1.58×10^{-5}	8.77×10^{-6}	1.75×10^{-5}	7.01×10^{-6}
$B_c^- \to h_{c1}K^{*-}$	$6.18 \pm 0.37 \pm 0.06 \times 10^{-5}$	1.23×10^{-4}	6.75×10^{-5}	1.28×10^{-4}	6.84×10^{-5}

(2) Similar to the ($\Delta C = 1, \Delta S = -1$) mode, the dominant decay channels have branching ratios of $\mathcal{O}(10^{-6})$ in the CKM-suppressed ($\Delta b = 1$, $\Delta C = 1$, $\Delta S = 1$) mode, i.e., $\mathrm{Br}(B_c^- \to D_1^- \chi_{c1}) = 7.48 \pm 0.92 \pm 0.50 \times 10^{-6}$, $\mathrm{Br}(B_c^- \to \bar{D}_1^0 a_1^-) = 3.41 \pm 0.20^{+1.30}_{-0.77} \times 10^{-6}$, $\mathrm{Br}(B_c^- \to \underline{D}_1^- \chi_{c1}) = 1.60 \pm 0.15 \pm 0.04 \times 10^{-6}$, and $\mathrm{Br}(B_c^- \to D_1^- h_{c1}) = 1.79 \pm 0.11 \pm 0.01 \times 10^{-6}$. However, branching ratios for $B_c^- \to a_1^- \underline{\bar{D}}_1^0$ and $B_c^- \to \underline{D}_1^- h_{c1}$ decays are of $\mathcal{O}(10^{-7})$.

(3) Decay channels in the CKM-suppressed ($\Delta b = 1$, $\Delta C = -1$, $\Delta S = -1$) and ($\Delta b = 1$, $\Delta C = -1$, $\Delta S = 0$) modes remain highly suppressed, with $\mathrm{Br}(B_c^- \to \underline{D}_{s1}^- \underline{\bar{D}}_1^0) = 1.76 \pm 0.15^{+0.55}_{-0.34} \times 10^{-6}$. The $B_c^- \to \underline{\bar{D}}_1^0 D_{s1}^-$ and $B_c^- \to \underline{\bar{D}}_{s1}^- \bar{D}_1^0$ decays have branching ratios $\mathcal{O}(10^{-7})$.

(4) As noticed in the previous case, the longitudinal helicity amplitudes have larger magnitudes in comparison to the transverse components for most of the decay channels. However, decay channels involving a $c\bar{c}$ meson in the final state show a transverse H_--component dominance.

It may also be noted that the effective variation in N_c leads to the change in amplitude and, hence, the branching ratios of these decays. The branching ratios of color-favored class I decays show a ~6% variation in the central value and color-suppressed class II decays show a variation of ~30%. However, class III decays involving both color-favored and color-suppressed diagrams show a variation from 7% to 15%.

We wish to emphasize that with remarkable improvements in experiments and sophisticated instrumentation, branching ratios of the order of (10^{-6}) could be measured precisely [39] at the LHC, LHC-b, and Super-B factories in the near future. Therefore, these results may provide the necessary information for the phenomenological study of B_c meson physics.

Since there is no experimental information available at present for such decays, we compare our results with other theoretical works (see Table XVII). There are several theoretical models—such as the Bethe-Salpeter approach [25], the relativistic quark model [13,23], the nonrelativistic quark model [15], etc.—which give their predictions for $B_c \to VA$ decays with charmonium in the final state. We find that the results given by these different models are comparable, with some exceptions. In Table XVII, the listed branching ratios are obtained by using $a_1 = 1.12$ in the referred models. It may be noted that H. F. Fu *et al.* [24] also predicted the branching ratios of a few decay modes, namely $B_c^- \to h_{c1}D^{*-}/\chi_{c1}D^{*-}/J/\psi D_{s1}^-/J/\psi \underline{D}_{s1}^-$. Their predictions are lager than our results by an order magnitude, except for $B_c^- \to \chi_{c1}D^{*-}$, which is comparable to our prediction. In addition to these, H. F. Fu *et al.* [24] predicted the branching ratios of $B_c^- \to D_{s1}^- \phi^0/\underline{D}_{s1}^- \phi^0/\underline{D}_{s1}^- K^{*0}$ decays based on contributions from penguin diagrams, which we ignore in the present analysis. We wish to remark here that for $B_c \to AA$ decays, theoretical predictions for only four decay channels are available for comparison, i.e., $B_c^- \to h_{c1}D_{s1}^-/h_{c1}\underline{D}_{s1}^-/\chi_{c1}D_{s1}^-/\chi_{c1}\underline{D}_{s1}^-$ [24]. In addition, the branching ratios predicted in the present work are small compared to the results given by Ref. [24].

VII. SUMMARY AND CONCLUSIONS

In the present work we have calculated $B_c \to A$ transition form factors using the ISGW II model framework. Consequently, we have predicted the branching ratios of

$B_c \to VA/AA$ decays. We have used flavor-dependent $B_c \to V$ transition form factors in the BSW Model framework. Also, we have calculated the helicity components corresponding to different polarization amplitudes in $B_c \to VA/AA$ decays. We draw the following conclusions:

(1) In the case of the $B_c \to VA$ mode, CKM-enhanced ($\Delta b = 1$, $\Delta C = 1$, $\Delta S = 0$) dominant decays are $B_c^- \to J/\psi a_1^-$, $B_c^- \to \rho^- \chi_{c1}$, and $B_c^- \to \rho^- h_{c1}$, while the dominant decays in ($\Delta b = 1$, $\Delta C = 0$, $\Delta S = -1$) are $B_c^- \to D_s^{*-} \chi_{c1}$, $B_c^- \to D_s^{*-} h_{c1}$, $B_c^- \to J/\psi D_{s1}^-$, and $B_c^- \to J/\psi \underline{D}_{s1}^-$. Their branching ratios range from 10^{-3}–10^{-11}.

(2) Branching ratios of CKM-enhanced modes in the case of $B_c \to AA$ decays are smaller by an order of magnitude compared to those in $B_c \to VA$ decays. The dominant decays are $B_c^- \to \chi_{c1} a_1^-$, $B_c^- \to h_{c1} a_1^-$, and $B_c^- \to D_{s1}^- \chi_{c1}$. The branching ratios range from 10^{-4}–10^{-10}.

(3) In CKM-suppressed modes, the branching ratios are smaller by an order of magnitude for both $B_c \to VA$ and $B_c \to AA$ decays. The branching ratios for the dominant decays $B_c^- \to J/\psi K_1^-$, $B_c^- \to J/\psi \underline{K}_1^-$, and $B_c^- \to J/\psi D_1^-$ are of order of magnitude (10^{-4}).

(4) In general, branching ratios of $B_c \to VA$ decays involving axial-vector mesons $A(^3P_1)$ in the final state are larger compared to the decays involving axial-vector mesons $A(^1P_1)$ in the final state.

(5) For most of the decays, the magnitude of the helicity component of the longitudinal polarization amplitude is larger compared to the transverse amplitudes. However, in $B_c \to AA$ decays the transverse-polarization-amplitude-dominance has been observed for channels involving a $c\bar{c}$ meson in the final state.

Since LHC and LHC-b are expected to accumulate data for more than 10^{10} B_c events per year, we hope that predicted BRs will soon be measured in these experiments.

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