# Effective superpotential in the supersymmetric Chern-Simons theory with matter

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We develop a superfield approach to the effective potential in the supersymmetric Chern-Simons theory coupled to matter and, in the Landau gauge, calculate the one-loop Kählerian effective potential and give some qualitative prescriptions for the two-loop one.

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It is well known that the low-energy effective dynamics of a quantum field theory is characterized by the effective potential [1]. Its study is especially relevant in the context of supersymmetric theories since the effective (super)potential allows us to obtain important information about supersymmetry and/or gauge symmetry breaking. In four space-time dimensions, the methodology of studying the effective superpotential has been well developed in Refs. [2,3] and then successfully applied in many examples, including the Wess-Zumino model, both in commutative [3] and noncommutative [4] cases, general chiral superfield model [5], super-Yang-Mills theory [6] and higher-derivative theories [7].

Recently, a great deal of attention has been devoted to studies of the supersymmetric Chern-Simons theory with matter. The reason is the fact that $N = 6$ and $N = 8$ supersymmetric Chern-Simons theories are finite and conformal invariant, which allows for studying the $AdS_4/CFT_3$ correspondence [8]. Earlier, different aspects of the supersymmetric Chern-Simons theories were studied in Ref. [9].

In three space-time dimensions, the first attempt to formulate a superfield approach to the study of the supersymmetric effective potential was carried out in Refs. [10,11] (some preliminary discussions of this approach were presented in Refs. [12,13]; see also Ref. [14] for some issues related to extended supersymmetry in these theories). Additional studies of the effective potential including a component analysis of the Coleman-Weinberg and Wess-Zumino models were performed in Ref. [15].

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In this work, we consider the superfield version of the supersymmetric Coleman-Weinberg model, that is, the Chern-Simons theory coupled to a self-interacting massless scalar matter, which is the simplest example of a three-dimensional supersymmetric gauge theory with matter, where all couplings are dimensionless. Employing an adequate method, devised by us, we will calculate the one-loop and discuss the two-loop superpotentials. Throughout the paper, we follow the notations and conventions adopted in Ref. [16].

The action of the Coleman-Weinberg theory looks like

$$S[\Phi, A^\alpha] = \int d^5z\Big[A^\alpha W_\alpha - \frac{1}{2}(D^\alpha - igA^\alpha)\Phi(D_\alpha + igA_\alpha)\bar{\Phi} + V(\Phi) + \frac{1}{2\xi}\int d^5z(D^\alpha A_\alpha)(D^\beta A_\beta)\Big], \quad (1)$$

where $\Phi$ is a (complex) scalar superfield and $V(\Phi) = \frac{\lambda}{2}(\Phi\bar{\Phi})^2$ is a classical potential. The $A_\alpha$ is a gauge superfield, with $W_\alpha = \frac{1}{2}D^\beta D_\alpha A_\beta$ the corresponding gauge-invariant superfield strength. The last term of the expression (1) is the gauge-fixing action. Since the theory is Abelian, the ghosts completely decouple, and their action will be omitted.

We will start by evaluating the superfield effective action, within the methodology of the loop expansion [17]. To do it, we make a shift $\Phi \to \Phi + \phi$ in the superfield $\Phi$ (the shift for the $\bar{\Phi}$ will be similar), where now $\Phi$ is a background (super)field, and $\phi$ is a quantum one. Through this study, the gauge field $A_\alpha$ is taken to be purely quantum since our aim consists of studying quantum corrections depending only on the scalar fields. As a result, the classical action (1) takes the form

 

$$S[\Phi;\phi,A^\alpha]=S[\Phi,A^\alpha]|_{A^\alpha=0}+\int d^5z\Big[\frac{1}{2}A_\alpha\Big(D^\beta D^\alpha+\frac{1}{\xi}D^\alpha D^\beta\Big)A_\beta+\phi D^2\bar\phi+\frac{\lambda}{2}(\Phi^2\bar\phi^2+4\Phi\bar\Phi\phi\bar\phi+\bar\Phi^2\phi^2)$$
$$+\frac{ig}{2}(\Phi A^\alpha D_\alpha\bar\phi-\bar\Phi A^\alpha D_\alpha\phi)+\frac{ig}{2}(\phi A^\alpha D_\alpha\bar\Phi-\bar\phi A^\alpha D_\alpha\Phi)+\lambda\Big(\phi^2\bar\phi\,\bar\Phi+\bar\phi^2\phi\Phi+\frac{1}{2}(\phi\bar\phi)^2\Big)$$
$$+\frac{ig}{2}(\phi A^\alpha D_\alpha\bar\phi-\bar\phi A^\alpha D_\alpha\phi)-\frac{g^2}{2}A^\alpha A_\alpha(\Phi\bar\Phi+\Phi\bar\phi+\phi\bar\Phi+\phi\bar\phi)\Big]. \quad (2)$$

Here, we eliminated the linear terms in quantum fields since they produce only irrelevant, one-particle-reducible contributions. The effective action $\Gamma[\Phi]$ is defined by the expression

$$\exp(i\Gamma[\Phi])=\int D\phi DA^\alpha\exp(iS[\Phi,\phi,A^\alpha])|_{1PI}, \quad (3)$$

where the subscript 1PI stands for one-particle-irreducible supergraphs. The general structure of the effective action can be cast in a form similar to the four-dimensional case [2,3]:

$$\Gamma[\Phi]=\int d^5zK(\Phi)+\int d^5zF(D^\alpha\Phi D_\alpha\Phi,D^2\Phi;\Phi), \quad (4)$$

where the $K(\Phi)$ is the Kählerian effective potential which depends only on the superfield $\Phi$ but not on its derivatives. The $F$ term is called auxiliary fields' effective potential whose key property is its vanishing in the case when all derivatives of the superfields are equal to zero (within this paper, we will not discuss it). We restrict ourselves to the Kählerian effective potential.

We will work within a loop expansion for the effective action $\Gamma$,

$$\Gamma[\Phi]=S[\Phi]+\Gamma^{(1)}[\Phi]+\Gamma^{(2)}[\Phi]+\cdots, \quad (5)$$

and for the Kählerian potential $K$,

$$K(\Phi)=V(\Phi)+\sum_{L=1}^{\infty}K_L(\Phi), \quad (6)$$

that is, the tree-order Kählerian effective potential is $K^{(0)}(\Phi)=V(\Phi)=\frac{\lambda}{2}(\Phi\bar\Phi)^2$.

For the background fields equal to zero, the free propagators of the scalar and gauge superfields corresponding to the action (1) are

$$\langle A^\alpha(z_1)A^\beta(z_2)\rangle=\frac{i}{4\Box}(D^\beta D^\alpha+\xi D^\alpha D^\beta)\delta^5(z_1-z_2);$$
$$\langle\phi(z_1)\bar\phi(z_2)\rangle=-\frac{i}{\Box}D^2\delta^5(z_1-z_2). \quad (7)$$

The key point of this paper consists of the development of a methodology for the background dependent propagators and their use for the calculation of the effective action. While in the four-dimensional superfield theories this formalism has been well developed and successfully applied in a number of papers, see, for example, Refs. [3,5–7], in the three-dimensional superfield theories, it has been used only in Ref. [10] for the purely scalar superfield model. Also, in the works [11], a simplified form of this methodology based on the imposition of restrictions on the structure of the background scalar field ($\Phi=\phi_1-\theta^2\phi_2$) was used, whereas in the present paper, the general form $\Phi=\phi_1+\theta^\alpha\psi_\alpha-\theta^2\phi_2$ is used, with no restrictions except for the condition $D_\alpha\Phi=0$, leading to the Kählerian effective potential; actually, we do not employ the component expansion of $\Phi$. Moreover, we generalize the powerful technique of summation over one-loop diagrams (which has been intensively applied in the four dimensions [5,7]) for the three-dimensional case, specially, for the supersymmetric gauge theories, which is done here for the first time. We expect that this methodology can be efficiently applied to more sophisticated theories.

It will be convenient to obtain the effective propagators on the base of summation of different sequences of free propagators. First of all, we can take into account the situation with the triple gauge-scalar vertices. Including such a vertex into a diagram, with one background scalar leg, produces a fragment of the diagram as shown in Fig. 1.

In that figure, the bold line is for the background field, and the factor $D^\beta D^\alpha+\xi D^\alpha D^\beta$ originates from the gauge propagator. The indices $\alpha$ and $\beta$ are the indices of gauge fields contracted into this propagator. We see that if we move the derivative $D_\alpha$, originated from the interaction vertex, to the propagator of the gauge fields proportional to $D^\beta D^\alpha+\xi D^\alpha D^\beta$, we will annihilate its gauge-independent part, and the gauge-dependent part is proportional to $\xi$; by imposing the gauge $\xi=0$ (Landau gauge [16]), the calculations are simplified, removing from considerations all diagrams with the vertices $(\Phi A^\alpha D_\alpha\bar\phi-\bar\Phi A^\alpha D_\alpha\phi)$.

Let us find the one-loop Kählerian effective potential. Taking into account the conclusion made in the previous paragraph, we can find that in the Landau gauge, there are

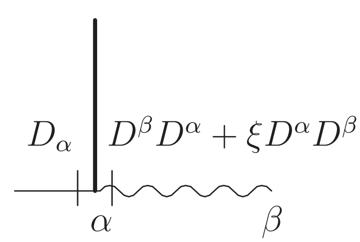


FIG. 1. A fragment of a Feynman diagram involving triple vertices.

two types of contributions to the Kählerian effective potential: in the first of them, all supergraphs involve only the gauge propagators, and in the second of them, all supergraphs involve only matter propagators. It is equivalent to write down two contributions to the one-loop effective action as two traces of the logarithms:

$$\Gamma_1^{(1)} = \frac{i}{2}\,\mathrm{Tr}\ln[D^\beta D^\alpha + \frac{1}{\xi}D^\alpha D^\beta + g^2 C^{\alpha\beta}\Phi\bar\Phi];$$
$$\Gamma_2^{(1)} = -\frac{i}{2}\,\mathrm{Tr}\ln\begin{pmatrix} M & D^2+m \\ D^2+m & \bar M\end{pmatrix}, \tag{8}$$

where $m = 2\lambda\Phi\bar\Phi$, $\bar M = \lambda\Phi^2$, $M = \lambda\bar\Phi^2$. The plus sign in $\Gamma_1^{(1)}$ arose due to the fermionic statistics of $A_\alpha$. The $\Gamma_1^{(1)}$, at the $\xi = 0$ limit, can be explicitly obtained via the expansion of the trace of the logarithm: since the operator inverse to $\mathcal{O}^{\alpha\beta} = D^\beta D^\alpha + \frac{1}{\xi}D^\alpha D^\beta$ is $G_{\beta\gamma} = \frac{i}{4\Box}(D_\gamma D_\beta + \xi D_\beta D_\gamma)\delta^5(z-z')$ [i.e., $\mathcal{O}^{\alpha\beta}G_{\beta\gamma} = -i\delta^\alpha_\gamma\delta^5(z-z')$], we can write $\Gamma_1^{(1)}$, at $\xi = 0$, as

$$\Gamma_1^{(1)} = \frac{i}{2}\mathrm{Tr}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{g^2\Phi\bar\Phi}{4\Box}\right)^n \times D^{\alpha_2}D_{\alpha_1}D^{\alpha_3}D_{\alpha_2}\ldots D^{\alpha_1}D_{\alpha_n}\delta_{12}|_{\theta_1=\theta_2}. \tag{9}$$

Here, we took into account that each relevant scalar-gauge vertex looks like $-\frac{1}{2}g^2\Phi\bar\Phi A^\alpha A_\alpha$, and $\langle A_\alpha(z_1)A^\beta(z_2)\rangle = \frac{i}{4\Box}D^\beta D_\alpha\delta_{12}$. Using the commutator for the supercovariant derivatives $[D^\gamma, D_\beta] = 2\delta^\gamma_\beta D^2$ together with the property $D^\alpha D_\beta D_\alpha = 0$, one finds that $D^{\alpha_2}D_{\alpha_1}D^{\alpha_3}D_{\alpha_2}\ldots D^{\alpha_1}D_{\alpha_n}\delta_{12}|_{\theta_1=\theta_2} = 2^n\Box^{(n-1)/2}$, for $n = 2l+1$; instead, if $n = 2l$ is even, this expression vanishes. Hence, we have

$$\Gamma_1^{(1)} = \frac{i}{2}\int d^3x_1 d^2\theta\sum_{l=0}^{\infty}\frac{1}{2l+1}\left(\frac{g^2\Phi\bar\Phi}{4\Box}\right)^{2l+1} \times 2^{2l+1}\Box^l\delta^3(x_1-x_2)|_{x_1=x_2}. \tag{10}$$

Then, by carrying out the Fourier transform ($\Box \to -k^2$), we arrive at the following contribution to the Kählerian effective potential (as usual, the corresponding effective action can be restored from the relation $\Gamma_1^{(1)} = \int d^5z K_1^{(1)}$):

$$K_1^{(1)} = -i\int\frac{d^3k}{(2\pi)^3}\sum_{l=0}^{\infty}\frac{(-1)^l}{2l+1}\left(\frac{\mu^2}{2k^2}\right)^{2l+1}(k^2)^l, \tag{11}$$

where $\mu^2 = g^2\Phi\bar\Phi$. To do the summation, we can consider

$$\frac{dK_1^{(1)}}{d\mu^2} = -\frac{i}{2}\int\frac{d^3k}{(2\pi)^3k^2}\sum_{l=0}^{\infty}\frac{(-1)^l}{2^{2l+1}}\left(\frac{\mu^2}{\sqrt{k^2}}\right)^{2l}. \tag{12}$$

This expression can be rearranged, summed and integrated with use of the dimensional regularization as

$$\frac{dK_1^{(1)}}{d\mu^2} = \frac{1}{4}\int\frac{d^3k}{(2\pi)^3k^2}\sum_{l=0}^{\infty}\left(-\frac{\mu^4}{4k^2}\right)^l = \frac{1}{4}\int\frac{d^3k}{(2\pi)^3}\frac{1}{k^2+\frac{1}{4}\mu^4} = -\frac{\mu^2}{32\pi}. \tag{13}$$

Here, we performed a Wick rotation which yields $k_E^2 = k^2$. All this allows us to integrate the Eq. (12) for $K_1^{(1)}$ and write down the following contribution to the one-loop effective action

$$\Gamma_1^{(1)} = -\int d^5z\frac{(g^2\Phi\bar\Phi)^2}{64\pi}. \tag{14}$$

This expression, in *its functional structure*, is similar to the results of Ref. [10]. Indeed, it is finite, polynomial and does not involve any logarithmlike dependence.

At the same time, we can find the contribution from the purely matter sector:

$$\Gamma_2^{(1)} = -\frac{i}{2}\,\mathrm{Tr}\ln\left(D^2\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} + \mathcal{M}\right), \tag{15}$$

where

$$\mathcal{M} = \begin{pmatrix} M & m \\ m & \bar M\end{pmatrix}.$$

One can elaborate this expression via expansion in a power series, which yields

$$\Gamma_2^{(1)} = \frac{1}{2}\sum_{n=0}^{\infty}\frac{1}{2n+1}\mathrm{Tr}\int\frac{d^3k_E}{(2\pi)^3}\frac{\tilde{\mathcal{M}}^{2n+1}}{(k^2)^{n+1}}, \tag{16}$$

where

$$\tilde{\mathcal{M}} = \mathcal{M}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} = \begin{pmatrix} m & M \\ \bar M & m\end{pmatrix}.$$

The explicit result can be obtained via the diagonalization of the matrix $\tilde{\mathcal{M}}$. As the matrix is Hermitian, its diagonal form can be obtained by calculating its eigenvalues which are $\lambda_{1,2} = m \pm \sqrt{M\bar M}$. This means that

$$\Gamma_2^{(1)} = \frac{1}{2}\sum_{n=0}^{\infty}\frac{1}{2n+1}\int\frac{d^3k}{(2\pi)^3}\frac{\lambda_1^{2n+1}+\lambda_2^{2n+1}}{(k^2)^{n+1}}. \tag{17}$$

This sum can be evaluated in the same way as above, yielding

$$\Gamma_2^{(1)} = \frac{1}{8\pi}\int d^5z(\lambda_1^2+\lambda_2^2) = \frac{1}{4\pi}\int d^5z(M\bar M + m^2). \tag{18}$$

Hence, the complete one-loop Kählerian effective potential can be read off from the sum of Eqs. (14) and (18):

$$K^{(1)} = -\frac{(g^2\Phi\bar\Phi)^2}{64\pi} + \frac{5\lambda^2(\Phi\bar\Phi)^2}{8\pi}. \tag{19}$$

As it was already noted, it is finite and polynomial.

Now, let us turn to the two-loop approximation which we restrict to some qualitative remarks.

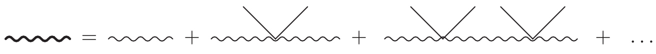

FIG. 2. Calculation of the "dressed" propagator.

First, we obtain the effective propagator of $A_\alpha$ field via the sum depicted at Fig. 2.

To do it, we start with the quadratic action of the $A_\alpha$ field:

$$S_2 = \frac{1}{2}\int d^5z A_\alpha\left[\left(D^\beta D^\alpha + \frac{1}{\xi}D^\alpha D^\beta\right) + \mu^2 C^{\alpha\beta}\right]A_\beta, \quad (20)$$

which can be read off from Eq. (2). As we already noted, after the calculation, we must impose $\xi = 0$. Then, we employ the identity

$$C^{\alpha\beta} = -\frac{1}{2D^2}(D^\alpha D^\beta - D^\beta D^\alpha), \quad (21)$$

which straightforwardly follows from the well-known relation $D^\alpha D^\beta = i\partial^{\alpha\beta} - C^{\alpha\beta}D^2$ (which implies $[D^\alpha, D^\beta] = -2C^{\alpha\beta}D^2$). To obtain the background-dependent propagator from Eq. (20), we must invert the operator

$$\mathcal{O}^{\alpha\beta} = D^\beta D^\alpha + \frac{1}{\xi}D^\alpha D^\beta + C^{\alpha\beta}\mu^2$$
$$= D^\beta D^\alpha\left(1 + \frac{\mu^2}{2D^2}\right) + D^\alpha D^\beta\left(\frac{1}{\xi} - \frac{\mu^2}{2D^2}\right). \quad (22)$$

It is easy to verify that if $\mathcal{O}^{\alpha\beta} = AD^\beta D^\alpha + BD^\alpha D^\beta$, then the corresponding propagator, that is, $G_{\beta\gamma}$ defined such that $\mathcal{O}^{\alpha\beta}G_{\beta\gamma} = -i\delta^\alpha_\gamma\delta^5(z - z')$, looks like

$$G_{\beta\gamma} = (G_1 D_\beta D_\gamma + G_2 D_\gamma D_\beta)\delta^5(z - z'), \quad (23)$$

where

$$G_1 = -\frac{i}{4B\Box}; \qquad G_2 = -\frac{i}{4A\Box}. \quad (24)$$

Replacing the values $A = 1 + \frac{\mu^2}{2D^2}$ and $B = \frac{1}{\xi} - \frac{\mu^2}{2D^2}$, see Eq. (22), we find

$$G_{\beta\gamma}(z_1, z_2) = \frac{i}{4\Box}\left(\frac{\xi D_\beta D_\gamma}{1 - \frac{\xi M^2}{2D^2}} + \frac{D_\gamma D_\beta}{1 + \frac{\mu^2}{2D^2}}\right)\delta^5(z_1 - z_2). \quad (25)$$

Imposing the Landau gauge $\xi = 0$, we get

$$\langle A^\alpha(z_1)A^\beta(z_2)\rangle = \frac{i}{4\Box + 2\mu^2 D^2}D^\beta D^\alpha\delta^5(z_1 - z_2). \quad (26)$$

The next step consists of obtaining the background-dependent propagator for the superfields $\phi$, $\bar\phi$. To do it, we consider the quadratic action of only these quantum fields. As we have already noted before, it looks like

$$S_2[\phi, \bar\phi] = \frac{1}{2}\int d^5z (\phi \quad \bar\phi)\begin{pmatrix} M & D^2 + m \\ D^2 + m & \bar M \end{pmatrix}\begin{pmatrix}\phi \\ \bar\phi\end{pmatrix}. \quad (27)$$

Therefore, the matrix propagator is

$$\begin{pmatrix} \langle\phi(z_1)\phi(z_2)\rangle & \langle\phi(z_1)\bar\phi(z_2)\rangle \\ \langle\bar\phi(z_1)\phi(z_2)\rangle & \langle\bar\phi(z_1)\bar\phi(z_2)\rangle \end{pmatrix}$$
$$= -i\begin{pmatrix} \bar M & -(D^2 + m) \\ -(D^2 + m) & M \end{pmatrix}$$
$$\times \frac{1}{M\bar M - (D^2 + m)^2}\delta^5(z_1 - z_2). \quad (28)$$

Thus, we have found all background-dependent propagators.

The interaction part of the action (2), after absorbing some terms into the propagator and removing irrelevant terms mentioned above, looks like

$$S_{\rm int}[\Phi; \phi, A^\alpha] = \int d^5z\Big[\lambda\Big(\phi^2\bar\phi\,\bar\Phi + \bar\phi^2\phi\Phi + \frac{1}{2}(\phi\bar\phi)^2\Big)$$
$$+ \frac{ig}{2}(\phi A^\alpha D_\alpha\bar\phi - \bar\phi A^\alpha D_\alpha\phi)$$
$$+ g^2 A^\alpha A_\alpha(\Phi\bar\phi + \phi\bar\Phi + \phi\bar\phi)\Big]. \quad (29)$$

Using the vertices from this expression, we must calculate the contributions from the diagrams depicted in Fig. 3. We will not calculate these diagrams exactly, but the dimensional analysis and a straightforward inspection of their contributions show that the two-loop Kählerian effective potential has the following form:

$$K^{(2)} = (\Phi\bar\Phi)^2\left(c_1 + c_2\ln\frac{\Phi\bar\Phi}{\mu_r}\right). \quad (30)$$

Here, $c_1$ is a function of couplings involving the factor $\frac{1}{\epsilon}$, $\epsilon = d - 3$, where $d$ is a spacetime dimension within the dimensional regularization prescription, and $c_2$ is a function of couplings, which, however, is finite. The $\mu_r$ is a renormalization scale.

To obtain the renormalized effective Kählerian potential, after adding the corresponding counterterms, we impose the following normalization condition:

$$\frac{\partial^2 K}{\partial(\Phi\bar\Phi)^2}\Big|_{\Phi=\bar\Phi=v} = \lambda, \quad (31)$$

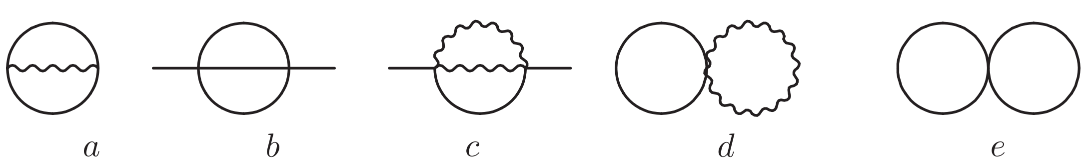


FIG. 3. The two-loop Feynman supergraphs.

where $v$ is a mass scale, and $K = K^{(0)} + K^{(1)} + K^{(2)}$, where $K^{(0)} = \frac{\lambda}{2}(\Phi\bar{\Phi})^2$, and $K^{(1)}$ and $K^{(2)}$ given by Eqs. (19) and (30), respectively, is the complete Kählerian effective potential up to two-loop order. Using this condition to eliminate the dependence on $\mu_r$, we arrive at the following renormalized Kählerian effective potential:

$$K_R = \frac{\lambda}{2}(\Phi\bar{\Phi})^2 + \frac{1}{8\pi}\left(5\lambda^2 - \frac{1}{8}g^4\right)(\Phi\bar{\Phi})^2 - c_2(\Phi\bar{\Phi})^2\left[3 - \ln\frac{\Phi\bar{\Phi}}{v^2}\right]. \quad (32)$$

We see that after the two-loop calculations, a mass scale has been generated, breaking thus the scale invariance just as it occurs in the Coleman-Weinberg model [1].

In summary, in this paper, we presented a superfield method for the calculation of the effective potential in three-dimensional supersymmetric gauge field theories. We succeeded to obtain explicit expressions for the Kählerian effective potential (which depends on superfield $\Phi$ but not on its derivatives) up to two loops. We found that, in the two-loop order, a mass scale is generated; thus, the scale invariance and superconformal symmetry are broken. One must emphasize the difference of our methodology from the one used in Ref. [11]. While in that paper, the calculations were performed for a strongly restricted background field; here, we have done the calculations without any restriction on its structure and also did not use any component expansion. Nevertheless, the functional dependence of the effective action on the background fields is the same as in Ref. [11], being given by Eq. (32). To carry out an exact calculation of the two-loop effective potential for this arbitrary background, however, one must calculate the contributions of the supergraphs depicted in Fig. 3, where the background-dependent propagators are given by Eqs. (26) and (28). Such a calculation is extremely complicated from the technical viewpoint. In principle, our approach can be directly generalized for higher loops, and also for theories with extended supersymmetry and noncommutativity. Also, due to the similarity between the structure of two-dimensional and three-dimensional supersymmetry algebras, we expect that this approach can be also applied for the study of the two-dimensional superfield theories.

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