PHYSICAL REVIEW D **87,** 024026 (2013)

Interacting scalar fields in the context of effective quantum gravity

Artur R. Pietrykowski*

Bogolubov Laboratory of Theoretical Physics, JINR, Dubna 141980, Moscow Region, Russia

(Received 1 October 2012; published 14 January 2013)

A four-dimensional scalar field theory with quartic and of higher-power interactions suffers the triviality issue at the quantum level. This is due to coupling constants that, contrary to the physical expectations, seem to grow without a bound with energy. Since this problem concerns the high- energy domain, interaction with a quantum gravitational field may provide a natural solution to it. In this paper we address this problem considering a scalar field theory with a general analytic potential having $\mathbb{Z}_2$ symmetry and interacting with a quantum gravitational field. The dynamics of the latter is governed by the cosmological constant and the Einstein-Hilbert term, both being the lowest and next-to-lowest terms of the effective theory of quantum gravity. Using the Vilkovisky-DeWitt method we calculate the one-loop correction to the scalar field effective action. We also derive the gauge-independent one-loop beta functions for all the scalar field couplings in the minimal subtraction scheme. We find that the leading gravitational corrections act in the direction of asymptotic freedom. Moreover, assuming both the Newton and cosmological constants have nonzero fixed point values, we find asymptotically free Halpern-Huang potentials.

DOI: 10.1103/PhysRevD.87.024026 PACS numbers: 04.60.Gw, 11.10.Gh, 11.10.Hi, 11.10.Jj

I. INTRODUCTION

The interacting scalar field theory in four spacetime dimensions is a basic constituent of perhaps the best experimentally corroborated theory of particle physics, that is the standard model. In this model the Higgs particle is described by a four-component scalar field interacting with itself according to the quartic operator. It brings about a mass upon all the fermions in the standard model as well as the part of the gauge bosons obeying $SU(2)$ symmetry through the Higgs mechanism. A scalar field theory is also very important for cosmology, where it serves to describe the dynamics of a very early stage of cosmological evolution, the inflationary era. However, the leading-order quantum corrections to the quartic coupling, which depend on the energy scale, revealed it to be not as physically meaningful as the ultraviolet domain. The arguments came from the one-loop beta function for which a solution is given by a relation between the momentum transfer dependent quartic coupling $\tilde{\lambda}(p^2)$ and the renormalized one λ_R at some arbitrarily chosen renormalization point. It turns out that it increases with momentum transfer and at a finite value, the effective coupling becomes infinite. It is usually argued that this divergence of the effective coupling takes place at large momenta, where the effective coupling is $\tilde{\lambda}(p^2) > 1$, which is far beyond the applicability of the leading- order approximation. For this value, a sum of all orders should be taken into account. However, investigation of the N-component scalar field theory with $O(N)$ symmetry for large N showed that the beta function does not depend on N and has the same algebraic form as in the one-loop approximation [1], even though it is a function of the effective coupling rather then the renormalized one. Since this is the nonperturbative result, one concludes that for the theory to be physically meaningful it is required that $\lambda_R = 0$. This means that in the large-N limit, a scalar field theory is a free field theory. Although it is not a rigorous proof, it nevertheless represents a strong premise that in general there might be no physically meaningful interacting scalar field theory in four spacetime dimensions. Theory with this property is said to be trivial, and nonexistence of an interacting theory is referred to as the triviality issue. A conjecture of triviality for scalar field theory, first put forward by Wilson [2] and examined within the functional renormalization group (RG) [2,3], has been further supported by the large body of evidence in the Monte Carlo RG, high-temperature expansion, and numerical simulations (for review see Ref. [4]). An interesting discovery was made by Halpern and Huang in Ref. [5]. They considered scalar field theory with $O(N)$ symmetry in “local potential approximation” with a general, $\mathbb{Z}_2$-symmetric potential that admits a Taylor expansion form. In the space of all couplings, using the Wilson's RG method, they examined small perturbations about the free field theory fixed point (FP), termed the Gaussian FP. What they found is a continuum class of nontrivial directions, along which the theory is asymptotically free. Potentials along these directions are nonpolynomial and reveal an exponential growth for large field values. This one-loop result offers a way to avoid the triviality issue. However, it was questioned by Morris [6] as leading to wrong scaling in the large field limit as well as to singular potentials at some value of the field in all but the Gaussian FP, even though his arguments might implicitly assume a polynomial form of potentials [7]. Despite the doubts cast on the validity of this result, these nontrivial directions

*pietrie@theor.jinr.ru

were further investigated by many authors in various contexts [8]. The triviality issue for scalar field theory and QED was recently considered from the point of view of the exact RG in Ref. [9]. Requiring that any physical theory have a derivative expansion as well as a continuation from Euclidean to Minkowski space, it was found that there are no physically acceptable nontrivial FPs; the only one is the Gaussian FP.

The problem of triviality in an interacting scalar field theory may be rescued if non-Abelian gauge fields are incorporated. This phenomenon was first demonstrated in Ref. [10] in the case of Yang-Mills theory with the $O(N)$ symmetry group interacting with scalar fields. It was found that the theory is asymptotically free in both sectors, provided that $N \geq 6$. It is therefore conceivable that if gravitational interactions are taken into account, the triviality issue may find its natural solution at the energies close to the Planck scale. However, in the pioneering papers [11,12], it was revealed that the quantum field theory of gravity based on the Einstein-Hilbert action is nonrenormalizable. Owing to Wilson's new look at the renormalization, it was realized that the renormalizable theories are but a low- energy manifestation of some underlying fundamental theory that should reveal itself in the form of new interactions when a fundamental heavy mass threshold is approached. This new perspective was first implemented in gravity by Donoghue [13]. In this approach the cosmological constant and the Einstein-Hilbert term are the lowest and next-to-lowest terms in an energy expansion of the full theory of gravity given in the form of a covariant power series of interactions, each of which is formed with all possible contractions of the Riemann tensors to a given power. The requirement of covariance makes the theory invariant with respect to the underlying diffeomorphism symmetry.[1] Thus a way to study quantum effects at the low-energy domain of quantum gravity has been opened. This novel point of view was utilized recently by Robinson and Wilczek [17] to examine the effects of quantum gravity on the interaction of Yang-Mills gauge fields. It has been shown that a gravitational correction to running Yang-Mills coupling act in the direction of asymptotic freedom independently of whether the symmetry group is non-Ableian or $U(1)$. If true, this would solve the triviality issue in the case of QED, a gauge theory with the $U(1)$ symmetry group. The fact that the correction was quadratic in the loop momentum cutoff led many authors to question its validity due to its gauge [18] and regularization [19] dependence. Making use of the geometric Vilkovisky-DeWitt formulation of the effective action and taking into account the cosmological constant, Toms [20] found the gravitational correction to the Maxwell theory in the minimal subtraction (MS) scheme. For a positive cosmological constant, the correction renders the Maxwell theory asymptotically free. It makes the QED a nontrivial theory. A gauge-independent power law gravitational correction has been found by Toms [21] through the Vilkovisky-DeWitt effective action in conjunction with the Schwinger "proper time" method to deal with divergent loop momentum integrals. Although different in form, this gravitational correction leads to the same conclusions as those drawn by Robinson and Wilczek [17]. This result has also been derived by Ho *et al.* [22] in the momentum subtraction scheme and corrected by Tang and Wu [23] in the loop regularization scheme. The power law correction has been criticized by may authors. Mavromatos and Ellis [24] argued that this correction is redundant and thus unphysical from the point of view of the equivalence theorem. Anber *et al.* [25] and Anber and Donoghue [26] pointed out that the power law corrections in general lead to violation of crossing symmetry and therefore are not universal. As such, they cannot appropriately account for the quantum effects due to gravity. The only exception from this rule is the scalar field. Toms has recently also critically reexamined the role of the power law corrections [27] and has come to the conclusion that these corrections have no physical meaning. Nielsen [28] has shown in detail that the quadratic corrections depend on the gauge, even though they are calculated in the Vilkovisky-DeWitt formalism. The gravitational contribution to the running of Yang-Mills couplings has also been examined within the asymptotic safety scenario by Daum *et al.* [29]. Using the Euclidean and scale-dependent effective action (termed the effective average action) about the flat background metric, they found a gravitational correction quadratic in IR cutoff. The correction turned out to be of the same sign as the result of Robinson and Wilczek and, hence, the conclusions. This result has been reexamined by Folkers *et al.* [30], where requiring that the self-energy diagrams obey certain symmetries has produced the zero result. These studies imply that the status of the power law gravitational corrections is rather obscure from the physical point of view. Thus the only gravitational correction contributing the running coupling involves the cosmological constant as found in Ref. [20].

As for the scalar field, a generalization of the RG methods to nonrenormalizable theories proposed by Kazakov [31] was enhanced and used by Barvinsky *et al.* [32], where a scalar field nonminimally coupled to gravity was considered. Assuming the scalar field potential and nonminimal coupling function have an exponential form for the large field value, it was possible to solve RG equations in such a way that a resulting theory appeared to be asymptotically free. However, the solution yields a form of the potential that is unbounded from below and therefore unphysical. The method used in the studies was recently questioned by Steinwachs and Kamenshchik [33].

[1] Excellent reviews on the effective field theory may be found in Ref. [14] and on its application to gravity in Ref. [15] and, most recently, [16].

The effect of quantum gravity on the interaction of minimally coupled scalar fields was also studied by Griguolo and Percacci [34] by means of the effective average action. Taking the flat background metric and the scalar field potential in the broken phase, they calculated the one-loop gravitational corrections to running of the quartic coupling and the vacuum expectation value of the scalar field. At the high-energy region, the gravitational correction to the beta function is found to be quadratic in the cutoff and positive. This implies that the triviality problem persists. This result was further reexamined in Ref. [35] in the context of an asymptotic safety scenario [14] within Einstein-Hilbert truncation as an extension to the nonperturbative study of quantum gravity [36]. They considered stability of the system about the Gaussian matter FP, where the Newton coupling constant and the cosmological constant both have nonzero FP values contrary to all the scalar field ones. Within the five-coupling truncation, it was found that due to the gravitational correction the quartic operator becomes irrelevant, whereas the nonminimal operator $\varphi^2 R$ becomes relevant. This result coincides with the one obtained earlier in Ref. [34]. The analysis has been recently repeated and extended to the arbitrary form of the potential including the nonpolynomial one as well as the nonminimal coupling function in Ref. [37]. In the case of polynomial potentials, the result found in Ref. [34] was rederived. Moreover, investigation of the stability matrix about Gaussian matter FP revealed its bidiagonal block structure and that each block is related to the other by a recursion relation. Hence, the entire stability matrix is determined by the first diagonal and the second diagonal block, both involving solely the gravitational couplings. The eigenvalues were obtained only for the truncated potential up to mass operator with a positive real part. An infinite number of couplings was not considered due to requirements of the asymptotic safety scenario, which restricts the number of couplings at the FP to be finite. It has to be mentioned here that the results obtained by means of the effective average action are gauge [38] and regulator [39] dependent. The study of the influence of quantized gravitational fields on the renormalization of a scalar field quartic coupling within the perturbative effective field theory has been recently undertaken by Rodigast and Schuster [40]. From the Feynman diagrams they have derived the leading order of the gravitational correction to the beta function in the harmonic gauge that makes the scalar field theory asymptotically free. This study was extended to include the cosmological constant and the nonminimal coupling to gravity by MacKay and Toms [41].[2] The computations have been performed within the Vilkovisky-DeWitt effective action. As a result, a gauge-independent gravitational contribution to the scalar field and mass renormalizations has been found. The gravitational correction to the beta function for quartic coupling was not considered there. Finally, a very recent study of φ^4 theory in a symmetry broken phase and gravity system within the effective approach was undertaken by Chang *et al.* [43]. It revealed inconsistencies in a renormalization of the Higgs sector which is due to the gravitational corrections. This analysis, however, will not be addressed here.

[2] For earlier study of this system, see Ref. [42].

In the present work we continue a search for the solution of the triviality issue by encompassing quantum gravitational fluctuations. As we have seen from the above paragraphs, this is best achieved by analyzing contributions to the RG beta functions that dictate the running of effective couplings. Although a form of the contributions is determined by means of the perturbation theory, it captures features that exceed the perturbative approach. In what follows we consider a single-component scalar field theory coupled to gravity. Since we work within the effective theory, we assume that both sectors, the scalar field and the gravity, have the lowest and next-to-lowest, i.e., the two-derivative term in the energy expansion. In the case of a scalar field, it corresponds to the potential and the kinetic term, whereas in the case of the gravitational sector this corresponds to the cosmological constant and the Einstein-Hilbert term. The scalar field potential is assumed to have an arbitrary, though analytic and $\mathbb{Z}_2$-symmetric form. Our objective is to compute the one-loop corrections to the effective action and derive from it the form of RG beta functions. Since computations are performed about the flat background metric in Euclidean space, we confine the theory to be minimally coupled to gravity.[3] The flat background metric is not a solution of Einstein equations with a cosmological constant and/or a nonconstant scalar field. Therefore, we perform our computations off the mass shell. In order to obtain gauge-independent results, we employ the Vilkovisky-DeWitt geometric approach to the effective action [44]. Although a lack of universality of quantum corrections pointed out by Anber *et al.* [25,26] is not a concern here, we use the minimal subtraction (MS) scheme [45] to avoid a possible gauge dependence [28]. Hence, any quantum corrections are logarithmic in momenta. We determine the RG beta functions for all the nonderivative scalar field couplings along with the corresponding gravitational corrections. This enables us to assess whether the gravitational corrections improve the high-energy behavior of the scalar field couplings. Furthermore, owing to the Vilkovisky-DeWitt formalism and assuming that both gravitational couplings, the Newton and the cosmological, take the nonzero FP values, it is possible to look for asymptotically free trajectories for all the scalar field couplings. This exploration is inspired by the Halpern-Huang

[3] In a general background, however, terms with scalar field nonminimally coupled to gravity are required for reasons of renormalizability.

discovery described in the first paragraph. The paper is organized as follows. In Sec. II we introduce a formalism of the gauge independent effective action that we use in subsequent computations. In Sec. III we perform detailed computations of the one-loop correction to the Vilkovisky-DeWitt effective action. We thus compare the obtained results with those known in the literature. Section IV is devoted to a study of RG equations for scalar field couplings. The summary and conclusions are given in Sec. V.

II. GEOMETRIC APPROACH TO THE EFFECTIVE ACTION

Standard formulation of the quantum effective action for theories with gauge symmetry turn out to be problematic from the point of view of applicability to the theories with gauge symmetry. The first obstacle derives from the fact that once a gauge condition is imposed on a variables of functional integration ϕ to render $S_{,ij}[\phi]$ invertible on the whole configuration field space, the resulting effective action, being a function of the mean field $\bar{\phi}$, is no longer invariant under the gauge symmetry transformation. This is because the gauge fixing also breaks the symmetry of the mean field. In order to keep the gauge invariance of the effective action manifested, DeWitt proposed [46] to parametrize the gauge-fixing condition for variables of integration $\chi^\alpha[\phi]$ with some unspecified external gauge field φ that subjects background gauge transformation rules such that the new gauge-fixing term with $\chi^\alpha[\phi;\varphi]$ for quantum fields is background-gauge invariant. However, this modification worked successfully at the one-loop approximation. The extension to higher loops was proposed by 't Hooft in Ref. [47] and further developed by Boulware, Abbot, and Hart [48]. The resulting effective action was gauge invariant. However, cases where the equations of motion are not satisfied, viz., off the mass shell ($J_i \neq 0$, see below), appeared to depend on the way the DeWitt's gauge-fixing term was chosen. Perhaps the easiest way to observe this dependence explicitly is to consider the one-loop approximation to the effective action. It is obtained through iterative solution of the following equation for the background field effective action:

$$\begin{aligned}\Gamma[\bar{\phi};\varphi] = -\log\int \mathcal{D}\phi\,\mathcal{M}[\phi;\varphi]\exp\Big\{&-S[\phi]\\ &-\frac{1}{2\xi}\chi^\alpha[\phi;\varphi]v_{\alpha\beta}[\varphi]\chi^\beta[\phi;\varphi]\\ &+(\phi^i-\bar{\phi}^i)\frac{\delta\Gamma[\bar{\phi};\varphi]}{\delta\bar{\phi}^i}\Big\},\end{aligned} \tag{2.1a}$$

where ξ is a positive real parameter. The second term fixes the gauge of the field ϕ. In order to avoid gauge fixing of the background field φ, the gauge condition must assume a specific form in this method, namely

$$\chi^\alpha[\phi;\varphi] = \chi^\alpha_{,i}[\varphi](\phi^i - \varphi^i). \tag{2.1b}$$

The rest of the quantities in Eq. (2.1a) are defined as follows,

$$\mathcal{M}[\phi;\varphi] \equiv \det Q^\alpha{}_\beta[\phi;\varphi](\det v_{\alpha\beta}[\varphi]/\xi)^{\frac{1}{2}}, \tag{2.1c}$$

where

$$Q^\alpha{}_\beta[\phi;\varphi] \equiv \chi^\alpha_{,\beta}[\varphi] = \chi^\alpha_{,i}[\varphi]K^i{}_\beta[\phi] \tag{2.2}$$

is the Fadeev-Popov ghost operator and

$$\begin{aligned} W[J;\varphi] &= -\Gamma[\bar{\phi};\varphi] + J_i\bar{\phi}^i,\\ \bar{\phi}^i \equiv \langle\phi^i\rangle_J &= \frac{\delta W[J;\varphi]}{\delta J_i},\\ \frac{\delta\Gamma[\bar{\phi};\varphi]}{\delta\bar{\phi}^i} &= J_i.\end{aligned}$$

The measure defined in Eq. (2.1c) contains the determinants of the ghost operator and of $v_{\alpha\beta}[\varphi]$, which is a nonsingular matrix that derives from smearing with a Gaussian weight the Dirac delta functional inserted into the integral by the Fadeev-Popov procedure. The classical action $S[\phi]$ is invariant under the action of the gauge group G on configuration field space $\mathcal{F}$, which can be expressed by the infinitesimal gauge transformation, namely

$$\delta_\varepsilon\phi^i = K^i{}_\alpha[\phi]\delta\varepsilon^\alpha \Rightarrow S_{,i}[\phi]K^i{}_\alpha[\phi] = 0, \tag{2.3}$$

for any $\phi \in \mathcal{F}$. In case the gauge group G is non-Abelian, its generators $K^i{}_\alpha[\phi]$ for nonsupersymmetric theories fulfill the following relation:

$$K^i{}_{[\alpha|,j}[\phi]K^j{}_{|\beta]}[\phi] = \frac{1}{2}f^\gamma{}_{\alpha\beta}[\phi]K^i{}_\gamma[\phi], \tag{2.4}$$

where $f^\gamma{}_{\alpha\beta}[\phi]$ are the structure functions of G. Square brackets denote antisymmetrization with respect to α and β [cf. (A2)]. It is assumed that the generators are linear, i.e., $K^i{}_{\alpha,jk} = 0$, a condition that embraces the Yang-Mills theory as well as the gravity theory. The structure functions in the two theories are structure constants. The equation for the effective action in Eq. (2.1a) can be solved iteratively. The loop expansion proceeds by changing the variable of integration $\phi = \varphi + \eta$ and developing the classical action about the background field configuration φ. In the end of the computations, one takes the limit $\varphi \to \bar{\phi}$, the result of which is equivalent to the standard effective action but without the obstacles the original formulation suffered. Within this limit the effective action is invariant with respect to the transformation $\delta_\varepsilon\varphi = K[\varphi]\cdot\delta\varepsilon$ [48]. In the case that the gauge group is not compact, one also has to provide some method of regularization that brings the quantity $f^\mu{}_{\alpha\mu}[\varphi]$ to zero. Otherwise, the effective action would not be gauge invariant with respect to background and quantum gauge transformations as well. Solving iteratively Eq. (2.1a) up to first order, we obtain the one-loop effective action that takes the form

$$\Gamma[\varphi]=S[\varphi]+\frac{1}{2}\mathrm{logdet}\Big(S_{,ij}[\varphi]+\frac{1}{\xi}\chi^{\alpha}_{,i}[\varphi]v_{\alpha\beta}[\varphi]\chi^{\beta}_{,j}[\varphi]\Big)-\mathrm{logdet}Q^{\alpha}{}_{\beta}[\varphi]. \tag{2.5}$$

That this effective action depends on the gauge off the mass shell, i.e., $J_i^{(0)}=S_{,i}\neq 0$, can be seen by considering the way it alters if we impose the new gauge condition that differs infinitesimally from the one we had begun with, namely

$$\chi'^{\alpha}[\phi;\varphi]=\chi^{\alpha}[\phi;\varphi]+\delta\chi^{\alpha}[\phi;\varphi].$$

The resulting difference between the old and the new one-loop effective action amounts to

$$\delta_{\chi}\Gamma[\varphi]=G^{ij}[\varphi]S_{,k}[\varphi]K^{k}{}_{\alpha,i}[\varphi]Q^{-1\alpha}{}_{\beta}[\varphi]\delta\chi^{\beta}_{,j}[\varphi], \tag{2.6}$$

where G is the Green's function that is inverse to the operator defined as the argument of the first determinant in Eq. (2.5). This result evidently shows the dependence off the mass shell on the way the gauge condition is chosen.

It was Vilkovisky who first noticed [44] that the gauge dependence of the effective action may be traced back to the parametrization dependence of quantum fields. The parametrization dependence might be seen in the term containing coupling between the difference of mean and quantum fields and the external sources in Eq. (2.1a). If we redefine the variables of integration, then the new variables become, in general, nonlinear regular local functionals $\phi'=f[\phi]$ of the old ones. The effective action should be scalar with respect to transformations on the configuration field space which entails $\Gamma[\bar\phi]=\Gamma[\bar f[\phi]]$ and

$$(\bar\phi^i-\phi^i)\frac{\delta\bar f^j[\phi]}{\delta\bar\phi^i}\frac{\delta\Gamma[\bar f]}{\delta\bar f^j}=(\bar f^i[\phi]-f^i[\phi])\frac{\delta\Gamma[\bar f]}{\delta\bar\phi^i}.$$

However, except for the specific cases, this holds for a constant matrix $\delta\bar f^j[\phi]/\delta\bar\phi^j$. In general this matrix is a functional of ϕ, and this transformation rule is valid for ϕ^i infinitesimally close to $\bar\phi^i$. Moreover, in the loop expansion described above, the development of the classical action about the background field is not covariant with respect to the change of coordinates on the configuration field space $\mathcal{F}$. Therefore, the effective action is not a scalar, i.e., $\Gamma[\bar\phi]\neq\Gamma[\bar f[\phi]]$.

The above arguments reveal the necessity of placing the formalism of the effective action in a fully geometric setting. Therefore, one regards the field configuration space $\mathcal{F}$ as a differential manifold $\mathcal{M}$ endowed with a metric γ, that is $\mathcal{F}=(\mathcal{M},\gamma)$. Instead of using the difference of coordinates in the coupling to the external sources, which is a vector in the flat space, one uses a tangent vector to the geodesic, connecting the background field with the quantum field. This tangent vector is taken at the background field, which is a point of coupling to the external sources,

$$\gamma^{ij}[\varphi]\frac{\delta}{\delta\varphi^j}\sigma[\varphi;\phi]\equiv\sigma^i[\varphi;\phi]=-(s_2-s_1)\frac{\mathrm{d}\phi^i(s)}{\mathrm{d}s}\Big|_{s=s_1},$$

for $\phi^i(s_1)=\varphi^i$, $\phi^i(s_2)=\phi^i$. $\sigma[\phi,\phi']$ is the half square of the geodesic distance connecting the points ϕ and ϕ'. The important property of the quantity defined in the above equation is that it transforms as a vector at the background field φ and as a scalar at the quantum field ϕ [49]. In the vicinity of the background field, the tangent vector to the geodesic has the following expansion,

$$-\sigma^i[\varphi;\phi]\approx\phi^i-\varphi^i+\frac{1}{2}\Gamma^i_{jk}[\gamma](\phi^j-\varphi^j)(\phi^k-\varphi^k), \tag{2.7}$$

where the symbol in front of the terms of the second order in fields denotes the Christoffel connection built out of the metric γ and its derivative to be defined below. In flat configuration field space, it vanishes so that the above quantity reduces to the difference of the coordinates previously used to couple with the external sources.

This extension resolves the issue of a spurious quantum field coupling to the fixed external sources. The lack of covariance that is met if one develops the classical action about the background field in the course of the iterative solution for the effective action might be removed by means of the functional covariant derivatives replacing the usual ones. The covariant derivatives are accompanied by the Christoffel connection that depends on the metric γ of $\mathcal{F}$. However, the physical configuration space of the theory with a local gauge symmetry is a quotient space $\mathcal{F}/\mathsf{G}$. Its elements are equivalence classes that are orbits generated by the action of the local gauge group G on $\mathcal{F}$. Each member of the orbit of the group G which is a manifold itself is enumerated by a corresponding parameter ε^{α} that constitutes a local coordinate on this group manifold. Thus the orbit space $\mathcal{F}/\mathsf{G}$ along with the local gauge group G provide a configuration space $\mathcal{F}$ with a local product structure $\mathcal{F}/\mathsf{G}\times\mathsf{G}$. From the geometric point of view, this orbit space is a submanifold endowed with an induced metric from the full configuration space metric γ. Therefore, the covariant derivatives on the physical configuration space should be accompanied by the Christoffel connection evaluated on the metric of the orbit space. If we denote the displacement of the field coordinate in the direction of an orbit as $\mathrm{d}\phi^i_{\parallel}=K^i{}_{\alpha}[\phi]\mathrm{d}\varepsilon^{\alpha}$, then the one along the space of orbits can be found from the condition $\gamma_{ij}[\phi]\mathrm{d}\phi^i_{\perp}\mathrm{d}\phi^i_{\parallel}=0$. Hence, the metric decomposes to

$$\gamma_{ij}[\phi]\mathrm{d}\phi^i\mathrm{d}\phi^j=\gamma^{\perp}_{ij}[\phi]\mathrm{d}\phi^i_{\perp}\mathrm{d}\phi^j_{\perp}+N_{\alpha\beta}[\phi]\mathrm{d}\varepsilon^{\alpha}\mathrm{d}\varepsilon^{\beta}, \tag{2.8}$$

where

$$N_{\alpha\beta}\equiv K^i{}_{\alpha}\gamma_{ij}K^j{}_{\beta},\qquad N_{\alpha\lambda}N^{\lambda\beta}=\delta^{\beta}_{\alpha}. \tag{2.9}$$

A tensor field $\gamma^{\perp}$ is a metric on $\mathcal{F}/\mathsf{G}$, and $N_{\alpha\beta}$ is the metric on G. The former is obtained by projection of the configuration space metric γ onto the orbit space, namely

$$\gamma^{\perp}_{ij} \equiv P^k_i \gamma_{kl} P^l_j, \qquad \gamma^{\perp}_{jk}\gamma^{ki}_{\perp} = P^i_j,$$

where the projector takes the form

$$P^i_j \equiv \delta^i_j - K^i{}_\alpha N^{\alpha\beta} K^k{}_\beta \gamma_{kj}. \tag{2.10}$$

Due to the terms containing N^{-1}, this metric is nonlocal. The physical configuration space connection may be found from the condition of the compatibility of the covariant derivative with the metric on $\mathcal{F}/\mathsf{G}$ that is $\nabla\gamma^{\perp} = 0$ [50]. The resulting Christoffel connection constructed by means of the metric $\gamma^{\perp}$ reads

$$\Gamma^i_{jk}[\gamma^{\perp}] \equiv \frac{1}{2}\gamma^{il}_{\perp}(\gamma^{\perp}_{jl,k} + \gamma^{\perp}_{kl,j} - \gamma^{\perp}_{jk,l}) = P^i_l \bar{\Gamma}^l_{jk}, \tag{2.11}$$

where the symbol on the right-hand side of the above equation, which we will refer to as the orbit space connection, has the following form

$$\bar{\Gamma}^{\,i}_{\,jk} \equiv \Gamma^i_{jk}[\gamma] + T^i_{jk}[\gamma;\phi] + \cdots. \tag{2.12}$$

The first term is the Christoffel connection on $\mathcal{F}$ and the second one denoted by T^i_{jk} is the nonlocal contribution that may be found in Ref. [44]. As one may infer from the formula in Eq. (2.11), the expression for $\bar{\Gamma}$ is not gauge independent, which is indicated by the ellipsis. It is given up to terms proportional to the generators of the gauge group. However, these terms do not contribute because any covariant derivative of the classical action with the orbit space connection Eq. (2.12) is orthogonal to the symmetry directions generated by vector fields K [44]. Moreover, due to the nonlocal part of the connection, the covariant derivative of the generator yields

$$\nabla_i K^j{}_\alpha = -\frac{1}{2} K^j{}_\gamma f^\gamma{}_{\alpha\beta} N^{\beta\lambda} K^k{}_\lambda \gamma_{ki} \propto K^j{}_\gamma. \tag{2.13}$$

The above property is crucial for the proof of gauge invariance of the effective action and of its gauge independence. The Vilkovisky-DeWitt effective action for the theories with a symmetry group is defined as a limit in $\varphi \to \bar{\phi}$ of the following formula

$$\begin{aligned}\Gamma[\bar{\phi};\varphi] \equiv -\log\int \mathcal{D}\phi\,\hat{\mathcal{M}}[\phi;\varphi]\exp\Big\{&-S[\phi] \\ &-\frac{1}{2\xi}\chi^\alpha[\phi;\varphi]\upsilon_{\alpha\beta}[\varphi]\chi^\beta[\phi;\varphi] \\ &+(\sigma^i[\varphi;\bar{\phi}] - \sigma^i[\varphi;\phi])\frac{\delta\Gamma[\bar{\phi};\varphi]}{\delta\sigma^i[\varphi;\bar{\phi}]}\Big\},\end{aligned} \tag{2.14}$$

where the measure is related to (2.1c) by $\hat{\mathcal{M}}[\phi;\varphi] \equiv (\det\gamma_{ij}[\phi])^{1/2}\mathcal{M}[\phi;\varphi]$ and $\sigma^i[\varphi;\bar{\phi}] = \langle\sigma^i[\varphi;\phi]\rangle$. A functional fixing the gauge χ is not confined to have a specific form as in the case of the standard background field effective action (2.1b) nor must it be covariant with respect to the background field. The only condition it should satisfy is $\chi[\varphi;\varphi] = 0$, so as not to contribute the zeroth and first order of iterative solution to the Eq. (2.14).

After the limit $\varphi \to \bar{\phi}$ is taken, the resulting effective action $\Gamma[\bar{\phi};\varphi] \to \Gamma_{\mathrm{VD}}[\bar{\phi}]$ has an altered form of coupling to the geodesic tangent vector field, namely

$$\lim_{\varphi\to\bar{\phi}} \frac{\delta\Gamma[\bar{\phi};\varphi]}{\delta\sigma^i[\varphi;\bar{\phi}]} = -C^{-1j}{}_i[\bar{\phi}]\Gamma_{\mathrm{VD},j}[\bar{\phi}],$$

where $C^i_j[\bar{\phi}] \equiv \langle\nabla_j\sigma^i[\bar{\phi};\phi]\rangle|_{\phi=\bar{\phi}}$. To solve the functional equation for the effective action, one must first determine the form of $C^{-1i}{}_j$, which in turn requires the knowledge of the effective action. Thus one has to solve iteratively two coupled functional equations. This complication is irrelevant at the one loop as $C^{-1i}{}_j$ is a Kronecker delta, and at higher loops it may be circumvented by the method discussed by Rheban [51]. It may be proved that this effective action is gauge invariant and gauge independent off the mass shell [50,52]. This assertion is valid, likewise, for the standard formulation, provided the trace of structure constant $f^\alpha{}_{\beta\alpha}$ vanishes. In case of the noncompact gauge groups (e.g., metric theories of gravity with the group of diffeomorphisms as a gauge group) this is accomplished by means of a suitable regularization. The most popular one is the dimensional regularization [45]. This obstacle is usually ignored when the ''physical'' cutoff regularization is used. However, it may result in the gauge parameter dependence of the final result, which was recently exemplified by Nielsen in the Einstein-Maxwell theory in Ref. [28].

Iterative solution of the effective action Eq. (2.14) proceeds in a similar manner as in the previous case. This time, however, the change of variables of integration is equivalent to the change of a coordinate system in $\mathcal{F}$. Due to the coupling of a tangent geodesic vector field to the external sources in Eq. (2.14), the most suitable new coordinates are geodesic normal coordinates $\sigma^i[\varphi;\phi]$. The expansion of the classical action about the background field is performed in an explicit covariant way, where, up to the terms needed at the one loop, it takes the form

$$\begin{aligned}S[\phi] = S[\varphi] &- \nabla_i S[\varphi]\sigma^i[\varphi;\phi] \\ &+ \frac{1}{2}\nabla_i\nabla_j S[\varphi]\sigma^i[\varphi;\phi]\sigma^j[\varphi;\phi] + \mathcal{O}((\sigma^i)^3).\end{aligned}$$

The one-loop geometric counterpart of the Eq. (2.5) is the Vilkovisky-DeWitt one-loop effective action ($\Gamma_{\mathrm{VD}}[\varphi] = S[\varphi] + \Gamma_{\mathrm{VD}}{}^{(1L)}[\varphi]$)

$$\begin{aligned}\Gamma_{\mathrm{VD}}{}^{(1L)}[\varphi] = \frac{1}{2}\mathrm{logdet}\Big(\nabla_i\nabla_j S[\varphi] + \frac{1}{\xi}\chi^\alpha_{,i}[\varphi]\upsilon_{\alpha\beta}[\varphi]\chi^\beta_{,j}[\varphi]\Big) \\ - \mathrm{logdet}Q^\alpha{}_\beta[\varphi] + \mathcal{N}[\gamma,\upsilon],\end{aligned} \tag{2.15}$$

where

$$\mathcal{N}[\gamma,\upsilon,\xi] \equiv -\frac{1}{2}\mathrm{logdet}(\upsilon_{\alpha\beta}[\varphi]/\xi) - \frac{1}{2}\mathrm{logdet}\gamma_{ij}[\varphi],$$

and the last term comes from changing variables of integration $\phi \to \sigma$. Replacement of a functional derivative with a covariant one in the expression in Eq. (2.6) and using the property in Eq. (2.13) shows that this effective action is independent of the gauge by virtue of Eq. (2.3). The formula in Eq. (2.15) involves nonlocal expressions, which is due to the second part of orbit space connection in Eq. (2.12). This nonlocal part makes computations hardly feasible. Therefore in practice one chooses the orthogonal gauge [53] defined as

$$\chi^\alpha[\phi;\varphi] = v^{\alpha\beta}[\varphi]K^i{}_\beta[\varphi]\gamma_{ij}[\varphi]\sigma^j[\varphi;\phi] = 0. \tag{2.16}$$

In the vicinity of φ, where according to Eq. (2.7) terms of higher order may be neglected, this gauge condition amounts to the Landau-DeWitt gauge, provided that $v_{\alpha\beta}[\varphi] = c_{\alpha\beta}$ for a constant matrix c and the limit $\xi \to 0$ is taken. In this gauge, the covariant derivative reduces to the local one with the Christoffel connection. If one is able to find a new chart in which the Christoffel connection vanishes, then the result obtained in the gauge-independent effective action is equivalent to that obtained in the standard background field effective action (2.1a) [53]. In the case of gravity, there are no such coordinates, and the two results are incomparable. Within this limit, the Gaussian functional with gauge fixing term in Eq. (2.14) shrinks to the functional Dirac delta. The resulting effective action has as a variable of integration solely the fields $\phi_\perp$, which are nonlocal themselves. To avoid this obstacle one may instead perform computations with the covariant derivatives on the entire $\mathcal{F}$ in the one-loop effective action and in the end take the limit $\xi \to 0$. Thus the one-loop correction to Eq. (2.15) reads

$$\begin{aligned}\Gamma_{\mathrm{VD}}{}^{(1L)} = \frac{1}{2}\lim_{\xi\to 0}\mathrm{logdet}\Big(S_{;ij} + \frac{1}{\xi}\gamma_{im}K^m{}_\alpha c^{\alpha\beta}K^n{}_\beta\gamma_{nj}\Big)\\ - \mathrm{logdet}N_{\alpha\beta} + \cdots,\end{aligned} \tag{2.17}$$

where we have omitted φ dependence and dots stand for missing $\mathcal{N}[\gamma, c, \xi]$. The semicolon denotes covariant differentiation with respect to the Christoffel connection given in the first term on the right-hand side of Eq. (2.12). The ghost part in this gauge amounts to the determinant of the metric on the group space defined in Eq. (2.8). In what follows we will apply the above-described formalism to compute the one-loop effective action for the theory of scalar fields minimally coupled to gravity.

III. ONE-LOOP EFFECTIVE ACTION FOR GRAVITY AND SCALAR FIELD SYSTEM

Equipped with a well established geometrical apparatus to deal with quantum field theories possessing gauge symmetries, we may address the question of low-energy influence of quantum gravitational degrees of freedom carried by gravitons on a scalar field defined with an arbitrary but analytic potential. Since the fundamental scale for the theory of gravity is the Planck scale, the gravitational dynamics in a low-energy limit is governed by the lowest and next-to-lowest term from the infinite series of interactions defining the effective field theory of gravity [13]. Therefore, a mentioned physical system for this energy limit is described by the following n-dimensional Euclidean version of the action,

$$\begin{aligned}S[g,\varphi] = -\frac{1}{\kappa^2}\int \mathrm{d}^n x\sqrt{g}R(g)\\ + \int \mathrm{d}^n x\sqrt{g}\Big[\frac{1}{2}g^{\mu\nu}(\partial_\mu\varphi)(\partial_\nu\varphi) + U(\varphi)\Big],\end{aligned} \tag{3.1}$$

where $R(g)$ is the Ricci scalar and $\kappa \equiv \sqrt{16\pi G}$. We assume that the potential of the scalar field has the following general form,

$$U(\varphi) = \sum_{n=0}^{\omega}\frac{\lambda_{2n}}{(2n)!}\varphi^{2n}, \tag{3.2a}$$

where couplings are defined as

$$\lambda_0 = 2\Lambda/\kappa^2, \qquad \lambda_2 = m^2/2, \qquad \lambda_4 = \lambda, \tag{3.2b}$$

where Λ is the cosmological constant. In what follows, it will be convenient to redefine the scalar field in such a manner that will enable us to treat both the gravitational and the scalar fields on equal footing. This can be attained by the following substitution, $\varphi \to \varphi/\kappa$, which renders the scalar field dimensionless. This redefinition produces an overall factor $1/\kappa^2$ in the action, i.e., $S[g,\varphi] \to S[g,\varphi]/\kappa^2$. Since we are interested in gravitational corrections to coupling constants at the one-loop level, we develop the action (3.1), which now depends on variables of integration $S[g^q, \varphi^q]$, about the background field configuration $\varphi^i = (g_{\mu\nu}(x), \varphi(x))$ up to terms quadratic in fluctuations $\eta^i = \kappa(h_{\mu\nu}(x), \phi(x))$, which are implemented by the substitution $(g^q_{\mu\nu}, \varphi^q) = (g_{\mu\nu}, \varphi) + \kappa(h_{\mu\nu}, \phi)$. The resulting background- dependent action for fluctuations reads

$$\begin{aligned}\frac{1}{2\kappa^2}\eta^i S_{,ij}[g,\varphi]\eta^j = \int \mathrm{d}^n x\sqrt{g}\mathcal{L}^{(2)}(x),\\ \mathcal{L}^{(2)} = \mathcal{L}^{(2)}_E + \mathcal{L}^{(2)}_\phi + \mathcal{L}^{(2)}_{\mathrm{int}},\end{aligned} \tag{3.3}$$

where ($\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$)

$$\begin{aligned}\mathcal{L}^{(2)}_E = \frac{1}{2}h_{\mu\nu}[-G^{\mu\nu,\alpha\beta}\Box + X^{\mu\nu,\alpha\beta}_\varphi + X^{\mu\nu,\alpha\beta}_g]h_{\alpha\beta}\\ -\frac{1}{2}C^2_\mu(h),\end{aligned} \tag{3.4a}$$

$$\mathcal{L}^{(2)}_\varphi = \frac{1}{2}\phi[-\Box + V''(\varphi)]\phi, \quad V(\varphi) \equiv \kappa^2 U(\varphi), \tag{3.4b}$$

$$\mathcal{L}^{(2)}_{\mathrm{int}} = -h_{\mu\nu}Q^{\alpha|\mu\nu}\nabla_\alpha\phi + h_{\mu\nu}\Big(\frac{1}{2}V'(\varphi)g^{\mu\nu}\Big)\phi. \tag{3.4c}$$

The prime in $V'(\varphi)$ denotes the derivative with respect to φ. The other symbols used above are defined as follows:

$$G^{\mu\nu,\alpha\beta} \equiv \frac{1}{4}(g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha} - g^{\mu\nu}g^{\alpha\beta}), \tag{3.5a}$$

$$\begin{aligned} X_\varphi^{\mu\nu,\alpha\beta} \equiv & -G^{\mu\nu,\alpha\beta}\left[\frac{1}{2}(\partial\varphi)^2 + V(\varphi)\right] \\ & -\frac{1}{2}g^{(\mu(\alpha}(\partial^{\beta)}\varphi)(\partial^{\nu)}\varphi) - \frac{1}{4}g^{\mu\nu}(\partial^\alpha\varphi)(\partial^\beta\varphi) \\ & -\frac{1}{4}g^{\alpha\beta}(\partial^\mu\varphi)(\partial^\nu\varphi), \end{aligned} \tag{3.5b}$$

$$Q^{\alpha|\mu\nu} \equiv g^{\alpha(\mu}\partial^{\nu)}\varphi - \frac{1}{2}g^{\mu\nu}\partial^\alpha\varphi, \tag{3.5c}$$

$$C_\mu(h) \equiv \nabla^\nu h_{\nu\mu} - \frac{1}{2}\partial_\mu h^\alpha{}_\alpha. \tag{3.5d}$$

The curl braces around indices denote the symmetrization (see Eq. (A2) in Appendix A). The matrix X_g contains a combination of Riemann tensor, Ricci tensor, and Ricci scalar, all defined on the background metric. In what follows we will, for simplicity, take this metric to be flat so this quantity will vanish. The above derivation constitutes preliminary computations to determine a standard one-loop effective action and in consequence to find a renormalization of coupling constants due to interaction of the scalar field with gravitons. However, the flat background metric is not a solution to the Einstein equations of motion derived from Eq. (3.1). From the previous section it is known that the standard effective action is not gauge independent if these equations of motion are not satisfied. Therefore, in order to avoid problems of gauge dependence related to off-shell effective action, we will perform computations by means of the Vilkovisky-DeWitt geometric formalism described in the previous section.

The fundamental quantity in the Vilkovisky-DeWitt formalism is a metric of configuration field space $\mathcal{F}$, which must be a local. It is usually chosen from a second-order term in the expansion of a classical action about some field configuration where it accompanies the d'Alembertian of highest power acting on fluctuations about this field configuration.[4] For the action in Eq. (3.1) after a field redefinition, as described below Eq. (3.2a), the metric tensor, as may be inferred form Eqs. (3.3), (3.4a), and (3.4b), takes the form

$$\begin{aligned} ds^2 &= \gamma_{ij}[\varphi]d\varphi^i d\varphi^j \\ &= \frac{1}{\kappa^2}\int d^nx\int d^nx'\sqrt{g}G^{\mu\nu,\rho\sigma}\delta(x,x')dg_{\mu\nu}(x)dg_{\rho\sigma}(x') \\ &\quad + \frac{1}{\kappa^2}\int d^nx\int d^nx'\sqrt{g}\delta(x,x')d\varphi(x)d\varphi(x'), \end{aligned} \tag{3.6}$$

where $\delta(x, x')$ is a density at the point x and scalar at x'. This metric tensor may be used to determine the orbit space connection as described in Sec. II. However, in order to facilitate computations hampered by the nonlocal part of the orbit space connection (2.12), one chooses the orthogonal gauge defined in Eq. (2.16) in which the nonlocal part decouples. In the vicinity of the background field configuration, the orthogonal gauge amounts to the Landau-DeWitt gauge. The classical action has the diffeomorphism symmetry, i.e.,

$$\delta_\varepsilon g_{\{\mu\nu;x\}} = -2\nabla_{(\mu}\varepsilon_{\nu)}(x), \quad \delta_\varepsilon\varphi_{\{x\}} = -(\partial_\alpha\varphi(x))\varepsilon^\alpha(x). \tag{3.7}$$

Thus with these generators, the gauge we choose takes the form

$$\chi_\alpha = K^i{}_\alpha\gamma_{ij}\eta^j = \int d^nx\sqrt{g}[C^\alpha(h) - b(\partial^\alpha\varphi)\phi]. \tag{3.8}$$

Above we introduced a parameter b that in principle can assume any value. The most popular choice is $b = 0$. The Landau-DeWitt gauge requires us to take $b = 1$ for this parameter, and we will choose this value in the end of the computations. Leaving this parameter unspecified enables us to follow the gauge dependence of the resulting effective action. However, as was anticipated in the end of Sec. II, in order to obtain the Vilkovisky-DeWitt one-loop effective action, we must choose the Landau-DeWitt gauge. Using the above general form of gauge, the gauge-breaking term can be reorganized to yield

$$\begin{aligned} &\frac{1}{2\xi}\eta^i\gamma_{ik}K^k{}_\alpha c^{\alpha\beta}K^l{}_\beta\gamma_{lj}\eta^j \\ &= \int d^nx\sqrt{g}\left(\frac{1}{2\xi}C_\alpha^2 - \frac{b}{\xi}(\partial_\alpha h_{\mu\nu})Q^{\alpha|\mu\nu}\phi + \frac{b}{2\xi}\phi(\partial\varphi)^2\phi\right). \end{aligned} \tag{3.9}$$

Due to this gauge, we are left with the local part of the connection which is a Christoffel symbol constructed by means of the metric on the full field space. According to the definition (2.11) components of the Christoffel connection for the metric (3.6) are

$$\begin{aligned} \Gamma^{\{\mu\nu;y\},\{\rho\sigma;z\}}_{\{\alpha\beta;x\}}[\gamma] = \Big[& -\delta^{\mu\nu,\rho\sigma}_{\alpha\beta} + \frac{1}{4}(g^{\rho\sigma}\delta^{\mu\nu}_{\alpha\beta} + g^{\mu\nu}\delta^{\rho\sigma}_{\alpha\beta}) \\ & + \frac{1}{n-2}g_{\alpha\beta}G^{\mu\nu,\rho\sigma}\Big](x)\delta(x,y)\delta(x,z), \end{aligned} \tag{3.10a}$$

$$\Gamma^{\{\mu\nu;y\},\{z\}}_{\{x\}}[\gamma] = \frac{1}{4}g^{\mu\nu}(x)\delta(x,y)\delta(x,z), \tag{3.10b}$$

$$\Gamma^{\{y\},\{z\}}_{\{\alpha\beta;x\}}[\gamma] = \frac{1}{n-2}g_{\alpha\beta}(x)\delta(x,y)\delta(x,z), \tag{3.10c}$$

where the multi-index delta symbols are defined in Appendix A. These connections, along with the first functional derivatives of the action that they are contracted with, give the additional contribution to the effective action. As was mentioned earlier, the Vilkovisky-DeWitt formalism is defined off the mass shell. It also does not depend on the

[4] In general the choice of a configuration space metric in the theory of gravity is ambiguous, as there is a one-parameter family of ultralocal metrics invariant along the directions generated by generators (2.4). However, there is a method [44] to make a unique choice of the metric: it must come from the highest derivative term of the classical action, and the group space metric (2.9) must be nondegenerate. This choice has been adopted in this paper. For consequences resulting from other choices of metric, see, e.g., Refs. [54,55] and corresponding discussion in Ref. [56].

background field. Hence, we may take the background metric to be flat, although it is not a solution of the equation of motion with cosmological constant. Thus in all the above formulas, we put $g_{\mu\nu} \to \delta_{\mu\nu}$. Resulting first derivatives of the action with redefined fields take the form

$$S^{,\{\mu\nu;x\}}|_{\varphi^i} = \delta^{\mu\nu}\left[\frac{1}{4}(\partial\varphi)^2 + \frac{1}{2}V(\varphi)\right] - \frac{1}{2}(\partial^\mu\varphi)(\partial^\nu\varphi),$$
$$S^{,\{x\}}|_{\varphi^i} = -\partial^2\varphi + V'(\varphi),$$
$$(\partial^2 \equiv \delta^{\mu\nu}\partial_\mu\partial_\nu),$$

where $\varphi^i = (\delta_{\mu\nu}, \varphi)$. Combining the above equations with the Christoffel connections given in Eqs. (3.10a)–(3.10c), we get the following Vilkovisky-DeWitt counterpart of the action for fluctuations in Eq. (3.3), supplemented with the gauge-fixing term (3.9), namely

$$\frac{1}{2}\eta^i\left(S_{,ij} - a\Gamma^k_{ij}S_{,k} + \frac{1}{\xi}\gamma_{ik}K^k{}_\alpha c^{\alpha\beta}K^l{}_\beta\gamma_{lj}\right)\eta^j = \int d^n x[\tilde{\mathcal{L}}^{(2)}(x) + \mathcal{L}^{(2)}_{\rm GF}(x)].$$

In the above formula we have introduced an additional parameter to be able to compare the results between the standard one-loop effective action ($a=0$) and the Vilkovisky-DeWitt modified one ($a=1$). The quantities from Eqs. (3.4a)–(3.4c), altered due to insertion of both connection and gauge fixing as well as a rearrangement because of requirements of Hermicity of the whole operator, read

$$\tilde{\mathcal{L}}^{(2)} + \mathcal{L}^{(2)}_{\rm GF} = \frac{1}{2}h_{\mu\nu}[-D^{\mu\nu,\alpha\beta}(\xi,\partial)\partial^2 + \tilde{X}^{\mu\nu,\alpha\beta}]h_{\alpha\beta} \tag{3.11a}$$
$$+ \frac{1}{2}\phi[-\partial^2 + Y(\xi)]\phi \tag{3.11b}$$
$$+ \zeta(\partial_\alpha h_{\mu\nu})Q^{\alpha|\mu\nu}\phi - \zeta h_{\mu\nu}Q^{\alpha|\mu\nu}(\partial_\alpha\phi) + \frac{1}{4}h_{\mu\nu}Z^{\mu\nu}(\xi)\phi, \qquad \zeta \equiv \frac{1}{2}\left(1 - \frac{b}{\xi}\right). \tag{3.11c}$$

The quantities in the above operator are defined as follows:

$$D^{\mu\nu,\alpha\beta}(\xi,\partial) \equiv \mathcal{G}^{\mu\nu,\alpha\beta} - \left(1 - \frac{1}{\xi}\right)\left[\delta^{\mu\nu,\alpha\beta}_{\rho\sigma} - \frac{1}{2}\delta^{\mu\nu}\delta^{\alpha\beta}_{\rho\sigma} - \frac{1}{2}\delta^{\alpha\beta}\delta^{\mu\nu}_{\rho\sigma} + \frac{1}{4}\delta^{\mu\nu}\delta^{\alpha\beta}\delta_{\rho\sigma}\right]\frac{\partial^\rho\partial^\sigma}{\partial^2}, \tag{3.12a}$$

$$\tilde{X}^{\mu\nu,\alpha\beta} \equiv \left(\frac{a}{2} - 1\right)\left[\frac{1}{2}\mathcal{G}^{\mu\nu,\alpha\beta}(\partial\varphi)^2 - \delta^{(\mu(\alpha}(\partial^{\beta)}\varphi)(\partial^{\nu)}\varphi)\right] + \frac{1}{4}\left(\frac{a}{2} - 1\right)[\delta^{\mu\nu}(\partial^\alpha\varphi)(\partial^\beta\varphi) + \delta^{\alpha\beta}(\partial^\mu\varphi)(\partial^\nu\varphi)]$$
$$- \left[1 - a + \frac{na}{2(n-2)}\right]\mathcal{G}^{\mu\nu,\alpha\beta}V(\varphi), \tag{3.12b}$$

$$Y(\xi) \equiv \left(\frac{b}{\xi} - \frac{a}{4}\right)(\partial\varphi)^2 + V''(\varphi) - \frac{na}{2(n-2)}V(\varphi), \tag{3.12c}$$

$$Z^{\mu\nu}(\xi) \equiv 2\left(1 + \frac{b}{\xi}\right)\partial^\mu\partial^\nu\varphi - \left(1 - a + \frac{b}{\xi}\right)\delta^{\mu\nu}\partial^2\varphi + (2-a)\delta^{\mu\nu}V'(\varphi). \tag{3.12d}$$

In order to obtain the form of the one-loop correction in Eq. (2.17), the above formula must be completed with the ghost Lagrangian. In the Landau-DeWitt gauge (3.8) by virtue of the definition given in Eq. (2.8), it takes the form

$$\bar{\vartheta}^\alpha N_{\alpha\beta}\vartheta^\beta = \int d^n x\bar{\vartheta}^\rho[-\delta_{\rho\sigma}\partial^2 - b(\partial_\rho\varphi)(\partial_\sigma\varphi)]\vartheta^\sigma. \tag{3.13}$$

The next step that we will take in the course of determining the gravitational renormalization of scalar field couplings is the expansion of a determinant that results from a functional integration of Eqs. (3.11a)–(3.11c) and (3.13) as described in the previous section.

A. The functional determinant and its expansion

To find the leading quantum gravitational corrections to the running of scalar coupling constants, we need to compute the one-loop divergences to the kinetic term and all the vertices in the theory. Although for their derivation it is sufficient to confine oneself to only a few terms that contribute to the renormalization of the corresponding operator, we will extend computations to the full scalar sector of the one-loop effective action. This will enable us to compare the Vilkovisky-DeWitt method to the standard effective action results off the mass shell obtained in Refs. [11,32]. Instead of using the algorithm by Barvinsky and Vilkovisky in Ref. [57] to derive the result, we will use a more straightforward one that does not make use of the Ward identity. It will allow us to follow the factor $1/\xi$ that should cancel in the end of the computations so that the final result would at most depend on the positive power of the gauge parameter ξ. This will enable us to send this parameter to zero, which is required by the Landau-DeWitt gauge. Explicit computation will allow us to verify the applicability of this gauge-independent method to the nonrenormalizable theory first attempted in Ref. [57] in the case of pure Einstein gravity and by other authors in different contexts in Refs. [42,54,55], including a recent study on the full form of the orbit space connection in the case of the Einstein-Maxwell system undertaken in Ref. [28]. The derivation of the one-loop effective action for nonminimal coupling of the scalar field theory to gravity, including the gravitational sector, will be given elsewhere in another context [58].

In the previous subsection we have determined the form of the functional operator. After functional integration over fluctuations the we get the determinant of this operator that contains a full information about the one-loop divergence structure of the scalar sector of the theory. In order to extract this information, we will expand the latter quantity in a series of a growing number of background field-dependent vertices defined in Eqs. (3.5c) and (3.12b)–(3.12d) and keep only those terms that are divergent in four space dimensions. The functional determinant, up to infinite constant terms, reads

$$\begin{aligned}\frac{1}{2}\,\mathrm{logdet}\Big(S_{;ij}+\frac{1}{\xi}\gamma_{ik}K^{k}{}_{\alpha}c^{\alpha\beta}K^{l}{}_{\beta}\gamma_{lj}\Big)-\mathrm{logdet}N_{\alpha\beta}\\ =-\log\Big\langle\exp\Big\{-\frac{1}{2}h^{A}\tilde{X}_{AB}h^{B}-\frac{1}{2}\phi^{a}Y_{ab}\phi^{b}\\ +\zeta h^{A}Q_{Aa}\phi^{a}-\zeta\phi^{a}Q^{T}_{aA}h^{A}-\frac{1}{4}h^{A}Z_{Aa}\phi^{a}\Big\}\Big\rangle_{0}\\ -\log\langle\exp\{-b\bar{\vartheta}^{\alpha}(\partial\varphi\partial\varphi)_{\alpha\beta}\vartheta^{\beta}\}_{\alpha\beta}\rangle_{0}+\cdots,\end{aligned}$$

where $i=\{A,a\}$ and $A=\{\mu\nu;x\}$, $a=\{x\}$. The ellipsis denotes the infinite constant part. We have introduced the following notation:

$$h^{A}Q_{Aa}\phi^{a}\equiv\int\mathrm{d}^{n}x\,h_{\mu\nu}(x)Q^{\alpha|\mu\nu}(x)\partial_{\alpha}\phi(x),$$

$$\phi^{a}Q^{\mathrm{T}}_{aA}h^{A}\equiv\int\mathrm{d}^{n}x\,\phi(x)Q^{\alpha|\mu\nu}(x)\partial_{\alpha}h_{\mu\nu}(x).$$

The average is taken with the Gaussian weighting functional of the massless free field theory (which is indicated by subscript 0), defined by kinetic terms of quantum fields in Eqs. (3.11a), (3.11b), and (3.13). Expanding the exponent under the functional integral, averaging with the Gaussian functional, making use of the Wick's theorem, and finally expanding the logarithm, we arrive at the explicit form of the divergent part of the ξ-dependent effective action.

In what follows we address the evaluation of the nonghost as well as the ghost divergent parts of the above functional determinants. The divergent parts are extracted by means of the dimensional regularization method (DimReg), where they appear as pole terms in ϵ about the physical dimension of integrals over virtual particle momenta evaluated in arbitrary complex dimension n, i.e., for $\epsilon=4-n$. The advantage of this method is that it regularizes the quadratic divergences to zero that would appear if momentum cutoff regularization on virtual particles momenta was used. This solves the formal problem of gauge noninvariance of the functional integral measure that is met in gauge theories with noncompact gauge group such as the group of diffeomorphisms in gravity, which was mentioned in Sec. II. Moreover, it will allow us to extract genuine quantum gravitational corrections that in the perturbative regime contribute to the renormalization of the scalar field couplings as was discussed in detail in Ref. [27]. Therefore, in the computations, we confine ourselves to the terms proportional to $1/\epsilon$.

1. Computations of the nonghost part

The entire nonghost part has the following divergent contribution:

$$\frac{1}{2}\mathrm{logdet}\Big(S_{;ij}+\frac{1}{\xi}\gamma_{ik}K^{k}{}_{\alpha}c^{\alpha\beta}K^{l}{}_{\beta}\gamma_{lj}\Big)$$

$$=\frac{1}{2}\tilde{X}_{AB}G^{AB}+\frac{1}{2}Y_{ab}G^{ab}-\frac{1}{2}\zeta^{2}Q_{Ab}G^{AB}Q_{Ba}G^{ab}-\frac{1}{2}\zeta^{2}Q^{\mathrm{T}}_{bA}G^{AB}Q^{\mathrm{T}}_{aB}G^{ab}\tag{3.14a}$$

$$+\zeta^{2}Q^{\mathrm{T}}_{bA}G^{AB}Q_{Ba}G^{ab}+\frac{1}{4}\zeta Q_{Ba}G^{ab}Z_{bA}G^{AB}-\frac{1}{4}\zeta Q^{\mathrm{T}}_{bA}G^{AB}Z_{Ba}G^{ab}\tag{3.14b}$$

$$-\frac{1}{32}Z_{bA}G^{AB}Z_{Ba}G^{ab}-\frac{1}{4}\tilde{X}_{DA}G^{AB}\tilde{X}_{BC}G^{CD}-\frac{1}{4}Y_{ab}G^{bc}Y_{cd}G^{da}\tag{3.14c}$$

$$+\frac{1}{2}\zeta^{2}\tilde{X}_{DA}G^{AB}Q^{\mathrm{T}}_{aB}G^{ab}Q^{\mathrm{T}}_{bC}G^{CD}-\zeta^{2}\tilde{X}_{DA}G^{AB}Q^{\mathrm{T}}_{aB}G^{ab}Q_{Cb}G^{CD}+\frac{1}{2}\zeta^{2}\tilde{X}_{DA}G^{AB}Q_{Ba}G^{ab}Q_{Cb}G^{CD}\tag{3.14d}$$

$$+\frac{1}{2}\zeta^{2}Y_{da}G^{ab}Q^{\mathrm{T}}_{bA}G^{AB}Q^{\mathrm{T}}_{cB}G^{cd}-\zeta^{2}Y_{da}G^{ab}Q^{\mathrm{T}}_{bA}G^{AB}Q_{Bc}G^{cd}+\frac{1}{2}\zeta^{2}Y_{da}G^{ab}Q_{Ab}G^{AB}Q_{Bc}G^{cd}\tag{3.14e}$$

$$\begin{aligned}&-\frac{1}{4}\zeta^{4}Q^{\mathrm{T}}_{aD}G^{ab}Q^{\mathrm{T}}_{bA}G^{AB}Q^{\mathrm{T}}_{cB}G^{cd}Q^{\mathrm{T}}_{dC}G^{CD}-\frac{1}{4}\zeta^{4}Q_{Da}G^{ab}Q_{Ab}G^{AB}Q_{Bc}G^{cd}Q_{Cd}G^{CD}\\ &+\zeta^{4}Q_{Da}G^{ab}Q^{\mathrm{T}}_{bA}G^{AB}Q^{\mathrm{T}}_{cB}G^{cd}Q^{\mathrm{T}}_{dC}G^{CD}+\zeta^{4}Q_{Da}G^{ab}Q_{Ab}G^{AB}Q^{\mathrm{T}}_{cB}G^{cd}Q_{Cd}G^{CD}-\frac{1}{2}\zeta^{4}Q_{Da}G^{ab}Q_{Ab}G^{AB}Q^{\mathrm{T}}_{cB}G^{cd}Q^{\mathrm{T}}_{dC}G^{CD}\\ &-\frac{1}{2}\zeta^{4}Q_{Da}G^{ab}Q^{\mathrm{T}}_{bA}G^{AB}Q^{\mathrm{T}}_{cB}G^{cd}Q_{Cd}G^{CD}-\frac{1}{2}\zeta^{4}Q_{Da}G^{ab}Q^{\mathrm{T}}_{bA}G^{AB}Q_{Bc}G^{cd}Q^{\mathrm{T}}_{dC}G^{CD}+\mathrm{o.t.},\end{aligned}\tag{3.14f}$$

where "o.t." indicates some other terms that do not contribute the divergent part and are omitted. The above symbols denote the two-point correction functions for graviton, scalar, and ghost fields, respectively, defined as

$$G^{AB} \equiv \langle h^A h^B \rangle_0, \qquad G^{ab} \equiv \langle \phi^a \phi^b \rangle_0.$$

Their momentum space representations take the forms, respectively,

$$G^{(h)}_{\mu\nu,\rho\sigma}(p) = \left[\mathcal{G}^{-1}_{\mu\nu,\rho\sigma} - \left(4(1-\xi) - 4\xi \frac{M^2}{p^2}\right) \delta^{\alpha\beta}_{\mu\nu,\rho\sigma} \frac{p_\alpha p_\beta}{p^2} \right] \times (p^2 + M^2)^{-1}, \tag{3.15a}$$

and

$$G^{(\phi)}(p) = (p^2 + M^2)^{-1}. \tag{3.15b}$$

$\mathcal{G}^{-1}$ is the inverse of the graviton metric from Eq. (3.5a) defined in Eq. (A3) and M^2 is the IR regulator. Although there is no need for this regulator since there is a mass term in the theory, from the RG analysis point of view, it is convenient to regard this mass term as a perturbation vertex. The graviton propagator in Eq. (3.15a) owes its form to the manner in which we have introduced the IR regulator. Namely, we have modified the kinetic part of the operator in Eq. (3.11a) as follows $-h^A D_{AB}(\infty) h^B = -h^A_\perp D_{AB}(1) h^B_\perp \to h^A_\perp [-D_{AB}(1) + \delta_{AB} M^2] h^B_\perp$, where the explicit form of $D_{AB}(\xi)$ is given in Eq. (3.12a) for $(A, B) = (\{x, \alpha\beta\}, \{y, \mu\nu\})$. $h^A_\perp \equiv P^A_B h^B$ and P^A_B is the projector on the orbit space, a generic form of which is defined in Eq. (2.10). In the end of the computations, we take $M \to 0$. All the computations were performed with the aid of the CADABRA software [59].

Evaluation of the first two parts is straightforward and we find the following pole term:

$$\left[\frac{1}{2}\tilde{X} + \frac{1}{2}Y\right]_{\text{div}} = -\frac{M^2}{\hat{\epsilon}} \int d^4x \left[V''(\varphi) + \left(-\frac{a}{4} + \frac{b}{\xi}\right)(\partial\varphi)^2 - (6 + a + 4\xi) V(\varphi) \right], \tag{3.16a}$$

where $\hat{\epsilon} \equiv (4\pi)^2 \epsilon$. For the sake of conciseness we have shortened notation, discarding propagators and indices as follows $X \equiv X_{AB} G^{AB}$. This term is regulator dependent and in the limit of vanishing M, there is no contribution from this part. The third trace from Eq. (3.14a) is more involved. Explicitly, it takes the form

$$Q_{Ab} G^{AB} Q_{Ba} G^{ab} = \int d^n x \int d^n x' Q^{\lambda|\mu\nu}(x) \langle h_{\mu\nu}(x) h_{\rho\sigma}(x') \rangle \times Q^{\kappa|\rho\sigma}(x') \partial_{\kappa'} \partial_\lambda \langle \phi(x') \phi(x) \rangle.$$

Its evaluation can be performed in the momentum space, making use of the formulas (3.15a) and (3.15b), the Feynman parameters method, and the averaging-over directions. The divergences from virtual particles in the loop after some algebra yield the following contribution

$$QQ|_{\text{div}} = \frac{2}{\hat{\epsilon}} \left[-\xi M^2 \left(\frac{1}{6} \operatorname{Tr}\{Q^\mu \mathbb{1}_{(2)} Q_\mu\} + \frac{1}{3} \operatorname{Tr}\{Q_\alpha \mathbb{1}^{\alpha\beta}_{(3)} Q_\beta\} \right) + T \right],$$

where

$$\operatorname{Tr}\{AB\} \equiv \int d^n x \operatorname{tr}\{A(x) B(x)\}.$$

The second term takes the form

$$T \equiv \frac{1}{3} \operatorname{Tr}\{(\partial_\mu Q^\mu) \mathcal{G}^{-1} (\partial_\nu Q^\nu)\} - \frac{1}{12} \operatorname{Tr}\{(\partial^\lambda Q^\mu) \mathcal{G}^{-1} (\partial_\lambda Q_\mu)\} - (1-\xi) \left(\frac{1}{2} \operatorname{Tr}\{(\partial_\mu Q^\mu) \mathbb{1}_{(2)} (\partial_\nu Q^\nu)\} + \frac{1}{6} \operatorname{Tr}\{(\partial_\alpha Q^\mu) \mathbb{1}^{\alpha\beta}_{(3)} (\partial_\beta Q_\mu)\} - \frac{2}{3} \operatorname{Tr}\{(\partial_\mu Q^\mu) \mathbb{1}^{\alpha\beta}_{(3)} (\partial_\alpha Q_\beta)\} - \frac{1}{12} \operatorname{Tr}\{(\partial_\lambda Q^\mu) \mathbb{1}_{(2)} (\partial^\lambda Q_\mu)\} - \frac{1}{6} \operatorname{Tr}\{(\partial_\lambda Q_\alpha) \mathbb{1}^{\alpha\beta}_{(3)} (\partial^\lambda Q_\beta)\} \right).$$

$\mathbb{1}_{(n)}$ are defined in Appendix A. Computation of T shows that the sum of traces compensate one another to yield no pole terms i.e., $T = 0$. Thus the only contribution comes from the regulator-dependent term. After some algebra one finally gets

$$\frac{1}{2} \zeta^2 QQ|_{\text{div}} = \frac{M^2}{\hat{\epsilon}} \xi \zeta^2 \int d^4x (\partial\varphi)^2. \tag{3.16b}$$

Within the limit $M \to 0$, we have no contribution from this part. Similar computations for the first trace in Eq. (3.14b) yield the set of traces over discreet indices different than the above. However, it eventually amounts to the same result. As for the first term in Eq. (3.14b), its pole part reads

$$\zeta^2 Q^{\mathrm{T}} Q|_{\text{div}} = \frac{1}{\hat{\epsilon}} \int d^4x [2M^2 \xi \zeta^2 (\partial\varphi)^2 + \zeta^2 \xi (\partial^2\varphi)^2]. \tag{3.16c}$$

Evaluation of the rest of the terms in Eqs. (3.14b) and (3.14c) proceeds in the same manner as sketched above. What we find for the second term of Eq. (3.14b) is

$$\frac{1}{4} \zeta QZ|_{\text{div}} = \frac{1}{\hat{\epsilon}} \int d^4x \left[-\zeta \left(\frac{3a}{2} + \frac{b}{2} + \xi\left(\frac{1}{2} - a\right) \right) (\partial^2\varphi)^2 + \zeta \left(3 - \frac{3a}{2} - \xi(2-a) \right) (\partial\varphi)^2 V''(\varphi) \right], \tag{3.16d}$$

and for the third term of Eq. (3.14b),

$$-\frac{1}{4} \zeta Q^{\mathrm{T}} Z|_{\text{div}} = \frac{1}{\hat{\epsilon}} \int d^4x \left[\zeta \left(\frac{b}{2} + \xi\left(\frac{1}{2} + \frac{a}{2}\right) \right) (\partial^2\varphi)^2 + \zeta\xi \left(-1 + \frac{a}{2} \right) (\partial\varphi)^2 V''(\varphi) \right]. \tag{3.16e}$$

As for the traces in Eq. (3.14c), their evaluation is straightforward, and one finally finds

$$\frac{1}{32}ZZ|_{\rm div}=\frac{1}{\hat{\epsilon}}\int d^4x\bigg[\left(\frac{ab}{4}-\frac{b}{2}-\frac{b}{4\xi}(b+3a)-\frac{1}{4}\xi\right)(\partial^2\varphi)^2+\left(3-\frac{9a}{4}-\xi+\frac{3a}{4}\xi\right)(V'(\varphi))^2+\left(\frac{3}{2}-\frac{9a}{4}-\frac{b}{2}+\frac{ab}{4}+\frac{3b}{4\xi}(2-a)-\xi\left(\frac{1}{2}-\frac{3a}{4}\right)\right)(\partial\varphi)2V''(\varphi)\bigg], \tag{3.16f}$$

for the first term, and the second one amounts to

$$-\frac{1}{4}\tilde{X}\tilde{X}|_{\rm div}=\frac{1}{\hat{\epsilon}}\int d^4x\bigg[\left(\frac{3a}{8}-\frac{1}{2}\right)(1+\xi+\xi^2)(\partial\varphi)^4-(3+2\xi^2)V^2(\varphi)\bigg], \tag{3.16g}$$

whereas the third trace boils down to

$$-\frac{1}{4}YY|_{\rm div}=\frac{1}{\hat{\epsilon}}\int d^4x\bigg[aV''(\varphi)V(\varphi)-\frac{1}{2}\left(\frac{a}{4}-\frac{b}{\xi}\right)^2(\partial\varphi)^4-a\left(\frac{a}{4}-\frac{b}{\xi}\right)(\partial\varphi)^2V(\varphi)-\frac{a}{2}V^2(\varphi)+\left(\frac{a}{4}-\frac{b}{\xi}\right)(\partial\varphi)^2V''(\varphi)-\frac{1}{2}(V''(\varphi))2\bigg]. \tag{3.16h}$$

Computation of the next few traces is slightly more complicated than those above. Therefore, we present a more detailed derivation of them. The first trace in Eq. (3.14d) after averaging over directions in momentum space and extracting the divergent part can be cast into the form

$$\begin{aligned}\tilde{X}QQ|_{\rm div}&=\frac{2}{\hat{\epsilon}}\bigg[\frac{1}{4}\delta_{\alpha\beta}{\rm Tr}\{Q^\alpha G^{-1}\tilde{X}G^{-1}Q^\beta\}-8(1-\xi)\mathbb{I}^{(2)}_{\alpha\beta,\mu\nu}{\rm Tr}\{\tilde{X}G^{-1}Q^\alpha Q^\beta\mathbb{1}^{\mu\nu}_{(3)}\}\\&\quad+16(1-\xi)^2\mathbb{I}^{(3)}_{\alpha\beta,\mu\nu,\rho\sigma}{\rm Tr}\{\tilde{X}\mathbb{1}^{\mu\nu}_{(3)}Q^\alpha Q^\beta\mathbb{1}^{\rho\sigma}_{(3)}\}\bigg]\\&=-\frac{1}{\hat{\epsilon}}\int d^4x\bigg[\xi^2\frac{3}{8}(a-2)(\partial\varphi)^4+\xi^2(\partial\varphi)^2V(\varphi)\bigg],\end{aligned}$$

where symbols $\mathbb{I}^{(n)}_{\mu_1\nu_1,\ldots,\mu_n\nu_n}$ are defined in Appendix A. The rest of the terms in Eq. (3.14d) have the same form modulo sign that comes from the different distribution of the derivatives. Accounting for the sign in front of the individual term, the final result for the set of traces reads

$$\left[\frac{1}{2}\zeta^2\tilde{X}Q^{\rm T}Q^{\rm T}-\cdots\right]_{\rm div}=\frac{1}{\hat{\epsilon}}\int d^4x\bigg[\frac{3}{2}(2-a)\zeta^2\xi^2(\partial\varphi)^4-4\zeta^2\xi^2(\partial\varphi)^2V(\varphi)\bigg], \tag{3.16i}$$

where the dots stand for the rest of the terms in Eq. (3.14d). The same remarks may be directly applied to the subsequent set of traces. Namely, computations of the first trace in Eq. (3.14e) amount to

$$YQ^TQ^T|_{\rm div}=\frac{2}{\hat{\epsilon}}\int d^4x\bigg[\left(b-\frac{a}{4}\xi\right)(\partial\varphi)^4-a\xi V(\varphi)(\partial\varphi)^2+\xi V''(\varphi)(\partial\varphi)^2\bigg].$$

Taking into account the different distribution of derivatives that affect the sign in front of the individual traces in Eq. (3.14e), their sum yields

$$\left[\frac{1}{2}\zeta^2YQ^{\rm T}Q^{\rm T}-\cdots\right]_{\rm div}=\frac{1}{\hat{\epsilon}}\int d^4x[\zeta^2(4b-a\xi)(\partial\varphi)^4-4a\xi\zeta^2V(\varphi)(\partial\varphi)^2+4\xi\zeta^2V''(\varphi)(\partial\varphi)^2]. \tag{3.16j}$$

As for the last set of traces given in Eq. (3.14f), proceeding in a similar manner as in previous two sets of traces, we may confine to the first one. After some momentum space computations and extracting, the divergent part may be cast into the following form:

$$\begin{aligned}QQQQ|_{\rm div}&=-\frac{2}{\hat{\epsilon}}[\mathbb{I}^{(2)}_{\alpha\beta,\gamma\lambda}{\rm Tr}\{Q^\alpha Q^\beta G^{-1}Q^\gamma Q^\lambda G^{-1}\}-8(1-\xi)\mathbb{I}^{(3)}_{\alpha\beta,\gamma\lambda,\mu\nu}{\rm Tr}\{Q^\alpha Q^\beta G^{-1}Q^\gamma Q^\lambda\mathbb{1}^{\mu\nu}_{(3)}\}\\&\quad+16(1-\xi)^2\mathbb{I}^{(4)}_{\alpha\beta,\gamma\lambda,\mu\nu,\rho\sigma}\times{\rm Tr}\{Q^\alpha Q^\beta\mathbb{1}^{\mu\nu}_{(3)}Q^\gamma Q^\lambda\mathbb{1}^{\rho\sigma}_{(3)}\}]\\&=\frac{2}{\hat{\epsilon}}\xi^2\int d^4x(\partial\varphi)^4.\end{aligned}$$

We find that the rest of them have the same abstract value, though different signs. Taking into account the sign of each trace contributing to the sum, we find the final result for the set of traces in Eq. (3.14f) [dots represent the rest of them],

$$[QQQQ-\cdots]_{\rm div}=-\frac{8\zeta^4\xi^2}{\hat{\epsilon}}\int d^4x(\partial\varphi)^4. \tag{3.16k}$$

2. Computations of the ghost part

The above computations pertain to the nonghost part. The ghost part of the one-loop effective action may be developed as follows,

$$-\log\det N_{\alpha\beta}|_{\rm div}=-b(\partial\varphi\partial\varphi)_{\alpha\beta}G^{\alpha\beta}_{\rm gh}+\frac{b}{2}(\partial\varphi\partial\varphi)_{\alpha\beta}G^{\beta\gamma}_{\rm gh}(\partial\varphi\partial\varphi)_{\gamma\delta}G^{\delta\alpha}_{\rm gh}, \tag{3.17}$$

where the above symbol $G_{\rm gh}$ is defined along with its momentum space representation as

$$G_{\rm gh}^{\alpha\beta} \equiv \langle \vartheta^\alpha \bar{\vartheta}^\beta \rangle_0, \qquad G_{\rm gh}^{\alpha\beta}(p) = \delta^{\alpha\beta}(p^2 + M^2)^{-1}.$$

Evaluation of the divergent part of the ghost determinant is straightforward. The first term of its expansion given in Eq. (3.17) contributes to the infrared regulator only. The second trace yields nonzero contributions in the $M \to 0$ limit. Thus a total ghost contribution takes the form

$$-\log\det N_{\alpha\beta}|_{\rm div} = \frac{1}{\hat{\epsilon}} \int \mathrm{d}^4x [2bM^2(\partial\varphi)^2 + b^2(\partial\varphi)^4]. \tag{3.18}$$

B. The pole part of the effective action

Assembling all the results obtained in Eqs. (3.16a)–(3.16k) and in Eq. (3.18), we arrive at the final form of the functional determinant. Retrieving the canonical dimension of the background field $\varphi \to \kappa\varphi$ entails appropriate replacement of the potential and its derivatives with respect to φ, namely $V(\varphi) \to \kappa^2 U(\varphi)$, $V'(\varphi) \to \kappa U'(\varphi)$, $V''(\varphi) \to U''(\varphi)$. Its explicit form reads

$$\begin{aligned} \Gamma_{\rm VD}{}^{(1L)}[\varphi] = {}& \lim_{\xi\to 0} \frac{M^2}{(4\pi)^2\epsilon} \int \mathrm{d}^4x \left\{ -U''(\varphi) + \left[\frac{a}{4} + 2b + \xi - \frac{b}{\xi}(1-b) \right] \kappa^2(\partial\varphi)^2 + (6 + a + 4\xi)\kappa^2 U(\varphi) \right\} \\ & + \frac{1}{(4\pi)^2\epsilon} \int \mathrm{d}^4x \left\{ -\frac{1}{2}(U''(\varphi))^2 + A\kappa^2(\partial^2\varphi)^2 + B\kappa^4 U^2(\varphi) + C\kappa^2(U'(\varphi))^2 + D\kappa^2 U''(\varphi)U(\varphi) \right. \\ & \left. + E\kappa^2(\partial\varphi)^2 U''(\varphi) + F\kappa^4(\partial\varphi)^2 U(\varphi) + G\kappa^4(\partial\varphi)^4 \right\}, \end{aligned} \tag{3.19a}$$

where the coefficients in front of individual terms are defined as follows:

$$\begin{aligned} & A = -b - \frac{1}{2}ab - \frac{3}{4}a + \frac{3}{4}a\xi, \qquad D = a, \qquad B = -3 - \frac{1}{2}a - 2\xi^2, \\ & E = 3 - \frac{11}{4}a - b - \frac{1}{2}ab + \left(\frac{3}{2}a - 1\right)\xi + \frac{1}{\xi}b(b-1), \qquad C = 3 - \frac{9}{4}a - \xi + \frac{3}{4}a\xi, \\ & F = -\frac{1}{4}a + 2ab - b^2 + \frac{1}{\xi}ab(1-b) + \xi(2b - a) - \xi^2, \\ & G = -\frac{1}{2} + \frac{11}{32}a + \left(2 - \frac{9}{4}b\right)b + \frac{1}{2}ab\left(1 - \frac{3}{4}b\right) + \left(-\frac{1}{2} + \frac{1}{8}a + \frac{1}{2}b + \frac{3}{4}ab\right)\xi - \frac{1}{4}\xi^2 \\ & \qquad + \frac{b}{\xi}\left(\frac{1}{4}a - 2b - \frac{1}{4}ab + 2b^2\right) + \frac{b^2}{\xi^2}\left(-\frac{1}{2} + b - \frac{1}{2}b^2\right). \end{aligned} \tag{3.19b}$$

It should be noted that in the above result, some of the coefficients of operators depend on the inverse of ξ, preventing us form taking the zero limit required to obtain the Vilkovisky-DeWitt one-loop correction as prescribed in Eq. (2.17). However, if we let the parameter b be such that $b^2 = b$, which entails either $b = 0$ or $b = 1$, then all the terms with $1/\xi$ compensate one another. Note that this result could be obtained if there were no configuration space connections at all, as may be checked by setting the parameter $a = 0$ in Eqs. (3.16a)–(3.16k)). The presence of the gauge parameter ξ and b in Eq. (3.19a) is a consequence of neglect of the nonlocal part of the orbit space connection given in Eq. (2.12), which if taken into account would also remove the terms associated with these parameters. In order to make up for this lack of the nonlocal part of the connection, we have to make the gauge parameter zero, as prescribed in the end of the previous section, as well as set the parameters $a = 1$ and $b = 1$. This procedure leads to the gauge- independent and gauge-invariant one-loop effective action. Thus it is another explicit example for applicability of the Vilkovisky-DeWitt formalism to the nonrenormalizable theory, at least at the one-loop level.

Before we proceed, it is interesting to compare the result in Eq. (3.19a) for $M = 0$ with those obtained by means of the standard effective action technique in various gauges. Those results are juxtaposed in Table I. The first row represents a set of values for the gauge parameters used in the exact renormalization group approach to the scalar field theory nonminimally coupled to gravity where the beta functions for the system have been obtained [35].

TABLE I. Comparison of the one-loop corrections to the standard effective action ($a = 0$) in various gauges parametrized by ξ and b [see Eq. (3.9)] with the Vilkovisky-DeWitt effective action ($a = 1$). A-G are coefficients of the one-loop correction given in Eq. (3.19a).

$(a\|\xi, b)$	A	B	C	D	E	F	G
$(0\|0, 0)$	0	-3	3	0	3	0	$-\frac{1}{2}$
$(0\|1, 0)$	0	-5	2	0	2	-1	$-\frac{5}{4}$
$(0\|1, 0)$	-1	-5	2	0	1	0	-1
$(0\|0, 1)$	-1	-3	3	0	2	-1	$-\frac{3}{4}$
$(1\|0, 1)$	$-\frac{9}{4}$	$-\frac{7}{2}$	$\frac{3}{4}$	1	$-\frac{5}{4}$	$\frac{3}{4}$	$-\frac{9}{32}$

The results in the second row of the table may be compared with those of Refs. [32,60], where the one-loop effective action for the quantum gravity, nonminimally and minimally coupled scalar field was considered. Direct comparison reveals a coincidence in the abstract values of the coefficients in Refs. [32,60] (up to an erroneous coefficient B in the latter paper) compared with those displayed in the above table. An overall sign difference comes from the different approach, namely Lorentzian in Ref. [32] and Euclidean in the present paper. The gauge in the third row of Table I was addressed in Ref. [11] (see also Ref. [61]), where the system of the scalar field minimally coupled to the quantized gravitational field with $V(\varphi) = 0$ was examined. We find that the G coefficient coincides with that obtained in Ref. [11], although there is a discrepancy in the A coefficient. Finally, the case in the fifth row of Table I was recently considered in Ref. [41] for the massive scalar field with quartic interaction and nonminimal coupling to gravity. In order to enable this comparison and for the sake of further discussion, we adopt the potential in the form given in Eq. (3.2a) and confine our considerations up to φ^4 and $(\partial\varphi)^2$ terms. Reinstating the original definition of the scalar field which is implemented by replacing $\varphi \to \kappa\varphi$, the resulting Vilkovisky-DeWitt one-loop effective action reads

$$\Gamma_{\mathrm{VD}}{}^{(1L)}[\varphi] = -\frac{1}{(4\pi)^2\epsilon}\int \mathrm{d}^4x\Big[\mathsf{A}(\partial^2\varphi)^2 + \mathsf{B}\frac{1}{2}(\partial\varphi)^2 + \mathsf{C}\frac{1}{2}m^2\varphi^2 + \mathsf{D}\frac{\lambda}{4!}\varphi^4\Big],$$

where

$$\mathsf{A} = \frac{9}{4}\kappa^2,$$
$$\mathsf{B} = 14\kappa^2\Lambda + (1 - 2\Lambda m^{-2})\lambda - \frac{5}{2}\kappa^2 m^2,$$
$$\mathsf{C} = -3\kappa^2\Lambda + \frac{5}{2}\kappa^2 m^2,$$
$$\mathsf{D} = 3\lambda + (14\Lambda - 13m^2)\kappa^2 + 21m^4\kappa^4\lambda^{-1}.$$

From the comparison of the above coefficients with Ref. [41] in the case of vanishing nonminimal coupling and taking into account the different definition of gravitational coupling [the relation is $\kappa^2 = \tilde{\kappa}^2/2$, where LHS denotes the definition given below Eq. (3.1)]. Other than a misprint in the Λ-accompanied factor in Eq. (37) of this paper,[5] we find a full agreement up to the term φ^2. The coefficient of the quartic coupling is missing there.

The one-loop correction to the effective action given in Eq. (3.19b) is related to the one-loop counterterm by the equation $\Delta S|_{1L} = -\Gamma_{\mathrm{VD}}{}^{(1L)}$. If we take the limit $M \to 0$ and adopt the potential to have the form given in Eq. (3.2a), then the bare action reads

$$S_B[\varphi] = S[\varphi] + \Delta S[\varphi],$$

where the counterterm takes the form

$$\Delta S[\varphi] = \int \mathrm{d}^4x\Big[Z_\varphi^{(1)}\frac{1}{2}(\partial\varphi)^2 + \sum_{n=0}^{\omega} Z_{\varphi^{2n}}^{(1)}\frac{1}{(2n)!}\lambda_{2n}\varphi^{2n} + \sum_{n=1}^{\omega} Z^{(1)}_{(\partial\varphi)^2\varphi^{2n}}(\partial\varphi)^2\varphi^{2n} + Z^{(1)}_{(\partial\varphi)^4}(\partial\varphi)^4\Big].$$

The coefficients in front of operators are related to the corresponding renormalization constants by the equation

$$Z_{\mathcal{O}}(g,\epsilon) = 1 + \sum_{\nu\geq 1} Z_{\mathcal{O}}^{(\nu)}(g)\epsilon^{-\nu},$$

where

$$Z_{\mathcal{O}}^{(\nu)}(g) = \sum_{r\geq 1} z_{\mathcal{O}}^{(\nu|rL)}(g),$$

and $\mathcal{O} = \{\varphi, \varphi^{2n}, (\partial\varphi)^2\varphi^{2n}, (\partial\varphi)^4\}$. The form of the first two one-loop renormalization constants may be inferred from the Eq. (3.19b) and read

$$(4\pi)^2 z_\varphi^{(1|1L)} = -2E\kappa^2\lambda_2 - 2F\kappa^4\lambda_0 \tag{3.20a}$$

and

$$(4\pi)^2 z_{\varphi^{2n}}^{(1|1L)} = \frac{1}{\lambda_{2n}}\sum_{k=0}^{n}\binom{2n}{2k}\Big\{\frac{1}{2}\lambda_{2(k+1)}\lambda_{2(n-k+1)} - \Big(C\frac{2(n-k)}{2k+1} + D\Big)\kappa^2\lambda_{2(k+1)}\lambda_{2(n-k)} - B\kappa^4\lambda_{2k}\lambda_{2(n-k)}\Big\}, \tag{3.20b}$$

respectively. The rest of the one-loop renormalization constants can be readily inferred from the mentioned formula. However, as they are not to be further utilized we will keep them implicit. Having evaluated the form of the counterterm and one-loop renormalization constants, we can derive out of it equations for running couplings in the theory under consideration.

IV. RUNNING SCALAR FIELD COUPLINGS IN THE MS SCHEME

Let us address the question of how couplings in the action (3.1) with the general form of the potential given in Eq. (3.2a) change with respect to the energy scale. In a full effective theory, the set of couplings consists of derivative and nonderivative ones. Since we have restricted the effective action to the lowest energy terms of the entire effective action as in Eq. (3.1), in what follows we consider solely the nonderivative and the two derivative parts of the one- loop correction given in Eq. (3.19a). Keeping in mind the remarks given in the Introduction, a scaling of couplings will be derived in the MS scheme [45]. In this scheme the bare fields and the coupling constants are related to the renormalized ones via the following formulas

[5]This Λ-accompanied factor in Ref. [41], according to the definition of B in Eq. (35), in the limit ω, $\nu \to 1$ and α, $\xi_{\mathrm{nmc}} \to 0$ is equal to $-3/8$ instead of $-1/2$, given there in Eq. (37).

$$\begin{aligned}\varphi_B(\epsilon) &= \mu^{1-\epsilon/2}\varphi(\mu,\epsilon)Z_\varphi^{1/2}(g,\epsilon),\\ (\lambda_{2n})_B(\epsilon) &= \mu^{4-2n+(n-1)\epsilon}g_{2n}(\mu,\epsilon)Z_{g_{2n}}(g,\epsilon),\end{aligned} \tag{4.1a}$$

which holds for $n = 1, 2, \ldots, \omega$, where $Z_{g_{2n}} \equiv Z_{\varphi^{2n}}/Z_\varphi^n$ and for gravitational coupling

$$\kappa_B^2(\epsilon) = \mu^{\epsilon-2}g_\kappa(\mu,\epsilon)Z_\kappa(g,\epsilon), \tag{4.1b}$$

where in the above formula we have introduced the *dimensionless* couplings g_i and field. As we have not computed a quantum correction to the gravitational coupling, its renormalization constant is equal to one which entails a vanishing beta function for this coupling. Remaining renormalization constants for couplings may be found from comparison of the simple pole terms in the second line of Eq. (4.1a), and what we finally get is $Z_{g_{2n}}^{(1)} = Z_{\varphi^{2n}}^{(1)} - nZ_\varphi^{(1)}$, where the explicit forms of one-loop parts of $Z_\varphi^{(1)}$ and $Z_{\varphi^{2n}}^{(1)}$ are given in Eqs. (3.20a) and (3.20b). The running of parameters g_{2n} and the anomalous dimension $\gamma_\varphi(g)$ of the scalar field may be found from the condition that the bare couplings in Eqs. (4.1a) and (4.1b) should not depend on μ which, barring the running of gravitational coupling and taking the limit $\epsilon \to 0$, amounts to the following formulas in the MS scheme,

$$\beta_{2n}(g) = [-(4-2n) + \gamma_{g_{2n}}(g)]g_{2n}, \tag{4.2a}$$

for $n = 1, 2, \ldots, \omega$, where the second term in the above equation, to which we further refer as the anomalous dimensions for the scalar field couplings $\gamma_{g_{2n}}$, and the anomalous dimension of a scalar field γ_φ take the general form

$$\gamma_\alpha(g) \equiv \left(1 - \frac{3}{2}\delta_{\alpha,\varphi}\right)\sum_{j\in\{\kappa,0,2n\}} a_j g_j \frac{\partial Z_\alpha^{(1)}(g)}{\partial g_j}, \tag{4.2b}$$

for $\alpha = \varphi$, g_{2n}. In the above equations a_j is a coefficient multiplying the DimReg parameter ϵ in an exponent of RG mass parameter μ in Eqs. (4.1a) and (4.1b) and $\delta_{\alpha,\varphi}$ is the Kronecker delta. By virtue of Eqs. (3.20a) and (3.20b), these formulas boil down to simple relations between corresponding anomalous dimensions and coefficients of the simple poles of renormalization constants. Hence the explicit form of the one-loop anomalous dimensions for the scalar field couplings from Eq. (4.2a) reads

$$\begin{aligned}\gamma_{g_{2n}}^{(1L)}(g)g_{2n} = &\frac{1}{(4\pi)^2}\sum_{k=0}^{n}\binom{2n}{2k}\Big[\frac{1}{2}g_{2(k+1)}g_{2(n-k+1)}\\ &-\left(C\frac{2(n-k)}{2k+1}+D\right)g_\kappa g_{2(k+1)}g_{2(n-k)}\\ &-Bg_\kappa^2 g_{2k}g_{2(n-k)}\Big]\\ &+2n\frac{1}{(4\pi)2}(Eg_\kappa g_2 + Fg_\kappa^2 g_0)g_{2n},\end{aligned} \tag{4.3a}$$

and for the one-loop anomalous dimension of the field, one obtains

$$\gamma_\varphi^{(1L)}(g) = \frac{1}{(4\pi)^2}(Eg_\kappa g_2 + Fg_\kappa^2 g_0). \tag{4.3b}$$

The first term of the formula (4.3a) is a pure nonlinear scalar field part of the one-loop correction to the beta functions. In the absence of gravitational interactions vanishing of the beta function yields the FP. Apart from the mass parameter, all the scalar field couplings obtain a positive contribution from quantum corrections and, therefore, the only FP in this case is the one where all the couplings vanish. This FP is a free field theory or Gaussian infrared FP.[6] In order to asses whether this FP is stable or unstable with respect to the RG flow one usually examines a flow of small perturbations about the FP, determined by means of linearized RG equations at this FP. However, in the MS scheme the lowest one-loop order of the anomalous dimension of the coupling constant is quadratic in the couplings and therefore at the Gaussian FP yields no information about its stability.

As for the gravitational contribution to beta functions, let us first restrict ourselves to the polynomial potential containing all up to quartic interaction. The results for different methods and gauges are summarized in Table II. The last row represents the gauge-independent gravitational corrections to the beta functions. Recall that

TABLE II. Comparison of the one-loop gravitational corrections to the beta functions for mass parameter g_2 and quartic coupling g_4 obtained in standard ($a = 0$) and Vilkovisky-DeWitt effective action (1) and gauges (ξ, b). Notation is the following $\beta_{2n} = \beta_{2n}^0 + \Delta\beta_{2n}/(4\pi)^2$, where $\beta_{2n}^0 = \beta_{2n}(g_\kappa = 0, g_0 = 0)$ denotes the beta function for pure nonlinear scalar field theory, whereas $\Delta\beta_{2n}/(4\pi)^2$ represents the gravitational correction to it.

$(a\|\xi, b)$	$\Delta\beta_2^{(1L)}$
(0|0, 0)	$6g_0g_2g_\kappa^2$
(0|1, 0)	$8g_0g_2g_\kappa^2$
(0|1, 1)	$-2g_2^2g_\kappa + 10g_0g_2g_\kappa^2$
(0|0, 1)	$-2g_2^2g_\kappa + 4g_0g_2g_\kappa^2$
(1|0, 1)	$-(g_0g_4 + 5g_2^2)g_\kappa + \frac{17}{2}g_0g_2g_\kappa^2$
	$\Delta\beta_4^{(1L)}$
(0|0, 0)	$-12g_2g_4g_\kappa + (6g_0g_4 + 18g_2^2)g_\kappa^2$
(0|1, 0)	$-8g_2g_4g_\kappa + (6g_0g_4 + 30g_2^2)g_\kappa^2$
(0|1, 1)	$-12g_2g_4g_\kappa + (10g_0g_4 + 30g_2^2)g_\kappa^2$
(0|0, 1)	$-16g_2g_4g_\kappa + (2g_0g_4 + 18g_2^2)g_\kappa^2$
(1|0, 1)	$-18g_2g_4g_\kappa + (10g_0g_4 + 21g_2^2)g_\kappa^2$

[6]There are also other possible fixed points apart from the Gaussian one that are parametrized by the mass parameter g_2 as may be inferred from Eq. (4.3a) for $\beta_2 = 0$ setting $g_0 = g_\kappa = 0$ and applying the solution to subsequent equations with vanishing beta functions. Although it provides an infinite continuum number of FPs-a fixed line-the potentials have singularities at some value of the field for all but zero mass parameters [6] and therefore the only physically acceptable FP is the Gaussian FP [7].

according to Eq. (3.2b) and the rescaling $\lambda_0 = \mu^4 g_0$, we have $g_0 g_\kappa = g_\Lambda$. Since a cosmological constant is an additional gravitational coupling, we see that the leading gravitational corrections enter the beta function for both mass and quartic coupling with a negative sign. In the case of positive cosmological constants, the two contributions give rise to a decrease of the effective couplings. On the other hand, the next-to-leading term which is of the form $\sim g_0 g_\kappa^2 = 2\Lambda\kappa^2$ produces the opposite effect. At low energy this term is negligible as compared to the leading contribution. At high energies, i.e., $g_\kappa \sim (\mu/M_P)^2 \sim 1$, it becomes important and competes with the two negative contributions. In this case, however, the prediction that hinges on the one-loop beta function becomes unreliable, since higher-order gravitational interactions from the series defining the effective theory like R^2 must be taken into account. Hence, we conclude that the net effect of the gravitational contribution in the adopted approximation gives rise to an asymptotically free trend of running couplings. On the other hand, in the region of small couplings in the coupling space, that is, in the perturbative region, this contribution is small compared to the pure scalar field one-loop correction, which will dominate the running of scalar field couplings. These remarks may be extended to the case of an arbitrary number of scalar field couplings. The only difference is that now the beta function for the quartic coupling acquires a positive contribution from the nonrenormalizable coupling g_6 in a pure scalar one-loop correction and a negative contribution to the leading gravitational one which may be found in Appendix B. The total one-loop contribution to beta functions for nonrenormalizable couplings has a similar structure as the mass parameter and quartic coupling beta functions. In the region of small couplings, it is dominated by a canonical dimension term and as such governs the RG flow in the vicinity of the Gaussian FP.

Before we proceed, let us note that if we set $g_0 = 0$, then the second row corresponds to the result found by Rodigast and Schuster in Ref. [40]. Although their result was obtained by computing appropriate Feynman diagrams, it is tantamount to that obtained by means of the standard effective action in the harmonic gauge as we have done above. Taking into account a different definition for the gravitational constant ($\kappa^2 = \tilde{\kappa}^2/2$ entails $g_\kappa = \tilde{g}_\kappa/2$), a direct comparison with Ref. [40] shows that both forms of gravitational correction coincide.

A. The scalar field Gaussian fixed point

The set of equations (4.2a) also admits a Gaussian FP. Nevertheless, it is interesting to note whether it admits a scalar field Gaussian FP (SGFP) as well, with nonzero gravitational couplings at the FP. Analysis of the gauge-independent form of RG equations (see Appendix B) reveals that, up to leading order in gravitational correction, there is a FP solution where all but the g_{0*} and g_{2*} couplings vanish. However, it turns out to be unstable against the addition of the next-order gravitational correction. If we include the next-to-leading gravitational correction, the only nonzero FP coupling appears to be g_{0*}. Indeed, if we put $g_2 = 0$ then the vanishing of beta functions entails the vanishing of all the rest of the scalar field couplings without the need for specifying the gravitational coupling. This FP seems to be sought SGFP, although with the value $g_{0*} = 8(4\pi)^2/7g_{\kappa*}^2$, which is entirely out of reach of the perturbation theory approach. However, this FP may appear spurious as well, since if the next-order corrections are added, it might appear unstable. Nevertheless, it is likely that a genuine FP for $g_0 \neq 0$ does exist, for if we equate all the scalar field couplings to zero, the only contribution will be that from gravitational coupling, which at each order, say nth, will enter with a power of $(g_0 g_\kappa^2)^n$, where $g_0 g_\kappa^2 = 2\kappa^2\Lambda$ is a dimensionless combination of gravitational couplings. Given a flipping of the sign of gravitational coupling with each order [as it happens at first and second order, see Eq. (4.3a)], it is conceivable that taking into account a complete series of loop contributions we will eventually obtain an entire beta function with its zero in the vicinity of the Gaussian FP, which would then be a non-Gaussian FP for both cosmological and Newton coupling parameters as the asymptotic safety scenario suggests.[7] As it was mentioned earlier, we have not calculated the beta function for the gravitational coupling. Therefore, it enters the RG equations as a small parameter. Let us assume that both gravitational couplings have nonzero values at the SGFP. Such a situation takes place, e.g., in the asymptotic safety scenario [35]. Given nonzero FP values for the gravitational couplings, we are able to examine the directions of the RG flow in the vicinity of the SGFP. We consider that the gauge-independent form of the beta functions, when linearized about the SGFP, yields the stability matrix that amounts to

$$\frac{\partial \beta_{2n}}{\partial g_{2m}}(g_{\kappa*}, g_{0*}, 0) = \left[2n - 4 + \frac{1}{(4\pi)^2}\left(7 + \frac{3}{2}n\right) g_{\kappa*}^2 g_{0*}\right]\delta_m^n - \frac{1}{(4\pi)^2} g_{\kappa*} g_{0*} \delta_{m-1}^n, \tag{4.4}$$

for $n = 1, 2, \ldots, \omega$. Let us consider two cases—a finite number of scalar field couplings $\omega < \infty$ and an infinite number of scalar field couplings $\omega = \infty$.

(a) *The case of a finite number of couplings* ($\omega < \infty$). Assuming a finite number of scalar field vertex operators, it is possible to diagonalize the above stability matrix, eigenvalues of which are its

[7] The non-Gaussian FP was indeed found in the asymptotic safety scenario in Einstein-Hilbert truncation [36,62].

diagonal elements. Depending on the sign, these eigenvalues pinpoint a direction in which an operator relative to a given eigenvalue flows in the course of the RG flow. These operators that are attracted to the FP are termed relevant, whereas those repelled from it are irrelevant. There are also marginal operators that correspond to a zero eigenvalue. As one may infer from the diagonal elements of Eq. (4.4) for $g_{0*} > 0$, the gravitational correction reduces the number of relevant vertex operators. In particular, a quartic operator being classically marginal becomes irrelevant due to gravitational correction. Thus the only relevant operator appears to be the mass operator.

(b) *The case of an infinite number of couplings* ($\omega = \infty$). As for the infinite number of couplings, it is possible to diagonalize the stability matrix in Eq. (4.4). This time, however, off-diagonal terms also contribute the eigenvalue. It is worth mentioning that these terms derive from the configuration space connection and are absent in the standard background field approach. The form of the stability matrix resembles that obtained in the Wilson RG method in Ref. [5]. Therefore, making use of Eq. (4.4), it is possible to find a scalar field potential that has te required properties of being physically nontrivial. Solving the eigenvalue problem for small disturbances about the SGFP enables us to cast Eq. (4.4) into a form

$$u_{2n+2} = \left[(4\pi)^2(2n-4-\theta) + \left(7+\frac{3}{2}n\right)g_{0*}g_{\kappa*}^2\right] \times \frac{u_{2n}}{g_{0*}g_{\kappa*}},$$

for $n = 1, 2, \ldots,$ where $u_i \equiv g_i - g_{i*}$ and θ is an eigenvalue. This is the recursion relation that starting from u_2 allows one to express all the couplings in terms of u_2, θ and the fixed point values of gravitational couplings. Since at SGFP $g_{2n*} = 0$ for $n > 0$, this recursion relates all scalar field couplings to g_2. Explicitly,

$$g_{2n} = \frac{\alpha^{n-1}\Gamma(a+n)}{a\Gamma(a)} g_2, \qquad n > 0,$$

where $\Gamma(x)$ is the Euler special function, and

$$a \equiv \frac{-4(4\pi)^2 + 7g_{0*}g_{\kappa*}^2 - (4\pi)^2\theta}{2(4\pi)^2 + \frac{3}{2}g_{0*}g_{\kappa*}^2}, \qquad \alpha \equiv \frac{4(4\pi)^2 + 3g_{0*}g_{\kappa*}^2}{2g_{0*}g_{\kappa*}}. \tag{4.5}$$

The potential defined in Eq. (3.2a) after making use of identity

$$(2n)! = 2^{2n}n!\Gamma\left(n+\frac{1}{2}\right)/\Gamma\left(\frac{1}{2}\right)$$

may be rewritten as follows

$$U_a(\varphi) = g_0 + \frac{g_2}{\alpha a}\left[M\left(a, \frac{1}{2}, \alpha\varphi^2\right) - 1\right], \tag{4.6}$$

where $M(a, b, x)$ is the Kummer's function [63]. Thus we have found a class of potentials, termed following Halpern and Huang [5] "eigenpotentials." Their shape is determined by the value of two parameters: a and the mass parameter g_2. The latter, in turn, is related to two gravitational parameters, g_κ and g_0, which are seen if we complete the stability matrix given in Eq. (4.4) with entries for $n = 0$ that take the form

$$\frac{\partial\beta_0}{\partial g_m}(g_{\kappa*}, g_{0*}, 0) = -4\delta_m^0 + \frac{1}{(4\pi)^2}[7g_{0*}g_{\kappa*}^2\delta_m^0 + 7g_{0*}^2g_{\kappa*}\delta_{m-\kappa}^0 - g_{0*}g_{\kappa*}\delta_{m-1}^0].$$

There is also a component $\partial\beta_\kappa/\partial g_m$. However, within the assumed approximation in this paper, this component is not known. The eigenvalue problem yields an additional recursion relation,

$$g_2 = \alpha a u_0 + 7g_{0*}u_\kappa. \tag{4.7}$$

This recursion relation will be modified if we include $\partial\beta_\kappa/\partial g_m$, which may be solved for u_0 in terms of g_κ, θ, and possibly g_2. In order to find a physically nontrivial potential, the eigenvalue must be negative which implies that the two gravitational couplings are attracted to their FP. Hence, a corresponding scalar field theory will be asymptotically free. Since the shape of the potential is determined by the parameter a it is interesting to investigate whether there are values of a corresponding to potentials that enable symmetry breaking. Requirements for the potential to have this property include $U'(0) < 0$ and $U(\varphi) > 0$ for $\varphi \gg 1$. For large φ the Kummer's function behaves like $M(a, b, x) \sim \Gamma(b)x^{a-b}e^x/\Gamma(a)$. When applied to Eq. (4.6), these requirements entail the following conditions on g_2:

$$g_2 a < 0 \quad \wedge \quad g_2/\Gamma(a) > 0.$$

Since g_2 is related to gravitational couplings u_0 and u_κ as in Eq. (4.7), there are many possibilities to fulfill these nonequalities. Let us consider one of them and assume for simplicity that $g_2 > 0$. This implies that $a < 0$ and according to properties of the gamma function, we get $a \in (-2k, -2k+1)$ for $k > 0$. From Eq. (4.5) for $\theta < 0$, one may infer that a must fall at most into the interval $a \in (-2, -1)$. If we take the FP value for g_0 obtained from vanishing of the beta function for $n = 0$ in Eq. (4.2a) which amounts $g_{0*} = 8(4\pi)^2/7g_{\kappa*}^2$ then we obtain $a = 7(4 - \theta)/28$, a value that falls outside the mentioned interval. However, this value for a derives from the one-loop approximation to the beta function. Nevertheless, it is conceivable that for the full beta function $a < 0$ and therefore belongs to this interval.

V. SUMMARY AND CONCLUSIONS

In this paper we reconsidered quantum gravitational corrections to renormalization of the scalar field couplings and the effect they have on their running, which had been touched upon earlier in different contexts by many authors [22,25,32–35,37,40,41,64]. The reason we undertook this task was to investigate whether the influence of quantum gravitational fluctuations is capable of resolving the problem of triviality in an interacting quantum scalar field theory. We searched for these corrections within the effective field theory approach to quantum gravity and confined ourselves to a cosmological constant and Ricci scalar. A scalar field potential is assumed to have a $\mathbb{Z}_2$-symmetric and analytic form. As we performed computations in the flat background metric, all the operators with nonminimally coupled scalar fields to the gravity were discarded. This choice of the background metric requires off the mass shell computations, for the flat metric is not a solution to the Einstein equations with the cosmological constant and/or a scalar field theory. This choice for the background metric was adopted recently by Rodigast and Schuster [40] in the derivation of a gravitational contribution to the beta function for the quartic scalar field coupling by means of the Feynman diagram technique. However, a sign of the beta function, which determines the direction of a running, may vary depending on the chosen gauge. This usually occurs in off the mass shell computations. In order to avoid a possible gauge dependence, we used a geometric formulation of the method of background field, namely the Vilkovisky-DeWitt effective action. Using this method we derived the gauge-independent beta functions for all dimensionless coupling parameters of the theory defined in the MS scheme. Since we restricted ourselves to the flat background, the beta function for the Newton coupling parameter $g_\kappa \propto \kappa^2$ assumed zero value. The analysis of the system of RG equations for the scalar field couplings revealed that the leading-order gravitational corrections act in the direction of asymptotic freedom. This result was first found in Ref. [40], although in a different form due to the harmonic gauge. In addition to the gravitational contribution related to the Newton constant, there is the one associated with the cosmological constant. In the case of quartic coupling, the presence of the cosmological constant modifies the asymptotically free trend. A positive cosmological constant enhances the effect of the leading gravitational correction. As for the rest of the scalar field couplings, the structure of gravitational contributions is similar to the case of quartic coupling. It reveals the asymptotically free trend strengthened by the positive cosmological constant. In the region of coupling space, where the perturbation theory applies, the dominating contribution to these beta functions comes from their canonical dimensions. Thus their running does not change much in the presence of gravitational interactions in this region.

We also found that RG equations admit another FP with nonzero FP values solely for both gravitational couplings Λ and κ^2 that is the scalar field Gaussian FP (SGFP). Since we did not determine the form of the beta function for κ^2, this coupling entered the computations as a free parameter. In order to examine what consequences it may have, we assumed it to take a nonzero value at the FP. This, in view of found RG equations, entails a nonzero FP value for Λ. We found through examination of the stability matrix at SGFP that for a finite number of scalar field vertex operators, gravitational corrections render them more irrelevant. Specifically, a quartic operator being marginal in the absence of gravitational interactions is made irrelevant due to gravitational contribution. These conclusions were also found in Ref. [35], where the theory of scalar field nonminimally coupled to gravity was explored within the effective average action.

We also considered the case of infinite many scalar field interactions examined earlier in a pure interacting scalar field theory by Halpern and Huang in Ref. [5]. The reason for this was to explore possible nontrivial directions with respect to the RG flow in the space of all scalar field coupling parameters in the presence of gravitational interactions. In order to do this, we looked for the solution of linearized RG equations for small disturbances about the SGFP. The stability matrix found in this way is bidiagonal. The second diagonal comes from the Vilkovisky-DeWitt configuration space connection and is absent in the stability matrix derived within standard formulation of the background field method. Owing to the bidiagonal form, the eigenvalue problem boiled down to the recursion relation for all the couplings. As a result, we found a class of potentials termed eigenpotentials [cf. Eq. (4.6)], parametrized by the eigenvalue and depending merely on the two gravitational couplings. In order for the scalar field theory to be nontrivial, the eigenvalue must be negative. Hence, the theory with the eigenpotential corresponding to this eigenvalue is asymptotically free. The shape of the eigenpotentials is entirely determined by some parameter a [cf. Eq. (4.5)], which is a linear function of the eigenvalue and nonlinear function of FP values of both gravitational coupling parameters g_κ and g_0. The most appealing eigenpotentials are those that admit the symmetry breaking. This substantially constrains the set of possible values for the shape parameter a. In the case considered in this paper, it is confined to a certain open interval of the negative part of $\mathbb{R}$. Taking the FP value of g_0 found in this one-loop approximation to β_0, the shape parameter is positive. If taken at face value this would imply that the theory with nontrivial eigenpotentials does not admit the symmetry breaking shapes. However, this may not be the case if we take the FP value of g_0 obtained from the full beta function. Thus we found a class of scalar field potentials—gravitationally modified Halpern-Huang potentials—that are nonpolynomial and that have features making an interacting scalar field theory nontrivial, provided that there exists a nonzero fixed point value for the two gravitational couplings, namely the Newton constant and the cosmological constant. Nonperturbative studies of Einstein quantum gravity [36,62] indicate that nonzero FP values for the two gravitational couplings may indeed exist. Interestingly, this result was derived within the MS scheme. Nevertheless, it has a universal validity, as the FP's as well as eigenvalues do not depend on a specific definition of coupling constants. Since this result hinges on a continuum rather than on a quantized eigenvalue, the remarks and the caveats mentioned in Sec. I also apply.

The analysis performed in this paper does not allow for operators with scalar fields nonminimally coupled to gravity, which is acceptable in adopted approximation, i.e., flat background metric. However, in curved spacetime nonminimal coupling to gravity is required for reasons of renormalizability. From this point of view, investigations just performed are pertaining to the subspace of the full coupling parameter space. It is therefore interesting to examine how the presence of nonminimal couplings affects the triviality issue when considered in the framework of Vilkovisky-DeWitt effective action. Specifically, it is interesting to examine whether in the case of infinite many scalar field couplings it is possible to find potentials with nontrivial properties at high energies. Another problem that has not been addressed here and that is important for the results obtained in this paper is a possible configuration space metric dependence of the Vilkovisky-DeWitt effective action. Here we have followed Vilkovisky's prescription [44] to specify a value of a parameter enumerating a family of the configuration space metrics in the gravitational sector. However, if we used a different, ultralocal metric, this could effect the RG beta functions. These tasks will be undertaken in a separate paper [58].

ACKNOWLEDGMENTS

I am deeply indebted to Professor Z. T. Haba from Wroclaw University, Professor E. A. Ivanov and Professor D. I. Kazakov from BLTP, JINR, and Dr. M. R. Piatek from Szczecin University for careful reading of the manuscript and for many valuable comments and critical remarks. I am grateful for kind hospitality and interesting conversation to Dr. F. S. Nogueira from Free University of Berlin, where the early stage of this work was presented during the lecture. I am also thankful to Professor J. M. Pawlowski from Heidelberg University for enlightening discussion on some aspects of the Vilkovisky-DeWitt formalism. Finally, I am grateful to the referees for many valuable remarks.

APPENDIX A: THE DEFINITIONS AND NOTATION

Evaluation of momentum integrals with explicit indices in integrated components of momenta results in multi-index generalized deltas defined below. The meaning of $\mathbb{1}_{(n)}$ employed in Sec. III is the following:

$$\begin{aligned}(\mathbb{1}_{(1)})_{\alpha\beta} &\equiv \delta_{\alpha\beta},\\ (\mathbb{1}_{(2)})_{\alpha\beta,\mu\nu} &\equiv \delta_{\alpha\beta,\mu\nu} \equiv \delta_{\alpha(\mu}\delta_{\nu)\beta},\\ (\mathbb{1}_{(3)})_{\alpha\beta,\mu\nu,\rho\sigma} &\equiv \delta_{\alpha\beta,\mu\nu,\rho\sigma} \equiv \delta_{\alpha\beta,(\mu(\rho}\delta_{\sigma)\nu)},\\ (\mathbb{1}_{(n)})_{\alpha_1\beta_1,\alpha_2\beta_2,\ldots,\alpha_n\beta_n} &\equiv \delta_{\alpha_1\beta_1,\alpha_2\beta_2,\ldots,\alpha_n\beta_n},\end{aligned} \tag{A1}$$

where the indices embraced with round brackets are to be symmetrized. For a tensor, any two indices are symmetrized (antisymmetrized) if

$$A_{\substack{\mu_1\ldots(\alpha|\ldots|\beta)\ldots\mu_n\\ \mu_1\ldots[\alpha|\ldots|\beta]\ldots\mu_n}} \equiv \frac{1}{2}(A_{\mu_1\ldots\alpha\ldots\beta\ldots\mu_n} \pm A_{\mu_1\ldots\beta\ldots\alpha\ldots\mu_n}). \tag{A2}$$

Curl (square) brackets denote symmetrization (antisymmetrization), which corresponds to the plus (minus) sign. Vertical bars indicate that indices in between them should remain on their positions. The representation of the last formula in the above expression in terms of the Kronecker delta is highly nontrivial and will not be given here. For the sake of brevity, we introduce doubled index $i_l \equiv (\mu_l\nu_l)$. The above-defined quantities satisfy the following identities:

$$(\mathbb{1}_{(n)})_{i_1,\ldots,i_m} \equiv \delta_{i_1,\ldots,i_{k-1},i_k,i_{k+1},\ldots,i_n} = \delta_{i_k,j}\delta^{j}_{i_1,\ldots,i_{k-1},i_{k+1},\ldots,i_n},$$

where $\delta_{(i_1,\ldots,i_n)} = \delta_{i_1,\ldots,i_n}$ and

$$\delta^{i_k}\delta_{i_1,\ldots,i_{k-1},i_k,i_{k+1},\ldots,i_n} = \delta_{i_1,\ldots,i_{k-1},i_{k+1},\ldots,i_n}.$$

The DeWitt configuration space metric defined in Eq. (3.5a) and its inverse can be written in the flat n-dimensional Euclidean spacetime as

$$G^{i,j} = \frac{1}{4}(2\delta^{i,j} - \delta^i\delta^j), \quad G^{-1}_{i,j} = 2\delta_{i,j} - \frac{2}{n-2}\delta_i\delta_j. \tag{A3}$$

In Sec. III we have also introduced symbols $\mathbb{I}^{(n)}$ for $n = 2, 3, 4$, resulting from the averaging over angles of momenta with free indices in the momentum integrals. Making use of Eq. (A1) as well as doubled index notation, these symbols can be given in the following concise form:

$$\mathbb{I}^{(2)}_{i_1,i_2} \equiv \frac{1}{24}(\delta_{i_1}\delta_{i_2} + 2\delta_{i_1,i_2}), \tag{A4}$$

$$\mathbb{I}^{(3)}_{i_1,i_2,i_3} \equiv \frac{1}{192}(\delta_{i_1}\delta_{i_2}\delta_{i_3} + 2\delta_{i_1,i_2}\delta_{i_3} + 2\delta_{i_3,i_1}\delta_{i_2} + 2\delta_{i_2,i_3}\delta_{i_1} + 8\delta_{i_1,i_2,i_3}) \tag{A5}$$

and

$$\mathbb{I}^{(4)}_{i_1,i_2,i_3,i_3} \equiv \frac{1}{1920}(\delta_{i_1}\delta_{i_2}\delta_{i_3}\delta_{i_4} + 2\delta_{i_1,i_2}\delta_{i_3}\delta_{i_4} + 2\delta_{i_4,i_1}\delta_{i_2}\delta_{i_3} + 2\delta_{i_3,i_4}\delta_{i_1}\delta_{i_2} + 2\delta_{i_2,i_3}\delta_{i_4}\delta_{i_1} + 2\delta_{i_1,i_3}\delta_{i_2}\delta_{i_4} + 2\delta_{i_2,i_4}\delta_{i_1}\delta_{i_3} + 4\delta_{i_1,i_2}\delta_{i_3,i_4} + 4\delta_{i_1,i_3}\delta_{i_2,i_4} + 4\delta_{i_1,i_4}\delta_{i_2,i_3} + 8\delta_{i_1,i_2,i_3}\delta_{i_4} + 8\delta_{i_4,i_1,i_2}\delta_{i_3} + 8\delta_{i_3,i_4,i_1}\delta_{i_2} + 8\delta_{i_2,i_3,i_4}\delta_{i_1} + 16\delta_{i_1,i_2,i_3,i_4}). \quad (A6)$$

In particular the last symbol in Eq. (A6) takes the explicit form

$$\delta_{\alpha\beta,\gamma\lambda,\mu\nu,\rho\sigma} \equiv \delta_{(\sigma|(\alpha}\delta_{\beta)(\gamma}\delta_{\lambda)(\mu}\delta_{\nu)|\rho)} + \delta_{(\nu|(\alpha}\delta_{\beta)(\gamma}\delta_{\lambda)(\rho}\delta_{\sigma)|\mu)} + \delta_{(\sigma|(\alpha}\delta_{\beta)(\mu}\delta_{\nu)(\gamma}\delta_{\lambda)|\rho)}. \quad (A7)$$

APPENDIX B: THE GAUGE-INDEPENDENT BETA FUNCTIONS FOR THE SCALAR-GRAVITY SYSTEM

In this appendix we present the explicit form of the beta functions obtained in Eqs. (4.2a) and (4.3a) and discussed in Sec. IV for gauge-independent values of coefficients given in Table I, i.e., $(a|\xi, b) = (1|0, 1)$ for $n = 5$ couplings. The full beta functions β_{2n} can be split into two parts, the one for a pure scalar field theory $\beta^0_{2n}(g)$ and that coming from gravitational corrections $\Delta\beta_{2n}(g)/16\pi^2$, namely

$$\beta_{2n}(g) = \beta^0_{2n}(g) + \Delta\beta_{2n}(g)/16\pi^2,$$

where corresponding terms assume the following explicit forms:

$$\beta^0_2 = -2g_2 + \frac{1}{16\pi^2}g_2g_4, \qquad \Delta\beta_2(g) = -(g_0g_4 + 5g_2^2)g_\kappa + \frac{17}{2}g_0g_2g_\kappa^2,$$

$$\beta^0_4 = \frac{1}{16\pi^2}(g_2g_6 + 3g_4^2), \qquad \Delta\beta_4(g) = -(g_0g_6 + 18g_2g_4)g_\kappa + (10g_0g_4 + 21g_2^2)g_\kappa^2,$$

$$\beta^0_6 = 2g_6 + \frac{1}{16\pi^2}(g_2g_8 + 15g_4g_6), \qquad \Delta\beta_6(g) = -\left(g_0g_8 + \frac{65}{2}g_2g_6 + 30g_4^2\right)g_\kappa + \left(\frac{23}{2}g_0g_6 + 105g_2g_4\right)g_\kappa^2,$$

$$\beta^0_8 = 4g_8 + \frac{1}{16\pi^2}(g_2g_{10} + 28g_4g_8 + 35g_6^2), \qquad \Delta\beta_8(g) = -(g_0g_{10} + 51g_2g_8 + 182g_4g_6)g_\kappa + (13g_0g_8 + 196g_2g_6 + 245g_4^2)g_\kappa^2,$$

$$\beta^0_{10} = 6g_{10} + \frac{1}{16\pi^2}(g_2g_{12} + 45g_4g_{10} + 210g_6g_8), \qquad \Delta\beta_{10}(g) = -\left(g_0g_{12} + \frac{147}{2}g_2g_{10} + 435g_4g_8 + 399g_6^2\right)g_\kappa + \left(\frac{29}{2}g_0g_{10} + 315g_2g_8 + 1470g_4g_6\right)g_\kappa^2.$$

$$\vdots \qquad\qquad \vdots$$

The beta function for the cosmological constant reads

$$\beta_0 = -4g_0 + \frac{1}{32\pi^2}\left(g_2^2 - g_0g_2g_\kappa + \frac{7}{2}g_0^2g_\kappa^2\right).$$

[1] L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3320 (1974); S. R. Coleman, R. Jackiw, and H. D. Politzer, Phys. Rev. D **10**, 2491 (1974).
[2] K. G. Wilson, Phys. Rev. B **4**, 3174 (1971); K. G. Wilson and J. B. Kogut, Phys. Rep. **12**, 75 (1974).
[3] A. Hasenfratz and P. Hasenfratz, Nucl. Phys. **B270**, 687 (1986).
[4] D. J. E. Callaway, Phys. Rep. **167**, 241 (1988).
[5] K. Halpern and K. Huang, Phys. Rev. Lett. **74**, 3526 (1995); Phys. Rev. D **53**, 3252 (1996).
[6] T. R. Morris, Phys. Rev. Lett. **77**, 1658 (1996).
[7] K. Halpern and K. Huang, Phys. Rev. Lett. **77**, 1659 (1996).
[8] K. Halpern, Phys. Rev. D **57**, 6337 (1998); H. Gies, Phys. Rev. D **63**, 065011 (2001); V. Branchina, Phys. Rev. D **64**, 043513 (2001); K. Huang, H.-B. Low, and R.-S. Tung, Classical Quantum Gravity **29**, 155014 (2012); Int. J. Mod. Phys. A **27**, 1250154 (2012).
[9] O. J. Rosten, J. High Energy Phys. 07, (2009) 019.
[10] T. P. Cheng, E. Eichten, and L.-F. Li, Phys. Rev. D **9**, 2259 (1974).

[11] G. 't Hooft and M. Veltman, Ann. Inst. Henri Poincaré, A **20**, 69 (1974).
[12] M. H. Goroff and A. Sagnotti, Nucl. Phys. **B266**, 709 (1986).
[13] J. F. Donoghue, Phys. Rev. D **50**, 3874 (1994).
[14] S. Weinberg, in *General Relativity: An Einstein Centenary Survey*, edited by S. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1979); H. Georgi, Annu. Rev. Nucl. Part. Sci. **43**, 209 (1993); A. Pich, arXiv: hep-ph/9806303.
[15] J. F. Donoghue, arXiv:gr-qc/9512024; C. P. Burgess, Living Rev. Relativity **7**, 5 (2004), http://www.livingreviews.org/lrr-2004-5.
[16] J. F. Donoghue, AIP Conf. Proc. **1483**, 73 (2012).
[17] S. P. Robinson and F. Wilczek, Phys. Rev. Lett. **96**, 231601 (2006).
[18] A. R. Pietrykowski, Phys. Rev. Lett. **98**, 061801 (2007); D. Ebert, J. Plefka, and A. Rodigast, Phys. Lett. B **660**, 579 (2008).
[19] D. J. Toms, Phys. Rev. D **76**, 045015 (2007); J. C. C. Felipe, L. C. T. Brito, M. Sampaio, and M. C. Nemes, Phys. Lett. B **700**, 86 (2011).
[20] D. J. Toms, Phys. Rev. Lett. **101**, 131301 (2008); Phys. Rev. D **80**, 064040 (2009).
[21] D. J. Toms, Nature (London) **468**, 56 (2010).
[22] H.-J. He, X.-F. Wang, and Z.-Z. Xianyu, Phys. Rev. D **83**, 125014 (2011).
[23] Y. Tang and Y.-L. Wu, J. High Energy Phys. 11 (2011) 073; Commun. Theor. Phys. **57**, 629 (2012).
[24] J. Ellis and N. E. Mavromatos, Phys. Lett. B **711**, 139 (2012).
[25] M. M. Anber, J. F. Donoghue, and M. El-Houssieny, Phys. Rev. D **83**, 124003 (2011).
[26] M. M. Anber and J. F. Donoghue, Phys. Rev. D **85**, 104016 (2012).
[27] D. J. Toms, Phys. Rev. D **84**, 084016 (2011).
[28] N. Nielsen, Ann. Phys. (Amsterdam) **327**, 861 (2012).
[29] J.-E. Daum, U. Harst, and M. Reuter, J. High Energy Phys. 01 (2010) 084; U. Harst and M. Reuter, J. High Energy Phys. 05 (2011) 119.
[30] S. Folkerts, D. F. Litim, and J. M. Pawlowski, Phys. Lett. B **709**, 234 (2012).
[31] D. I. Kazakov, Theor. Math. Phys. **75**, 440 (1988).
[32] A. O. Barvinsky, A. Y. Kamenshchik, and I. P. Karmazin, Phys. Rev. D **48**, 3677 (1993).
[33] C. F. Steinwachs and A. Y. Kamenshchik, Phys. Rev. D **84**, 024026 (2011).
[34] L. Griguolo and R. Percacci, Phys. Rev. D **52**, 5787 (1995).
[35] R. Percacci and D. Perini, Phys. Rev. D **68**, 044018 (2003).
[36] M. Reuter, Phys. Rev. D **57**, 971 (1998).
[37] G. Narain and R. Percacci, Classical Quantum Gravity **27**, 075001 (2010).
[38] S. Falkenberg and S. D. Odintsov, Int. J. Mod. Phys. A **13**, 607 (1998).
[39] W. Souma, arXiv:gr-qc/0006008.
[40] A. Rodigast and T. Schuster, Phys. Rev. Lett. **104**, 081301 (2010).
[41] P. T. Mackay and D. J. Toms, Phys. Lett. B **684**, 251 (2010).
[42] S. D. Odintsov, Europhys. Lett. **10**, 287 (1989); Theor. Math. Phys. **82**, 45 (1990).
[43] H.-R. Chang, W.-T. Hou, and Y. Sun, Phys. Rev. D **85**, 124025 (2012).
[44] G. A. Vilkovisky, Nucl. Phys. **B234**, 125 (1984).
[45] G. 't Hooft, Nucl. Phys. **B61**, 455 (1973).
[46] B. S. DeWitt, Phys. Rev. **162**, 1195 (1967).
[47] G. 't Hooft, in *Proceedings of XIIth Winter School of Theoretical Physics*, Vol. 1, edited by B. Jancewicz (Wroclaw University Press, Wroclaw, Poland, 1976).
[48] D. G. Boulware, Phys. Rev. D **23**, 389 (1981); L. F. Abbott, Nucl. Phys. **B185**, 189 (1981); C. F. Hart, Phys. Rev. D **28**, 1993 (1983).
[49] B. S. DeWitt and R. W. Brehme, Ann. Phys. (N.Y.) **9**, 220 (1960).
[50] P. Ellicott, G. Kunstatter, and D. J. Toms, Ann. Phys. (N.Y.) **205**, 70 (1991).
[51] A. Rebhan, Nucl. Phys. **B298**, 726 (1988).
[52] A. Rebhan, Nucl. Phys. **B288**, 832 (1987).
[53] E. S. Fradkin and A. A. Tseytlin, Nucl. Phys. **B234**, 509 (1984).
[54] S. R. Huggins, G. Kunstatter, H. P. Leivo, and D. J. Toms, Phys. Rev. Lett. **58**, 296 (1987); Nucl. Phys. **B301**, 627 (1988).
[55] S. D. Odintsov, Phys. Lett. B **262**, 394 (1991).
[56] M. Y. Kalmykov and D. I. Kazakov, Phys. Lett. B **404**, 253 (1997).
[57] A. O. Barvinsky and G. A. Vilkovisky, Phys. Rep. **119**, 1 (1985).
[58] A. R. Pietrykowski (to be published).
[59] K. Peeters, arXiv:hep-th/0701238; Comput. Phys. Commun. **176**, 550 (2007).
[60] Yu. V. Gryzov, A. Yu. Kamenshchik, and I. P. Karmazin, Russ. Phys. J. **35**, 201 (1992).
[61] M. T. Grisaru, P. van Nieuwenhuizen, and C. C. Wu, Phys. Rev. D **12**, 3203 (1975).
[62] I. Donkin and J. M. Pawlowski, arXiv:1203.4207.
[63] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables* (Dover, New York, 1964).
[64] G. Narain and C. Rahmede, Classical Quantum Gravity **27**, 075002 (2010).