

Two-loop supersymmetric QCD corrections to Higgs-quark-quark couplings in the generic MSSM

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In this article we compute the two-loop supersymmetric QCD corrections to Higgs-quark-quark couplings in the generic minimal supersymmetric standard model (MSSM) generated by diagrams involving squarks and gluinos. We give analytic results for the two-loop contributions in the limit of vanishing external momenta for general supersymmetry (SUSY) masses valid in the MSSM with general flavor structure. Working in the decoupling limit ($M_{\text{SUSY}} \gg v$) we resum all chirally enhanced corrections (related to Higgs-quark-quark couplings) up to order $\alpha_s^{(n+1)} \tan^n \beta$. This resummation allows for a more precise determination of the Yukawa coupling and Cabibbo-Kobayashi-Maskawa elements of the MSSM superpotential necessary for the study of Yukawa coupling unification. The knowledge of the Yukawa couplings of the MSSM superpotential in addition allows us to derive the effective Higgs-quark-quark couplings entering flavor changing neutral current processes. These effective vertices can in addition be used for the calculation of Higgs decays into quarks as long as $M_{\text{SUSY}} > M_{\text{Higgs}}$ holds. Furthermore, our calculation is also necessary for consistently including the chirally enhanced self-energy contributions into the calculation of flavor changing neutral current processes in the MSSM beyond leading order. At two-loop order, we find an enhancement of the supersymmetry threshold corrections, induced by the quark self-energies, of approximately 9% for $\mu = M_{\text{SUSY}}$ compared to the one-loop result. At the same time, the matching scale dependence of the effective Higgs-quark-quark couplings is significantly reduced.

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I. INTRODUCTION

In the MSSM diagrams with sfermions and gauginos as virtual particles generate important loop corrections to Higgs-quark-quark couplings. After the spontaneous breaking of $SU(2)_L \otimes U(1)_Y$ at the electroweak scale, the Higgs fields acquire their vacuum expectation values (VEVs), and the genuine vertex corrections to Higgs-quark-quark couplings also generate chirality changing quark self-energies (or self-masses). Thus, there is a one to one correspondence between loop corrections to three-point Higgs-quark-quark functions and quark self-energies: The correction to a Higgs-quark-quark coupling is given by the corresponding chirality-changing self-energy divided by the VEV of the involved Higgs field.

This means that we can simplify the calculation of three-point functions by reducing the problem to the calculation of two-point functions (self-energies). In this way, the self-energy contributions to quark masses can be directly related to effective Higgs-quark-quark couplings, which allow for an efficient calculation of the effective Higgs vertices.

The quark self-energies also modify the relation between the Yukawa couplings of the minimal supersymmetric standard model (MSSM) superpotential and the quark masses (extracted from low-energy observables). Especially if $\tan \beta$ (the ratio of the VEVs of the two Higgs fields) is large, these contributions are generically very large and can be of order one [1–4]. In an analogous way, also the relation between the

Cabibbo-Kobayashi-Maskawa (CKM) matrix of the superpotential and the physical one is altered (by chargino-squark diagrams in the MSSM with minimal flavour violation (MFV) [5–9] and in addition by squark-gluino diagrams in the general MSSM [10,11]). Because of these corrections the physical quark masses and the measured CKM elements no longer equal the ones that appear in the MSSM superpotential. One says that these relations are modified by so-called threshold corrections, i.e., by the decoupling of heavy particles. Since in Higgs decays Higgs mediated flavor changing neutral currents (FCNCs) (like $B_{s(d)}$ mixing and $B_{s(d)} \rightarrow \mu^+ \mu^-$) and in Higgsino vertices the Yukawa couplings (of the superpotential) and not the physical quark masses enter, a precise knowledge of these quantities and thus of the threshold corrections is necessary. Furthermore, in grand unified theory models with Yukawa coupling unification not the effective Yukawa coupling of the SM, but rather the Yukawas of the superpotential unify, and the supersymmetry (SUSY) threshold corrections must be taken into account in order to judge whether they actually do unify [12,13]. In conclusion, it is desirable to know the relation between the parameters of the MSSM superpotential and the physical, i.e., measurable quantities, very precisely.

Having the relation between the Yukawa couplings (CKM elements) of the superpotential and the physical quark masses (physical CKM elements) at hand, one can calculate the effective Higgs couplings entering FCNC processes that include the SUSY loop corrections. This is most easily achieved by matching the MSSM on the two-Higgs-doublet model of type three (2HDM III). The loop-induced couplings of quarks to the “wrong” Higgs field,

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i.e., to the Higgs that is not involved in the Yukawa term in the superpotential, induce flavor-changing neutral Higgs couplings after switching to the physical basis in which the quark mass matrices are diagonal in flavor space. These effective Higgs couplings can be expressed entirely in terms of the physical masses and self-energies depending on MSSM parameters. Here a complication arises because these self-energies must be calculated using the Yukawa couplings and the CKM elements of the superpotential, which must have been determined previously in the process of renormalization by including the loop corrections, i.e., by resumming the threshold corrections. This problem can be solved analytically in the decoupling limit of the generic MSSM in which the self-energies are at most linear in the Yukawa couplings [14].

The importance of these threshold corrections and thus of the chirally enhanced self-energies motivates their calculation at next-to-leading order (NLO) in α_s . In the MSSM with MFV these corrections have been calculated in Refs. [15–19]. Here we want to extend this analysis to the MSSM with generic sources of flavor violation and resum all chirally enhanced effects using the results of Refs. [7, 10, 14, 20]. In addition, working in the approximation of vanishing external momenta, we are able to give relatively simple analytic expressions for the self-energies, and therefore also the resummation of all chirally enhanced corrections can be (and is) performed analytically.

After discussing the quark self-energies (and their connection to Higgs-quark-quark couplings in the decoupling limit of the MSSM) in the next section, we derive the relations between the MSSM Yukawa couplings and the quark masses at LO in Sec. III. As the main result of this article we calculate the supersymmetric QCD (SQCD) contribution to the chirality-changing self-energy at the two-loop level in Sec. IV. In Sec. V we discuss the topics of Sec. III at NLO. In Sec. IV we derive the effective Higgs-quark-quark couplings and conclude in Sec. VII. Various appendices summarize the relevant one-loop results.

II. QUARK SELF-ENERGIES, EFFECTIVE LAGRANGIAN, AND THE DECOUPLING LIMIT

As described in the Introduction, there is a one to one correspondence between chirality changing self-energies and Higgs-quark-quark couplings: In the decoupling limit of the MSSM ($M_{\text{SUSY}} > v$ and $M_{\text{SUSY}} > p$, where p is the external momentum) chirality changing self-energies are proportional to one power of a VEV only, and the corrections to the Higgs-quark-quark couplings can be obtained by dividing the corresponding self-energy by the VEV of the Higgs field involved. Thus, as long as the momentum flowing through the Higgs is small compared to the SUSY masses and the SUSY masses are heavier than the electroweak VEV, the decoupling limit is a valid approximation. In this approximation the calculation of the Higgs-quark-quark three-point function can be reduced to the calculation of quark self-energies. For this reason we will consider

the quark self-energies in this section in some detail and discuss the decoupling limit. The analysis is valid independent of the loop order (concerning α_s corrections) at which the self-energies are calculated.

In general, it is possible to decompose any quark (or any fermion) self-energy into chirality-flipping and chirality-conserving parts in the following way:

$$\begin{aligned} \Sigma_{fi}^q(p) &= (\Sigma_{fi}^{qLR}(p^2) + \not{p}\Sigma_{fi}^{qRR}(p^2))P_R + (\Sigma_{fi}^{qRL}(p^2) \\ &+ \not{p}\Sigma_{fi}^{qLL}(p^2))P_L. \end{aligned} \quad (1)$$

Note that the chirality-flipping parts $\Sigma_{fi}^{qRL,LR}$ have dimension mass, while the chirality conserving parts $\Sigma_{fi}^{qLL,RR}$ are dimensionless.

In the following we will be interested in the contributions to Eq. (1) that involve heavy SUSY particles. The reason for this is that only these contributions lead to the threshold corrections entering the relation between the quark masses and the Yukawa couplings of the MSSM superpotential. It is convenient to work in an effective field theory in which the part of the effective Lagrangian containing mass terms and kinetic terms for the quarks is given by

$$\begin{aligned} \mathcal{L}_{qq}^{\text{eff}} &= -(v_q Y_{\text{tree}}^{q_i^*} \delta_{fi} + C_{fi}^{qRL}) O_{fi}^{qRL} - (v_q Y_{\text{tree}}^{q_i} \delta_{fi} + C_{fi}^{qLR}) O_{fi}^{qLR} \\ &+ (\delta_{fi} - C_{fi}^{qRR}) O_{fi}^{qRR} + (\delta_{fi} - C_{fi}^{qLL}) O_{fi}^{qLL}, \end{aligned} \quad (2)$$

with the operators defined as

$$\begin{aligned} O_{fi}^{qRL} &= \bar{q}_f P_L q_i, & O_{fi}^{qLL} &= i \bar{q}_f \not{p} P_L q_i, \\ O_{fi}^{qLR} &= \bar{q}_f P_R q_i, & O_{fi}^{qRR} &= i \bar{q}_f \not{p} P_R q_i. \end{aligned} \quad (3)$$

Throughout this paper, the Wilson coefficients in the effective Lagrangian (2) (or, equivalently, the operators) are renormalized in the $\overline{\text{MS}}$ scheme. The final results for the Wilson coefficients will be written as an expansion in g_s , where g_s is meant to be the $\overline{\text{MS}}$ renormalized strong coupling constant of the effective theory, running with six (quark) flavors.

In Eq. (2) the term $-v_q Y_{\text{tree}}^{q_i} \delta_{fi}$ denotes the part of the Wilson coefficient of the operator O_{fi}^{qRL} that is induced at tree level by the Yukawa coupling of the MSSM superpotential. The running of $v_q Y_{\text{tree}}^{q_i}$ (and also that of C_{fi}^{qRL}) is the same as the one of the quark mass in the SM (in the $\overline{\text{MS}}$ scheme). At the matching scale m_{SUSY} , $Y_{\text{tree}}^{q_i}$ is just the Yukawa coupling Y^q of the MSSM superpotential.¹ Note that $Y_{\text{tree}}^{q_i}$ is not the

¹The matching calculation for $Y_{\text{tree}}^{q_i}$ is most easily done by using the $\overline{\text{MS}}$ scheme, both on the MSSM side and on the effective theory side. When working up to order α_s , we get at the matching scale m_{SUSY} $Y_{\text{tree}}^{q_i} = Y^{q_i}$, where Y^{q_i} denotes the Higgs-quark-quark coupling of the MSSM in the $\overline{\text{MS}}$ scheme. However, it is well known that one should use the $\overline{\text{DR}}$ scheme on the MSSM side, such that supersymmetry is preserved. This can be achieved by the shift $Y^{q_i} = (1 + \frac{\alpha_s}{4\pi} C_F) Y_{\overline{\text{DR}}}^{q_i}$. This issue will be considered in more detail in Sec. V. The matching condition then reads $Y_{\text{tree}}^{q_i} = (1 + \frac{\alpha_s}{4\pi} C_F) Y_{\overline{\text{DR}}}^{q_i}$.

effective Yukawa coupling of the SM, which instead is obtained from the physical quark mass [see Eq. (11)].

The Wilson coefficients $C_{f_i}^{qLR,LR}$ and $C_{f_i}^{qLL,RR}$ in Eq. (2) contain the effects of heavy particles only. Self-energy diagrams involving no heavy SUSY particles, i.e., ordinary QCD corrections containing only quarks and gluons, do

$$\left. \begin{aligned} C_{f_i}^{qLR} &= \frac{\alpha_s}{2\pi} W_{f_s}^{\tilde{q}} W_{i+3,s}^{\tilde{q}*} C_F m_{\tilde{g}} \left(\frac{x_s^2 \ln(x_s^2)}{1-x_s^2} \right) \\ C_{f_i}^{qLL}(0) &= -\frac{\alpha_s}{4\pi} W_{f_s}^{\tilde{q}} W_{i_s}^{\tilde{q}*} C_F \left(\ln(x_s^2) + \frac{3-4x_s^2+x_s^4+(4x_s^2-2x_s^4)\ln(x_s^2)}{2(1-x_s^2)^2} \right) \\ Y_{\text{tree}}^q &= Y^q \end{aligned} \right\} \text{at LO in } \alpha_s. \quad (4)$$

Further, in the following we will focus on the nondecoupling pieces of Eq. (1), i.e., those contributions that do not vanish in the limit $M_{\text{SUSY}} \rightarrow \infty$ (which also includes $\mu \rightarrow \infty$). In contrast, all parts that vanish in this limit are called decoupling. There are two different kinds of decoupling contributions concerning self-energies (or effective Higgs-quark couplings):

- (i) The first kind of decoupling effects is related to the expansion of the self-energies in powers of p^2/M_{SUSY}^2 . This expansion is certainly possible in on-shell configurations because the SUSY particles are known to be much heavier than the external quarks. In this series, higher order contributions are clearly suppressed for all light quarks, and even for the top quark, nondecoupling corrections are only of the order $m_t^2/M_{\text{SUSY}}^2 \leq 4\%$ with respect to the leading term. Thus, higher orders in p^2/M_{SUSY}^2 can be safely neglected as long as the external momentum p^2 is small, which is the case for all low-energy flavor observables.
- (ii) The second kind of decoupling effect is related to the mixing matrices (and also the physical masses) of the MSSM particles (squarks and charginos/neutralinos), which appear because the mass matrices of the SUSY particles are not diagonal in a weak basis. These mixing matrices and mass eigenvalues can be expanded in powers of v/M_{SUSY} , and also in this case it turns out that the decoupling limit (i.e., the leading order v/M_{SUSY}) for realistic values of SUSY masses² is an excellent approximation to the full expressions [20]. Beyond the decoupling limit higher dimensional operators involving several Higgs fields would appear.

From dimensional analysis we see that all nondecoupling contributions are contained in $\Sigma_{f_i}^{qRR,LL}$ and $\Sigma_{f_i}^{qLR,RL}$ evaluated at $p^2 = 0$. Furthermore, the nondecoupling part

not contribute to the Wilson coefficients in the matching procedure, because they are the same on the full side (the MSSM) and on the effective side (the 2HDM III or the SM). At the matching scale m_{SUSY} we find for the Wilson coefficients of Eq. (2), using the results for $\Sigma_{q_f q_i}^{\tilde{g}LL}(0)$ and $\Sigma_{q_f q_i}^{\tilde{g}LR}(0)$ given in Eq. (A2):

of $\Sigma_{f_i}^{qRR,LL}(p^2 = 0)$ is independent of a VEV, while $\Sigma_{f_i}^{qLR,RL}(p^2 = 0)$ is linear in v . Thus, in the following we will work in the limit $\Sigma_{f_i}^{qRR,LL}(p^2 = 0)$, $\Sigma_{f_i}^{qLR,RL}(p^2 = 0)$, and only keep the leading term in v that is equivalent to considering operators up to dimension 4 only. This simplification allows us to perform an analytic resummation of all chirally enhanced effects as developed in Ref. [14].

There is a fundamental difference between $\Sigma_{f_i}^{qLR,RL}$ and $\Sigma_{f_i}^{qRR,LL}$ (and thus also between $C_{f_i}^{qLR,RL}$ and $C_{f_i}^{qRR,LL}$) even though both pieces do not decouple. We explain this issue at one-loop order: $\Sigma_{f_i}^{qRR,LL}$ enters always proportional to the quark mass itself into the renormalization of the Yukawa coupling and CKM elements and thus has the same generic size as an ordinary QCD loop correction (it is of order α_s). Furthermore, as we will see later, the $\Sigma_{f_i}^{qRR,LL}$ even do not contribute to effective Higgs-quark-quark couplings at the one-loop level [23]. On the other hand, $\Sigma_{f_i}^{qLR,RL}$ can be ‘‘chirally enhanced’’ by a factor of $\tan\beta$ [24] or $A_{ij}^f/(Y_{ij}^f M_{\text{SUSY}})$ [10], which can compensate for the loop factor. Because of this possible enhancement, $\Sigma_{f_i}^{qLR,RL}$ generates the most important contribution to the threshold corrections between Yukawa couplings and quark masses. The resulting Wilson coefficients $C_{f_i}^{qLR,RL}$ can even be of order one, i.e., numerically as large as the corresponding physical quantities (m_{q_i} in the flavor-conserving case or $V_{f_i} \times \max[m_{q_i}, m_{q_i}]$ in the flavor-changing one). Furthermore, concerning flavor-changing neutral Higgs couplings, $\Sigma_{f_i}^{qLR,RL}$ even constitutes the leading order, since these couplings are first generated at the one-loop level.

Because the gluino contribution to $\Sigma_{f_i}^{qLR,RL}$ involves the strong coupling constant, it is the numerically dominant contribution to the threshold corrections modifying the relations between the quark masses and the Yukawa coupling. Regarding flavor changes, in the MSSM with MFV only the chargino contribution enters the renormalization of the CKM matrix, but once there are sizable nonminimal sources of flavor violation, again the gluino contribution

²The new results of the CMS Collaboration [21] and the ATLAS experiment [22] require that squark and gluino masses are at least of the order of 1 TeV.

becomes dominant. The neutralino contribution is in most regions of parameter space suppressed (except if the gluino is much heavier than the other SUSY particles). Thus we consider the gluino contribution in this article. The calculation of the chargino- and neutralino-induced contributions to the threshold corrections and the effective Higgs-quark-quark couplings is work in progress [25].

From the arguments given above we see that at any loop order (concerning α_s corrections) the chirality-flipping quark self-energy containing at least one gluino and one squark as virtual particles is always proportional to one³ off-diagonal element Δ_{ij}^{qLR} of the squark mass matrix that, in the super-CKM basis, is given by

$$\begin{aligned}\Delta_{ij}^{dLR} &= -v_d A_{ij}^d - v_u A_{ij}^{d'} - v_u \mu Y_{ij}^{\tilde{d}}, \\ \Delta_{ij}^{uLR} &= -v_u A_{ij}^u - v_d A_{ij}^{u'} - v_d \mu Y_{ij}^{\tilde{u}},\end{aligned}\quad (5)$$

with $\Delta_{ij}^{qRL} = \Delta_{ji}^{qLR*}$. Note the presence of the tilde in the Yukawa couplings $Y_{ij}^{\tilde{q}}$. This refers to the fact that a squark-squark-Higgs coupling is involved, while Y_{ij}^q entering the Wilson coefficient $Y_{\text{tree}}^{q_i}$ in Eq. (2) is a quark-quark-Higgs coupling. Of course, both of these couplings are *a priori* equal in the MSSM owing to supersymmetry and could be identified with each other from the beginning if the calculations of the chirality-flipping quark self-energies would be performed in the $\overline{\text{DR}}$ scheme, in which supersymmetry is preserved. However, we decided to work out in an intermediate step the SQCD two-loop corrections to the self-energies in the $\overline{\text{MS}}$ scheme, i.e., in dimensional regularization followed by modified minimal subtraction rather than using dimensional reduction. At this level, the two couplings Y_{ij}^q and $Y_{ij}^{\tilde{q}}$ are different and therefore have to be distinguished in the notation. We will discuss this in more detail in Sec. V.

The elements Δ_{ij}^{qLR} generate chirality-enhanced effects with respect to the tree-level quark masses if they involve the large VEV v_u ($\tan\beta$ enhancement for the down quark) or a trilinear $A^{(i)q}$ term [$A_{ij}^{(i)q}/(Y_{ij}^q M_{\text{SUSY}})$ enhancement].

A. Decomposition of quark self-energy contributions

We diagonalize the full 6×6 squark mass matrices in the following way⁴:

$$W_{\text{dec}}^{\tilde{q}\dagger} \mathcal{M}_{\tilde{q}}^2 W_{\text{dec}}^{\tilde{q}} = \text{diag}(m_{\tilde{q}_1}^2, m_{\tilde{q}_2}^2, m_{\tilde{q}_3}^2, m_{\tilde{q}_4}^2, m_{\tilde{q}_5}^2, m_{\tilde{q}_6}^2), \quad (6)$$

where $m_{\tilde{q}_s}$ ($s=1, \dots, 6$) denote the physical squark masses.

In the decoupling limit, i.e., to leading order in v/M_{SUSY} , the chirality-flipping elements Δ_{ij}^{qLR} can be neglected in the determination of the squark mixing

matrices $W_{\text{dec}}^{\tilde{q}}$ and the physical squark masses $m_{\tilde{q}_s}^2$. The down (up) squark mass matrices are then block diagonal and diagonalized by the mixing matrices Γ_{DL}^{ij} , Γ_{DR}^{ij} (Γ_{UL}^{ij} , Γ_{UR}^{ij}) in the following way:

$$\begin{aligned}W_{\text{dec}}^{\tilde{q}\dagger} \mathcal{M}_{\tilde{q}}^2 W_{\text{dec}}^{\tilde{q}} &= \text{diag}(m_{\tilde{q}_1}^2, m_{\tilde{q}_2}^2, m_{\tilde{q}_3}^2, m_{\tilde{q}_4}^2, m_{\tilde{q}_5}^2, m_{\tilde{q}_6}^2), \\ W_{\text{dec}}^{\tilde{q}} &= \begin{pmatrix} \Gamma_{QL} & 0 \\ 0 & \Gamma_{QR} \end{pmatrix}.\end{aligned}\quad (7)$$

The 3×3 matrices Γ_{QL}^{ij} and Γ_{QR}^{ij} ($Q = U, D$) take into account the flavor mixing in the left-left and right-right sector of sfermions, respectively. It is further convenient to introduce the abbreviations

$$\Lambda_{mij}^{qLL} = \Gamma_{QL}^{im} \Gamma_{QL}^{jm*} \quad (q = u, d), \quad \Lambda_{mij}^{qRR} = \Gamma_{QR}^{im} \Gamma_{QR}^{jm*}, \quad (8)$$

where $i, j, m = 1, 2, 3$, and the index m is not summed over.

On the other hand, left-right mixing of squarks is not described by a mixing matrix, but rather treated perturbatively in the form of two-point $\tilde{q}_i^R - \tilde{q}_j^L$ vertices governed by the couplings Δ_{ji}^{qLR} , i.e., by what is called mass insertions [28].

For the relations between the Yukawa couplings and the quark masses (to be discussed in Sec. III) and for the effective Higgs-quark-quark vertices (see Sec. VI) it is necessary to decompose $C_{ii}^{dLR,RL}$ according to its Y^d dependence as

$$C_{ii}^{dLR} = C_{ii\tilde{Y}_i}^{dLR} + \epsilon_i^d v_u Y_{ii}^{\tilde{d}}, \quad (9)$$

where, as the notation implies, $C_{ii\tilde{Y}_i}^{dLR}$ is independent of a Yukawa coupling. Note that we did the decomposition with respect to the Yukawa coupling $Y_{ii}^{\tilde{d}}$, as C_{fi}^{dLR} can only involve $Y_{ii}^{\tilde{d}}$ but not Y_{fi}^d [see Eq. (5)].

For the discussion of the effective Higgs-quark-quark vertices in Sec. VI we also need a decomposition of Σ_{ji}^{qLR} and thus of C_{ji}^{qLR} into its holomorphic and nonholomorphic parts.⁵ In the decoupling limit (and in the approximation $m_q = 0$) all holomorphic self-energies are proportional to A terms. Thus we denote the holomorphic part of the Wilson coefficient as C_{jiA}^{fLR} , while the nonholomorphic part (which can be induced by the μ term or by an A' term) is denoted as $C_{ji}^{f'qLR}$. This means that we have the relation

$$C_{ji}^{qLR} = C_{jiA}^{qLR} + C_{ji}^{f'qLR}. \quad (10)$$

³More precisely, in the decoupling limit Σ_{fi}^{qLR} is linear in Δ_{ij}^{dLR} , while beyond the decoupling limit it contains all add powers of Δ_{ij}^{dLR} .

⁴Note that our mixing matrices $W_{\text{dec}}^{\tilde{q}}$ correspond to the Hermitian conjugate of the matrices Γ_Q defined in Refs. [26,27].

⁵With (non)holomorphic we mean that the loop-induced Higgs coupling is to the (opposite) same Higgs doublet as involved in the corresponding Yukawa coupling of the MSSM superpotential.

III. RELATIONS BETWEEN QUARK MASSES AND YUKAWA COUPLINGS AT LEADING ORDER IN α_s

Let us discuss the renormalization⁶ of quark masses and Yukawa couplings induced by nondecoupling self-energy contributions to the Wilson coefficients $C_{ji}^{qLR,RL}$ and $C_{ji}^{qLL,RR}$ in the MSSM. For this purpose we focus on the flavor-conserving case, but we will return to the flavor-changing one in Sec. VI. As it turns out, flavor-changing self-energies only contribute to the relation between quark masses and Yukawa couplings at higher orders in the perturbative diagonalization of the quark mass matrices.

For the renormalization and the inclusion of the threshold corrections it is very important to distinguish between the Yukawa couplings of the MSSM superpotential Y^q and the “effective” Yukawa couplings of the SM (or the 2HDM of type III) $Y_{\text{eff}}^q = m_{q_i}/v_q$. At the matching scale M_{SUSY} the running quark mass m_{q_i} of the SM is related to the Yukawa coupling of the MSSM in the following way:

$$v_q Y_{\text{eff}}^{q_i} = m_{q_i} = (v_q Y_{\text{tree}}^{q_i} + C_{ii}^{qLR}) \times \left(1 + \frac{1}{2}(C_{ii}^{qLL} + C_{ii}^{qRR})\right). \quad (11)$$

The term $\frac{1}{2}(C_{ii}^{qLL} + C_{ii}^{qRR})$ originates from rendering the kinetic terms of the effective theory diagonal or, equivalently, in the full theory from the Lehmann-Symanzik-Zimmermann factor that originates for the truncation of the external legs.

As discussed in the last section, only Σ_{ii}^{qLR} (or equivalently C_{ii}^{qLR} in the effective theory) can be chirally enhanced. If we restrict ourselves to this term, we recover (in the decoupling limit in which C_{ii}^{qLR} is proportional to one power of Y^{d_i} at most) the well-known resummation formula for $\tan\beta$ -enhanced corrections, with an additional correction attributable to the A terms [15] (and possibly the A' terms). The resummation formula at leading order is given by⁷

$$Y_{d_i} = \frac{m_{d_i} - C_{ii}^{dLR(1)}}{v_d} = \frac{m_{d_i} - C_{ii}^{dLR(1)}}{v_d(1 + \tan\beta\epsilon_i^{d(1)}),} \quad (12)$$

with $\epsilon_i^{d(1)}$ and $C_{ii}^{dLR(1)}$ defined through Eq. (9). The superscript (1) denotes the fact that a corresponding quantity is calculated at the one-loop order.

⁶Throughout this article, renormalization is understood not only as the process of removing divergences but also as the altering of the relations between different quantities induced by loop contributions.

⁷For large flavor-changing elements also a contribution involving two self-energies can be important for the renormalization of the light quark masses [29]. In this case the resummation formula reads for $i = 1$:

$$Y_{d_1} = \frac{m_{d_1} - C_{11}^{dLR} - \frac{C_{13}^{dLR} C_{31}^{dLR}}{m_{d_3}}}{v_d(1 + \tan\beta\epsilon_1^d)}.$$

IV. CALCULATION OF THE WILSON COEFFICIENT C_{fi}^{qLR} AT NLO

In this section we describe the calculation of the two-loop contribution to C_{fi}^{qLR} , discuss the issue of renormalization, show the expected reduction of the matching scale dependence, and discuss the decoupling limit in which only one coupling to a VEV of a Higgs field is involved. To be specific, we describe in the following the calculation and the results for the down quark, i.e., C_{fi}^{dLR} , and mention at the very end how C_{ij}^{uLR} can be obtained.

In the following we write the Wilson coefficient C_{fi}^{dLR} as

$$C_{fi}^{dLR} = C_{fi}^{dLR(1)} + C_{fi}^{dLR(2)} + \dots, \quad (13)$$

where $C_{fi}^{dLR(1)}$ and $C_{fi}^{dLR(2)}$ denote the one- and two-loop contributions, respectively. We perform the two-loop matching calculation (order α_s^2) for the Wilson coefficient C_{fi}^{dLR} in $D = (4 - 2\epsilon)$ dimensions, using dimensional regularization, both for the full theory (MSSM) and for the effective theory in Eq. (2). The complete list of genuine 1-particle irreducible (PI) two-loop diagrams contributing in the full theory is shown in Fig. 1 (generated with FeynArts [30,31]).

As the first two diagrams (involving squark tadpoles) give rise to some subtle points concerning renormalization, we ignore them in this subsection and take into account their impact on C_{fi}^{dLR} only in the next subsection.

A. Matching calculation for $C_{fi}^{dLR(2)}$ ignoring tadpoles

In the full theory we first calculate the 1-PI two-loop diagrams (diagrams 3–16 in Fig. 1) in the approximation $m_q = 0$ and $p = 0$, but to all orders in v/m_{SUSY} (using exact diagonalization of the squark mass matrices). All diagrams except diagram 16 can be calculated by naively setting $m_q = 0$ and $p = 0$. Diagram 16, however, leads to two contributions: the *hard* contribution, which amounts to the naive limit of vanishing quark masses and external momenta of the full two-loop diagram, and the *soft* contribution, which amounts to the same limit but only for the heavy one-loop subdiagram [32]. As the soft contribution is identical to the one-loop gluon correction to $-iC_{fi}^{dLR(1,D)}O_{fi}^{dLR}$ in the effective theory,⁸ this contribution drops out in the matching for $C_{fi}^{dLR(2)}$. As this soft contribution is the only one that is infrared singular, this means in particular that $C_{fi}^{dLR(2)}$ is free of infrared problems, as it should be.

We then add the counterterm contributions in the full theory, which are induced by the renormalization of the parameters $m_{\tilde{q}_s}^2$, $m_{\tilde{g}}$, and α_s in the corresponding one-loop

⁸ $C_{fi}^{dLR(1,D)}$ is the one-loop Wilson coefficient in $D = (4 - 2\epsilon)$ dimensions, i.e., $C_{fi}^{dLR(1,D)} = \Sigma_{d_j d_i}^{\tilde{d}LR}(0)$; see Eq. (A2).

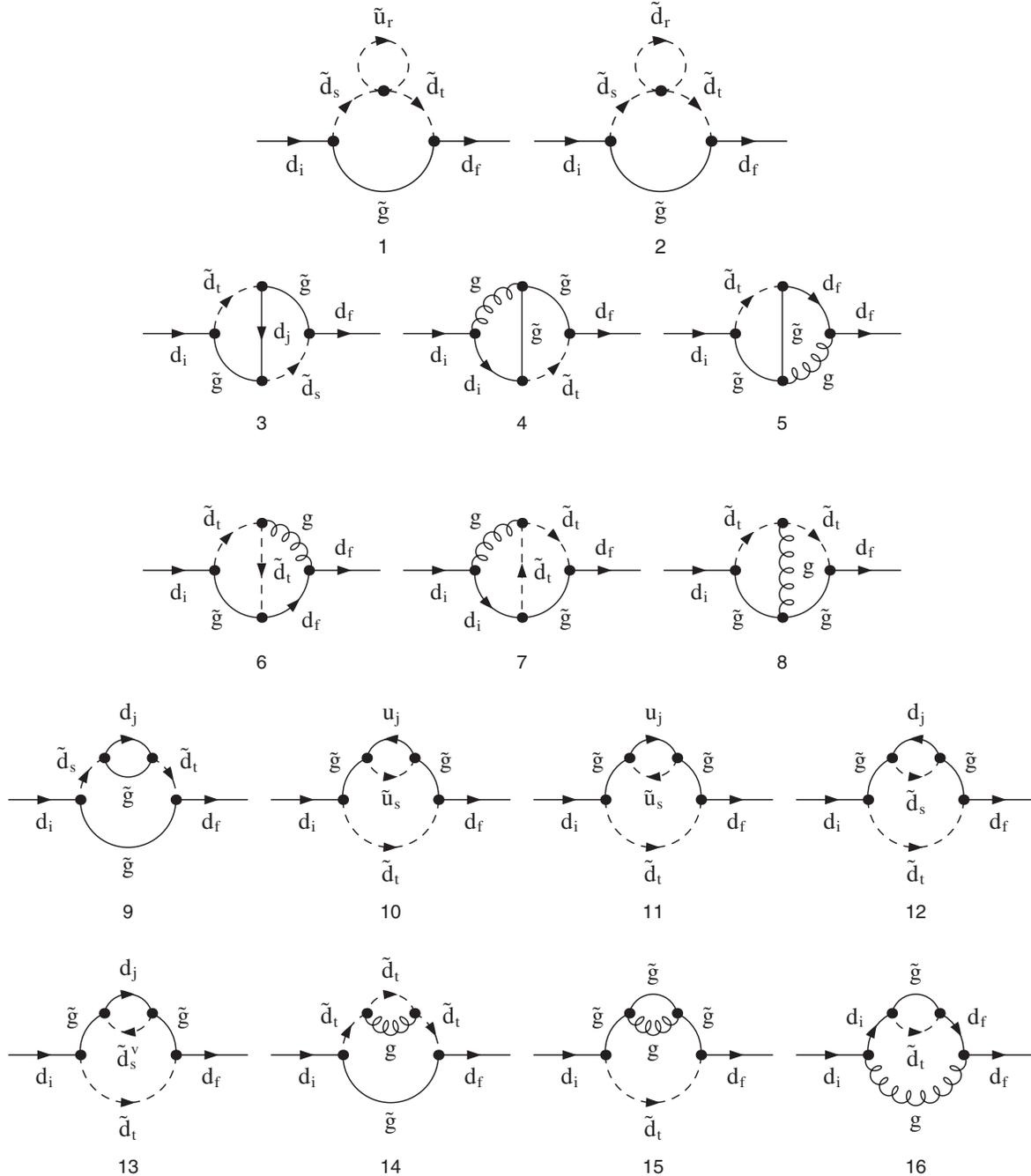


FIG. 1. Genuine 1-PI two-loop diagrams involving squarks and gluinos necessary for the calculation of $\Sigma_{fi}^{dLR(2)}$.

result (where at this level of the calculation these three parameters are renormalized in the $\overline{\text{MS}}$ scheme). The explicit expressions are listed in Sec. A3a. In one of these counterterm contributions the squark-mass counterterm $\delta m_{\tilde{q}_s}^2$ enters. Of course, when ignoring the tadpole diagrams in this section, the tadpole contribution to $\delta m_{\tilde{q}_s}^2$ also has to be ignored.

Besides the renormalization of the parameters in the full theory, we also have to attach one-loop wave function renormalization constants for the external quark legs to

the corresponding one-loop result. These wave function renormalization constants have two contributions: One from a self-energy with a gluon-quark loop, and another one from a gluino-squark loop. The first one is also present in the effective theory and consequently drops out in the determination of $C_{fi}^{dLR(2)}$, while the second one contributes. Since we perform the renormalization in the $\overline{\text{MS}}$ scheme, only the divergent pieces of $\Sigma_{d_j d_i}^{\tilde{g}LL,RR}$ enter $C_{fi}^{dLR(2)}$ while the finite part gives rise to $C_{fi}^{dLL,RR}$.

We now turn to the effective theory. Here, we have to work out one-loop QCD corrections to $-iC_{fi}^{dLR(1,d)}O_{fi}^{dLR}$, i.e., the 1-PI diagram, attach the wave function renormalization constants and take into account the effect of the ($\overline{\text{MS}}$) renormalization constant δZ_O of the operator O_{fi}^{dLR} . While the first two get canceled against contributions in the full theory (as already mentioned above), the effect of the renormalization constant of the operator enters the matching condition for $C_{fi}^{dLR(2)}$.

Putting things together, we get the following (schematic) matching equation:

$$\begin{aligned} & -i\delta Z_O C_{fi}^{dLR(1,d)} - iC_{fi}^{dLR(2)} \\ & = D_3 + \dots + D_{15} + D_{16}^{\text{hard}} - i[\text{CT}_{m_{\tilde{g}}} + \text{CT}_{m_{\tilde{q}_s}} + \text{CT}_{\alpha_s}] \\ & \quad - i\frac{1}{2}[\delta Z_{2,f}^{\text{heavy}} + \delta Z_{2,i}^{\text{heavy}}]C_{fi}^{dLR(1,d)}. \end{aligned} \quad (14)$$

Here $\text{CT}_{m_{\tilde{g}}}$, $\text{CT}_{m_{\tilde{q}_s}}$, and CT_{α_s} stand for the contributions induced by the insertions of the corresponding counterterms into the one-loop diagram, and D_i represents the contribution stemming from diagram i of Fig. 1. As already mentioned, we did our two-loop calculation in dimensional regularization. So far the parameters $m_{\tilde{g}}$, $m_{\tilde{q}_s}$, and α_s appearing in the full theory were renormalized according to the $\overline{\text{MS}}$ scheme. Also the various Z factors appearing in Eq. (14) are renormalized in the $\overline{\text{MS}}$ scheme. The result for $C_{fi}^{dLR(2)}$ we get at this level corresponds to the sum of the first five terms on the right-hand side of Eq. (23). When giving the explicit expressions for these terms, we freely made use of the unitarity properties of the $W^{\tilde{q}}$ mixing matrices.

We should be more precise concerning g_s (or α_s). In our calculation of the full theory side g_s stands for $g_{s,Y}$, i.e., for the strong coupling constant of the Yukawa type of the full MSSM renormalized in the $\overline{\text{MS}}$ scheme. As we want to express the final result for the Wilson coefficient $C_{fi}^{dLR(2)}$ in terms of $g_{s,\overline{\text{MS}}}^{(6)}$, i.e., by the strong coupling constant of the SM in the $\overline{\text{MS}}$ scheme running with six flavors, we make use of the relation [33,34]

$$\begin{aligned} \alpha_{s,Y}(\mu) = & \left\{ 1 + \frac{\alpha_s}{4\pi} \frac{1}{3} \left[(n_f + 6) \ln(x_\mu^2) - \sum_{s=1}^6 (\ln(x_s) + \ln(y_s)) \right. \right. \\ & \left. \left. + 4C_A - 3C_F \right] \right\} \alpha_{s,\overline{\text{MS}}}^{(6)}(\mu). \end{aligned} \quad (15)$$

Actually, this relation summarizes three steps: first, the transition from $g_{s,Y}$ in the $\overline{\text{MS}}$ scheme to g_s of the full MSSM in the $\overline{\text{DR}}$ scheme; second, the decoupling of the SUSY particles, leading to g_s running with six (quark) flavor in the $\overline{\text{DR}}$ scheme; third, the transition to $g_{s,\overline{\text{MS}}}^{(6)}$. Equation (15) leads to the additional piece $C_{fi}^{(2),\text{shift}\alpha_s}$ in Eq. (23).

In principle we should have performed our calculation (of the full theory side) using dimensional reduction, which preserves supersymmetry, followed by modified minimal subtraction. The corresponding result for $C_{fi}^{dLR(2)}$ can be reconstructed by also shifting the parameter $m_{\tilde{q}_s}$ and $m_{\tilde{g}}$ from the $\overline{\text{MS}}$ scheme to the $\overline{\text{DR}}$ scheme in the expression for $C_{fi}^{dLR(1)}$. As only $m_{\tilde{g}}$ gets such a shift at the relevant order in α_s , we denote this contribution in Eq. (23) as $C_{fi}^{(2),m_{\tilde{g}}\overline{\text{MS}} \rightarrow m_{\tilde{g}}\overline{\text{DR}}}$.

This completes the derivation of the matching condition for $C_{fi}^{dLR(2)}$ when ignoring the tadpole contribution (i.e., diagrams 1 and 2). Note that we performed our calculation using the expression for the gluon propagator in an arbitrary R_ξ gauge and found a gauge-invariant result for $C_{fi}^{dLR(2)}$.

B. The squark tadpole

The diagrams containing a squark-tadpole self-energy as a subdiagram require close examination. Diagram 1 vanishes, but the squark-tadpole contained in diagram 2 contains a divergence that enforces a renormalization of both the physical squark masses and the trilinear couplings of squarks to the Higgs field (the Yukawa couplings and the A terms). Thus it has to be decomposed into the corresponding two parts.

Let us first consider the decoupling limit in which the expressions are simpler but the structure of the divergences is the same as in the full theory because higher powers (two or more) of Δ_{ij}^{qLR} generate finite contributions only. In the decoupling limit Eq. (A14) simplifies to

$$\begin{aligned} & -\frac{\alpha_s}{4\pi} C_F \left(\delta_{st} m_{\tilde{q}_s}^2 - 2 \sum_{i,j=1}^3 (\delta_{i'+3,s} \Gamma_{QR}^{ii'\star} \Delta_{ij}^{qRL} \Gamma_{QL}^{jj'} \delta_{j't} \right. \\ & \left. + \delta_{i's} \Gamma_{QL}^{ii'\star} \Delta_{ij}^{qLR} \Gamma_{QR}^{jj'} \delta_{j'+3,t}) \right) \frac{1}{\epsilon} + \text{finite}. \end{aligned} \quad (16)$$

Here we clearly see that to render the first term in Eq. (16) finite, which is flavor diagonal [corresponding to Fig. 2(b)], a renormalization of the squark masses is necessary. On the other hand, for canceling the divergence of the second term in Eq. (16) [corresponding to Fig. 2(a)], which is proportional to Δ_{ij}^{qLR} , a counterterm to the Yukawa coupling and the A term contained in Δ_{ij}^{qLR} is necessary. The latter point can be seen as follows: In the decoupling limit the amputated chirality-changing squark two-point function for $\tilde{q}_j^L \rightarrow \tilde{q}_i^R$ is given, at lowest order in α_s , by

$$\Gamma_{QR}^{ii'\star} \Delta_{ij}^{qRL} \Gamma_{QL}^{jj'}. \quad (17)$$

From this we can read off the common renormalization constant Z_Y of the Yukawa couplings $Y^{\tilde{q}_i}$ and the A_{ij}^q and the A_{ij}^{lq} terms, obtaining in the minimal subtraction scheme ($\overline{\text{DR}}$ or $\overline{\text{MS}}$)

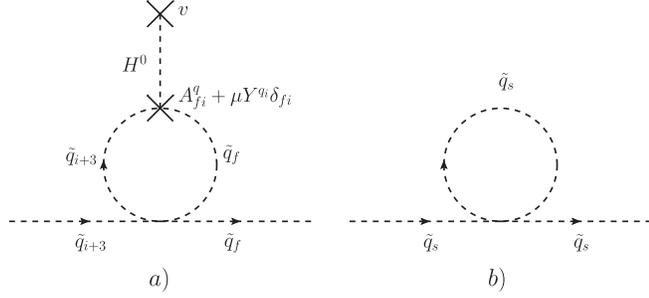


FIG. 2. Decomposition of the squark tadpole that is contained in diagram 2 of Fig. 1 as a subdiagram: In the decoupling limit the squark tadpole is either proportional to one element Δ_{ij}^{qLR} (a) or independent of Δ_{ij}^{qLR} (b). In the first case, it connects left-handed with right-handed squarks, while in the second case it is flavor and chirality conserving (proportional to δ_{st}). The divergence of the piece proportional to Δ_{ij}^{qLR} is absorbed by the counterterms to Y^q and A^q while the divergence of the piece stemming from (b) is canceled by a squark mass counterterm.

$$Z_Y = 1 - \frac{\alpha_s}{4\pi} 2C_F \frac{1}{\epsilon}. \quad (18)$$

In fact, it turns out that this renormalization of the Yukawa couplings is necessary for maintaining supersymmetry with respect to the Yukawa coupling involved quark-quark-Higgs coupling and the one of the squark-squark-Higgs coupling.

C. Result for C_{fi}^{dLR} retaining all powers of v/M_{SUSY}

For the Wilson coefficient C_{fi}^{dLR} of the two-quark operator $\bar{q}_f P_R q_i$, we write the general decomposition

$$C_{fi}^{dLR} = C_{fi}^{dLR(1)} + C_{fi}^{dLR(2)} \equiv \frac{\alpha_s}{4\pi} C_{fi}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_{fi}^{(2)}. \quad (19)$$

From Eq. (A2) we directly obtain

$$C_{fi}^{(1)} = \sum_{t=1}^6 \left(4m_{\tilde{g}} C_F W_{ft}^{\tilde{d}} W_{i+3,t}^{\tilde{d}\star} \frac{x_t^2 \ln(x_t)}{1-x_t^2} \right). \quad (20)$$

Here we introduced the abbreviations

$$x_t = m_{\tilde{d}_t} / m_{\tilde{g}}, \quad (21)$$

and for later convenience we also define

$$y_t = m_{\tilde{u}_t} / m_{\tilde{g}}, \quad x_\mu = \mu / m_{\tilde{g}}, \quad (22)$$

where μ is the renormalization scale.

According to the detailed description in the previous subsections, we decompose the Wilson coefficient $C_{fi}^{(2)}$ into various pieces:

$$C_{fi}^{(2)} = C_{fi}^{(2),1} + C_{fi}^{(2),2} + C_{fi}^{(2),3} + C_{fi}^{(2),4} + C_{fi}^{(2),\mu} + C_{fi}^{(2),\text{shift}\alpha_s} + C_{fi}^{(2),m_{\tilde{g}}\overline{m_S} \rightarrow m_{\tilde{g}}\overline{m_{\text{DR}}}} + C_{fi}^{(2),TP}. \quad (23)$$

We freely made use of the unitarity of the mixing matrices $W^{\tilde{q}}$ and obtain

$$C_{fi}^{(2),1} = \sum_{j=1}^3 \sum_{s,t=1}^6 \left\{ 2W_{ft}^{\tilde{d}} W_{j+3,t}^{\tilde{d}\star} W_{i+3,s}^{\tilde{d}\star} W_{j+3,s}^{\tilde{d}} m_{\tilde{g}} C_F (2C_F - C_A) \frac{1}{(1-x_s^2)(1-x_t^2)} \left[(1-x_s^2)^2 \text{Li}_2(1-x_s^2) - (1-x_t^2)^2 \text{Li}_2(1-x_t^2) \right. \right. \\ \left. \left. + (x_s^2 - x_t^2)^2 \text{Li}_2(1-x_t^2/x_s^2) - 4x_t^2(x_t^2 - x_s^2) \ln(x_s) \ln(x_t) + 6x_s^2(x_t^2 - 1) \ln(x_s) - 6x_t^2(x_s^2 - 1) \ln(x_t) + 2x_t^2(x_s^2 - 1) \ln^2(x_t) \right. \right. \\ \left. \left. + 2(x_s^4 + x_t^4 - 3x_t^2 x_s^2 + x_s^2) \ln^2(x_s) \right] \right\} + \sum_{t=1}^6 \left\{ 4W_{ft}^{\tilde{d}} W_{i+3,t}^{\tilde{d}\star} m_{\tilde{g}} C_F (2C_F - C_A) \frac{x_t^2}{(1-x_t^2)^2} \left[(1-2x_t^2) \ln^2(x_t) - 2(1-x_t^2) \ln(x_t) \right] \right\}, \quad (24)$$

$$C_{fi}^{(2),2} = \sum_{s,t=1}^6 \left\{ \frac{W_{ft}^{\tilde{d}} W_{i+3,t}^{\tilde{d}\star} \text{tr} m_{\tilde{g}} C_F}{(1-x_t^2)^2} \left[4(1-x_s^2)(-x_s^2 + 2x_t^2 - 1) \text{Li}_2(1-x_s^2) - 4(x_s + x_t)^2 (x_s - x_t)^2 \text{Li}_2(1-x_s^2/x_t^2) \right. \right. \\ \left. \left. - 4x_t^2((1+x_t^2) \ln(x_t) + 1 - x_t^2)(x_s^2 - 2(1-x_s^2)^2 \ln(x_s)) - \frac{1}{3} [48(\ln(x_t) \ln(x_s) x_s^2 (x_t^2 - x_s^2 - x_t^4) \right. \right. \\ \left. \left. + \ln^2(x_t) x_t^2 (x_t^2 - x_s^2) - \ln(x_t) x_t^4 - x_t^2) + 24(\ln(x_t) \ln(x_s) x_s^4 x_t^2 (1+x_t^2) + \ln(x_s) x_s^2 x_t^2 (x_t^2 - 1) \right. \right. \\ \left. \left. + \ln(x_t) x_t^2 + \ln^2(x_t) x_s^4) + 12(\ln(x_t) x_s^2 x_t^2 (1-x_t^2) + \ln(x_s) x_s^4 (1-x_t^4) - x_s^2 x_t^2) + 6x_s^2 + 6x_s^2 x_t^4 + 30x_t^4 + 18] \right] \right\}, \quad (25)$$

$$C_{fi}^{(2),3} = C_{fi}^{(2),2}(x_s \rightarrow y_s), \quad (26)$$

$$C_{fi}^{(2),4} = \sum_{i=1}^6 \left\{ 2W_{f,t}^{\bar{d}} W_{i+3,t}^{\bar{d}\star} m_{\bar{g}} C_F \left[(-3C_A + 2C_F) \text{Li}_2(1-x_t^2) + \frac{1}{3(1-x_t^2)(1+x_t^2)} [\text{trn}_f((24\ln^2(x_t) - 12)x_t^4 + (24\ln(x_t) + 12)x_t^2) + 6C_A((3\ln^2(x_t) - 11\ln(x_t) + 9)x_t^4 + (3\ln(x_t) - 14)x_t^2 + 5) + 3C_F(-2\ln(x_t) + 1)x_t^4 - (12\ln^2(x_t) - 12\ln(x_t) + 7)x_t^2 + 8\ln(x_t) + 8)] \right] \right\}, \quad (27)$$

$$C_{fi}^{(2),\mu} = \sum_{i=1}^6 \left\{ -4W_{f,t}^{\bar{d}} W_{i+3,t}^{\bar{d}\star} m_{\bar{g}} C_F \frac{\ln(x_\mu^2)}{(1-x_t^2)^2} \left[\text{trn}_f(-2x_t^2((2\ln(x_t) - 1)x_t^2 + 1)) + C_A(3x_t^2((2\ln(x_t) - 1)x_t^2 + 1)) + \frac{C_F}{2}(-4\ln(x_t) - 1)x_t^4 + (2\ln(x_t) + 3)x_t^2 - 8\ln(x_t) - 4) \right] \right\}, \quad (28)$$

$$C_{fi}^{(2),\text{shift}\alpha_s} = \sum_{i=1}^6 \left\{ -\frac{4W_{f,t}^{\bar{d}} W_{i+3,t}^{\bar{d}\star} m_{\bar{g}} C_F x_t^2 \ln(x_t)}{3(1-x_t^2)} \left[\sum_{s=1}^6 (\ln(x_s) + \ln(y_s)) - 4C_A + 3C_F \right] + \frac{4(n_f + 6)W_{f,t}^{\bar{d}} W_{i+3,t}^{\bar{d}\star} m_{\bar{g}} C_F x_t^2 \ln(x_t)}{3(1-x_t^2)} \ln(x_\mu^2) \right\}, \quad (29)$$

$$C_{fi}^{(2),m_{\bar{s}}\overline{\text{MS}} \rightarrow m_{\bar{s}}\overline{\text{DR}}} = -2 \sum_{i=1}^6 \left\{ W_{f,t}^{\bar{d}} W_{i+3,t}^{\bar{d}\star} m_{\bar{g}} C_F C_A \frac{(1+x_t^2)(1-x_t^2+2x_t^2 \ln(x_t))}{(1-x_t^2)^2} \right\}, \quad (30)$$

$$C_{fi}^{(2),TP} = -2m_{\bar{g}} C_F^2 \sum_{i=1}^6 \left\{ W_{f,t}^{\bar{d}} W_{i+3,t}^{\bar{d}\star} \frac{x_t^2}{(1-x_t^2)^2} (1-x_t^2+2\ln(x_t))(1-2\ln(x_t)+\ln(x_\mu^2)) \right\} - 8m_{\bar{g}} C_F^2 \sum_{j,j'=1}^3 \sum_{s,t,s'=1}^6 \left[W_{fs}^{\bar{d}} (W_{j'+3,s}^{\bar{d}\star} W_{j'+3,t}^{\bar{d}} W_{jt}^{\bar{d}\star} W_{js'}^{\bar{d}} + W_{j's}^{\bar{d}\star} W_{jt}^{\bar{d}} W_{j+3,t}^{\bar{d}\star} W_{j+3,s'}^{\bar{d}}) W_{i+3,s'}^{\bar{d}\star} \times \frac{x_t^2(2\ln(x_t) - \ln(x_\mu^2) - 1)(x_s^2 x_{s'}^2 \ln(\frac{x_t}{x_s}) + x_s^2 \ln(x_s) - x_{s'}^2 \ln(x_{s'}))}{(x_s^2 - x_{s'}^2)(x_s^2 - 1)(x_{s'}^2 - 1)} \right]. \quad (31)$$

In the MSSM we have

$$C_A = 3, \quad C_F = 4/3, \quad \text{tr} = 1/2, \quad \text{and } n_f = 6. \quad (32)$$

To summarize, Eqs. (20) and (23) contain the full result for the Wilson coefficient C_{fi}^{dLR} where the A terms, the Yukawa coupling, and the squark and the gluino masses of the MSSM are renormalized in the $\overline{\text{DR}}$ scheme, while g_s stands for the strong coupling constant of the SM in the $\overline{\text{MS}}$ scheme, running with six flavors. The effective operators, or equivalently the Wilson coefficients, are understood to be renormalized according to the $\overline{\text{MS}}$ scheme.

So far, we discussed the derivations of C_{fi}^{dLR} . The corresponding result C_{fi}^{dLR} for up quarks can be obtained by replacing $W^{\bar{d}}$ with $W^{\bar{u}}$ and exchanging x and y .

D. Reduction of the matching scale dependence at NLO

The purpose of our NLO calculation is also the reduction of the matching scale dependence of the effective Higgs couplings that can serve as an estimate of the theory uncertainty. This reduction not only is an improvement

achieved by our NLO calculation but also serves as an additional check of its correctness.

As we will see in the next section, the quantity directly related to the Higgs couplings is \hat{C}_{fi}^{qLR} defined as

$$\hat{C}_{fi}^{qLR} = C_{fi}^{qLR(1)} + C_{fi}^{qLR(2)} + \frac{1}{2} (C_{ff}^{qLL} C_{fi}^{qLR(1)} + C_{fi}^{qLR(1)} C_{ii}^{qLL}) + O(\alpha_s^2, \alpha_s^3 \tan\beta). \quad (33)$$

We use in the following the decomposition $\hat{C}_{fi}^{qLR} = \hat{C}_{fi}^{qLR(1)} + \hat{C}_{fi}^{qLR(2)}$. At LO in our counting of α_s and $\tan\beta$ we have $\hat{C}_{fi}^{qLR(1)} = C_{fi}^{qLR(1)}$.

\hat{C}_{fi}^{qLR} (and thus also $\hat{C}_{fi}^{qLR(1)}$) at a fixed low scale μ_{low} is obtained from \hat{C}_{fi}^{qLR} at the matching scale μ_0 via

$$\hat{C}_{ij}^{qLR}(\mu_{\text{low}}) = U(\mu_{\text{low}}, \mu_0) \hat{C}_{ij}^{qLR}(\mu_0). \quad (34)$$

This evolution is the same as for the quark masses in the SM. The explicit NLO expression can be taken, e.g., from Eq. (4.81) in Ref. [35]. It is this expression that we use for

the numerical study in Sec. [IV E](#) when doing the evolution to the low scale μ_{low} .

However, for showing analytically the reduced matching scale dependence, it is sufficient to assume that the scale μ_{low} is close to the matching scale μ_0 so that it is not necessary to resum large logarithms. In this case the evolution matrix $U(\mu_{\text{low}}, \mu_0)$ can be expanded as

$$U(\mu_{\text{low}}, \mu_0) \approx 1 + \alpha_s(\mu_0) \frac{\gamma_m^{(0)}}{8\pi} \ln\left(\frac{\mu_0^2}{\mu_{\text{low}}^2}\right), \quad \gamma_m^{(0)} = 6C_F. \quad (35)$$

At LO \hat{C}_{ij}^{qLR} depends only implicitly on the renormalization scale via the scale dependence of various parameters. For small changes of the original matching scale μ_0 to a new matching scale μ , we get

$$\frac{\hat{C}_{ij}^{qLR(1)}(\mu)}{\hat{C}_{ij}^{qLR(1)}(\mu_0)} \approx 1 + \frac{\alpha_s(\mu_0)}{4\pi} (\beta_0 + S) \ln\left(\frac{\mu_0^2}{\mu^2}\right). \quad (36)$$

The contribution involving β_0 comes from expressing $\alpha_s(\mu)$ in terms of $\alpha_s(\mu_0)$, while the one involving S is attributable to the corresponding manipulation of the squark and gluino masses, the Yukawa couplings, and the A (and A') terms. Together with Eqs. (34) and (35) the variation of the matching scale leads to the following ratio:

$$\frac{U(\mu_{\text{low}}, \mu) \hat{C}_{ij}^{qLR(1)}(\mu)}{U(\mu_{\text{low}}, \mu_0) \hat{C}_{ij}^{qLR(1)}(\mu_0)} \approx 1 + \frac{\alpha_s(\mu_0)}{4\pi} \left(\beta_0 + S - \frac{\gamma_m^{(0)}}{2} \right) \ln\left(\frac{\mu_0^2}{\mu^2}\right). \quad (37)$$

The explicit μ dependence proportional to α_s in this ratio has to be compensated when going to NLO.

The piece of $\hat{C}_{ij}^{qLR(2)}$ with explicit scale dependence [with contributions from Eqs. (28), (29), and (31), and from Eq. (33) through C_{ff}^{qLL} and C_{ii}^{qLL}], can be compactly written as

$$\hat{C}_{ij}^{qLR(2),\mu}(\mu) = \frac{\alpha_s(\mu)}{4\pi} \left[S - 2 \text{tr}n_f + 3C_A - 3C_F + \frac{n_f}{3} + 2 \right] \hat{C}_{ij}^{qLR(1)}(\mu) \ln(x_\mu^2). \quad (38)$$

Using this information, we finally get at NLO

$$\begin{aligned} & \frac{U(\mu_{\text{low}}, \mu) \hat{C}_{ij}^{qLR}(\mu)}{U(\mu_{\text{low}}, \mu_0) \hat{C}_{ij}^{qLR}(\mu_0)} \\ & \approx 1 + \frac{\alpha_s(\mu_0)}{4\pi} \left(\beta_0 + S - \frac{\gamma_m^{(0)}}{2} - S + 2 \text{tr}n_f - 3C_A \right. \\ & \quad \left. + 3C_F - \frac{n_f}{3} - 2 \right) \ln\left(\frac{\mu_0^2}{\mu^2}\right) \\ & = 1, \end{aligned} \quad (39)$$

as expected.

E. Numerics

In this section we study the numerical importance of our two-loop corrections and the reduced matching scale dependence compared to the one-loop result.

The matching scale dependence, as shown in Fig. 3 for SUSY masses of 1 TeV, is significantly reduced as expected from the previous subsection. Note that the relative importance of the NLO result is to a very good approximation independent of the size of Δ_{ij}^{qLR} .

The relative importance of the two-loop contribution to $\hat{C}_{ij}^{qLR}(\mu)$ is shown in Fig. 4 as a function of the matching scale μ . For SUSY masses of 1 TeV the α_s^2 corrections lead to a constructive contribution of approximately 9% compared to the one-loop result that is in agreement with Ref. [16]. Again, the relative importance of the NLO result is to a very good approximation independent of the size of Δ_{ij}^{qLR} .

F. Transition to the decoupling limit

While the two-loop contributions calculated in this section are obtained in the approximation $p = m_q = 0$, the

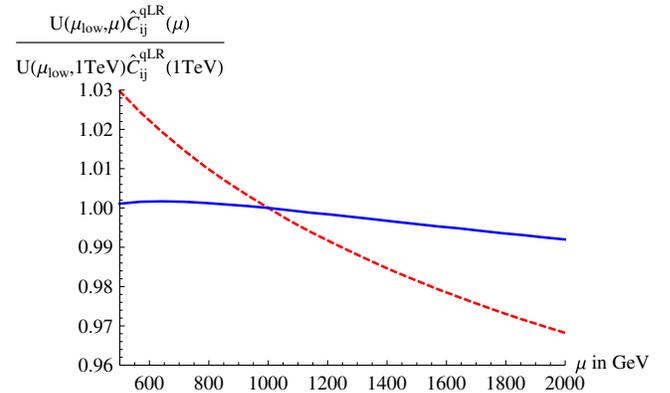


FIG. 3 (color online). Dependence on the matching scale μ of the one-loop and two-loop results for $\hat{C}_{fi}^{qLR}(\mu_{\text{low}})$, using $M_{\text{SUSY}} = 1$ TeV and $\mu_{\text{low}} = m_W$. Red dashed line: matching done at LO; blue solid line: matching done at NLO matching. As expected, the matching scale dependence is significantly reduced. For the one-loop result, \hat{C}_{fi}^{qLR} is understood to be $C_{fi}^{qLR(1)}$ (see text).

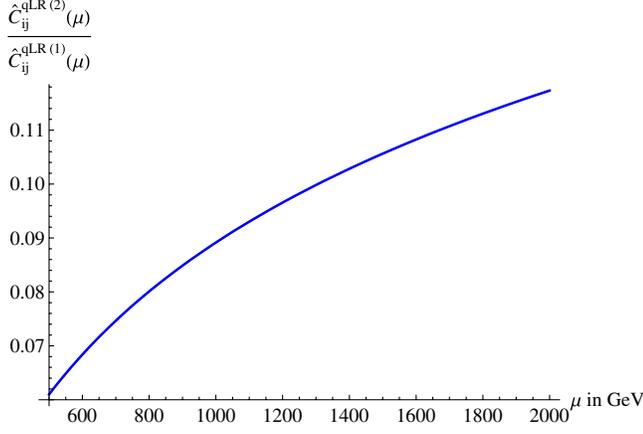


FIG. 4 (color online). Relative importance of the two-loop corrections as a function of the matching scale μ . We see that the two-loop contribution is approximately +9% of the one-loop contribution for $\mu = M_{\text{SUSY}} = 1$ TeV. \hat{C}_{fi}^{qLR} is defined in Eq. (33).

results given in Sec. IV C still contain all powers v/M_{SUSY} implicitly via the squark mixing matrices $W^{\tilde{q}}$ and the physical squark masses $m_{\tilde{q}}$ involved. The transition to the decoupling limit, in which all chirally enhanced corrections can be resummed analytically, can be done by the following prescription.

In all parts of the genuine two-loop contributions listed above [Eqs. (24)–(30)] only two mixing matrices occur, except in Eqs. (24) and (31). Equation (24) contains the following combinations of mixing matrices and a loop function f , which depends on squarks masses $m_{\tilde{q}_s}$ and $m_{\tilde{q}_t}$,

$$\sum_{s,t=1}^6 \sum_{j=1}^3 W_{ft}^{\tilde{d}} W_{j+3,t}^{\tilde{d}\star} W_{i+3,s}^{\tilde{d}\star} W_{j+3,s}^{\tilde{d}} f(x_s^2, x_t^2). \quad (40)$$

Note that in the decoupling limit, the squark with index s in Eq. (40) must be a linear combination of right-handed squark only, since otherwise at least two chirality changes (two insertions of Δ_{ij}^{dLR}) would be necessary. Thus we can replace

$$W_{i+3,s}^{\tilde{d}\star} W_{j+3,s}^{\tilde{d}} \rightarrow \Gamma_{DR}^{ik\star} \Gamma_{DR}^{jk} = \Lambda_{kji}^{dRR} \quad \text{and} \quad x_s^2 \rightarrow x_{Rk}^2, \quad (41)$$

where k only runs from 1 to 3 and we defined

$$x_{L(R)k}^2 = \frac{m_{\tilde{q}_k}^{2L(R)}}{m_{\tilde{g}}^2}. \quad (42)$$

The resulting expression

$$\sum_{t=1}^6 \sum_{k,j=1}^3 W_{ft}^{\tilde{d}} W_{j+3,t}^{\tilde{d}\star} \Lambda_{kji}^{dRR} f(x_{Rk}^2, x_t^2) \quad (43)$$

can now be expanded in powers of v/M_{SUSY} , which amounts at leading order to the replacement

$$\begin{aligned} & \sum_{t=1}^6 W_{ft}^{\tilde{d}} W_{j+3,t}^{\tilde{d}\star} f(\dots, x_t^2) \\ & \rightarrow \sum_{m,n,j',j''=1}^3 \Lambda_{mfj''}^{dLL} \Delta_{j''j'}^{dLR} \Lambda_{nj'j}^{dRR} \frac{f(\dots, x_{Lm}^2) - f(\dots, x_{Rn}^2)}{m_{\tilde{q}_m}^2 - m_{\tilde{q}_n}^2}, \end{aligned} \quad (44)$$

where the dots represent possible additional dependences on squark masses. Now we apply Eq. (44) to Eq. (43) and use

$$\sum_{j=1}^3 \Lambda_{mfj}^{d(LL)RR} \Lambda_{nji}^{d(LL)RR} = \Lambda_{mfi}^{d(LL)RR} \delta_{mn}. \quad (45)$$

The final result for Eq. (40) in the decoupling limit is then

$$\sum_{m,n,j',j''=1}^3 \Lambda_{mfj''}^{dLL} \Delta_{j''j'}^{dLR} \Lambda_{nj'i}^{dRR} \frac{f(x_{Rn}^2, x_{Lm}^2) - f(x_{Rn}^2, x_{Rn}^2)}{m_{\tilde{q}_m}^2 - m_{\tilde{q}_n}^2}. \quad (46)$$

For Eq. (31) a similar procedure works. It contains the following combination of mixing matrices with a loop function depending on three different squark masses with the indices s , t , and s'

$$\begin{aligned} & \sum_{s,t,s'=1}^6 \sum_{j,j'=1}^3 W_{fs}^{\tilde{d}} (W_{j+3,s}^{\tilde{d}\star} W_{j+3,t}^{\tilde{d}} W_{jt}^{\tilde{d}\star} W_{js'}^{\tilde{d}} \\ & + W_{js}^{\tilde{d}\star} W_{j't}^{\tilde{d}} W_{j+3,t}^{\tilde{d}\star} W_{j+3,s'}^{\tilde{d}}) W_{i+3,s'}^{\tilde{d}\star} f(x_s^2, x_t^2, x_{s'}^2). \end{aligned} \quad (47)$$

Note that the first term in Eq. (47) vanishes in the decoupling limit since it necessarily involves multiple chirality flips. For the second term two replacements analogous to Eq. (41) have to be performed, and after using two times the relation in Eq. (45) the decoupling limit of Eq. (47) reads

$$\sum_{m,n,j',j''=1}^3 \Lambda_{mfj''}^{dLL} \Delta_{j''j'}^{dLR} \Lambda_{nj'i}^{dRR} \frac{f(x_{Lm}^2, x_{Lm}^2, x_{Rn}^2) - f(x_{Lm}^2, x_{Rn}^2, x_{Rn}^2)}{m_{\tilde{q}_m}^2 - m_{\tilde{q}_n}^2}. \quad (48)$$

This result involves the same combination of mixing matrices as the one in Eq. (46).

To all other parts of C_{fi} the rule in Eq. (44) can be applied directly to obtain the corresponding expression in the decoupling limit.

V. RELATIONS BETWEEN QUARK MASSES AND THE MSSM YUKAWA COUPLINGS AT NLO

Beyond one-loop Eqs. (11) and (12) for the determination of Y^d can easily be generalized to higher loop orders because the chirality changing self-energy (and also the resulting Wilson coefficient) is still proportional to one element Δ_{ij}^{dLR} in the decoupling limit, as shown in Sec. IV F. However, since we are dealing with order one corrections, we must specify how we count contributions at

higher loop orders in α_s . $C_{fi}^{qLR(1)}$ is proportional to $\alpha_s \tan\beta$, and $C_{fi}^{qLR(2)}$ is proportional to $\alpha_s^2 \tan\beta$. Here, $\tan\beta$ stands schematically for a chiral enhancement factor, also including $A_{ij}^q/(Y_{ij}^q M_{\text{SUSY}})$. We will count $\alpha_s \tan\beta$ as order one and thus $\alpha_s^2 \tan\beta$ as order α_s . Since $C_{fi}^{qLL,RR}$ is not chirally enhanced, the only relevant term in our approximation (of order α_s) is the one-loop contribution. Thus, $C_{fi}^{qLL,RR}$ is always understood to be the one-loop contribution proportional to α_s .

To derive the relation between the quark masses and the Yukawa couplings of the MSSM superpotential at NLO, we also need to specify the renormalization scheme used for the matching procedure. Let us explicitly denote the renormalization scheme for the quantities in the matching condition Eq. (11) (at the scale m_{SUSY}), which is important at NLO:

$$\begin{aligned} v_q Y_{\text{eff}}^{q_i \overline{\text{MS}}} &= m_{q_i}^{\overline{\text{MS}}} \\ &= (v_q Y_{\text{tree}}^{q_i \overline{\text{MS}}} + C_{ii \overline{\text{MS}}}^{qLR(1)} + C_{ii \overline{\text{MS}}}^{qLR(2)}) \\ &\quad \times \left(1 + \frac{1}{2}(C_{ii}^{qLL} + C_{ii}^{qRR})\right). \end{aligned} \quad (49)$$

Again, $Y_{\text{tree}}^{q_i \overline{\text{MS}}}$ is the Wilson coefficient induced via the Yukawa coupling of the MSSM. This means at the matching scale it is given by

$$\begin{aligned} Y_{\text{tree}}^{q_i \overline{\text{MS}}}(\mu_{\text{SUSY}}) &= \frac{Y_{\text{tree}}^{q_i}}{M_{\text{SUSY}}}(\mu_{\text{SUSY}}) \\ &= \left(1 + \frac{\alpha_s}{4\pi} C_F\right) Y_{\text{DR}}^{q_i}(\mu_{\text{SUSY}}). \end{aligned} \quad (50)$$

In our counting in α_s and $\tan\beta$ the renormalization scheme for C_{ii}^{qLL} and C_{ii}^{qRR} is irrelevant. Note that the quark mass m_{q_i} is understood to be evaluated at the matching scale. Further, one should recall from the last section that despite the fact that we renormalized C_{ii}^{qLR} in the $\overline{\text{MS}}$ scheme, it contains parameters given in the $\overline{\text{DR}}$ scheme, e.g., $Y_{\text{tree}}^{q_i} = Y_{\text{DR}}^{q_i}$. Since we are interested in $Y_{\text{tree}}^{q_i}$, the Yukawa coupling of the MSSM superpotential, we must express $Y_{\text{tree}}^{q_i \overline{\text{MS}}}$ in Eq. (49) in terms of $Y_{\text{DR}}^{q_i}$ via Eq. (50) so that we can solve for $Y_{\text{DR}}^{q_i}$.

In conclusion, we arrive at the NLO generalization (order $\alpha_s^2 \tan\beta$) of Eq. (11):

$$Y_{\text{DR}}^{q_i} = \frac{m_{d_i}^{\overline{\text{MS}}} - \hat{C}_{ii \cancel{X}_i}^{dLR}}{v_d(1 + \frac{\alpha_s}{4\pi} C_F + \tan\beta \hat{\epsilon}_i^d)}, \quad (51)$$

with \hat{C}_{fi}^{qLR} defined in Eq. (33) and the corresponding equation for $\hat{\epsilon}_i^d$. Here $\epsilon_i^{d(1)}$ and $\epsilon_i^{d(2)}$ are defined in direct analogy to Eq. (13). Further, the Wilson coefficients appearing here are assumed to be in the decoupling limit. Equation (51) constitutes the NLO determination of the Yukawa coupling

of the superpotential. When later inserting the Yukawa coupling into the Wilson coefficients, one has to use this relation.⁹

The electroweak contributions (involving charginos and neutralinos) to the relation between the quark masses and the Yukawa couplings are in most regions of the parameter space subleading compared to the strong contributions. However, the LO electroweak corrections are easily as large as the NLO SQCD corrections and should be included in a numerical analysis. This can be achieved by simply adding the corresponding contributions to $\hat{C}_{ii \cancel{X}_i}^{dLR}$ and $\hat{\epsilon}_i^d$ in Eq. (51).

VI. EFFECTIVE HIGGS VERTICES

To derive the effective Higgs-quark-quark couplings,¹⁰ we have to assume that the external momenta (flowing through the Higgs-quark-quark vertex) are much smaller than the masses of the virtual SUSY particles running in the loop. This assumption limits the applicability of the resulting Feynman rules. If m_{H^0} , m_{A^0} , $m_{H^\pm} \ll M_{\text{SUSY}}$ (H^0 , A^0 , and H^\pm denote the neutral CP -even, CP -odd, and the charged Higgs boson, respectively), the effective Feynman rules can be used for the calculation of all flavor observables (also if the Higgs is propagating in a loop) and for processes with a Higgs on the mass shell. If the hierarchy m_{H^0} , m_{A^0} , $m_{H^\pm} \ll M_{\text{SUSY}}$ is not satisfied, the effective Higgs vertices can still be used for processes in which the momentum flow through the Higgs-quark-quark vertex is small compared to M_{SUSY} , which is true for all low-energy flavor observables with tree-level Higgs exchange (like $B_{d,s} \rightarrow \mu^+ \mu^-$, $B^+ \rightarrow \tau^+ \nu$ or the double Higgs penguin contributing to $\Delta F = 2$ processes).

As discussed in the Introduction, we use an effective field theory approach in our study of the Higgs-quark-quark couplings, which simplifies the calculations significantly. This means that we match the MSSM on the 2HDM of type III at the scale M_{SUSY} rather than calculating the Higgs-quark-quark coupling within the MSSM.

Let us first consider the effective Lagrangian of a general 2HDM (including Higgs-quark-quark couplings and kinetic terms):

⁹The generalization to the CKM matrix can be achieved following the procedure of Refs. [7,10,11].

¹⁰In principle also the renormalization of the Higgs potential should be addressed. Our derivation of chirally enhanced flavor effects does not depend on the specific relations between Higgs self-couplings and their masses. Since no chirally enhanced effects occur in the Higgs sector, it is consistent to use the tree-level values for the Higgs parameters. However, one can as well use the NLO values for the Higgs masses and mixing angles, which might be even better from the numerical point of view.

$$\begin{aligned}
\mathcal{L}^{\text{eff}} = & \bar{Q}_{fL}^a ((Y_{fiew}^{\text{dtree}} + E_{fi}^{\text{dew}}) \epsilon_{ba} H_d^{b*} - E_{fi}^{\prime\text{dew}} H_u^a) d_{iR} \\
& + \bar{Q}_{fL}^a ((Y_{fiew}^{\text{utree}} + E_{fi}^{\text{uew}}) \epsilon_{ab} H_u^{b*} - E_{fi}^{\prime\text{uew}} H_d^a) u_{iR} \\
& + \bar{d}_{fR} i \not{\partial} (\delta_{fi} - R_{fi}^{\text{dew}}) d_{iR} + \bar{u}_{fR} i \not{\partial} (\delta_{fi} - R_{fi}^{\text{uew}}) u_{iR} \\
& + \bar{Q}_{fL}^a i \not{\partial} (\delta_{fi} - L_{fi}^{\text{qew}}) Q_{iL}^a, \quad (52)
\end{aligned}$$

where adding the Hermitian conjugate of the terms involving Higgs fields is implicitly meant. The Higgs doublets are defined as

$$H_d = \begin{pmatrix} H_d^1 \\ H_d^2 \end{pmatrix} = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \quad H_u = \begin{pmatrix} H_u^1 \\ H_u^2 \end{pmatrix} = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}. \quad (53)$$

In Eq. (52) a, b denote $SU(2)_L$ indices and ϵ_{ab} is the two-dimensional antisymmetric tensor with $\epsilon_{12} = -1$. We introduced the holomorphic couplings E_{fi}^{qew} , the nonholomorphic couplings $E_{fi}^{\prime\text{qew}}$ ($q = u, d$), and the contributions to the kinetic terms $R_{fi}^{d,uew}$ and L_{fi}^{qew} . Here the superscript

“ew” refers to the fact that these terms are given in a weak-interaction eigenbasis. In Eq. (52) we already anticipated the MSSM where the terms $E_{fi}^{(\prime)\text{qew}}$, L_{fi}^{qew} , and R_{fi}^{qew} are loop induced but Y_{fiew}^{utree} and Y_{fiew}^{dtree} are generated at tree level via the MSSM Yukawa couplings.¹¹

To connect the effective theory to the MSSM, we go to the super-CKM basis, in which the Yukawa couplings are diagonal, by rotating the fields

$$q_{jL,R} \rightarrow U_{ji}^{qL,R(0)} q_{iL,R}, \quad (54)$$

such that

$$U_{kf}^{qL(0)*} Y_{kjew}^{\text{qtree}} U_{ji}^{qR(0)} = Y_{tree}^{qi} \delta_{fi}. \quad (55)$$

We now break the electroweak symmetry and write the effective Lagrangian in component form:

$$\begin{aligned}
\mathcal{L}^{\text{eff}} = & \bar{u}_{fL} V_{jf}^{(0)} ((Y_{tree}^{dj} \delta_{ji} + E_{ji}^d) H_d^{2*} - E_{ji}^{\prime d} H_u^1) d_{iR} + \bar{d}_{fL} V_{jf}^{(0)*} ((Y_{tree}^{uj} \delta_{ji} + E_{ji}^u) H_u^{1*} - E_{ji}^{\prime u} H_d^2) u_{iR} - \bar{d}_{fL} ((Y_{tree}^{df} \delta_{fi} + E_{fi}^d) H_d^{1*} \\
& + E_{fi}^{\prime d} H_u^2) d_{iR} - \bar{u}_{fL} ((Y_{tree}^{uf} \delta_{fi} + E_{fi}^u) H_u^{2*} + E_{fi}^{\prime u} H_d^1) u_{iR} + \bar{d}_{fR} i \not{\partial} (\delta_{fi} - R_{fi}^d) d_{iR} + \bar{u}_{fR} i \not{\partial} (\delta_{fi} - R_{fi}^u) u_{iR} \\
& + \bar{d}_{fL} i \not{\partial} (\delta_{fi} - L_{fi}^d) d_{iL} + \bar{u}_{fL} i \not{\partial} (\delta_{fi} - L_{fi}^u) u_{iL} - \bar{d}_{fL} ((Y_{tree}^{df} \delta_{fi} + E_{fi}^d) v_d + E_{fi}^{\prime d} v_u) d_{iR} \\
& - \bar{u}_{fL} ((Y_{tree}^{uf} \delta_{fi} + E_{fi}^u) v_u + E_{fi}^{\prime u} v_d) u_{iR}, \quad (56)
\end{aligned}$$

where $V^{(0)} = U^{uL(0)\dagger} U^{dL(0)}$ is not the physical CKM matrix, but rather the CKM matrix generated by the misalignment of the Yukawa couplings. Adding the Hermitian conjugate of the mass terms and the terms involving Higgs fields is tacitly understood. The terms

$$\begin{aligned}
E_{fi}^q &= U_{kf}^{qL(0)*} E_{kj}^{\text{qew}} U_{ji}^{qR(0)}, \quad E_{fi}^{\prime q} = U_{kf}^{qL(0)*} E_{kj}^{\prime\text{qew}} U_{ji}^{qR(0)}, \\
R_{fi}^q &= U_{kf}^{qR(0)*} R_{kj}^{\text{qew}} U_{ji}^{qR(0)}, \quad L_{fi}^d = U_{kf}^{dL(0)*} L_{kj}^{\text{qew}} U_{ji}^{dL(0)}, \quad (57) \\
L_{fi}^u &= U_{kf}^{uL(0)*} L_{kj}^{\text{qew}} U_{ji}^{uL(0)} = V_{fk}^{(0)} L_{kj}^d V_{ij}^{(0)*}
\end{aligned}$$

are now given in the super-CKM basis. Note that this is the same basis as the one in which the effective Lagrangian of Eq. (2) is given (and the same basis in which we calculated the MSSM contributions to the Wilson coefficients). Thus, comparing the last four lines of Eq. (56) to Eq. (2) we have the following relation between the Wilson coefficients and the terms of the 2HDM III Lagrangian (at an arbitrary loop order):

¹¹In principle, without knowing anything about the MSSM, the holomorphic corrections could be absorbed into an effective Yukawa coupling (and also the corrections to the kinetic terms $R_{fi}^{d,uew}$ and L_{fi}^{qew} would not be physical). However, once we go back to the MSSM with the SUSY breaking terms as input parameters, the holomorphic corrections also become physical.

$$\begin{aligned}
E_{fi}^d &= \frac{C_{fiA}^{dLR}}{v_d}, \quad E_{fi}^{\prime d} = \frac{C_{fi}^{\prime dLR}}{v_u}, \quad E_{fi}^u = \frac{C_{fiA}^{uLR}}{v_u}, \\
E_{fi}^{\prime u} &= \frac{C_{fi}^{\prime uLR}}{v_d}, \quad L_{fi}^q = C_{fi}^{qLL}, \quad R_{fi}^q = C_{fi}^{qRR}. \quad (58)
\end{aligned}$$

Now we want to go to the physical basis with flavor-diagonal mass terms and canonical kinetic terms. As a first step we render the kinetic terms canonical by a field redefinition:

$$q_{iL} \rightarrow \left(\delta_{ij} + \frac{1}{2} L_{ij}^q \right) q_{jL}, \quad q_{iR} \rightarrow \left(\delta_{ij} + \frac{1}{2} R_{ij}^q \right) q_{jR}. \quad (59)$$

Consider now the quark mass matrices. The redefinition of the fields in Eq. (59) also leads to a shift in down-quark mass matrix so that it is now given by

$$\begin{aligned}
m_{fi}^d &= (\hat{E}_{fi}^d + \hat{Y}_{fi}^{\text{dtree}}) v_d + v_u \hat{E}_{fi}^{\prime d} = \hat{C}_{fi}^{\text{dLR}} + v_d \hat{Y}_{fi}^{\text{dtree}}, \\
m_{fi}^u &= (\hat{E}_{fi}^u + \hat{Y}_{fi}^{\text{utree}}) v_u + v_d \hat{E}_{fi}^{\prime u} = \hat{C}_{fi}^{\text{uLR}} + v_u \hat{Y}_{fi}^{\text{utree}}, \quad (60)
\end{aligned}$$

where we have defined

$$\begin{aligned}\hat{E}_{fi}^{(l)q} &= E_{fi}^{(l)q} + \frac{1}{2} \sum_{j=1}^3 (L_{fj}^q E_{ji}^{(l)q} + E_{fj}^{(l)q} R_{ji}^q), \\ \hat{Y}_{fi}^{q\text{tree}} &= Y_{\text{tree}}^{qi} \delta_{fi} + \frac{1}{2} \sum_{j=1}^3 (C_{fj}^{qLL} Y_{\text{tree}}^{qi} \delta_{ji} + Y_{\text{tree}}^{qf} \delta_{fj} C_{ji}^{qRR}), \\ \hat{C}_{fi}^{qLR} &= C_{fi}^{qLR} + \frac{1}{2} \sum_{j=1}^3 (C_{fj}^{qLL} C_{ji}^{qLR} + C_{fj}^{qLR} C_{ji}^{qRR}).\end{aligned}\quad (61)$$

Note that the quantities with a double hat contain also the contributions from flavor-changing left-left (LL) and right-right (RR) Wilson coefficients, while the quantities with one hat [see Eqs. (33) and (67)] contain only the flavor-conserving LL and RR Wilson coefficients.

We now diagonalize the quark mass matrices by a bi-unitary transformation

$$U_{kf}^{qL*} m_{kj}^q U_{ji}^{qR} = m_{q_i} \delta_{fi}, \quad (62)$$

where the rotation matrices

$$\begin{aligned}U^{qL} &= \begin{pmatrix} 1 & \frac{m_{12}^q}{m_{q_2}} & \frac{m_{13}^q}{m_{q_3}} \\ \frac{-m_{12}^{q*}}{m_{q_2}} & 1 & \frac{m_{23}^q}{m_{q_3}} \\ \frac{-m_{13}^{q*}}{m_{q_3}} + \frac{m_{12}^{q*} m_{23}^q}{m_{q_2} m_{q_3}} & \frac{-m_{23}^{q*}}{m_{q_3}} & 1 \end{pmatrix}, \\ U^{qR} &= \begin{pmatrix} 1 & \frac{m_{21}^{q*}}{m_{q_2}} & \frac{m_{31}^{q*}}{m_{q_3}} \\ \frac{-m_{21}^q}{m_{q_2}} & 1 & \frac{m_{32}^{q*}}{m_{q_3}} \\ \frac{-m_{31}^q}{m_{q_3}} + \frac{m_{21}^q m_{32}^q}{m_{q_2} m_{q_3}} & \frac{-m_{32}^{q*}}{m_{q_3}} & 1 \end{pmatrix}\end{aligned}\quad (63)$$

are obtained from a perturbative diagonalization of the quark mass matrix.¹²

Switching to the physical basis in which the quark mass matrices are diagonal, these rotations modify the effective Lagrangian as follows [20]:

$$\begin{aligned}\mathcal{L}^{\text{eff}} &= \bar{u}_{fL} U_{kf}^{uL*} V_{kk'}^{(0)*} \left(\frac{m_{kj}^d}{v_d} H_d^{2*} - \hat{E}_{k'j}^{ld} (H_u^1 + \tan(\beta) H_d^{2*}) \right) U_{ji}^{dR} d_{iR} + \bar{d}_{fL} U_{kf}^{dL*} V_{k'k}^{(0)*} \left(\frac{m_{kj}^u}{v_u} H_u^{1*} - \hat{E}_{k'j}^{lu} (H_d^2 + \cot(\beta) H_u^{1*}) \right) U_{ji}^{dR} u_{iR} \\ &\quad - \bar{d}_{fL} U_{kf}^{dL*} \left(\frac{m_{kj}^d}{v_d} H_d^{1*} + \hat{E}_{kj}^{ld} (H_u^2 - \tan(\beta) H_d^{1*}) \right) U_{ji}^{dR} d_{iR} - \bar{u}_{fL} U_{kf}^{uL*} \left(\frac{m_{kj}^u}{v_u} H_u^{2*} + \hat{E}_{kj}^{lu} (H_d^1 - \cot(\beta) H_u^{2*}) \right) U_{ji}^{uR} u_{iR} + \text{H.c.},\end{aligned}\quad (64)$$

where we skipped the mass terms and the kinetic terms. This can be further simplified by using the physical CKM matrix given by

$$V_{fi} = U_{jf}^{uL*} V_{jk}^{(0)} U_{ki}^{dL}. \quad (65)$$

In addition, we define the abbreviations

$$\bar{E}_{fi}^{lq} = U_{kf}^{qL*} \hat{E}_{kj}^{lq} U_{ji}^{qR} = \hat{E}_{fi}^{lq} - \begin{pmatrix} 0 & \hat{E}_{22}^{lq} \hat{c}_{12}^{qLR} & \hat{E}_{33}^{lq} (\hat{c}_{13}^{qLR} - \hat{c}_{12}^{qLR} \hat{c}_{23}^{qLR}) \\ \hat{E}_{22}^{lq} \hat{c}_{21}^{qLR} & 0 & \hat{E}_{33}^{lq} \hat{c}_{23}^{qLR} \\ \hat{E}_{33}^{lq} (\hat{c}_{31}^{qLR} - \hat{c}_{32}^{qLR} \hat{c}_{21}^{qLR}) & \hat{E}_{33}^{lq} \hat{c}_{32}^{qLR} & 0 \end{pmatrix}_{fi} \equiv \hat{E}_{fi}^{lq} - \Delta \hat{E}_{fi}^{lq}. \quad (66)$$

Note that in this expression only quantities with a single hat defined as

$$\hat{E}_{fi}^{(l)q} = E_{fi}^{(l)q} + \frac{1}{2} (L_{ff}^q E_{fi}^{(l)q} + E_{fi}^{(l)q} R_{ii}^q), \quad (67)$$

and \hat{c}_{ij}^{qLR} defined by combining Eq. (33) with

$$c_{ji}^{qLR} = \frac{C_{ji}^{qLR}}{\max\{m_q, m_{q_i}\}}, \quad (68)$$

enter. This is in agreement with the finding of Ref. [23] that the effect of the flavor-changing LL and RR self-energies drops out in the effective Higgs vertices.

Finally, to arrive at the effective Feynman rules, we project the fields H_u^0 and H_d^0 onto the physical components H^0 , h^0 , A^0 , and H^\pm as

¹²Note that these rotations are identical to the ones obtained in the diagrammatic approach (see Ref. [20] for details).

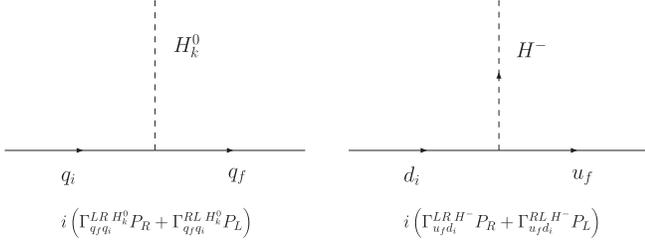


FIG. 5. Higgs-quark vertices with the corresponding Feynman rules. The couplings with exchanged chirality structure are obtained from Eq. (70) by using $\Gamma_{q_f q_i}^{LRH} = \Gamma_{q_i q_f}^{LRH^*}$.

$$\begin{aligned}
 H_u^0 &= \frac{1}{\sqrt{2}}(H^0 \sin\alpha + h^0 \cos\alpha + iA^0 \cos\beta), \\
 H_d^0 &= \frac{1}{\sqrt{2}}(H^0 \cos\alpha - h^0 \sin\alpha + iA^0 \sin\beta), \\
 H_u^{1*} &= \cos(\beta)H^-, \quad H_d^2 = \sin(\beta)H^-.
 \end{aligned} \tag{69}$$

Using Eqs. (65), (66), and (69), the effective Lagrangian in Eq. (64) leads to the following effective Higgs-quark-quark Feynman rules¹³ shown in Fig. 5 (note that the CKM matrix V in the charged Higgs coupling is the physical one):

$$\begin{aligned}
 \Gamma_{u_f u_i}^{LRH_k^0} &= x_u^k \left(\frac{m_{u_i}}{v_u} \delta_{fi} - \tilde{E}_{fi}^{\prime u} \cot\beta \right) + x_d^{k*} \tilde{E}_{fi}^{\prime u}, \\
 \Gamma_{d_f d_i}^{LRH_k^0} &= x_d^k \left(\frac{m_{d_i}}{v_d} \delta_{fi} - \tilde{E}_{fi}^{\prime d} \tan\beta \right) + x_u^{k*} \tilde{E}_{fi}^{\prime d}, \\
 \Gamma_{u_f d_i}^{LRH^\pm} &= \sum_{j=1}^3 \sin\beta V_{fj} \left(\frac{m_{d_i}}{v_d} \delta_{ji} - \tilde{E}_{ji}^{\prime d} \tan\beta \right), \\
 \Gamma_{d_f u_i}^{LRH^\pm} &= \sum_{j=1}^3 \cos\beta V_{jf}^* \left(\frac{m_{u_i}}{v_u} \delta_{ji} - \tilde{E}_{ji}^{\prime u} \tan\beta \right),
 \end{aligned} \tag{70}$$

where for $H_k^0 = (H^0, h^0, A^0)$ the coefficients x_q^k are given by

$$\begin{aligned}
 x_d^k &= \left(-\frac{1}{\sqrt{2}} \cos\alpha, \frac{1}{\sqrt{2}} \sin\alpha, \frac{i}{\sqrt{2}} \sin\beta \right), \\
 x_u^k &= \left(-\frac{1}{\sqrt{2}} \sin\alpha, -\frac{1}{\sqrt{2}} \cos\alpha, \frac{i}{\sqrt{2}} \cos\beta \right).
 \end{aligned} \tag{71}$$

It is important to keep in mind that the \hat{c}_{ij}^{qLR} in Eq. (66) must be calculated using the quantities Y^q and $V^{(0)}$ of the MSSM superpotential.

Note that without the nonholomorphic corrections $E_{ij}^{\prime q}$ the rotation matrices U^{qLR} would simultaneously

¹³Note that some of the Higgs-quark-quark couplings are suppressed by a factor $\cos\beta$ or $\sin\alpha$ stemming from the Higgs mixing matrices. If one decides to keep these suppressed couplings, one should be aware of the fact that they receive proper vertex corrections in which the suppression factor does not occur and which are thus $\tan\beta$ enhanced with respect to the tree-level couplings. Such enhanced corrections to the coupling of H^\pm to right-handed up quarks are important for $b \rightarrow s\gamma$ [36,37].

diagonalize the effective mass terms and the neutral Higgs couplings in Eq. (64). However, in the presence of nonholomorphic corrections this is no longer the case, and apart from a flavor-changing nonholomorphic correction a term proportional to a flavor-conserving nonholomorphic correction times a flavor-changing self-energy is also generated.

A. Effective Higgs-quark-quark vertices at NLO

The effective Higgs-quark-quark vertices at NLO in the MSSM are obtain in the following way: After inserting the definition for $\tilde{E}_{fi}^{\prime q}$ [see Eq. (66)] into Eq. (70) we express $\tilde{E}_{fi}^{\prime q}$ through $\hat{C}_{fi}^{(l)q}$ according to Eq. (58).

VII. CONCLUSIONS

In this article we computed the genuine two-loop SQCD corrections to the chirality-changing quark self-energies. In the limit where the external momentum and the quark mass are zero, we presented relatively simple analytic results without making further assumptions on the SUSY spectrum. Because of the one-to-one correspondence (in the decoupling limit) between chirality-changing quark self-energies and Higgs-quark-quark vertices, this is an efficient and elegant way of calculating at the same time not only effective Higgs vertices but also the Yukawa couplings and CKM elements of the MSSM superpotential in terms of the physical quark masses and the physical CKM matrix.

Our next-to-leading order results increase the values of Wilson coefficients C_{fi}^{qLR} of the operators $\bar{q}_f P_R q_i$ by approximately 9% compared to the values obtained at leading order. This means that, since at large $\tan\beta$ the threshold corrections to the Yukawa couplings are of order one, the effect on the down-quark Yukawa couplings of the two-loop corrections is $\mathcal{O}(10\%)$. At the same time the matching scale uncertainty of the effective Higgs-quark-quark couplings and of the corresponding Wilson coefficients is significantly reduced (see Fig. 3).

We resummed all chirally enhanced corrections modifying the relation between the quark masses and the Yukawa couplings of the MSSM superpotential up to order $\alpha_s^{n+1} \tan^n \beta$ [see Eq. (51)]. The resulting MSSM Yukawa couplings can be used for a precision study of Yukawa unification. Furthermore, using these Yukawa couplings, we derived effective Higgs-quark-quark vertices [see Eq. (70)] entering the calculation of FCNC processes and also of Higgs decays, as long as the momentum transfer is small compared to the SUSY scale.

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APPENDIX: ONE-LOOP RESULTS

Here we summarize various one-loop results necessary for the two-loop calculation of the chirality flipping self-energy (see Ref. [38] for details). Unless stated otherwise, all expressions appearing in this appendix were obtained in dimensional regularization. The matrices W^q diagonalize the squark mass matrices according to Eq. (6), and we use the definitions

$$x_s = \frac{m_{\tilde{d}_s}}{m_{\tilde{g}}}, \quad y_s = \frac{m_{\tilde{u}_s}}{m_{\tilde{g}}}, \quad x_\mu = \frac{\mu}{m_{\tilde{g}}}. \quad (\text{A1})$$

1. Self-energies

Here we give the explicit one-loop results for quark, gluino, and squark self-energies in dimensional regularization, where we put $D = 4 - 2\varepsilon$ and write the renormalization scale in the form $\mu e^{\gamma/2}/(\sqrt{4\pi})$. Our conventions are such that the calculation of the truncated self-energy diagrams give $-i\Sigma$.

a. Quark

The one-loop quark self-energies induced by gluinos and squarks are given by

$$\begin{aligned} \Sigma_{q_f q_i}^{\tilde{g}LR}(0) &= \frac{\alpha_s}{2\pi} W_{f_s}^{\tilde{q}} W_{i+3,s}^{\tilde{q}*} C_F m_{\tilde{g}} B_0(0; m_{\tilde{g}}^2, m_{q_s}^2) \\ &= \frac{\alpha_s}{2\pi} W_{f_s}^{\tilde{q}} W_{i+3,s}^{\tilde{q}*} C_F m_{\tilde{g}} \left(\frac{x_s^2 \ln(x_s^2)}{1-x_s^2} - \varepsilon \frac{x_s^2 (\ln^2(x_s^2) - 2 \ln(x_s^2) - 2 \ln(x_s^2) \ln(x_\mu^2))}{2(1-x_s^2)} + O(\varepsilon^2) \right), \\ \Sigma_{q_f q_i}^{\tilde{g}LL}(0) &= \frac{\alpha_s}{2\pi} W_{f_s}^{\tilde{q}} W_{i,s}^{\tilde{q}*} C_F B_1(0; m_{\tilde{g}}^2, m_{q_s}^2) \\ &= -\frac{\alpha_s}{4\pi} W_{f_s}^{\tilde{q}} W_{i,s}^{\tilde{q}*} C_F \left(\frac{1}{\varepsilon} + \ln(x_\mu^2) + \frac{3 - 4x_s^2 + x_s^4 + (4x_s^2 - 2x_s^4) \ln(x_s^2)}{2(1-x_s^2)^2} \right) + O(\varepsilon). \end{aligned} \quad (\text{A2})$$

Using unitarity, we can replace $B_0(0; m_{\tilde{g}}^2, m_{q_s}^2)$ by $[B_0(0; m_{\tilde{g}}^2, m_{q_s}^2) - B_0(0; m_{\tilde{g}}^2, 0)]$ in the first line of $\Sigma_{q_f q_i}^{\tilde{g}LR}(0)$. This we did when writing the explicit expression.

The ordinary gluon correction reads in Feynman gauge

$$\begin{aligned} \Sigma_{q_f q_i}^{gLL,RR}(p^2) &= \frac{\alpha_s}{4\pi} C_F (d-2) B_1(p^2; m_{q_i}^2, 0) \delta_{fi}, \\ &= \frac{\alpha_s}{4\pi} C_F \left(-\frac{1}{\varepsilon} + \frac{(p^2)^2 - m_{q_i}^4}{(p^2)^2} \ln\left(\frac{m_{q_i}^2 - p^2 - i0}{m_{q_i}^2}\right) - \frac{m_{q_i}^2}{p^2} + \ln\left(\frac{m_{q_i}^2}{\mu^2}\right) - 1 \right) \delta_{fi} + O(\varepsilon), \end{aligned} \quad (\text{A3})$$

$$\begin{aligned} \Sigma_{q_f q_i}^{gLR,RL}(p^2) &= \frac{\alpha_s}{4\pi} C_F d m_{q_i} B_0(p^2; m_{q_i}^2, 0) \delta_{fi}, \\ &= \frac{\alpha_s}{\pi} C_F m_{q_i} \left(\frac{1}{\varepsilon} + \frac{m_{q_i}^2 - p^2}{p^2} \ln\left(\frac{m_{q_i}^2 - p^2 - i0}{m_{q_i}^2}\right) - \ln\left(\frac{m_{q_i}^2}{\mu^2}\right) + \frac{3}{2} \right) \delta_{fi} + O(\varepsilon). \end{aligned} \quad (\text{A4})$$

Note that Eqs. (A3) and (A4) are given in dimensional regularization (not dimensional reduction).

b. Gluino

Here we assume that of the three gaugino masses the gluino mass is chosen to be real, which is always possible. For the gluino self-energy the part induced by a gluon reads

$$\Sigma_{\tilde{g}\tilde{g}}^g(p^2) = \frac{\alpha_s}{4\pi} C_A (d m_{\tilde{g}} B_0(p^2; m_{\tilde{g}}^2, 0) + \not{p}(d-2) B_1(p^2; m_{\tilde{g}}^2, 0)), \quad (\text{A5})$$

which decomposes for on-shell gluinos into

$$\Sigma_{\tilde{g}\tilde{g}}^{gLR,RL}(m_{\tilde{g}}^2) = \frac{\alpha_s}{\pi} C_A m_{\tilde{g}} \left(\frac{1}{\varepsilon} + \frac{3}{2} + \ln(x_\mu^2) \right) + O(\varepsilon), \quad \Sigma_{\tilde{g}\tilde{g}}^{gLL,RR}(m_{\tilde{g}}^2) = -\frac{\alpha_s}{4\pi} C_A \left(\frac{1}{\varepsilon} + 2 + \ln(x_\mu^2) \right) + O(\varepsilon), \quad (\text{A6})$$

where we inserted the explicit expressions for the loop functions. The part of the gluino self-energy with squarks and quarks as virtual particles in the approximation $m_q = 0$ is given by

$$\Sigma_{\tilde{g}\tilde{g}}^{q\tilde{q}LR,RL}(p^2) = 0, \quad \Sigma_{\tilde{g}\tilde{g}}^{q\tilde{q}LL,RR}(p^2) = \frac{\alpha_s}{4\pi} 2 \text{tr} \sum_{s=1}^6 (B_1(p^2; 0, m_{\tilde{d}_s}^2) + B_1(p^2; 0, m_{\tilde{u}_s}^2)), \quad (\text{A7})$$

where the latter reads explicitly for on-shell gluinos

$$\Sigma_{\tilde{g}\tilde{g}}^{q\tilde{q}LL,RR}(m_{\tilde{g}}^2) = -\frac{\alpha_s}{4\pi} 2\text{tr}\left[\left(\frac{1}{\epsilon} + 2 + \ln(x_\mu^2)\right)n_f - \frac{1}{2}\sum_{s=1}^6\left(x_s^2 + (1-x_s^2)^2 \ln\left(\frac{x_s^2-1-i0}{x_s^2}\right) + \ln(x_s^2) + (x_s \rightarrow y_s)\right)\right] + O(\epsilon), \quad (\text{A8})$$

with $n_f = 6$. The quantities $\Sigma_{\tilde{g}}^{LL}(m_{\tilde{g}}^2)$ and $\Sigma_{\tilde{g}}^{LR}(m_{\tilde{g}}^2)$ that appear in Eq. (A26) are defined as

$$\Sigma_{\tilde{g}}^{LL,LR}(m_{\tilde{g}}^2) = \Sigma_{\tilde{g}\tilde{g}}^{gLL,LR}(m_{\tilde{g}}^2) + \Sigma_{\tilde{g}\tilde{g}}^{q\tilde{q}LL,LR}(m_{\tilde{g}}^2). \quad (\text{A9})$$

c. Squark

For the squark self-energy we have

$$\Sigma_{\tilde{q}_s\tilde{q}_t} = \Sigma_{\tilde{q}_s\tilde{q}_t}^g + \Sigma_{\tilde{q}_s\tilde{q}_t}^{\tilde{g}q} + \Sigma_{\tilde{q}_s\tilde{q}_t}^{\tilde{q}}, \quad (\text{A10})$$

where the parts refer to the squark self-energy with gluon

$$\Sigma_{\tilde{q}_s\tilde{q}_t}^g(p^2) = \frac{\alpha_s}{4\pi} C_F (2(p^2 + m_{\tilde{q}_s}^2)B_0(p^2; m_{\tilde{q}_s}^2, 0) - A_0(m_{\tilde{q}_s}^2))\delta_{st}, \quad (\text{A11})$$

$$\Sigma_{\tilde{q}_s\tilde{q}_t}^{\tilde{g}q}(m_{\tilde{q}_s}^2) = 3\frac{\alpha_s}{4\pi} C_F m_{\tilde{q}_s}^2 \left(\frac{1}{\epsilon} - \ln(x_s^2) + \ln(x_\mu^2) + \frac{7}{3}\right)\delta_{st} + O(\epsilon), \quad (\text{A12})$$

the squark self-energy with quark and gluino

$$\begin{aligned} \Sigma_{\tilde{q}_s\tilde{q}_t}^{\tilde{g}q}(p^2) &= \frac{\alpha_s}{2\pi} C_F (A_0(m_{\tilde{g}}^2) + (m_{\tilde{g}}^2 - p^2)B_0(p^2; m_{\tilde{g}}^2, 0))\delta_{st}, \\ \Sigma_{\tilde{q}_s\tilde{q}_t}^{\tilde{q}q}(m_{\tilde{q}_s}^2) &= \frac{\alpha_s}{2\pi} C_F m_{\tilde{q}_s}^2 \left[\frac{2-x_s^2}{\epsilon} + 3 - 2x_s^2 + (2-x_s^2)\ln(x_\mu^2) + \left(\frac{1}{x_s^2} + x_s^2 - 2\right)\ln(1-x_s^2-i0)\right]\delta_{st} + O(\epsilon), \end{aligned} \quad (\text{A13})$$

and the squark tadpole self-energy of Fig. 2 [for up (down) type squarks only the diagram with internal up (down) squarks is nonzero]

$$\begin{aligned} \Sigma_{\tilde{q}_s\tilde{q}_t}^{\tilde{q}\tilde{q}} &= -\frac{\alpha_s}{4\pi} C_F (\delta_{st} A_0(m_{\tilde{q}_s}^2) - 2 \sum_{i,j=1}^3 \sum_{s'=1}^6 (W_{i+3s}^{\tilde{q}\tilde{q}*} W_{i+3s'}^{\tilde{q}} W_{j's'}^{\tilde{q}\tilde{q}*} W_{j't}^{\tilde{q}} + W_{is}^{\tilde{q}\tilde{q}*} W_{is'}^{\tilde{q}} W_{j+3s'}^{\tilde{q}\tilde{q}*} W_{j+3t}^{\tilde{q}}) A_0(m_{\tilde{q}_{s'}}^2)) \\ &= -\frac{\alpha_s}{4\pi} C_F \left[\delta_{st} m_{\tilde{q}_s}^2 \left(\frac{1}{\epsilon} + 1 - \ln(x_s^2) + \ln(x_\mu^2)\right) - 2 \sum_{i,j=1}^3 \sum_{s'=1}^6 (W_{i+3s}^{\tilde{q}\tilde{q}*} W_{i+3s'}^{\tilde{q}} W_{j's'}^{\tilde{q}\tilde{q}*} W_{j't}^{\tilde{q}} + W_{is}^{\tilde{q}\tilde{q}*} W_{is'}^{\tilde{q}} W_{j+3s'}^{\tilde{q}\tilde{q}*} W_{j+3t}^{\tilde{q}}) m_{\tilde{q}_{s'}}^2 \right. \\ &\quad \left. \times \left(\frac{1}{\epsilon} + 1 - \ln(x_s^2) + \ln(x_\mu^2)\right) + O(\epsilon) \right]. \end{aligned} \quad (\text{A14})$$

Note that $\Sigma_{\tilde{q}_s\tilde{q}_t}^{\tilde{q}\tilde{q}}$ is independent of the external momentum. The part proportional to δ_{st} in Eq. (A14) is due to Fig. 2(b), while the second part, which is proportional to at least one element Δ_{ij}^{qLR} , is generated by Fig. 2(a).

Note that in the sum of all contributions to the diagonal squark self-energy there is no divergence proportional to p^2 , and thus no wave-function renormalization is needed in order to render the diagonal squark two point function finite.

2. Loop functions

The one-loop functions $A_0(m^2)$, $B_0(p^2; m_1^2, m_2^2)$, and $B_1(p^2; m_1^2, m_2^2)$ in the previous paragraph are defined as

$$A_0(m^2) = \frac{16\pi^2}{i} \frac{\mu^{2\epsilon} e^{\gamma\epsilon}}{(4\pi)^\epsilon} \int \frac{d^d\ell}{(2\pi)^d} \frac{1}{[\ell^2 - m^2]}, \quad (\text{A15})$$

$$B_0(p^2; m_1^2, m_2^2) = \frac{16\pi^2}{i} \frac{\mu^{2\epsilon} e^{\gamma\epsilon}}{(4\pi)^\epsilon} \int \frac{d^d\ell}{(2\pi)^d} \times \frac{1}{[\ell^2 - m_1^2][(\ell + p)^2 - m_2^2]}, \quad (\text{A16})$$

$$B_1(p^2; m_1^2, m_2^2) p^\mu = \frac{16\pi^2}{i} \frac{\mu^{2\epsilon} e^{\gamma\epsilon}}{(4\pi)^\epsilon} \int \frac{d^d\ell}{(2\pi)^d} \times \frac{\ell^\mu}{[\ell^2 - m_1^2][(\ell + p)^2 - m_2^2]}. \quad (\text{A17})$$

Here $\epsilon = (4-d)/2$ is an artifact of dimensionally regularizing the ultraviolet divergent loop integral. The function $B_0(m_1^2, m_2^2)$, which also appears, is an abbreviation for $B_0(0; m_1^2, m_2^2)$. We give now relations among these functions and explicit versions for specific arguments

$$\begin{aligned}
A_0(m^2) &= m^2 \left[\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{m^2}\right) + 1 + \left(\frac{\pi^2}{12} + 1 + \ln\left(\frac{\mu^2}{m^2}\right) + \frac{1}{2} \ln^2\left(\frac{\mu^2}{m^2}\right)\right) \varepsilon + O(\varepsilon^2) \right], & B_0(m_1^2, m_2^2) &= \frac{A_0(m_1^2) - A_0(m_2^2)}{m_1^2 - m_2^2}, \\
C_0(m_1^2, m_2^2, m_2^2) &= \frac{\partial B_0(m_1^2, m_2^2)}{\partial m_2^2}, & B_0(p^2; m^2, 0) &= \frac{1}{\varepsilon} - \ln\left(\frac{m^2}{\mu^2}\right) + 2 + \frac{m^2 - p^2}{p^2} \ln\left(\frac{m^2 - p^2 - i0}{m^2}\right) + O(\varepsilon), \\
B_1(p^2; m^2, 0) &= \frac{1}{2p^2} [A_0(m^2) - (p^2 + m^2)B_0(p^2; m^2, 0)], \\
B_1(p^2; 0, m^2) &= \frac{1}{2p^2} [-A_0(m^2) - (p^2 - m^2)B_0(p^2; m^2, 0)], \\
B_1(0; m_1^2, m_2^2) &= -\frac{1}{2} \left(\frac{1}{\varepsilon} + \ln(x_\mu^2) + \frac{3 - 4x^2 + x^4 + (4x^2 - 2x^4)\ln(x^2)}{2(1 - x^2)^2} \right), \tag{A18}
\end{aligned}$$

with $x = m_2/m_1$.

3. One-loop renormalization and counterterms

a. One-loop counterterm diagrams

Squark-mass counterterm diagram:

$$\begin{aligned}
\Sigma_{q_f q_i}^{LR m_{\tilde{q}} \text{CT}} &= \frac{\alpha_s}{2\pi} C_F m_{\tilde{g}} \sum_{s=1}^6 \delta m_{\tilde{q}_s}^2 W_{f_s}^{\tilde{d}} W_{i+3s}^{\tilde{d}\star} C_0(m_{\tilde{q}_s}^2, m_{\tilde{q}_s}^2, m_{\tilde{g}}^2) \\
&= \frac{\alpha_s}{2\pi} C_F m_{\tilde{g}} \sum_{s=1}^6 W_{f_s}^{\tilde{d}} W_{i+3s}^{\tilde{d}\star} \frac{\delta m_{\tilde{q}_s}^2}{m_{\tilde{g}}^2} \frac{1}{(1 - x_s^2)^2} \left[\ln(x_s^2) + 1 - x_s^2 + \varepsilon \left(\ln(x_\mu^2)(1 - x_s^2 + \ln(x_s^2)) - \frac{1}{2} \ln^2(x_s^2) \right. \right. \\
&\quad \left. \left. + (\ln(x_s^2) - 1)x_s^2 + 1 \right) \right]. \tag{A19}
\end{aligned}$$

Gluino mass counterterm diagram:

$$\begin{aligned}
\Sigma_{q_f q_i}^{LR m_{\tilde{g}} \text{CT}} &= \frac{\alpha_s}{2\pi} C_F \delta m_{\tilde{g}} \sum_{s=1}^6 W_{f_s}^{\tilde{d}} W_{i+3s}^{\tilde{d}\star} (B_0(m_{\tilde{g}}^2, m_{\tilde{q}_s}^2) + 2m_{\tilde{g}}^2 C_0(m_{\tilde{q}_s}^2, m_{\tilde{g}}^2, m_{\tilde{g}}^2)) \\
&= \frac{\alpha_s}{2\pi} C_F \delta m_{\tilde{g}} \sum_{s=1}^6 W_{f_s}^{\tilde{d}} W_{i+3s}^{\tilde{d}\star} \left[\frac{-x_s^2((1 + x_s^2)\ln(x_s^2) + 2(1 - x_s^2))}{(1 - x_s^2)^2} - \varepsilon \frac{x_s^2}{2(1 - x_s^2)^2} [4(1 - x_s^2) + 2(1 + x_s^2)\ln(x_s^2) \right. \\
&\quad \left. - (1 + x_s^2)\ln^2(1 - x_s^2) + (4(1 - x_s^2) + 2(1 + x_s^2)\ln(x_s^2))\ln(x_\mu^2) \right]. \tag{A20}
\end{aligned}$$

α_s counterterm diagram:

$$\begin{aligned}
\Sigma_{q_f q_i}^{LR \alpha_s \text{CT}} &= \frac{\delta \alpha_s}{2\pi} C_F m_{\tilde{g}} \sum_{s=1}^6 W_{f_s}^{\tilde{d}} W_{i+3s}^{\tilde{d}\star} B_0(m_{\tilde{g}}^2, m_{\tilde{q}_s}^2) \\
&= \frac{\delta \alpha_s}{2\pi} C_F m_{\tilde{g}} \sum_{s=1}^6 W_{f_s}^{\tilde{d}} W_{i+3s}^{\tilde{d}\star} \left[\frac{x_s^2 \ln(x_s^2)}{1 - x_s^2} - \varepsilon \frac{x_s^2}{2(1 - x_s^2)} (\ln^2(x_s^2) - 2\ln(x_s^2) - 2\ln(x_s^2)\ln(x_\mu^2)) \right]. \tag{A21}
\end{aligned}$$

b. Renormalization of the Yukawa couplings in the MSSM

Because of supersymmetry, the renormalization of the Yukawa coupling in the quark-quark-Higgs vertex Y^{q_i} and the one in squark-squark-Higgs vertex $Y^{\tilde{q}_i}$ must be identical.¹⁴ Indeed, we explicitly find that the counterterms for these couplings are the same

$$Y^{q_i, \tilde{q}_i(0)} = Y^{q_i, \tilde{q}_i} + \delta Y^{q_i, \tilde{q}_i}, \quad \delta Y^{q_i, \tilde{q}_i} = -\frac{\alpha_s}{4\pi} \frac{2}{\varepsilon} C_F Y^{q_i, \tilde{q}_i}, \tag{A22}$$

which even holds in the $\overline{\text{MS}}$ scheme and in the $\overline{\text{DR}}$ scheme at the one-loop level.

¹⁴This also includes that the renormalization of the Yukawa coupling entering the squark mass matrices is the same as the renormalization of the quark-quark-Higgs coupling.

c. A-term renormalization

In the approximation $m_q = 0$ the SQCD renormalization of the A terms is the same as of the Yukawa coupling.¹⁵

d. Squark mass renormalization

We write the connection between the squares of bare and the renormalized squark masses as

$$(m_{\tilde{q}_i}^0)^2 = (m_{\tilde{q}_i})^2 + \delta m_{\tilde{q}_i}^2. \quad (\text{A23})$$

From Eqs. (A12)–(A14) and by taking into account that the second term of Eq. (A14) only renormalizes the Yukawa coupling (and the A, A' terms), we can easily read off $\delta m_{\tilde{q}_i}^2$. We obtain in the $\overline{\text{MS}}$ scheme

$$\delta m_{\tilde{q}_i}^2 = \frac{\alpha_s}{4\pi} C_F m_g^2 ((x_i^2 + 4) - x_i^2) \frac{1}{\epsilon}, \quad (\text{A24})$$

where the contribution proportional to $(x_i^2 + 4)$ comes from Eqs. (A12) and (A13) while the term $-x_i^2$ stems from the part of Eq. (A14) proportional to δ_{st} .

¹⁵If $m_q \neq 0$, the quark-gluino correction to A terms induced flavor-nondiagonal (divergent) corrections.

e. Gluino-mass renormalization

We decompose the gluino self-energy according to Eq. (1). Expressing the bare mass [marked with the superscript (0)] in terms of the physical one

$$m_{\tilde{g}}^0 = m_{\tilde{g}} + \delta m_{\tilde{g}}, \quad (\text{A25})$$

we get in the on-shell scheme

$$\delta m_{\tilde{g}} = -m_{\tilde{g}} \Sigma_{\tilde{g}}^{\text{LL}}(m_{\tilde{g}}^2) - \Sigma_{\tilde{g}}^{\text{LR}}(m_{\tilde{g}}^2). \quad (\text{A26})$$

For details see Ref. [38]. In the $\overline{\text{MS}}$ scheme only the divergence of the right-hand side enters; i.e., we get in this scheme

$$\delta m_{\tilde{g}} = -\frac{\alpha_s}{4\pi} m_{\tilde{g}} (3C_A - 2\text{tr}n_f) \frac{1}{\epsilon}. \quad (\text{A27})$$

f. Renormalization of g_s in the MSSM

In lowest order, the strong coupling constant involved in C_{fi}^{qLR} is Yukawa type. The relation between the bare and the renormalized version reads $g_{s,Y}^0 = (1 + \delta Z_{g_s,Y}) g_{s,Y}$, where the renormalization constant in the $\overline{\text{MS}}$ scheme is given by

$$\delta Z_{g_s,Y} = \frac{\alpha_s}{4\pi} \left[\text{tr}n_f - \frac{3}{2} C_A \right] \frac{1}{\epsilon}. \quad (\text{A28})$$

Note that at one loop the renormalization constant is the same for the $\overline{\text{MS}}$ scheme and the $\overline{\text{DR}}$ scheme.

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