Cosmic censorship: Formation of a shielding horizon around a fragile horizon

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The weak cosmic censorship conjecture asserts that spacetime singularities that arise in gravitational collapse are always hidden inside of black holes, invisible to distant observers. This conjecture, put forward by Penrose more than four decades ago, is widely believed to be one of the basic principles of nature. However, a complete proof of this hypothesis is still lacking and the validity of the conjecture has therefore remained one of the most important open questions in general relativity. In this study we analyze a gedanken experiment that is designed to challenge cosmic censorship by trying to overcharge a Reissner-Nordström black hole: a charged shell is lowered adiabatically into the charged black hole. The mass energy delivered to the black hole can be redshifted by letting the dropping point of the shell approach the black-hole horizon. On the other hand, the electric charge of the shell is not redshifted by the gravitational field of the black hole. It therefore seems, at first sight, that the charged shell is not hindered from entering the black hole, overcharging it and removing its horizon. However, in the present study we prove that the exposure of a naked singularity to distant observers is actually excluded due to the formation of a new (and larger) horizon around the original black hole. Moreover, we shall prove that this new horizon is already formed before the charged shell crosses the original black-hole horizon. This result, which seems to have been previously overlooked, guarantees the validity of the weak cosmic censorship conjecture in this type of gedanken experiments.

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The singularity theorems of Hawking and Penrose [1] reveal that gravitational collapse from smooth initial conditions may produce spacetime singularities, regions in which the known laws of physics break down. The utility of general relativity in describing gravitational phenomena in such extreme physical situations is maintained by the cosmic censorship principle [2–4]. The weak version of this hypothesis [the weak cosmic censorship conjecture (WCCC)] asserts that spacetime singularities that arise in gravitational collapse are always hidden inside of black holes (behind event horizons), invisible to distant observers.

The cosmic censorship principle is essential for preserving the predictability of Einstein's theory of gravity [2–4]. In fact, the principle has become one of the cornerstones of general relativity. However, a generic proof of the conjecture is still lacking. Thus, the validity of this principle has remained one of the most important open questions in general relativity, see e.g., Refs. [5–30] and references therein.

According to the WCCC, the destruction of a black-hole event horizon is ruled out because such process would expose the inner black-hole singularity to distant observers. For this reason, any physical process which is aimed to remove the black-hole horizon is expected to fail. For the advocates of the cosmic censorship conjecture the task remains to find out how such candidate processes eventually fail to remove the black-hole horizon.

One of the earliest attempts to remove the horizon of a black hole is due to Wald [5] who tried to overcharge a maximally charged Reissner-Nordström (RN) black hole by dropping into it a charged test particle whose charge-to-mass ratio is larger than unity. According to the uniqueness theorems [31–35], all spherically symmetric black-hole solutions of the Einstein-Maxwell equations are uniquely described by the RN metric

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1}dr^2 + r^2 d\Omega^2, \tag{1}$$

which is characterized by two conserved parameters: the gravitational mass M and the electric charge Q. The black-hole (event and inner) horizons are located at

$$r_\pm = M \pm (M^2 - Q^2)^{1/2}. \tag{2}$$

Thus, a black-hole solution must satisfy the relation

$$Q^2 \le M^2. \tag{3}$$

Maximally charged (extremal) black holes are the ones which saturate the condition (3). The RN spacetime with $M^2 < Q^2$ does not contain an event horizon and is therefore associated with a naked singularity rather than a black hole.

Wald [5] considered the specific case of a charged particle which starts falling towards the black hole from spatial infinity. Thus, the particle's energy at infinity was larger than (or equal to) its rest mass. It was shown [5] that this particular attempt to overcharge the black hole fails due to the Coulomb potential barrier which surrounds the charged black hole. A similar gedanken experiment was studied by Hubeny [14] who tried to overcharge a near-extremal RN black hole using a charged imploding shell.

It was found [14] that this attempt to remove the black-hole horizon also fails—the repulsive Coulomb interaction between the black hole and the shell and the coulomb self-repulsion of the shell itself both prevent the shell from overcharging the black hole.

In the present study we shall analyze a more dangerous version (from the point of view of the WCCC) of the overcharging gedanken experiment. This version consists of a charged object which is lowered slowly into the black hole. In this scenario, the energy delivered to the black hole [the part contributed by the rest mass of the object, see Eq. (4) below] can be redshifted by letting the dropping point of the object approach the black-hole horizon. On the other hand, the electric charge of the object is not redshifted by the black-hole gravitational field. The charge-to-energy ratio of a slowly descending charged object is therefore larger than the corresponding charge-to-energy ratios of the free falling (from infinity) objects considered in the original gedanken experiments [5,14]. Thus, the present version of the overcharging gedanken experiment poses a stronger challenge to the cosmic censorship conjecture.

We consider a spherical charged shell of rest mass m and electric charge q concentric with a charged RN black hole of mass M and electric charge Q. Our aim is to challenge the validity of the WCCC in the most dangerous situation—when the charge-to-energy ratio of the shell is as large as possible. We shall therefore consider a shell that is lowered slowly towards the charged black hole. Our plan is to lower the shell adiabatically (that is, with an infinitesimally small radial velocity) all the way down to the black-hole horizon. The mass energy of the shell would then be redshifted by the gravitational field of the black hole. This adiabatic process would therefore minimize the energy that is delivered to the black hole (for a given value of the shell's electric charge).

The total energy of the shell in the black-hole spacetime is given by [14,36]

$$\mathcal{E}(R) = m\left(1 - \frac{2M}{R} + \frac{Q^2}{R^2}\right)^{1/2} + \frac{qQ}{R} + \frac{q^2}{2R} - \frac{m^2}{2R}, \quad (4)$$

where R is the radius of the shell. Each term on the rhs of Eq. (4) has a clear physical interpretation:

(i) The first term represents the energy associated with the shell's rest mass (redshifted by the gravitational field of the black hole).

(ii) The second term represents the electrostatic interaction of the charged shell with the charged black hole.

(iii) The third term represents the electrostatic self-energy of the charged shell.

(iv) The fourth term represents the gravitational self-energy of the shell.

The generalized Birkhoff's theorem implies that the spacetime inside the shell is described by the RN metric (1) with parameters M and Q, whereas the spacetime outside the shell is described by the RN metric (1) with total mass $M + \mathcal{E}(R)$ and total electric charge $Q + q$.

Suppose the charged shell is indeed lowered adiabatically all the way down to the original black-hole horizon. In this case, the mass energy of the shell is completely redshifted by the gravitational field of the black hole and the energy and electric charge which are delivered to the black hole are given by $\Delta M = \mathcal{E}(R = r_+) = qQ/r_+ + q^2/2r_+ - m^2/2r_+$ and $\Delta Q = q$, respectively. If this scenario would have been possible, then charged shells with

$$r_+ - Q - m < q < r_+ - Q + m \quad (5)$$

could have overcharged the black hole [that is, could have violated the black-hole condition (3)], thereby violating the WCCC.

However, we shall now prove that the shell cannot be lowered adiabatically all the way down to the original black-hole horizon. In particular, we shall prove that a new (and larger) horizon is formed outside the original black-hole horizon (that is, outside r_+) already before the charged shell crosses the original black-hole horizon. The characteristic condition for the formation of a new horizon which engulfs both the original black hole and the descending charged shell is

$$1 - \frac{2[M + \mathcal{E}(R)]}{R} + \frac{(Q+q)^2}{R^2} = 0. \quad (6)$$

Substituting (4) into (6), one finds that a new horizon is formed when the radius of the shall reaches the limiting value

$$R \to r_{\mathrm{NH}} \equiv M + (M^2 - Q^2 + m^2)^{1/2}. \quad (7)$$

It is important to emphasize that the new horizon is formed outside [37] the original black hole already before the shell crosses the original horizon. The formation of the new shielding horizon outside the original black hole prevents the exposure of the inner singularity to distant observers. The newly formed horizon therefore guarantees the validity of the WCCC in this gedanken experiment.

We note that the radius (7) is the smallest possible radius of such newly formed horizons: a charged shell with a nonvanishing radial momentum has an energy that is larger than the one given by (4) and would therefore form a larger horizon [that is, even before [38] reaching the radius (7).]

It is worth reexamining a related gedanken experiment designed by Bekenstein and Rosenzweig [13] to challenge cosmic censorship [39]: suppose there exist two different types of local charges (for example, electric and magnetic charges). A RN spacetime with two different types of charges, $Q \in U(1)$ and $K \in U'(1)$, can have an event horizon only if

$$Q^2 + K^2 \le M^2. \quad (8)$$

Suppose the original black hole possesses a $U(1)$ charge Q but no $U'(1)$ charge. Thus, the original black hole is not endowed with a $U'(1)$ gauge field and an approaching shell of charge $k \in U'(1)$ encounters no electrostatic repulsion from the $U(1)$-charged black hole [see Eq. (9) below]. Thus, the charge-to-energy ratio of the shell is larger than the corresponding ratio considered in the former gedanken experiment [with only one type of local $U(1)$ charge]. Hence, this type of gedanken experiment seems to pose a greater challenge to the WCCC.

Bekenstein and Rosenzweig [13] considered a charged shell which starts falling towards the black hole from spatial infinity. They then concluded that the Coulomb self-repulsion of the shell is sufficient to guarantee the validity of the WCCC in their version of the gedanken experiment [13].

However, in our version of the gedanken experiment (which is more challenging from the point of view of the WCCC) the shell is lowered adiabatically towards the original black hole. As discussed above, in this case the mass energy of the shell is redshifted by the gravitational field of the black hole. As a consequence, the charge-to-energy ratio of the shell is larger than the corresponding charge-to-energy ratio considered in Ref. [13]. The total energy of the shell in the black-hole spacetime is now given by

$$\mathcal{E}(R) = m\left(1 - \frac{2M}{R} + \frac{Q^2}{R^2}\right)^{1/2} + \frac{k^2}{2R} - \frac{m^2}{2R}. \qquad (9)$$

Note, in particular, that the repulsion term qQ/R that appeared in (4) is absent now.

Suppose the $U'(1)$-charged shell is indeed lowered adiabatically all the way down to the original horizon of the $U(1)$-charged black hole. In this case, the mass energy of the shell is totally redshifted by the gravitational field of the black hole and the energy and electric charges which are delivered to the black hole are given by $\Delta M = \mathcal{E}(R = r_+) = k^2/2r_+ - m^2/2r_+$, $\Delta Q = 0$, and $\Delta K = k$. If this scenario would have been possible, then charged shells with

$$m^2 + 2r_+(r_+ - M - m) < k^2 < m^2 + 2r_+(r_+ - M + m) \qquad (10)$$

could have overcharged the black hole [that is, could have violated the black-hole condition (8)], thereby violating the WCCC.

However, it is easy to verify that a new and larger horizon is formed outside the original black-hole horizon (that is, outside r_+) already before the charged shell crosses the original black-hole horizon. The characteristic condition for the formation of a new horizon (which again engulfs both the original black hole and the descending charged shell) is

$$1 - \frac{2[M + \mathcal{E}(R)]}{R} + \frac{Q^2 + k^2}{R^2} = 0. \qquad (11)$$

Substituting (9) into (11), one finds that a new horizon is formed when the radius of the shall reaches the limiting value r_{NH} given by Eq. (7). We therefore recover our previous conclusion—the new shielding horizon, which is formed outside the original black hole, prevents the exposure of the inner singularity to distant observers. Cosmic censorship is therefore respected.

In summary, we have analyzed a gedanken experiment that was designed to challenge the cosmic censorship conjecture by trying to overcharge a black hole: a charged shell was lowered adiabatically towards a charged Reissner-Nordström black hole. The charge-to-energy ratio of the shell was made as large as possible by redshifting the energy associated with the rest mass of the shell. Thus, the present gedanken experiment is more challenging (from the point of view of the cosmic censorship conjecture) than former gedanken experiments considered in Refs. [5,13,14].

We have proved that when the shell approaches the original black-hole horizon (but has not yet crossed it!), a new and larger horizon is formed that engulfs both the original black hole and the descending charged shell, see Eq. (7).

The formation of the new horizon outside the original black hole before the shell crosses the original horizon, a fact which seems to have been previously overlooked, prevents the exposure of the inner singularity to distant observers. The newly formed shielding horizon therefore guarantees the validity of the WCCC in this type of gedanken experiments.

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[37] That is, $r_{\mathrm{NH}}>r_+$ [see Eqs. (2) and (7)].
[38] For example, a shell which starts falling from rest towards the black hole from spatial infinity would form a new horizon when reaching $r_{\mathrm{NH}}=M+m+[(M+m)^2-(Q+q)^2]^{1/2}$.
[39] Our version of the gedanken experiment would be more challenging (from the point of view of the WCCC) than the one considered in Ref. [13].