# Test of $SU(3)$ symmetry in hyperon semileptonic decays

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Existing analyses of baryon semileptonic decays indicate the presence of a small $SU(3)$ symmetry breaking effect in hyperon semileptonic decays. However, to provide evidence for $SU(3)$ symmetry breaking, one needs a relation similar to the Gell-Mann–Okubo baryon mass formula satisfied to within a few percent, showing evidence for $SU(3)$ symmetry breaking in the divergence of the vector current matrix element. In this paper, we shall present a similar Gell-Mann–Okubo relation for the hyperon semileptonic decay axial vector form factors. Using these relations and the measured axial vector current for vector current form factor ratios, we show that the amount of $SU(3)$ symmetry breaking in hyperon semileptonic decaysis 5–11%.

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## I. INTRODUCTION

The success of the Gell-Mann–Okubo (GMO) mass formula shows that $SU(3)$ is a good symmetry for strong interactions. This approximate symmetry can be incorporated into a QCD Lagrangian with $m_u$, $m_d \ll m_s$; with $m_s \ll \Lambda_{\rm QCD}$, we have an almost $SU(3)$-symmetric Lagrangian. At low energies, an effective chiral Lagrangian can be constructed with baryons coupled to the pseudoscalar meson octet, $\pi$, $K$, $\eta$ via a covariant derivative constructed with the derivative of the pseudoscalar meson field operator. This gives us the Goldberger-Treiman relation for the pion-nucleon coupling constant. This Lagrangian contains the axial vector current matrix elements and produces the axial vector form factors measured in baryon semileptonic decays. At zero order in $m_s$, the axial vector current form factors and the pseudoscalar baryon couplings are $SU(3)$ symmetric and are completely given by the two parameters $F$ and $D$ of the $F$ symmetric) and $D$ (symmetric) type coupling [1]. The success of the GMO formula which can be derived from this effective Lagrangian suggests that semileptonic hyperon decays can also be well described by the two $SU(3)$-symmetric $F$ and $D$ parameters as in the Cabibbo model [2], for which the agreement with experiments is quite good [3]. In general one expects some small $SU(3)$ symmetry breaking for the divergence of the vector current matrix element and the hyperon semileptonic decay axial vector current matrix elements which, unlike the vector current, are not protected by the Ademollo-Gatto theorem [4]. Using the precisely measured axial vector to vector form factor ratio $g_1/f_1$ for hyperon semileptonic decays [5], recent analyses [6–8] indicate the presence of a small $SU(3)$ symmetry breaking in hyperon semileptonic decays. However, to test $SU(3)$ symmetry, one needs a relation similar to the GMO baryon mass formula, which can be written as [3]

$$(3/4)\Delta M + (1/4)\Delta M' = (1/4)\Delta M'' + (3/4)\Delta M''', \tag{1}$$

with $\Delta M = m_\Lambda - m_N$, $\Delta M' = m_\Sigma - m_N$, $\Delta M'' = m_\Xi - m_\Sigma$, and $\Delta M''' = m_\Xi - m_\Lambda$. Numerically, the lhs of Eq. (1) is 0.1966 GeV while the rhs is 0.1867 GeV, showing evidence for $SU(3)$ breaking for the divergence of the vector current matrix elements which, in fact, gives the above GMO formula by equating the matrix elements of the divergence of the $\Delta S = 1$ vector current $\bar{u}\gamma_\mu s$ within the $V = 1$ multiplet. The baryon mass difference is given by $m_s\langle B'|\bar{u}s|B\rangle$, with $\bar{u}s$ a $V$-spin $V = 1$ scalar current in $SU(3)$ space. In the limit of neglecting the light current quark mass $m_{u,d}$, the lhs and the rhs of Eq. (1) are two matrix elements of the $V = 1$ $V$-spin multiplet $\langle V = 1, V_3 = 0|\bar{u}s|V = 1, V_3 = 1\rangle$ and $\langle V = 1, V_3 = -1|\bar{u}s|V = 1, V_3 = 0\rangle$; they would be equal but opposite in sign in the limit of $SU(3)$ symmetry. Similarly, in the limit of $SU(3)$ symmetry, for the axial vector current matrix elements, we have the equality of $\langle V = 1, V_3 = -1|\bar{u}\gamma_\mu\gamma_5 s|V = 1, V_3 = 0\rangle$ and $-\langle V=1, V_3=0|\bar{u}\gamma_\mu\gamma_5 s|V=1, V_3=1\rangle$; hence, the GMO-type relation for the axial vector current form factors in hyperon semileptonic decays. Another nontrivial relation for hyperon semileptonic decays is obtained from the equality of two matrix elements $\langle V=1, V_3=-1|\bar{u}\gamma_\mu\gamma_5 s|V=0, V_3=0\rangle$ and $\langle V=0, V_3=0|\bar{u}\gamma_\mu\gamma_5 s|V=1, V_3=1\rangle$. In the following, we will present a test of $SU(3)$ symmetry in semileptonic hyperon decays and an analysis of $SU(3)$ symmetry breaking using these relations. We show that the amount of $SU(3)$ symmetry breaking in hyperon semileptonic decays is 5–11%.

## II. TEST OF $SU(3)$ SYMMETRY IN HYPERON SEMILEPTONIC DECAYS

The traditional method to obtain the GMO mass formula is to assume that the $SU(3)$ symmetry breaking mass term in the baryon Lagrangian transforms like the eighth component of an $SU(3)$ octet. Nowadays, we know that in the standard model, $SU(3)$ symmetry breaking is given by the current quark mass term in the QCD Lagrangian with $m_{u,d} \ll m_s$. Instead of working with the quark mass term, we could obtain the GMO relation by considering the divergence of the $\Delta S = 1$ $V$-spin $V = 1$ vector current $\bar{u}\gamma_\mu s$ or the $U$-spin $U = 1$ vector current $\bar{d}\gamma_\mu s$ (putting $m_{u,d} = 0$ and neglecting isospin breaking). Considering the divergence of the $V = 1$ $\bar{u}\gamma_\mu s$ vector current, we have

$$\partial_\mu(\bar{u}\gamma_\mu s) = -im_s\bar{u}s. \tag{2}$$

Taking the matrix element of Eq. (2) between the baryons within a $V = 1$ multiplet, we see that the baryon mass difference is given by the $\bar{u}s$ scalar current form factor at the momentum transfer $q = 0$. Since the vector current form factor on the lhs has no first order $SU(3)$ breaking according to the Ademollo-Gatto theorem, in the limit of $SU(3)$ symmetry, the matrix element of $\bar{u}s$, like the I-spin symmetry for the matrix element of $\bar{u}d$, satisfies the $V$-spin symmetry relations from which one obtains the GMO relation. There could be first order $SU(3)$ symmetry breaking in the matrix element of $\bar{u}s$, which would be a violation of the GMO mass formula. In the limit of $SU(3)$ symmetry, we have

$$\left\langle\frac{1}{2}\Sigma^0 + \frac{\sqrt{3}}{2}\Lambda|\bar{u}s|\Xi^-\right\rangle = -\left\langle p|\bar{u}s|\frac{1}{2}\Sigma^0 + \frac{\sqrt{3}}{2}\Lambda\right\rangle, \tag{3}$$

$$\left\langle\frac{\sqrt{3}}{2}\Sigma^0 - \frac{1}{2}\Lambda|\bar{u}s|\Xi^-\right\rangle = \left\langle p|\bar{u}s|\frac{\sqrt{3}}{2}\Sigma^0 - \frac{1}{2}\Lambda\right\rangle, \tag{4}$$

where $|\Xi^-\rangle = |V = 1, V_3 = 1\rangle$, $|p\rangle = |V = 1, V_3 = -1\rangle$, $|\frac{1}{2}\Sigma^0 + \frac{\sqrt{3}}{2}\Lambda\rangle = |V = 1, V_3 = 0\rangle$, and $|\frac{\sqrt{3}}{2}\Sigma^0 - \frac{1}{2}\Lambda\rangle = |V = 0, V_3 = 0\rangle$. Equations (3) and (4) are the rotated $V$-spin versions of the two I-spin relations for the $\bar{u}d$ matrix elements:

$$\langle\Sigma^0|\bar{u}d|\Sigma^+\rangle = -\langle\Sigma^-|\bar{u}d|\Sigma^0\rangle, \tag{5}$$

$$\langle\Lambda|\bar{u}d|\Sigma^+\rangle = \langle\Sigma^-|\bar{u}d|\Lambda\rangle. \tag{6}$$

The above relations (3) and (4) are quite general and apply to matrix elements of any $SU(3)$ octet $\Delta S = 1$ operator, like the $\Delta S = 1$ axial vector current $\bar{u}\gamma_\mu s$ in hyperon semileptonic decays.

With $\langle B'|\partial_\mu(\bar{u}\gamma_\mu s)|B\rangle$ given by $(f_1)_{B\to B'}(m_{B'} - m_B)$, where $(f_1)_{B\to B'}$ is the vector form factor at $q^2 = 0$ momentum transfer in the vector current $\langle B'|\bar{u}\gamma_\mu s|B\rangle$ matrix element, we have

$$\begin{aligned}&[(1/4)(m_{\Xi^-} - m_{\Sigma^0}) + (3/4)(m_{\Xi^-} - m_\Lambda)]\\ &\quad = [(1/4)(m_{\Sigma^0} - m_p) + (3/4)(m_\Lambda - m_p)],\end{aligned} \tag{7}$$

$$\begin{aligned}&[(m_{\Xi^-} - m_{\Sigma^0}) - (m_{\Xi^-} - m_\Lambda))\\ &\quad = -[(m_{\Sigma^0} - m_p) - (m_\Lambda - m_p)].\end{aligned} \tag{8}$$

Equation (7) reproduces the GMO relation given in Eq. (1) mentioned above. Equation (8) reduces to a trivial identity with both its lhs and rhs equal to $-(m_{\Sigma^0} - m_\Lambda)$.

Experimentally, the lhs and rhs of Eq. (7) are 0.1867 GeV and 0.1966 GeV, respectively, showing a small amount of $SU(3)$ symmetry breaking effects, of the order $d = 0.05$, the ratio of the difference between the lhs and rhs to the average of the two quantities. One therefore expects a similar amount of symmetry breaking in hyperon semileptonic decays, appearing as a violation of the axial vector current GMO relations which are obtained easily by making a substitution $\bar{u}s \to \bar{u}\gamma_\mu\gamma_5 s$ in Eqs. (3) and (4). We have

$$\left\langle\frac{1}{2}\Sigma^0 + \frac{\sqrt{3}}{2}\Lambda|\bar{u}\gamma_\mu\gamma_5 s|\Xi^-\right\rangle = -\left\langle p|\bar{u}\gamma_\mu\gamma_5 s|\frac{1}{2}\Sigma^0 + \frac{\sqrt{3}}{2}\Lambda\right\rangle, \tag{9}$$

$$\left\langle\frac{\sqrt{3}}{2}\Sigma^0 - \frac{1}{2}\Lambda|\bar{u}\gamma_\mu\gamma_5 s|\Xi^-\right\rangle = \left\langle p|\bar{u}\gamma_\mu\gamma_5 s|\frac{\sqrt{3}}{2}\Sigma^0 - \frac{1}{2}\Lambda\right\rangle. \tag{10}$$

In terms of $(g_1/f_1)_{B\to B'}$, the axial vector current to vector current form factor ratios [9], we find

$$\begin{aligned}&(1/4)(g_1/f_1)_{\Xi^-\to\Sigma^0} + (3/4)(g_1/f_1)_{\Xi^-\to\Lambda}\\ &\quad = (1/4)(g_1/f_1)_{\Sigma^0\to p} + (3/4)(g_1/f_1)_{\Lambda\to p,}\end{aligned} \tag{11}$$

$$\begin{aligned}&(3/4)[(g_1/f_1)_{\Xi^-\to\Sigma^0} - (g_1/f_1)_{\Xi^-\to\Lambda}]\\ &\quad = -(3/4)[(g_1/f_1)_{\Sigma^0\to p} - (g_1/f_1)_{\Lambda\to p}].\end{aligned} \tag{12}$$

Since the measured $(g_1/f_1)_{B\to B'}$ terms contain first and second order $SU(3)$ breaking effects [$f_1$ has only second order $SU(3)$ breaking according to the Ademollo-Gatto theorem as mentioned above], there will be violation of the above relations by first and second order $SU(3)$ breaking terms. However, the violation due to second order $SU(3)$ breaking could be less important due to the possible cancellation of second order $SU(3)$ breaking effects in $(g_1/f_1)_{B\to B'}$. Thus, the validity of the above relations would depend essentially on first order $SU(3)$ symmetry breaking effects.

In the exact $SU(3)$ symmetry limit, the lhs and rhs of Eq. (11) are equal, as well as those in Eq. (12), given by $F$ and $D$, respectively. $F$ is just $(g_1/f_1)_{\Sigma^+\to\Sigma^0}$ and $-(g_1/f_1)_{\Sigma^0\to\Sigma^-}$, while $D$ is $\sqrt{3/2}(g_1)_{\Sigma^+\to\Lambda}$ and $\sqrt{3/2}(g_1)_{\Lambda\to\Sigma^-}$, as mentioned earlier. In the presence of $SU(3)$ symmetry breaking, the left- and right-hand sides of Eqs. (11) and (12) differ and are given by,

$$\begin{aligned}L_1 &= F + (1/4)d_{\Xi^-\to\Sigma^0} + (3/4)d_{\Xi^-\to\Lambda},\\ R_1 &= F + (1/4)d_{\Sigma^0\to p} + (3/4)d_{\Lambda\to p},\end{aligned} \tag{13}$$

$$\begin{aligned}L_2 &= D + (3/4)(d_{\Xi^-\to\Sigma^0} - d_{\Xi^-\to\Lambda}),\\ R_2 &= D - (3/4)(d_{\Sigma^0\to p} - d_{\Lambda\to p}).\end{aligned} \tag{14}$$

In our analysis, we define the $SU(3)$ breaking terms with respect to the neutron $\beta$ decay amplitude and our $D$, $F$ are pure $SU(3)$-symmetric parameters.

Since $\langle\Sigma^0|\bar{u}\gamma_\mu(1-\gamma_5)s|\Xi^-\rangle = \langle\Sigma^+|\bar{u}\gamma_\mu(1-\gamma_5)s|\Xi^0\rangle/\sqrt{2}$, $\langle p|\bar{u}\gamma_\mu(1-\gamma_5)s|\Sigma^0\rangle = \langle n|\bar{u}\gamma_\mu(1-\gamma_5)s|\Sigma^-\rangle/\sqrt{2}$, for both vector and axial vector current matrix elements, $(g_1/f_1)_{\Xi^-\to\Sigma^0} = (g_1/f_1)_{\Xi^0\to\Sigma^+}$ and $(g_1/f_1)_{\Sigma^0\to p} = (g_1/f_1)_{\Sigma^-\to n}$, one can use the measured values $(g_1/f_1)_{\Xi^0\to\Sigma^+}$ and $(g_1/f_1)_{\Sigma^-\to n}$ to test $SU(3)$ symmetry in hyperon semileptonic decays.

The differences $\Delta_1 = L_1 - R_1$ and $\Delta_2 = L_2 - R_2$ depend only on the symmetry breaking terms and are measures of $SU(3)$ symmetry breaking. We have

$$\Delta_1 = (1/4)(d_{\Xi^-\to\Sigma^0} - d_{\Sigma^0\to p}) + (3/4)(d_{\Xi^-\to\Lambda} - d_{\Lambda\to p}), \quad (15)$$

$$\Delta_2 = (3/4)(d_{\Xi^-\to\Sigma^0} - d_{\Xi^-\to\Lambda}) + (3/4)(d_{\Sigma^0\to p} - d_{\Lambda\to p}). \quad (16)$$

From the measured values in Table I, we have

$$L_1 = 0.490 \pm 0.05, \quad R_1 = 0.453 \pm 0.015, \quad \Delta_1 = 0.036 \pm 0.065, \quad (17)$$

$$L_2 = 0.720 \pm 0.075, \quad R_2 = 0.793 \pm 0.024, \quad \Delta_2 = -0.073 \pm 0.10, \quad (18)$$

showing on average an amount of $SU(3)$ breaking of 4% from $\Delta_1$ and 10% from $\Delta_2$ (ignoring experimental errors), to be compared with an amount of $SU(3)$ breaking of 5% in the $\langle B'|\bar{u}s|B\rangle$ matrix element from the GMO mass formula. For the $\Sigma^- \to \Lambda\ell\bar{\nu}$ decays, the measured value of $0.719 \pm 0.022$ for $\sqrt{3/2}(g_1)_{\Sigma^-\to\Lambda}$ differs also with $L_2$ and $R_2$ in Eq. (18), showing an $SU(3)$ breaking effect of 11% in $\Sigma^- \to \Lambda\ell\bar{\nu}$ decays.

Using the measured $(g_1/f_1)_{n\to p}$ and $(g_1/f_1)_{\Sigma^-\to n}$, we have

$$F = 0.464 - d_{\Sigma^-\to n}/2, \quad D = 0.805 + d_{\Sigma^-\to n}/2. \quad (19)$$

The $SU(3)$-symmetric fit of Ref. [11] produces an $SU(3)$ value $(g_1/f_1)_{\Sigma^-\to n} = -0.3178$, to be compared with the measured value of $-0.340 \pm 0.017$. This implies an $SU(3)$ breaking of 6.5%. This value is comparable with the calculations of Ref. [12] which give a 7.8% $SU(3)$ breaking. Including possible uncertainties in these values, we shall take $d_{\Sigma^-\to n} = -0.034$ for our determination of the symmetry breaking terms $d_{B\to B'}$ and $D$, $F$. From Eq. (19), we find

$$F = 0.464 + 017, \qquad D = 0.805 - 0.017. \quad (20)$$

Thus, the symmetry breaking for $(g_1/f_1)_{\Sigma^-\to n}$ makes a rather small contribution to $F$ and $D$. We note the importance of the small value for $d_{\Sigma^-\to n}$ used in the determination of $D$, $F$ with results close to the values obtained from the $SU(3)$-symmetric fit of Ref. [11]. More precisely, the fit of Ref. [11] looks like a zeroth order fit in $SU(3)$ breaking and ours is an improved determination of $D$, $F$ with $SU(3)$ symmetry breaking removed according to Eq. (20). From the data and the values determined above for $F$ and $D$, we now determine the $SU(3)$ breaking terms in hyperon semileptonic decays, using the above value $d_{\Sigma^-\to n} = -0.034$ and ignoring the experimental error of $\pm 0.05$ in the measured $(g_1/f_1)_{\Xi^0\to\Sigma^+}$ and $(g_1/f_1)_{\Xi^-\to\Lambda}$. We have

$$d_{\Xi^0\to\Sigma^+} = -0.06, \qquad d_{\Xi^-\to\Lambda} = 0.053 - 0.023,$$
$$d_{\Lambda\to p} = -0.015 - 0.011, \qquad d_{\Sigma^-\to\Lambda} = -0.070 - 0.028, \quad (21)$$

as shown in the last column of Table I. Though the symmetry breaking in $(g_1/f_1)_{\Xi^-\to\Lambda}$ is somewhat large (20%), the experimental error of the measured value is also large ($\pm$ 0.05); one needs a better measurement for a more accurate estimate of the symmetry breaking for $(g_1/f_1)_{\Xi^-\to\Lambda}$. In fact, with $F$ and $D$ obtained in Eq. (20), in the absence of a large symmetry breaking term $d_{\Xi^-\to\Lambda}$,

TABLE I. Vector and axial vector current form factors for baryon semileptonic decays in the Cabibbo model with the $SU(3)$ breaking term $d_{B\to B'}$, the measured axial vector to vector form factor ratio $g_1/f_1$, and the $SU(3)$ and measured values for $(g_1)_{\Sigma^-\to\Lambda}$. The last column is the estimated $d_{B\to B'}$.

| Decay | $f_1$ | $(g_1)_{SU(3)}$ | $(g_1/f_1)_{SU(3)+\mathrm{SB}}$ | $(g_1/f_1)_{\mathrm{exp}}$ [5,10] | $d_{B\to B'}$ (estimated) |
|---|---|---|---|---|---|
| $n\to p\ell\bar{\nu}$ | 1 | $F+D$ | $F+D$ | $1.2694 \pm 0.0028$ | |
| $\Lambda\to p\ell\bar{\nu}$ | $-\sqrt{3/2}$ | $-\sqrt{3/2}(F+D/3)$ | $F+D/3+d_{\Lambda\to p}$ | $0.718 \pm 0.015$ | $-0.015 - 0.011$ |
| $\Sigma^-\to n\ell\bar{\nu}$ | $-1$ | $-F+D$ | $F-D+d_{\Sigma^-\to p}$ | $-0.340 \pm 0.017$ | $-0.034$ (input) |
| $\Xi^-\to\Lambda^0\ell\bar{\nu}$ | $\sqrt{3/2}$ | $\sqrt{3/2}(F-D/3)$ | $F-D/3+d_{\Xi^-\to\Lambda}$ | $0.25 \pm 0.05$ | $0.053 - 0.023$ |
| $\Xi^0\to\Sigma^+\ell\bar{\nu}$ | 1 | $F+D$ | $F+D+d_{\Xi^0\to\Sigma^+}$ | $1.21 \pm 0.05$ | $-0.06$ (data) |
| $\Xi^-\to\Sigma^0\ell\bar{\nu}$ | $1/\sqrt{2}$ | $(1/\sqrt{2})(F+D)$ | $F+D+d_{\Xi^-\to\Sigma^0}$ | | |
| $\Sigma^-\to\Lambda\ell\bar{\nu}$ | 0 | $\sqrt{2/3}D$ | | $(g_1)_{\mathrm{exp}} =$ | $-0.070 - 0.028$ |
| $\Sigma^+\to\Lambda\ell\bar{\nu}$ | 0 | $\sqrt{2/3}D$ | | $0.587 \pm 0.016$ | $-0.070 - 0.028$ |

$(g_1/f_1)_{\Xi^-\to\Lambda}$ would be close to 0.20, a value at present not excluded by experiments.

In the above analysis, we consider $D$, $F$ the usual parameters of the $SU(3)$-symmetric part of $(g_1)_{B\to B'}$. $(g_1/f_1)_{n\to p} = F + D$ and $d_{B\to B'}$ are $SU(3)$ breaking terms, including first and second order $SU(3)$ breaking due to the $s$-quark mass, and contribute to the deviation of $L_1$, $R_1$ and $L_2$, $R_2$ from the $SU(3)$-symmetric $F$ and $D$, respectively. If one writes the first order $SU(3)$ breaking terms in $d_{B\to B'}$ as the induced terms produced by a term transforming as the 8-component of an $SU(3)$ octet [4],

$$\begin{aligned}\mathcal{L}_{\rm SB} = {}& a_0\mathrm{Tr}(\bar{B}B\lambda_i) + b_0\mathrm{Tr}(\bar{B}\lambda_i B) + a\mathrm{Tr}(\bar{B}B\{\lambda_i,\lambda_8\}) \\ & + b\mathrm{Tr}(\bar{B}\{\lambda_i,\lambda_8\}B) + c[\mathrm{Tr}(\bar{B}\lambda_i B\lambda_8) \\ & - \mathrm{Tr}(\bar{B}\lambda_8 B\lambda_i)] + g\mathrm{Tr}(\bar{B}B)\mathrm{Tr}(\lambda_i\lambda_8) \\ & + h[\mathrm{Tr}(\bar{B}\lambda_i)\mathrm{Tr}(B\lambda_8) + \mathrm{Tr}(\bar{B}\lambda_8)\mathrm{Tr}(B\lambda_i)], \end{aligned} \tag{22}$$

then the $SU(3)$ breaking terms from the $\underline{8}$ representations in the above expression will not produce a violation of the relations (11) and (12), like the $SU(3)$-symmetric $D$, $F$ terms. The $c$ terms are from the $\underline{10}$ and $\underline{10}^*$ representations and the $h$ terms are from the $\underline{27}$ representation. These terms will produce violations of the relations (11) and (12) and provide clear evidence for $SU(3)$ breaking in hyperon semileptonic decays. For example, in the analysis of Ref. [8], the $a$, $b$ terms in Eqs. (22) and (8a)–(8i) of the paper could be absorbed into $a_0 = D - F$ and $b_0 = D + F$ terms and, thus, do not contribute to $\Delta_1$ and $\Delta_2$. Assuming no isospin breaking, as in Ref. [4], by putting $\alpha = 0$, $\beta = 1$ in the expressions for $(g_1/f_1)_{B\to B'}$ in Eqs. (8a)–(8i) of Ref. [8], we have

$$\Delta_1 = h, \qquad \Delta_2 = 3c, \tag{23}$$

which allow a determination of $h$ and $c$ from the experimental values for $\Delta_1$ and $\Delta_2$. Note that in the notation of Ref. [12], $H_3 = 3c$ and $H_4 = h$, we have

$$\Delta_1 = H_4, \qquad \Delta_2 = H_3. \tag{24}$$

From Eqs. (17) and (18), we find

$$\begin{aligned} c &= -0.024 \pm 0.04, \\ H_3 &= -0.073 \pm 0.10, \\ h = H_4 &= 0.036 \pm 0.065, \end{aligned} \tag{25}$$

to be compared with the corresponding values $H_3 = -0.006$ and $H_4 = 0.037$, given in Ref. [12]. But there is a problem with this model. If we assume that there is no $SU(3)$ breaking in $(g_1/f_1)_{n\to p}$, we would have $b = c$ in the expression for $(g_1)_{n\to p}$ in Eq. (8a) of Ref. [8]; this implies that there would be no $SU(3)$ breaking in $(g_1/f_1)_{\Xi^-\to\Sigma^0}$, in contradiction with experiments. We note also that $d_{\Sigma^+\to\Lambda}$ would be large and positive, in contradiction with the value obtained from the $SU(3)$-symmetric fit of Ref. [10] and our result shown in Table I.

## III. CONCLUSION

In conclusion, we have shown that the GMO relations for the baryon mass difference are quite general and can be derived for the axial vector current matrix elements in hyperon semileptonic decays. With these GMO-type relations, we present evidence of an $SU(3)$ breaking, similar to that in the baryon mass difference. We then give an estimate for the $SU(3)$-symmetric $F$ and $D$ terms, as well as symmetry breaking terms, using the measured axial vector form factors. The small symmetry breaking effect we find also confirms the success of the Cabibbo model for hyperon semileptonic decays. Finally, these GMO relations could be used as experimental constraints on the $SU(3)$ symmetry breaking terms in theoretical calculations.


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