Semiclassical collapse with tachyon field and barotropic fluid

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The purpose of this paper is to extend the analysis presented in Goswami *et al.* [Phys. Rev. Lett. **96**, 031302 (2006)] by means of investigating how a specific type of loop (quantum) effect can alter the outcome of gravitational collapse. To be more concrete, a particular class of spherically symmetric spacetime is considered with a tachyon field ϕ and a barotropic fluid constituting the matter content; the tachyon potential $V(\phi)$ is assumed to be of the form ϕ^{-2}. Within inverse triad corrections, we then obtain, for a semiclassical description, several classes of analytical as well as numerical solutions. Moreover, we identify a subset whose behavior corresponds to an outward flux of energy, thus avoiding either a naked singularity or a black hole formation.

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I. INTRODUCTION

About forty years ago or so, Hawking and Penrose proved a series of powerful theorems that constitute the fundamental pillars for discussing the final fate of gravitational collapse [1–4]. Within a classical description, it may correspond to either a black hole or a naked singularity: If in a region of spacetime it occurs that there exist families of future-directed nonspacelike trajectories emerging from a past singularity and reaching distant observers, then a naked singularity emerges; but if no such families exist, trapped surfaces will be formed, the singularity will be hidden behind it, and a black hole results [3,4].

Different matter contents have therefore been investigated, with particular interest given to the so-called cosmic censorship conjecture [5]. In particular, it has generally been believed that the presence of pressure may prevent the formation of naked singularities, hence the interest in and study of barotropic fluids within this setting [6,7]. The use of scalar fields has also conveyed additional features for discussing gravitational collapse: Numerical [8] and analytical [9] solutions have then been retrieved—the published literature includes several cases, ranging from massless scalar fields to more elaborate settings by means of a wider class of potentials [10,11]. More recently, gravitational collapse involving tachyonic matter was considered in Ref. [12], exploring the noncanonical kinetic term and subsequent (anti)friction effects. The tachyon constitutes a manageable ingredient in bringing intrinsic string theory features into a gravitational collapse context as well as cosmology [12–14]; when joined by a "conventional" fluid, it allows us to investigate which will eventually dominate at different stages of the collapse and assert whether a standard (i.e., fluid-dominated) collapse will always prevail or other aspects could emerge. It is possible to establish initial conditions (and parameter ranges) when either the tachyon or the fluid becomes dominant, with a black hole or a naked singularity occurring, as well as solutions with a tracking behavior between tachyon and fluid. References [4,6–10,12,15–21] constitute a possible reading selection on scalar field collapse, covering several case studies.

In addition to the ingredients and settings mentioned in the previous paragraph, it is of pertinence to investigate whether elements from a quantum framework may alter the asymptotic states of gravitational collapse. Loop quantum gravity (LQG) [22,23] is one such element, where the formation of a naked singularity can be considered. In LQG, by means of a suitable operator (actually, the inverse volume) in some concrete configurations, the resulting effective equations provide significant differences with respect to the classical setting—in particular, concerning the late-time stages of gravitational collapse.

It was shown in Ref. [24] that inverse triad modifications replace a classical singularity with a bounce; in this case, the standard scalar field at the semiclassical limit begins to behave as exotic matter, namely a phantom field. Moreover, in Ref. [25], in the presence of a standard scalar field with a general potential, for a marginally bound gravitational collapse, this semiclassical analysis leads to an outward flux of energy. Subsequently, specific (anti)friction effects associated with different types of loop- (quantum-) induced terms, and with the different matter sources, have been explored in the literature [24–28]. However, it still remains an open issue whether induced corrections (inverse triad) resolve those singularities; results suggest that in some situations the naked singularity prevails [26], while others point to a different outcome [24,25,27,28].

In LQG, a tachyon scalar field has been proposed as a concrete example for investigating the initial singularity in

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the Universe [29,30]. It was found that the Universe can evolve through this singularity regularly [29]. Therefore, it is of interest to investigate whether a LQG-induced modification to the tachyon equation of motion can avoid the classical singularity that may arise at the final state of gravitational collapse. We need to widen the gravitational settings being explored, together with broadening the classes of matter being used, in order to fully investigate whether a gravitational collapse bears a nonsingular nature or not.

The above paragraphs support this paper's purpose—to enlarge the discussion on scalar field gravitational collapse by means of extending the scope analyzed in Ref. [25] and assembling herein three concrete elements: loop effects (inverse triad [22]), tachyonic dynamics [12,14], and (barotropic) fluid pressure [6,7]. In Sec. II, after providing the background scenario—more specifically, introducing the tachyon field as well as the barotropic fluid—within our choice of geometry, we address whether those loop modifications can modify the classical outcome of tachyon gravitational collapse by presenting a set of thorough numerical and analytical solutions. In Sec. III, we examine further a subset associated with an outward flux of energy. Our conclusions and a discussion of our results are presented in Sec. IV. In Appendix A, we complement our phase space discussion with a brief review of an additional tool for discussing the behavior of nearby critical points. Finally, in Appendix B, we employ a Hamilton-Jaccobi formulation to discuss the tachyon potential we use throughout this paper.

II. SEMICLASSICAL COLLAPSE

We follow the literature (see, e.g., Refs. [4,25,31]) and take for spacetime geometry a homogeneous interior spacetime, matched to a suitable (inhomogeneous) exterior geometry,[1] to provide the whole spacetime structure. More precisely, the interior spacetime is the marginally bound case ($k = 0$) [4,31] and is parametrized by a line element as follows:[2]

$$ds^2 = -dt^2 + a^2(t)dr^2 + R^2(t, r)d\Omega^2, \tag{2.1}$$

where t is the proper time for a falling observer whose geodesic trajectories are labeled by the comoving radial coordinate r, with $d\Omega^2$ being the standard line element on the unit two-sphere; we have set the units $8\pi G = c = 1$. In general, the equation for an apparent horizon in a spherical symmetric spacetime is given by $g^{ij}R_{,i}R_{,j} = 0$, in which using the Einstein equations gives us the equation of trapped surfaces as $\frac{F}{R} = 1$.

The quantity $F(R)$ is the mass function of the collapsing matter, with $R(t, r) = ra(t)$ being the area radius of the collapsing shell, which is a function of the comoving coordinates t and r. The spacetime is said to be trapped or untrapped if $\frac{F}{R} > 1$ or $\frac{F}{R} < 1$, respectively.

We will consider as matter content a spherically symmetric homogeneous tachyon field[3] together with a barotropic fluid. We use an inverse square potential for the tachyon field, given by (cf. Appendix B)[4]

$$V(\phi) = V_0\phi^{-2}, \tag{2.2}$$

where V_0 is a constant [13,14]. Note that we can consider the potential of Eq. (2.2) as two (mirror) branches upon the symmetry $\phi \to -\phi$, treating the $\phi > 0$ and $\phi < 0$ cases separately [12]. The equation of motion for the tachyon field (without loop modifications) can then be written as

$$\ddot{\phi} = -(1 - \dot{\phi}^2)\left[3H\dot{\phi} + \frac{V_{,\phi}}{V}\right], \tag{2.3}$$

with $H \equiv \frac{\dot{a}}{a}$. Since we are interested in a continuous collapsing scenario, we use $\dot{a} < 0$ (i.e., $H < 0$) henceforth, implying that the area radius of the collapsing shell, for a constant value of r, decreases monotonically.

We designate by ρ_b the energy density of the classical barotropic matter, whose pressure p_b, in terms of the barotropic parameter γ, satisfies the relation $p_b = (\gamma - 1)\rho_b \equiv w_b$, $\gamma > 0$, where $\rho_b = \rho_{0b}a^{-3(1+w_b)}$. The total energy density, without the loop elements $\rho = \rho_\phi + \rho_b$, is therefore given by

$$\rho = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} + \rho_b. \tag{2.4}$$

Let us now add a specific type of nonperturbative modification to the dynamics, motivated by LQG [22]. In the semiclassical framework of loop corrections of the inverse triad type [32], we have an effective energy density for the barotropic fluid given as $\rho_b^{\rm loop} = D(q)^{w_b}\rho_b$, or [33]

$$\rho_b^{\rm loop} = \rho_{0b}D^{(\gamma-1)}a^{-3\gamma}, \tag{2.5}$$

where $D(q)$ is the loop quantum parameter, given by [34]

$$D(q) = (8/77)^6 q^{3/2}\{7[(q+1)^{11/4} - |q-1|^{11/4}] - 11q[(q+1)^{7/4} - \mathrm{sgn}(q-1)|q-1|^{7/4}]\}^4, \tag{2.6}$$

with $q \equiv \mathsf{p}/\mathsf{p}_j = a^2/a_*^2$. This relation, for the classical regime $a \gg a_*$, implies that $D(q) \to \frac{1}{q}$; and for the

[1] E.g., in a collapsing configuration, in which the matter pressure vanishes at the boundary of two regions, it is always possible to match the interior to a vacuum Schwarzschild exterior [31].

[2] This particular setting has been of recent use, namely in investigations of loop (quantum) effects in gravitational collapse, involving a *standard* scalar field (cf. Ref. [25]).

[3] Please cf. footnote 2 regarding the use of a standard scalar field in Ref. [25], where no fluid is present.

[4] In Appendix B, we employ a Hamilton-Jaccobi formulation to discuss the tachyon potential. This analysis is to justify the use of an inverse square potential for the tachyon field.

semiclassical regime $a_i < a \ll a_*$, reduces to $D(q) = (\frac{12}{7}q)^4$. Here a_* is a critical scale at which the eigenvalue of the a^{-1} has a power-law dependance on a; $a_i \equiv \sqrt{\gamma}\ell_{\rm Pl}$ is the scale above which a classical continuous spacetime can be defined and below which the spacetime is discrete. In addition, $\gamma = 0.13$ is the Barbero-Immirzi parameter, and $\ell_{\rm Pl}$ is the Planck length.

Regarding the tachyon field, the semiclassical energy density can now be written as [35]

$$\rho_\phi^{\rm loop} = 3H^2 = \frac{V(\phi)}{\sqrt{1 - A^{-1}q^{-15}\dot{\phi}^2}}, \tag{2.7}$$

where the corresponding pressure, $p_\phi^{\rm loop}$, is given by

$$p_\phi^{\rm loop} = -\frac{V(\phi)}{\sqrt{1 - A^{-1}q^{-15}\dot{\phi}^2}}\left[1 + \frac{4\dot{\phi}^2}{Aq^{15}}\right]. \tag{2.8}$$

The equation of motion for ϕ in the semiclassical limit is

$$\ddot{\phi} - 12H\dot{\phi}\left[\frac{7}{2} - \frac{\dot{\phi}^2}{Aq^{15}}\right] + \left[Aq^{15} - \dot{\phi}^2\right]\frac{V_{,\phi}}{V} = 0, \tag{2.9}$$

where $A \equiv (12/7)^{12}$.

The total energy density will be given by the loop-modified energy density of the tachyon field plus that for the fluid, i.e., $\rho^{\rm loop} = \rho_\phi^{\rm loop} + \rho_{\rm b}^{\rm loop}$. As a consequence, the corresponding constraint equation follows as

$$3H^2 = \rho_\phi^{\rm loop} + \rho_{\rm b}^{\rm loop}. \tag{2.10}$$

The Raychaudhury equation becomes

$$2\dot{H} + 3H^2 = p_\phi^{\rm loop} - w_{\rm b}\left(1 - \frac{1}{3}\frac{d\ln D}{d\ln a}\right)\rho_{\rm b}^{\rm loop}. \tag{2.11}$$

Moreover, from Eq. (2.11), considering that $2\dot{H} + 3H^2 = -p_{\rm b}^{\rm loop}$, an effective equation of state in this semiclassical regime will be

$$w_{\rm b}^{\rm loop} = w_{\rm b}\left(1 - \frac{1}{3}\frac{d\ln D}{d\ln a}\right). \tag{2.12}$$

From Eq. (2.12) it is seen, if we rewrite $w_{\rm b}^{\rm loop}$ similarly to the classical expression, as $w_{\rm b}^{\rm loop} = (\tilde{\gamma} - 1)$, that we get

$$\tilde{\gamma} \simeq \frac{8 - 5\gamma}{3}. \tag{2.13}$$

In the semiclassical region, where $D(q) \ll 1$, from Eq. (2.11), we further have

$$2\dot{H} = \frac{4V(\phi)\dot{\phi}^2/(Aq^{15})}{\sqrt{1 - A^{-1}q^{-15}\dot{\phi}^2}} - \tilde{\gamma}\rho_{\rm b}^{\rm loop}. \tag{2.14}$$

In what follows, we will study our collapsing model using a dynamical system description [36] for Eqs. (2.9)–(2.14). We use a new time variable N (instead of the proper time t in the comoving coordinate system $\{t, r, \theta, \varphi\}$). In more concrete terms, we choose

$$N \equiv -\log q^{3/2}, \tag{2.15}$$

with q being defined in the interval $0 < N < \infty$; the limit $N \rightarrow 0$ corresponds to the initial condition of the collapsing system ($a \rightarrow a_*$), and the limit $N \rightarrow \infty$ corresponds to $a \rightarrow 0$. For any time-dependent function f,

$$\frac{df}{dN} = -\frac{\dot{f}}{3H}. \tag{2.16}$$

We further use a set of new dynamical variables:

$$X \equiv \frac{\dot{\phi}}{A^{\frac{1}{2}}q^{\frac{15}{2}}}, \qquad Y \equiv \frac{V}{3H^2}, \qquad Z \equiv A^{\frac{1}{2}}q^{\frac{15}{2}},$$
$$S \equiv \frac{\rho_{\rm b}^{\rm loop}}{3H^2}, \qquad \xi \equiv -\frac{V_{,\phi}}{V^{\frac{3}{2}}}, \qquad \Gamma \equiv \frac{VV_{,\phi\phi}}{(V_{,\phi})^2}, \tag{2.17}$$

in which S can also be written as

$$S = D^{w_{\rm b}}\left(\frac{\rho_{\rm b}}{3H^2}\right), \tag{2.18}$$

such that in the limit $D(q) \ll 1$, it reduces to

$$S \simeq \left(\frac{12}{7}\right)^{4w_{\rm b}} q^{4w_{\rm b}}\left(\frac{\rho_{\rm b}}{3H^2}\right). \tag{2.19}$$

An autonomous system of equations, in terms of the dynamical variables of Eq. (2.17), for Eqs. (2.9) and (2.14), is then retrieved:

$$\frac{dX}{dN} = X(4X^2 - 9) + \frac{1}{\sqrt{3}}\xi Z\sqrt{Y}(1 - X^2), \tag{2.20}$$

$$\frac{dY}{dN} = Y\left[4X^2 - \frac{\xi}{\sqrt{3}}XZ\sqrt{Y} - (\tilde{\gamma} + 4X^2)S\right], \tag{2.21}$$

$$\frac{dZ}{dN} = -5Z, \tag{2.22}$$

$$\frac{dS}{dN} = S(1 - S)(\tilde{\gamma} + 4X^2), \tag{2.23}$$

$$\frac{d\xi}{dN} = -\frac{1}{\sqrt{3}}\xi^2 XZ\sqrt{Y}\left(\Gamma - \frac{3}{2}\right). \tag{2.24}$$

For $V(\phi)$, as in Eq. (2.2), it brings $\xi = \pm 2/\sqrt{V_0}$ and $\Gamma = 3/2.$, i.e., as constants. The dynamical system then reduces to four differential equations with variables (X, Y, Z, S), namely Eqs. (2.20)–(2.23). Equation (2.10), in terms of the new variables, can be written as

TABLE I. Critical points and their properties.

Point	X	Y	Z	S	Ω_ϕ	γ_ϕ	Existence	Stability
A	0	1	0	0	1	0	All ξ; $\tilde{\gamma} \leq 0$ (i.e., $\gamma \geq \frac{8}{5}$)	Stable
							All ξ; $\tilde{\gamma} > 0$ (i.e., $\gamma < \frac{8}{5}$)	Saddle point
B^+	$\frac{3}{2}$	0	0	1	1	-9	All ξ, $\tilde{\gamma}$	Saddle point
B^-	$-\frac{3}{2}$	0	0	1	1	-9	All ξ, $\tilde{\gamma}$	Saddle point
C	0	0	0	1	0	0	All ξ; $\tilde{\gamma} = 0$ (i.e., $\gamma = \frac{8}{5}$)	Stable
							All ξ; $\tilde{\gamma} \neq 0$ (i.e., $\gamma \neq \frac{8}{5}$)	Saddle point

$$\frac{Y}{\sqrt{1-X^2}} + S = 1, \tag{2.25}$$

in which $Y \geq 0$ and $-1 \leq X \leq 1$ and $0 \leq S \leq 1$.

A discussion of the autonomous system of equations in Eqs. (2.20)–(2.23) requires us to identify the critical points (X_c, Y_c, Z_c, S_c); the properties of each critical point (and its associated stability features) are determined by the eigenvalues of the corresponding 4×4 Jacobi matrix $\mathcal{B}$. Setting therefore $(f_1, f_2, f_3, f_4)|_{(X_c,Y_c,Z_c,S_c)} = 0$, we can obtain them, where we have defined $f_1 \equiv dX/dN$, $f_2 \equiv dY/dN$, $f_3 \equiv dZ/dN$, $f_4 \equiv dS/dN$. The eigenvalues, defined at each fixed point (X_c, Y_c, Z_c, S_c), are then brought from

$$\mathcal{B} = \begin{pmatrix} \frac{\partial f_1}{\partial X} & \frac{\partial f_1}{\partial Y} & \frac{\partial f_1}{\partial Z} & \frac{\partial f_1}{\partial S} \\ \frac{\partial f_2}{\partial X} & \frac{\partial f_2}{\partial Y} & \frac{\partial f_2}{\partial Z} & \frac{\partial f_2}{\partial S} \\ \frac{\partial f_3}{\partial X} & \frac{\partial f_3}{\partial Y} & \frac{\partial f_3}{\partial Z} & \frac{\partial f_3}{\partial S} \\ \frac{\partial f_4}{\partial X} & \frac{\partial f_4}{\partial Y} & \frac{\partial f_4}{\partial Z} & \frac{\partial f_4}{\partial S} \end{pmatrix}_{|(X_c,Y_c,Z_c,S_c)}. \tag{2.26}$$

Physical solutions in the neighborhood of a critical point, $q_i^{\rm crit}$, can be extracted by making use of

$$q_i(t) = q_i^{\rm crit} + \delta q_i(t), \tag{2.27}$$

with the perturbation δq_i given by

$$\delta q_i = \sum_j^k (q_0)_i^j \exp(\zeta_j N), \tag{2.28}$$

where $q_i \equiv \{X, Y, Z, S\}$, and ζ_j are the eigenvalues of the Jacobi matrix; the $(q_0)_i^j$ are constants of integration. We have summarized the fixed points for the autonomous system and their stability properties in Table I.

Point (A): The eigenvalues of this fixed point are $\sigma_1 = -9$, $\sigma_2 = 0$, $\sigma_3 = -5$, and $\sigma_4 = \tilde{\gamma}$. It can be seen that, for $\tilde{\gamma} < 0$ (or $\gamma > \frac{8}{5}$), three eigenvalues are negative and one is 0. To gain insight about the stability of the system at this fixed point, we use the center manifold theorem.[5] [36]. Using this method, we see that (A) is a *stable* point. On the other hand, for $\tilde{\gamma} > 0$ (i.e., $\gamma < \frac{8}{5}$), one eigenvalue is 0, another is positive, and two others are negative; hence, a *saddle* point setting will be recovered. In the limit case in which $\tilde{\gamma} = 0$, a *stable* point behavior can likewise be shown to emerge (see Appendix A).

Point (B^+): The eigenvalues for this fixed point are $\sigma_1 = 18$, $\sigma_2 = 9$, $\sigma_3 = -5$, and $\sigma_4 = \tilde{\gamma} + 9$. For all values of $\tilde{\gamma}$, two characteristic values are positive and the others are negative. Thus, this fixed point is a *saddle* point.

Point (B^-): For this fixed point, the characteristic values are $\sigma_1 = 18$, $\sigma_2 = 9$, $\sigma_3 = -5$, and $\sigma_4 = \tilde{\gamma} + 9$, which are the same eigenvalues as the fixed point (B^+), and thus, similarly to (B^+), this is a *saddle* point.

Point (C): The eigenvalues are $\sigma_1 = -9$, $\sigma_2 = -\tilde{\gamma}$, $\sigma_3 = -5$, and $\sigma_4 = \tilde{\gamma}$. For all $\tilde{\gamma} \neq 0$, σ_2 and σ_4 always have opposite signs: This corresponds to a *saddle* point. For this case (i.e., $\gamma \neq \frac{8}{5}$), the power of the exponential term δS has a different sign with respect to the others, and assuming $\tilde{\gamma} > 0$, the term δS increases as N increases (i.e., a decreases). For the case $\tilde{\gamma} = 0$ (i.e., for the corresponding barotropic parameter $\gamma = 8/5$), σ_2 and σ_4 are 0, whereas the two others are negative. Using the center manifold theorem [cf. point (A)], it can be shown that this corresponds to a *stable* fixed point: Using Eq. (2.28), solutions in terms of the dynamical variables X, Y, Z, and S are given, respectively, by $\delta X \approx \exp(-9N) = (a/a_*)^{27}$, $\delta Y \approx \exp(-\tilde{\gamma}N) = (a/a_*)^{3\tilde{\gamma}}$, $\delta Z \approx \exp(-5N) = (a/a_*)^{15}$, and $\delta S \approx \exp(\tilde{\gamma}N) = (a/a_*)^{-3\tilde{\gamma}}$.

There are two solutions which are of particular interest concerning gravitational collapse whose asymptotic behaviors are stable at the late stages of the collapse (where scale factor is small): They are the case $\gamma > \frac{8}{5}$ [for the fixed point (A)] and the case $\gamma = \frac{8}{5}$ [for the fixed point (C)]. In the following subsections, we will discuss these two solutions.

A. Semiclassical tachyon-dominated solutions

Let us now study the behavior of the system near the asymptotic solution [for point (A)] when $\gamma \geq 8/5$. From Eq. (2.28), we can find the perturbation around the fixed point by using $X(t) = X_c + \delta X$, $Y(t) = Y_c + \delta Y$, $Z(t) = Z_c + \delta Z$, $S(t) = S_c + \delta S$, from which we can write

[5]The application of the center manifold theorem is a tool that can be employed in the case when the linearization of the autonomous system fails, with $\mathcal{B}$ having eigenvalues with zero real parts. Using this method, we can show that the critical point (A) is asymptotically *stable*. (See Appendix A for details of the analysis.)

$$X(t) \approx \left(\frac{a}{a_*}\right)^{27}, \qquad Y(t) \approx 1,$$
$$Z(t) \approx A^{1/2}\left(\frac{a}{a_*}\right)^{15}, \qquad S(t) \approx \left(\frac{a}{a_*}\right)^{-3\tilde{\gamma}}. \tag{2.29}$$

In this neighborhood, the effective energy density of the tachyon field is given by

$$\rho_\phi^{\rm loop} \approx V(\phi)\left(1 + \frac{1}{2}\frac{\dot{\phi}^2}{Aq^{15}}\right). \tag{2.30}$$

In addition, the energy density of barotropic matter is modified by the loop parameter $D(q)^{(\gamma-1)}$ [cf. Eq. (2.5)] and can be approximated as

$$\rho_{\rm b}^{\rm loop} \approx \left(\frac{a}{a_*}\right)^{(5\gamma-8)}. \tag{2.31}$$

For this solution, $\dot{\phi}$ decreases very fast—as $\dot{\phi} \propto (a/a_*)^{42}$—as the scale factor decreases; in this approximation, for very small values of a, the second term on the right-hand side of Eq. (2.30) evolves as $\dot{\phi}^2/Aq^{15} \propto (a/a_*)^{54}$ and decreases faster than $\dot{\phi}$, becoming negligible during the final stages of the collapse. We can then analyze it as follows.

This solution presents semiclassical effects that modify the energy density of the barotropic matter as well as the tachyon field: The energy density of the tachyon field is determined by Eq. (2.30), and the energy density of the barotropic matter is given by Eq. (2.31). This solution shows that the loop correction term $D(q)$ scales down the effect of the barotropic fluid and avoids its divergence towards the center of the star (i.e., for $\gamma > \frac{8}{5}$, $\rho_{\rm b}^{\rm loop}$ becomes negligible when a is close to the Planck scale). Then the collapsing matter content at this point is tachyon dominated, and the total energy density of the collapse is determined by the effective energy density of tachyonic matter, given by Eq. (2.30).

On the other hand, $Y \rightarrow 1$ at this point, with $V \approx 3H^2$ as well, implying that, in this regime (differently from its classical counterpart [12]) the potential of the tachyon field has the main role in determining the effective energy density of the system $\rho^{\rm loop} \approx V(\phi)$, where $\dot{\phi}^2/Aq^{15} \ll 1$. Substituting the potential with $V = V_0\phi^{-2}$ in $V \approx 3H^2$, we get $H(\phi) \simeq -\sqrt{V_0/3}|\phi|^{-1}$ for both the $\phi > 0$ and $\phi < 0$ branches. Integrating for $H(\phi)$, we can obtain

$$a^{42}(\phi) = 42\sqrt{\frac{V_0}{3}}\ln\left|\frac{\phi_f}{\phi}\right|, \tag{2.32}$$

where ϕ_f is a constant of integration. For initial conditions such as $\phi(0) = \phi_0$ and $a(0) = a_* \approx (\sqrt{V_0/3}\ln|\phi_f/\phi_0|)^{\frac{1}{42}}$, the tachyon field approaches a finite value as $\phi \rightarrow \phi_f$ when the scale factor is small at about the Planck scale. Thus, the potential of the tachyon field decreases from its initial value and approaches a finite value.

Then, using Eqs. (2.30) and (2.31), the total energy of the system in this regime reads

$$\rho^{\rm loop} \approx \frac{V_0}{\phi^2} + \left(\frac{a}{a_*}\right)^{(5\gamma-8)}. \tag{2.33}$$

When the scale factor is small, where $\phi \rightarrow \phi_f$, the effective energy density of the fluid [Eq. (2.31)] is very small, and thus the second term in Eq. (2.33) is negligible. Then the total energy of the system in this regime is given by the effective energy density of the tachyon field.

It should be noted that, since $\dot{\phi} \approx (a/a_*)^{42} > 0$, for a $\phi > 0$ branch, the tachyon field increases from its initial value $\phi_0 > 0$ and reaches its maximum at ϕ_f. From Eq. (2.33), it can be seen that when the tachyon field changes in the interval $\phi_0 < \phi < \phi_f$, the total energy density of the system decreases from its initial value $\rho_0^{\rm loop}$ and reaches its minimum and finite value at $\rho^{\rm loop} \rightarrow V_0/\phi_f^2$ for a very small a. Thus, in contrast to the classical counterpart [12], the total energy density does not blow up, becoming finite.

The total mass function in this regime can be approximated as

$$\frac{F^{\rm loop}}{R} \approx \frac{V_0}{\phi^2}r^2a^2 + \left(\frac{a}{a_*}\right)^{(5\gamma-6)}. \tag{2.34}$$

Since $\gamma \geq \frac{8}{5}$, and therefore, for very small values of s, the second term in Eq. (2.34) is negligible, the mass function behaves as $F^{\rm loop}/R \propto a^2$ and decreases towards the center. Moreover, in this region, the total pressure of the system is approximately given by

$$p^{\rm loop} \approx -V(\phi)\left(1 + \frac{9}{2}\frac{\dot{\phi}^2}{Aq^{15}}\right) + \left(\frac{8-5\gamma}{3}\right)\rho_{\rm b}^{\rm loop}, \tag{2.35}$$

which is negative for the semiclassical collapse. The effective pressure [Eq. (2.35)] evolves asymptotically such that $p^{\rm loop} \approx -V_0/\phi^2$ near the singularity. Thus, it remains finite towards the late-time stages of the collapse, inducing an outward flux of energy in the semiclassical regime.

A thorough numerical study allows the following to be additionally mentioned about the case $\gamma = \frac{8}{5}$: In Fig. 1, the semiclassical area radius (solid line) shows some deviation from what could be expected classically [12] in the early stages of the collapse; the energy density slowly converges to zero as the area radius gets smaller.

In order to further investigate curvature singularities, we can use scalar polynomials constructed out of the metric and the Riemann tensors. An appropriate example is provided by the Kretschmann scalar $\mathcal{K} = R_{abcd}R^{abcd}$ [37], which for the line element of Eq. (2.1), is given by $\mathcal{K} = 12[(\ddot{a}/a)^2 + (\dot{a}/a)^4]$. The right plot in Fig. 1 shows the semiclassical behavior of the Kretschmann scalar (solid line) as a function of proper time. Therein we observe that in the semiclassical regime, this quantity remains finite as the physical area radius decreases,

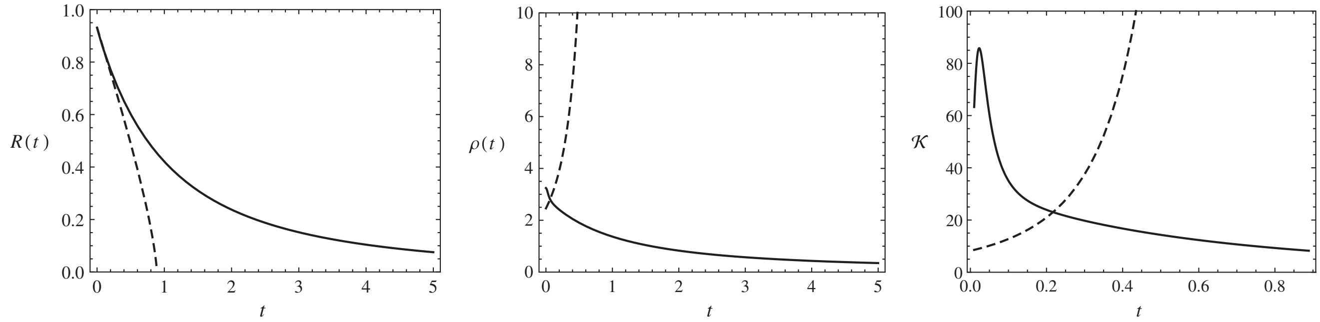

FIG. 1. Behaviors of the area radius, energy density, and Kretschmann scalars (with loop corrections; solid lines) compared against classical ones (dashed lines). We consider $t_i = 0$, $a(0) = a_*$, $V_0 = 1/3$, $\phi_0 = -0.6$, and $\gamma = \frac{8}{5}$.

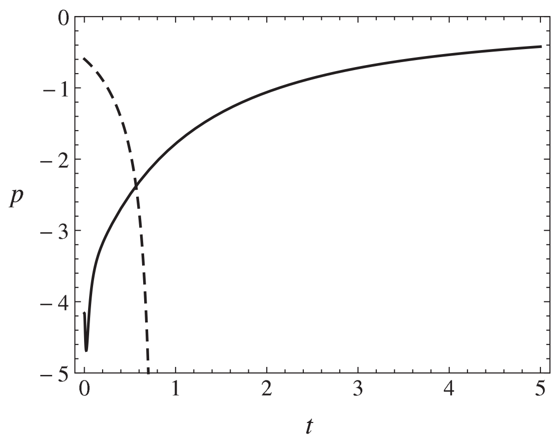

FIG. 2. Behavior of the effective pressure (with loop corrections; solid line) compared against the classical one (dashed line). We consider $t_i = 0$, $a(0) = a_*$, $V_0 = 1/3$, $\phi_0 = -0.6$, and $\gamma = \frac{8}{5}$.

consequently signaling the avoidance of a curvature singularity. This, together with the regularity of the energy density, seems to suggest that the corresponding spacetime of the setting in this subsection is regular as long as this specific semiclassical scenario is valid.

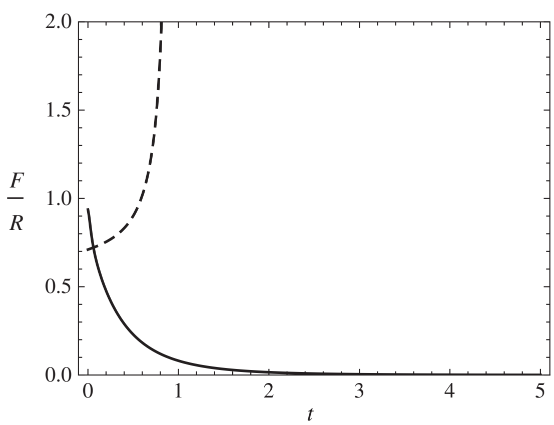

FIG. 3. Behavior of the mass function (with loop corrections; solid line) compared against the classical one (dashed lines). We consider $t_i = 0$, $a(0) = a_*$, $V_0 = 1/3$, $\phi_0 = -0.6$ and $\gamma = \frac{8}{5}$.

Figure 2 shows the semiclassical behavior of the effective pressure, indicating that the pressure remains negative during the semiclassical regime.

We further depict the semiclassical (solid line) behavior of the mass function in Fig. 3, showing that, from the early stages of the collapse, this quantity stays smaller than the area radius, and it converges to zero when it approaches the final stage of the collapse. Therefore, there are no trapped surfaces forming. Moreover, loop quantum corrections of the inverse triad type appear to induce an outward flux of energy at the final state of the collapse (cf. Fig. 4), which we will discuss in Sec. III.

Using Eqs. (2.7)–(2.8), the equation of state for the tachyon field, $w_\phi^{\rm loop} = p_\phi^{\rm loop}/\rho_\phi^{\rm loop}$, is given by $w_\phi^{\rm loop} = -(1 + 4X^2)$. We can further rewrite it as

$$w_\phi^{\rm loop} = -\left(1 + \frac{4\dot{\phi}^2}{Aq^{15}}\right). \tag{2.36}$$

Since $w_\phi^{\rm loop} < -1$, the effective equation of state behaves as a phantom matter for which the energy conditions are violated (see similar behavior for a standard scalar field in Ref. [24]). Satisfying the energy conditions is not expected in quantum gravity, but these conditions must be held in classical collapse (cf. Ref. [12]). Furthermore, in order to

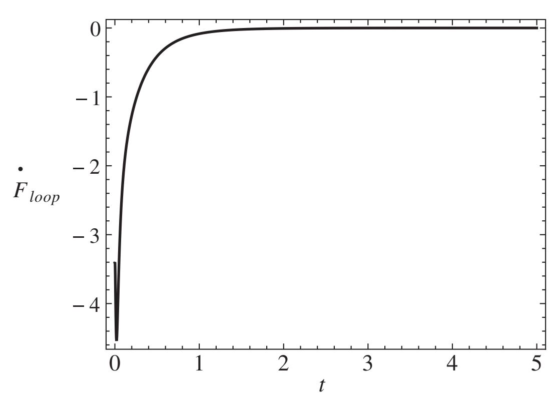

FIG. 4. Behaviors of the derivative of the mass function over time (with loop corrections). We considered $t_i = 0$, $a(0) = a_*$, $V_0 = 1/3$, $\phi_0 = -0.6$ and $\gamma = \frac{8}{5}$.

have a well-defined initial condition for our collapsing model, the initial data for the (classical featured) barotropic parameter γ must respect the energy conditions. So that the energy conditions will be satisfied for the barotropic matter in this case with the parameter $\gamma \geq \frac{8}{5}$, γ must hold the range $\gamma < 2$ (see Ref. [12] for a discussion on the energy conditions in the classical setting).

B. Semiclassical barotropic-dominated solutions

For the case with $\tilde{\gamma} = 0$ (or $\gamma = \frac{8}{5}$), near the fixed-point solution (C), the time derivative of the tachyon field vanishes towards the center (i.e., $\dot{\phi} \rightarrow 0$ for very small scale factor a), and hence, asymptotically the tachyon field and its potential remain constant. Furthermore, the energy density of the tachyon field in this regime is essentially dominated by the tachyon potential, i.e., $\rho_\phi^{\rm loop} \simeq V(\phi)$. On the other hand, $S \rightarrow 1$, implying that the total energy density of the collapse is dominated by the energy density of the barotropic matter as $\rho_{\rm b}^{\rm loop} = 3H^2$. From Eq. (2.5), we get $\rho^{\rm loop} \approx \rho_{\rm b}^{\rm loop} = \rho_{0{\rm b}}$, and thus $3H^2 = \rho_{0{\rm b}}$, which gives an expression $H = -(\rho_{0{\rm b}}/3)^{1/2} < 0$.

The ratio of the effective mass function to the area radius in this case can be obtained as $F^{\rm loop}/R = r^2\rho_{0{\rm b}}/3$, which, for any shell r, remains finite. For an adequate choice of $\rho_{0{\rm b}}$ in the the semiclassical regime, the effective mass function in this regime remains less than the area radius, and no trapped surface will form as the collapse proceeds.

The effective equation of state for the tachyon field in this case reads $w_\phi^{\rm loop} = -1$, which satisfies the energy condition. Moreover, the barotropic parameter satisfies the range $0 < \gamma = \frac{8}{5} < 2$, for which the energy conditions are satisfied as well.

III. OUTWARD FLUX OF ENERGY IN TACHYON-DOMINATED COLLAPSE

To complete the spacetime model in the semiclassical regime, the interior spacetime needs to be matched to a convenient exterior region. Since the effective pressure in the interior [Eq. (2.35)] is negative at the boundary, and furthermore, there are no trapped surfaces forming as the collapse evolves, such a region cannot be matched to a Schwarzschild exterior spacetime. The exterior region is therefore assumed to be a generalized Vaidya geometry [24,38] and can be matched to the interior at the boundary hypersurface Σ given by $r = r_b$. The generalized Vaidya exterior is given by [38]

$$ds_{\rm out}^2 = -\left(1 - \frac{2M(r_v, v)}{r_v}\right)dv^2 - 2dvdr_v + r_v^2 d\Omega^2, \tag{3.1}$$

where $M(r_v, v)$ is the usual Vaidya mass, with v and r_v being the retarded null coordinate and the Vaidya radius, respectively.

Let us designate the mass function at scales $a \gg a_*$ (i.e., in the classical regime), as F, whereas for $a < a_*$ (in the semiclassical regime) we use $F^{\rm loop}$. The mass loss is provided by the following expression:

$$\frac{\Delta F}{F} \equiv \frac{F - F^{\rm loop}}{F} = \left(1 - \frac{\rho^{\rm loop}}{\rho}\right). \tag{3.2}$$

In order to understand this, let us consider the geometry outside a spherically symmetric matter, as given by the Vaidya metric [Eq. (3.1)], with $v = t - r_v$ and $M(v)$ being the retarded null coordinate and the Vaidya mass, respectively. We can further take the relation $F^{\rm loop} = 2M(v)$ between the mass function and the Vaidya mass. Let us also assume the energy density of the flux to be measured locally by an observer with a four-velocity vector ξ^μ. Then the energy flux, as well as the energy density of radiation measured in this local frame, is given by $\sigma \equiv T_{\mu\nu}\xi^\mu\xi^\nu$, which, for only radially moving observers with the radial velocity ${\rm v} \equiv \xi^{r_v} = \frac{dr_v}{dt}$, becomes

$$\sigma \equiv -\frac{1}{(\gamma + {\rm v})^2}\left(\frac{1}{4\pi r_v^2}\frac{dM(v)}{dv}\right), \tag{3.3}$$

where $\gamma = (1 + {\rm v}^2 - 2M(v)/r_v)^{-1}$. The total luminosity for an observer with radial velocity v and for the radius r_v, is given by $L(v) = 4\pi r_v^2 \sigma$ [39]. Therefore, by substituting σ from Eq. (3.3) into the equation of luminosity, we can establish

$$L(v) = \frac{1}{(\gamma + {\rm v})^2}\frac{dM(v)}{dv}. \tag{3.4}$$

Then, from Eq. (3.4), the luminosity in terms of the mass function $F^{\rm loop}$ is

$$L(v) = \frac{\dot{F}^{\rm loop}}{2(\gamma + {\rm v})}. \tag{3.5}$$

For an observer being at rest (${\rm v} = 0$) at infinity ($r_v \rightarrow \infty$), the total luminosity of the energy flux can be obtained by taking the limit of Eq. (3.5):

$$L_\infty(v) = -\frac{\dot{F}^{\rm loop}}{2}. \tag{3.6}$$

As long as $dM/dv \leq 0$, the total luminosity of the energy flux is positive; since the effective energy density near the center decreases very slowly, the effective mass function given by Eq. (2.34) can be approximated as $F^{\rm loop} \approx a^2$ in the loop-modified regime, and is decreasing as the collapse evolves (see Fig. 4). Then, its time derivative is always negative, pointing to the positiveness of the luminosity; this indicates that there exists an energy flux radiated away from the interior spacetime, reaching the distant observer.

IV. CONCLUSIONS AND DISCUSSION

In this paper, we considered a particular class of spacetime to set up a gravitational collapse (cf. Ref. [25]) with a tachyon field and a barotropic fluid as matter content; the tachyon potential was assumed to be of an inverse square form. We investigated a semiclassical regime, characterized by inverse triad corrections (cf. Refs. [23,25]). We provided an analytical description discussing several asymptotic behaviors; these were also subject to a study involving a numerical appraisal, which added a clearer description of the dynamics.

In assembling these three elements (loop quantum effects [22], tachyonic dynamics [35], and barotropic fluids), our aim was to enlarge the discussion on scalar field gravitational collapse, extending the scope analyzed in Ref. [25], employing ingredients motivated by recent developments regarding a (high-energy) quantum gravity regime. We subsequently found a class of solutions which showed that the matter effective energy density remains finite as the collapse evolves. More precisely, using a dynamical system analysis, it was shown that the energy density of the tachyon is governed by its potential. Then the tachyonic energy density becomes regular during the collapse. On the other hand, the classical fluid with the energy density $\rho_{\rm b} \approx (a/a_*)^{-3\gamma}$ (which is singular at $a = 0$ for $\gamma > 0$) was modified in the loop quantum regime by a $D^{\gamma-1}$ factor, as $\rho_{\rm b}^{\rm loop} \approx (a/a_*)^{5\gamma-8}$. The phase space analysis allows stable solutions to exist only for the range of $\gamma \geq 8/5$. Indeed, the semiclassical effects prevent a fluid with the barotropic parameter $\gamma < 8/5$ from contributing to the gravitational collapse. Therefore, the barotropic energy density remains always finite and never blows up, as long as the semiclassical regime is valid.

Furthermore, the ratio of the effective mass function over the area radius stays less than 1; thus, no trapped surfaces form. In addition, a thorough numerical analysis showed that the Kretschmann scalar remains finite, suggesting the regularity of the geometry. This, together with the regularity of the energy density, indicated that the spacetime in this semiclassical (inverse-triad-corrected) collapse does not lead to any naked singularity formation as long as the semiclassical approximation holds. Moreover, those corrections induce an outward flux of energy at the final state of the collapse. This is consistent with the *standard* scalar field collapse (cf. Ref. [25]), wherein an inverse triad correction via loop quantum effect leads to a quantum evaporation of the naked singularity.

We also found that the tachyon in our semiclassical gravitational collapse would behave as phantom matter (see also Refs. [35,40]). A similar correspondence has also been found earlier in Ref. [24], with a massless scalar field with the usual canonical kinetic term.

Scalar field (and tachyon matter, in particular) gravitational collapse is still very much an open research subject [4,6–10,12,15–21]. In Ref. [25], a generic potential $V(\Phi)$ was considered for a standard scalar field Φ. By using a Hamilton-Jacobi formulation (see Appendix B), it is possible to find that the results of Ref. [25] regarding the divergence of the energy density of the collapsing system are not generic; in fact, they depend on the choice of the parameter b, if extending the analysis therein towards a potential of the form Φ^{2b} (with $b < 0$). In particular, for a potential of the inverse square form, the results in Ref. [25] are no longer satisfied. However, for a tachyon field, choosing a similar potential will lead to some differences (cf. Appendix IV), besides having the significant advantage of making our dynamical system tractable. So, it was worthwhile to assume an inverse square potential for our tachyonic system in order to investigate whether the collapsing spacetime is regular or not.

In the herein semiclassical regime, only modifications based on inverse triad corrections were employed. But there are other corrections imported from LQG, e.g., ‘‘holonomy’’ corrections [41,42]. Differently from the inverse triad modification, the corresponding effective Hamiltonian constraint leads to a quadratic density modification $H^2 \propto \rho(1 - \rho/\rho_c)$. It was shown that the quadratic density modification could dominate over the inverse volume correction [43]; this modification provides an upper limit ρ_c for the energy density of matter; whence for the tachyon matter, it predicts that the gravitational collapse would include a nonsingular bounce at the critical density $\rho_\phi = \rho_c$ [44]. However, more investigation is needed in order to study the behavior of the apparent horizon and its formation/avoidance during the collapsing scenario.

It should be noticed that the context in this paper is rather restricted, as the interior spacetime is assumed to be homogeneous, matched to the exterior generalized Vailya exterior. Nevertheless, the results obtained herein could still hold even in a general (inhomogeneous) setting [37], as it has been indicated in Ref. [33] that the negative pressures would exist for an arbitrary matter configuration.

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APPENDIX A: A BRIEF APPLICATION OF THE CENTER MANIFOLD THEOREM

In order to discuss the stability of (A) as (0,1,0,0), we have to determine the eigenvalues (and eigenvectors) of the matrix in Eq. (2.28). Using Eqs. (2.20)–(2.24) and (2.26), matrix $\mathcal{B}$ becomes

$$\mathcal{B}=\begin{pmatrix}-9 & 0 & \frac{\xi}{\sqrt{3}} & 0\\ 0 & 0 & 0 & -\tilde{\gamma}\\ 0 & 0 & -5 & 0\\ 0 & 0 & 0 & \tilde{\gamma}\end{pmatrix}. \tag{A1}$$

Eigenvalues of the matrix in Eq. (A1) are $\sigma_1=-9$, $\sigma_1=0$, $\sigma_1=-5$, and $\sigma_1=\tilde{\gamma}$. Therefore, all eigenvalues are real, but one is 0, and the rest are negative when $\gamma>8/5$; this implies that this is a nonlinear autonomous system with a nonhyperbolic point [36]. The asymptotic properties cannot be simply determined by linearization, so we need to resort to another method: the center manifold theorem [36]. For convenience, we transform the critical point $A(X_c=0, Y_c=1, Z_c=0, S_c=0)$ to $\tilde{A}(X_c=0, \tilde{Y}_c=Y_c-1=0, Z_c=0, S_c=0)$. The autonomous system of Eqs. (2.20)–(2.24) is rewritten as

$$\frac{dX}{dN}=X(4X^2-9)+\frac{1}{\sqrt{3}}\xi Z\sqrt{\tilde{Y}+1}(1-X^2), \tag{A2}$$

$$\frac{d\tilde{Y}}{dN}=-\frac{1}{\sqrt{3}}\xi XZ(\tilde{Y}+1)^{3/2}+(\tilde{Y}+1)\times[4X^2-(\tilde{\gamma}+4X^2)S], \tag{A3}$$

$$\frac{dZ}{dN}=-5Z, \tag{A4}$$

$$\frac{dS}{dN}=S(1-S)(\tilde{\gamma}+4X^2). \tag{A5}$$

Let $\mathcal{M}$ be a matrix whose columns are the eigenvectors of $\mathcal{B}$; whence, for the matrix of Eq. (A1), we obtain $\mathcal{M}$ as

$$\mathcal{M}=\begin{pmatrix}1 & 0 & \frac{\xi}{4\sqrt{3}} & 0\\ 0 & 1 & 0 & -1\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix} \tag{A6}$$

and $\mathcal{M}^{-1}$ its inverse matrix:

$$\mathcal{T}=\mathcal{M}^{-1}=\begin{pmatrix}1 & 0 & -\frac{\xi}{4\sqrt{3}} & 0\\ 0 & 1 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}. \tag{A7}$$

Using the similarity transformation $\mathcal{T}$, the matrix $\mathcal{B}$ can be rewritten as a block diagonal form:

$$\tilde{\mathcal{B}}\equiv\mathcal{T}\mathcal{B}\mathcal{T}^{-1}=\begin{pmatrix}-9 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & -5 & 0\\ 0 & 0 & 0 & \tilde{\gamma}\end{pmatrix}=\begin{pmatrix}\tilde{\mathcal{B}}_1 & 0\\ 0 & \tilde{\mathcal{B}}_2\end{pmatrix}, \tag{A8}$$

where $\tilde{\mathcal{B}}_1$ is the matrix whose eigenvalues all have zero real parts, and all eigenvalues of $\tilde{\mathcal{B}}_2$ have negative real parts. Changing the variables as bellow, we have

$$\begin{pmatrix}X'\\ Y'\\ Z'\\ S'\end{pmatrix}\equiv\mathcal{T}\begin{pmatrix}X\\ \tilde{Y}\\ Z\\ S\end{pmatrix}=\begin{pmatrix}X+\frac{\xi}{4\sqrt{3}}Z\\ \tilde{Y}+S\\ Z\\ S\end{pmatrix}. \tag{A9}$$

Then, the dynamical system of Eqs. (2.20)–(2.24) in terms of the new variables (X', Y', Z', S') becomes

$$\frac{dX'}{dN}=\frac{dX}{dN}+\frac{\xi}{4\sqrt{3}}\frac{dZ}{dN}=f_1'(X',Y',Z',S'), \tag{A10}$$

$$\frac{dY'}{dN}=\frac{d\tilde{Y}}{dN}+\frac{dS}{dN}=f_2'(X',Y',Z',S'), \tag{A11}$$

$$\frac{dZ'}{dN}=\frac{dZ}{dN}=-5Z', \tag{A12}$$

$$\frac{dS'}{dN}=S'(1-S')\Big[\tilde{\gamma}+4\Big(X'-\frac{\xi}{4\sqrt{3}}Z'\Big)^2\Big], \tag{A13}$$

where $f_1'(X',Y',Z',S')$ and $f_2'(X',Y',Z',S')$ are given by

$$f_1'\equiv\Big(X'-\frac{\xi}{4\sqrt{3}}Z'\Big)\Big[4\Big(X'-\frac{\xi}{4\sqrt{3}}Z'\Big)^2-9\Big]-\frac{5\xi}{4\sqrt{3}}Z' +\frac{\xi}{\sqrt{3}}Z'(Y'-S'+1)^{\frac{1}{2}}\Big[1-\Big(X'-\frac{\xi}{4\sqrt{3}}Z'\Big)^2\Big], \tag{A14}$$

$$f_2'\equiv\frac{1}{\sqrt{3}}\xi Z'(Y'-S'+1)^{3/2}\Big(X'-\frac{\xi}{4\sqrt{3}}Z'\Big) +4(Y'-S'+1)\Big(X'-\frac{\xi}{4\sqrt{3}}Z'\Big)^2 +S'(1-S')\Big[\tilde{\gamma}+4\Big(X'-\frac{\xi}{4\sqrt{3}}Z'\Big)^2\Big]. \tag{A15}$$

In particular, Eqs. (A10)–(A13) have the point of equilibrium $X_c'=Y_c'=Z_c'=S_c'=0$. Let us now suppose that we take initial data with $Z'=0$ and $S'=0$. Then $Z'(t)$ and $S'(t)$ are zero for all times, and we examine the stability (see Ref. [45] for more details) of the equilibrium $X_c'=0$ and $Y_c'=0$. The corresponding reduced system can now be written as

$$\frac{dY'}{dN}=4X'^2(1+Y'). \tag{A16}$$

Clearly (for initial data near the origin), $X'(t)$ converges exponentially to zero—say, approximately as $\exp(-9N)$. Since Y' is monotonous with N, the reduced system $dY'/dN\approx 4Y'\exp(-18N)$ will converge as N gets larger, but in general the limit is not zero. Thus, the critical point (A) is asymptotically *stable*.

On the other hand, for the case of the barotropic parameter $\gamma=8/5$, the set of eigenvalues of point (A) includes two negative real parts and two real parts that are 0. In order to analyze the stability of the system in this case, we can consider an additional condition for non-vanishing initial data $S' \neq 0$ where $Z' \approx 0$, so that the corresponding reduced system for $S'(t)$ can be further written from Eq. (A13) as

$$\frac{dS'}{dN} = 4S'(1 - S')X'^2. \tag{A17}$$

Since S' is monotonous with N, and $0 \leq S' \leq 1$, the reduced system $dS'/dN \approx 4S' \exp(-18N)$ will converge as N gets larger; therefore, the fixed point (A) for the case $\gamma = 8/5$ is asymptotically *stable*.

It should be noticed that, in the case of the parameter $\gamma = 8/5$, the eigenvalues of the two fixed points (A) and (C) become equal. Therefore, by employing a similar analysis for the fixed point (C), we can show that this fixed point is asymptotically *stable* for the case of the barotropic parameter $\gamma = 8/5$.

APPENDIX B: TACHYONIC POTENTIAL

In this appendix, we use a Hamilton-Jacobi formulation to discuss the tachyon potential.

1. Hamilton-Jacobi formulation with scalar field

Let us start with the gravitational collapse of a standard scalar field Φ with quantum induced modifications (inverse triad type) [25]. If Φ is a monotonically varying function of the proper time, then we can present a Hamilton-Jacobi equation [46]:

$$V(\Phi) = 3H^2 - \frac{D}{2} H_{\Phi}^2, \tag{B1}$$

where $H_\Phi \equiv \frac{\dot{\Phi}}{D} = \frac{1}{r}\frac{dH}{d\Phi}$, with r being given by $r \equiv \frac{1}{3}(\dot{D}/4HD - 3/2)$. Notice that in our model for the choice of $D(q)$, r takes the value $r = 1/6$. This formalism implies that the dynamics of the semiclassical period can be determined once the Hubble parameter $H(\Phi)$ has been specified as a function of the scalar field. Let us assume a Hubble parameter of the form $H(\Phi) = H_1 \Phi^b$ (where $H_1 < 0$ and $b < 0$ are constants), describing a collapsing model (see, e.g., Ref. [46]). Then we obtain the scale factor as $a^4(\Phi) = A_0 \Phi$, where $A_0 \equiv (2/3|b|)^{\frac{1}{2}} a_*^4 A^{-\frac{1}{6}}$. Therefore, the potential of the scalar field is given by Eq. (B1): $V(\Phi) \approx V_1 \Phi^{2b}$, where $V_1 = \text{const}$.

This solution shows that the scalar field remains finite and satisfies the range $0 < \Phi < \Phi_0$ during the collapse, with the initial data at $a(\Phi_0) = a_0$. Furthermore, as a decreases, the scalar field decreases towards the center. When the scale factor becomes very small and approaches the Planck scale, the scalar field Φ vanishes very fast. Then, as $\Phi \to 0$, the Hubble rate diverges, which corresponds to a curvature singularity. Notice that the modified equation of motion, for the case of this solution, reads $\ddot{\Phi} - 5H\dot{\Phi} + DV_{,\Phi} = 0$; the last term in this equation can be approximated as $DV_{,\Phi} \sim \Phi^{2b+1}$. Therefore, for the range of the parameter $b < -1/2$, this term increases towards the singularity and has an important role in the dynamics of the system. In addition, for the case $b = -1$, the solution corresponds to an inverse square potential for the scalar field, $V = V_1 \Phi^{-2}$, which also brings a singular final state for the collapsing model. It should be noticed that this further suggests that the results of Ref. [25], when taken in view of a general class of potentials, must be discussed with care. More precisely, in the presence of a potential with an inverse power of scalar field, the collapsing model in Ref. [25] may not be regular as long as the semiclassical effects are valid.

2. Hamilton-Jacobi formulation with tachyon field

If the tachyon field is a monotonically varying function of the proper time, then Eq. (2.7) can be written in a Hamilton-Jacobi form:

$$V^2(\phi) = 9H^4 \left[1 - \frac{1}{16\tilde{\beta}} a^{30} H_\phi^2\right], \tag{B2}$$

where $H_\phi \equiv \frac{2H_{,\phi}}{3H^2}$ [35]. Let us therefore assume a Hubble parameter of the form $H = H_1 \phi^b$ (see, e.g., Ref. [46]), where H_1 and b are arbitrary negative constants, describing a collapsing model. Then, integrating $H_{,\phi}$, the scale factor is obtained as a function of the tachyon field: $a^{30}(\phi) = B\phi^{2b+2}$, where $b \neq -1$ and B is a positive constant. Then, Eq. (B2) implies that the potential, as a function of the tachyon field, has the form $V(\phi) = \bar{V}_0 \phi^{2b}$, with the constant $\bar{V}_0$.

For $b = -1$, the Hubble parameter takes instead the form $H = H_1 \phi^{-1}$, for which the scale factor can be obtained as $a^{30}(\phi) = 180\tilde{\beta} H_1^2 \ln\phi$, where $1 < \phi < \phi_0$ and $\tilde{\beta}$ is a constant. The potential of the system can be established from Eq. (B2) as

$$V(\phi) = \frac{3H_1^2}{\phi^2}(1 - 5\ln\phi)^{\frac{1}{2}}. \tag{B3}$$

In this case, the initial value of the potential is given by $\bar{V}_0 = 3H_1^2(1 - 5\ln\phi)^{1/2}/\bar{\phi}_0^2$, as $a \to a_0$. On the other hand, the potential of the system at the semiclassical limit (i.e., $a \ll a_*$) behaves as an inverse square function of the tachyon field: $V(\phi) \simeq 3H_1^2/\phi^2$. This result implies that, at the semiclassical limit, the choice of an inverse square potential is a good approximation for the tachyon potential. Meanwhile, for a tachyon potential of the general form $V(\phi) \simeq \phi^b$, the dynamical system is much more complicated. If the potential is of inverse square form, it allows a three-dimensional autonomous system to be extracted, whereas for more general cases of the potential, the number of dimensions would become higher if the system is to remain autonomous. Therefore, in order to make the phase space analysis tractable, we assume an inverse square potential for the tachyon field. In addition, exact solutions can be found for a classical, purely tachyonic matter content (cf. Ref. [47]), for cases which combine tachyonic and barotropic fluids (cf. Refs. [12,13]), and for cases with the loop quantum correction terms (cf. Ref. [48]) with an inverse square potential.

Let us further add that an inverse square potential for a standard scalar field and for a tachyon field leads to two collapsing models with similar Hubble rates, but with differences as far as the collapse outcome is concerned. More concretely, for a standard scalar field collapse, we have shown that the collapse is expected to have a singular nature through the semiclassical regime, because the scalar field vanishes towards the collapsing center. Instead, for a similar tachyonic potential, the tachyon field decreases with the scale factor and remains finite and nonzero towards the final state of the collapse, as long as the semiclassical regime is valid. So, it is of interest to investigate, by employing a phase space analysis, whether the spacetime at the final stages of the collapse is regular or not with an inverse square potential, in the case of the tachyon field.

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[45] A description of the center manifold (cf. Ref. [36] for more details) is usually retrieved with the assistance of the functions $h(Y')$, $g(Y')$, and $f(Y')$, where $X' \equiv h(Y')$, $Z' = g(Y')$, and $S' = f(Y')$. Let us assume the solution for h, g, and f to be approximated arbitrarily closely as a Taylor series in Y'. Using this method, we can start with the simplest approximation, $h(Y') \approx 0$ and $f(Y') \approx 0$. Then the reduced system, obtained by substituting $h(Y') = \mathcal{O}(|Y|^2)$, $g(Y') \approx 0$, and $f(Y') \approx 0$ into Eqs. (A10)–(A12), is $dY'/dN = \mathcal{O}(|Y'|^4)$. Clearly, this cannot assist us in reaching any conclusion about the stability. Therefore, we calculate the coefficient h_2 in the series $h(Y') = h_2 Y'^2 + \mathcal{O}(|Y'|^3)$ and study the stability of the origin. The reduced system then reads

$$dY'/dN = 4h_2^2 Y'^4 + \mathcal{O}(|Y'|^5). \qquad \text{(B4)}$$

To get the value of h_2, we should solve the equation [36]

$$\mathcal{N}(h(Y')) \equiv (\partial h/\partial Y')f_2' - f_1' = 0. \qquad \text{(B5)}$$

But this leads to all coefficients being zero-valued in the series $h(Y')$. Considering instead $Z' \equiv g(Y') \neq 0$, this will not reach any result, because all coefficients g_i in the series $g(Y')$ are zero as well. Hence, we just focus on an analysis of the stability of this fixed point from a qualitative perspective for the reduced system in Eq. (A16).
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