Final state interaction contribution to the observed B_s decays into K^+K^- and π^+K^-

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Because at the tree level $B_s \to K^+K^-$ is Cabibbo triple suppressed, so its branching ratio should be smaller than that of $B_s \to \pi^+K^-$. The measurements present a reversed ratio as $R = \mathcal{B}(B_s \to \pi^+K^-)/\mathcal{B}(B_s \to K^+K^-) \sim 4.9/33$. Therefore, it has been suggested that the transition $B_s \to K^+K^-$ is dominated by the penguin mechanism, which is proportional to $V_{cb}V_{cs}^*$. In this work, we show that an extra contribution from the final state interaction to $B_s \to K^+K^-$ via sequential processes $B_s \to D_{(s)}^{(*)}\bar{D}_s^{(*)} \to K^+K^-$ is also substantial and should be superposed on the penguin contribution. Indeed, taking into account the final state interaction effects, the theoretical prediction on R is well consistent with the data.

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I. INTRODUCTION

It is generally believed that the hierarchy of the Cabibbo-Kabayashi-Maskawa (CKM) matrix elements determines the magnitudes of weak decays, and off-diagonal matrix elements would bring up an order suppression. For example, the ratio of $\mathcal{B}(B_s \to D_s^+\pi^-)/\mathcal{B}(B_s \to D_s^{\pm}K^{\mp}) = 3.2\times10^{-3}/3.0\times10^{-4}$ has been measured [1,2], and it is roughly determined by the ratio of the CKM matrix elements V_{ud}/V_{us}. Thus, one usually categorizes the weak decays according to their CKM structures as the Cabibbo favored; Cabibbo suppressed and even the Cabibbo double or triple suppressed. However, the newly observed modes, $B_s \to K^+K^-$ and $B_s \to \pi^+K^-$, obviously do not follow the rule, namely, at first look, the branching ratio of $B_s \to K^+K^-$ should be smaller than that of $B_s \to \pi^+K^-$ by $|V_{us}/V_{ud}|^2$; by contraries, the datum is $\mathcal{R} = \mathcal{B}(B_s \to \pi^+K^-)/\mathcal{B}(B_s \to K^+K^-) = 4.9\times10^{-6}/3.3\times10^{-5}$ [3–5]. If only the tree diagrams are taken into account, this reversion would compose an "anomaly." Descotes-Genon *et al.* [6] carefully analyzed the transitions of $B_s \to K^0\bar{K}^0$ and K^+K^- through flavor symmetries and QCD factorization, and pointed to a potential conflict between the QCD prediction on $B_s \to K^+K^-$ and the data. By analyzing the transition mechanism, Cheng and Chua [7] determined that the main contribution to $B_s \to K^+K^-$ comes from the penguin diagram, which is proportional to $V_{cb}V_{cs}^*$. Ali *et al.* also calculated the branching ratios of $B_s \to \pi^+K^-$ and $B_s \to K^+K^-$ in terms of the perturbative QCD (pQCD) to leading order (LO) and next-to-leading order (NLO) [8,9] and the authors of Ref. [10] did calculations in soft-collinear effective theory (SCET). Their results roughly were consistent with the data available then, so the conflict seemed resolved. Even though the theoretical uncertainties in all the calculations are not fully controlled, it is noted that the obtained central values are not sufficiently large to make up the data. It implies that there must be some mechanisms to remarkably enhance the branching ratio of $B_s \to K^+K^-$.

Looking at the central values they obtained and the resultant ratio of $\mathcal{B}(B_s \to \pi^+K^-)/\mathcal{B}(B_s \to K^+K^-)$, one can find that the calculated $\mathcal{B}(B_s \to \pi^+K^-)$ is close to the data; however, the central values of $\mathcal{B}(B_s \to K^+K^-)$ calculated in various approaches are smaller than the newly measured data [4,5].

Based on this observation, we suggest that the final state interaction (FSI) in B_s decays may greatly enhance the branching ratio of $B_s \to K^+K^-$ but not much for $B_s \to \pi^+K^-$. In fact, in the energy regions of b quark and c quark most such anomalies can be naturally explained by considering the role of FSI. For example, a simple quark diagram-analysis tells that the branching ratio of $D^0 \to K^0\bar{K}^0$ is almost zero, but its measured value is comparable with that of $D^0 \to K^+K^-$, which is large. This can be understood by considering the sequential process $D^0 \to K^+K^- \to K^0\bar{K}^0$ and the later step is a hadronic scattering [11].

The Particle Data Group (PDG) [3] tells us that the $D_s^{(*)+}D_s^{(*)-}$ are the dominant hadronic decay modes of B_s; therefore, it implies that the sequential decays $B_s \to D_s^{(*)+}D_s^{(*)-} \to K^+K^-$ would compose an important contribution to the observed $B_s \to K^+K^-$. The first step of $B_s \to D_s^{(*)+}D_s^{(*)-}$ is only suppressed by V_{bc} and the process

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does not suffer form a color suppression. Moreover, it is also an isospin-conserved mode; even though the weak interaction does not require isospin conservation, it still may be more favorable than the isospin violated ones. Thus, one can understand why $B_s \to D_s^{(*)+} D_s^{(*)-}$ is dominant. Then let us look at the next step. The FSI is a hadronic scattering process, which is completely governed by strong interaction, so that the isospin must be conserved. The isospin of $D_s^{(*)\pm}$ is zero, thus, the final state of the scattering is required to be zero. The isospin of K mesons is $1/2$, thus the $K\bar{K}$ states can be either isospin 0 or 1; therefore, the inelastic scattering $D_s^{(*)+} D_s^{(*)-} \to K^+ K^-$ is allowed. By contraries, the isospin of the pion is 1; thus, the system of $\pi^+ K^-$ does not contain an isospin 0 component, and thus the scattering $D_s^{(*)+} D_s^{(*)-} \to \pi^+ K^-$ is forbidden. Definitely, one can expect a substantial contribution from $B_s \to D_s^{(*)+} D_s^{(*)-} \to K^+ K^-$, which can greatly enhance the branching ratio of $B_s \to K^+ K^-$ in comparison with $B_s \to \pi^+ K^-$.

It is worth pointing out that, on the other hand, the decay $B_s \to \pi^+ K^-$ also receives a contribution from the FSI via $B_s \to D^{(*)+} D_s^{(*)-} \to \pi^+ K^-$. However, the first step process $B_s \to D^{(*)+} D_s^{(*)-}$ is Cabibbo double suppressed by $V_{cb} V_{cd}^*$, so is much less than $B_s \to D_s^{(*)+} D_s^{(*)-}$; therefore, the FSI does not contribute much to the branching ratio of $B_s \to \pi^+ K^-$.

Notice that the direct process $B_s \to K^+ K^-$ via penguin diagrams and the sequential process with FSI $B_s \to D_s^{(*)} D_s^{(*)} \to K^+ K^-$ have the same initial and final states; moreover, their amplitudes are of the same order of magnitude, so the two contributions interfere. In fact, their contributions are not directly experimentally, but theoretically distinguishable. Moreover, the strong scattering would have real and imaginary parts (see below, for the calculations of the triangle diagrams); thus, a phase is resulted. In most calculations of the FSI effects, only the absorptive (imaginary) part is kept; the reason is that one may argue that the absorptive part might be dominant or at most the dispersive and absorptive parts have close magnitudes. In that case, the absorptive part is imaginary while the penguin contribution is real, so that the two contributions can be added up at the rate level (amplitude square). Our final results indicate that while taking into account the new contribution, the theoretical predictions are indeed close to the new data.

In this work, we calculate the amplitudes of the decay channels of $B_s \to D_s^{(*)} (D^{(*)}) \bar{D}_s^{(*)}$ and the direct decay channels of $B_s \to K^+ (\pi^+) K^-$ by the factorization approach [12,13]. Then we use the effective SU(4) Lagrangian [14,15] to determine the vector-pseudoscalar-pseudoscalar and vector-vector-pseudoscalar vertices for calculating the FSI amplitude of $D_s^{(*)} (D^{(*)}) \bar{D}_s^{(*)} \to K^+ (\pi^+) K^-$. The paper is organized as follows: after the Introduction, we formulate the weak decays and the rescattering processes in Sec. II; our numerical results are presented along with all necessary inputs in Sec. III. The last section is devoted to discussion and conclusion.

II. THE THEORETICAL EVALUATIONS OF THE BRANCHING RATIOS

A. Weak decays in factorization approach

Because the measurements on the decay widths of $B_s \to D_s^{(*)+} D_s^{(*)-}$ and $B_s \to D^{(*)+} D_s^{(*)-} (B_s \to D^{(*)-} D_s^{(*)+})$ are not accurate yet, as for most of the above channels there are only upper bounds, so that we are going to directly calculate the transition amplitudes based on the quark diagrams. Even though this strategy might bring up certain theoretical uncertainties, it does not break our qualitative conclusion at all.

For calculating the transition amplitudes of $B_s \to D_s^{(*)+} (D^{(*)+}) D_s^{(*)-}$, one needs to employ the effective Hamiltonian at the quark level. With the operator product expansion, the effective Hamiltonian was explicitly presented in Ref. [16]. At the tree level, from the effective Lagrangian, one can notice that $B_s \to K^+ K^-$ is suppressed by $V_{ub} V_{us}^*$, i.e., triple Cabibbo suppressed, comparing with $B_s \to \pi^+ K^-$, which is double Cabibbo suppressed by V_{ub}. Therefore, Cheng and Chua decided that the direct transition $B_s \to K^+ K^-$ is obviously dominated by the penguin diagram whose CKM structure is $V_{cb} V_{cs}^*$. The transition $B_s \to D_s^{(*)+} D^{(*)-}$ is also double Cabibbo suppressed. Instead, $B_s \to D_s^{(*)+} D_s^{(*)-}$ is proportional to $V_{cb} V_{cs}^*$ and it is a tree process. Now let us compare the sequential processes $B_s \to D^{(*)} \bar{D}_s^{(*)} \to K^+ K^-$ with the penguin contribution. The two reactions are of the same CKM structure, but the penguin undergoes a loop suppression of about $\alpha_s/\pi \sim 0.06$, whereas the strong scattering $D_s^{(*)} \bar{D}_s^{(*)} \to \sum_i X_i$, where X_i stands as any possible final states allowed by symmetry and energy-momentum conservation. $K^+ K^-$ is only one of the possible channels, and its probability is proportional to $\langle D_s^{(*)} \bar{D}_s^{(*)} | H_{\mathrm{eff}} | K \bar{K} \rangle$, which is what we are going to calculate in this work. This is a suppression factor because the total probability to all channels is 1. Therefore, roughly, we notice that the penguin is loop suppressed and the sequential process is also suppressed by the probability; thus, the two modes compete and may have a similar order of magnitude. Concretely, we need to calculate them. The explicit calculation on the penguin contribution can be found in Refs. [7–9]; thus, we will use their numbers and only consider the contribution from the sequential processes.

Applying the effective Hamiltonian at the quark level to the hadron states, the hadronic matrix elements can be parametrized as [17]:

$$\langle 0|J_\mu|P(p_1)\rangle = -if_P p_{1\mu}, \quad \langle 0|J_\mu|V(p_1,\epsilon)\rangle = f_V\epsilon_\mu m_V,$$
$$\langle P(p_2)|J_\mu|B_s(p)\rangle = \left[P_\mu - \frac{m_{B_s}^2 - m_P^2}{q^2}q_\mu\right]F_1(q^2) + \frac{m_{B_s}^2 - m_P^2}{q^2}q_\mu F_0(q^2),$$
$$\langle V(p_2,\epsilon)|J_\mu|B_s(p)\rangle = \frac{i\epsilon^\nu}{m_{B_s}+m_V}\Big\{i\epsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta A_V(q^2) + (m_{B_s}+m_V)^2 g_{\mu\nu}A_1(q^2) - \frac{P_\mu P_\nu}{m_{B_s}+m_V}A_2(q^2)$$
$$- 2m_V(m_{B_s}+m_V)\frac{P_\nu q_\mu}{q^2}[A_3(q^2) - A_0(q^2)]\Big\}, \tag{1}$$

where $J_\mu = \bar{q}_1\gamma_\mu(1-\gamma_5)q_2$, $P_\mu = (p_1+p_2)_\mu$, and $q_\mu = (p_1-p_2)_\mu$.

With Eq. (1), we write down the amplitudes of $B_s \to D_s^{(*)+}D_s^{(*)-}$:

$$\mathcal{A}(B_s(p)\to D_s^+(p_1)D_s^-(p_2)) = -\frac{iG_F}{\sqrt{2}}V_{cb}V_{cs}^* a_1 f_{D_s}(m_{B_s}^2 - m_{D_s}^2)F_0^{B_sD_s}(p_1^2), \tag{2a}$$

$$\mathcal{A}(B_s(p)\to D_s^{*+}(p_1)D_s^{*-}(p_2)) = \frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^* a_1 f_{D_s^*}m_{D_s^*}\frac{ig^{\mu\nu}}{m_{B_s}+m_{D_s^*}}\Big\{i\epsilon_{\mu\nu\alpha\beta}(p_1+2p_2)^\alpha p_1^\beta A_V^{B_sD_s^*}(p_1^2)$$
$$+ (m_{B_s}+m_{D_s^*})^2 g_{\mu\nu}A_1^{B_sD_s^*}(p_1^2) - (p_1+2p_2)_\mu(p_1+2p_2)_\nu A_2^{B_sD_s^*}(p_1^2)$$
$$- \frac{(p_1+2p_2)_\mu p_{1\nu}}{p_1^2}[(m_{B_s}+m_{D_s^*})^2 A_1^{B_sD_s^*}(p_1^2) - (m_{B_s}^2 - m_{D_s^*}^2)A_2^{B_sD_s^*}(p_1^2)$$
$$- 2m_{D_s^*}(m_{B_s}+m_{D_s^*})A_0^{B_sD_s^*}(p_1^2)]\Big\}, \tag{2b}$$

and the amplitudes of $B_s \to D^{(*)+}D_s^{(*)-}$ read as

$$\mathcal{A}\,(B_s(p)\to D^+(p_1)D_s^-(p_2)) = -\frac{iG_F}{\sqrt{2}}V_{cb}V_{cd}^* a_1 f_D(m_{B_s}^2 - m_{D_s}^2)F_0^{B_sD_s}(p_1^2), \tag{2c}$$

$$\mathcal{A}(B_s(p)\to D^{*+}(p_1)D_s^{*-}(p_2)) = \frac{G_F}{\sqrt{2}}V_{cb}V_{cd}^* a_1 f_{D^*}m_{D^*}\frac{ig^{\mu\nu}}{m_{B_s}+m_{D_s^*}}\Big\{i\epsilon_{\mu\nu\alpha\beta}(p_1+2p_2)^\alpha p_1^\beta A_V^{B_sD_s^*}(p_1^2)$$
$$+ (m_{B_s}+m_{D_s^*})^2 g_{\mu\nu}A_1^{B_sD_s^*}(p_1^2) - (p_1+2p_2)_\mu(p_1+2p_2)_\nu A_2^{B_sD_s^*}(p_1^2)$$
$$- \frac{(p_1+2p_2)_\mu p_{1\nu}}{p_1^2}[(m_{B_s}+m_{D_s^*})^2 A_1^{B_sD_s^*}(p_1^2) - (m_{B_s}^2 - m_{D_s^*}^2)A_2^{B_sD_s^*}(p_1^2)$$
$$- 2m_{D_s^*}(m_{B_s}+m_{D_s^*})A_0^{B_sD_s^*}(p_1^2)]\Big\}, \tag{2d}$$

the amplitude of π^+K^- at the tree level is

$$\mathcal{A}^{\,\mathrm{direct}}(B_s(p)\to \pi^+(p_1)K^-(p_2)) = -\frac{iG_F}{\sqrt{2}}V_{ub}V_{ud}^* a_1 f_\pi(m_{B_s}^2 - m_K^2)F_0^{B_sK}(p_1^2), \tag{2e}$$

where a_1 is a proper combination of the Wilson coefficients in the effective Hamiltonian [16,18,19]. $F_0(q^2)$ and $A_{V,1,2}(q^2)$ are the form factors to be determined. In this work, due to lack of accurate data, we use the form factors obtained by fitting the data of the decays of the B meson. It is a reasonable approximation because the processes $B_s \to D_s^*$ and $B \to D^*$ have the same topological structure, and the flavor-SU(3) symmetry for light quarks (u, d, and s) would lead to the same form factors, namely, the difference between the form factors for different light-quark flavors would be proportional to an SU(3) breaking, which is small for the effective vertices as is well known. At least such small differences would not overtake the errors caused by experimental measurements and theoretical uncertainties for evaluating the nonperturbative effects.

Moreover, a symmetry analysis indicates that the sequential process $B_s \to D_s^{*+}D_s^-(D_s^+D_s^{*-}) \to K^+K^-$ is forbidden by angular-momentum conservation.

TABLE I. The parameters given in Ref. [17].

F	$F(0)$	a	b	F	$F(0)$	a	b
F_0^{BD}	0.67	0.65	0.00	$F_0^{B\pi}$	0.25	0.84	0.10
V^{BD^*}	0.75	1.29	0.45	$A_0^{BD^*}$	0.64	1.30	0.31
$A_1^{BD^*}$	0.63	0.65	0.02	$A_2^{BD^*}$	0.61	1.14	0.52

Taking the three-parameter form, the form factors are written as [17]

$$F(q^2) = \frac{F(0)}{1 - a\frac{q^2}{m_B^2} + b\left(\frac{q^2}{m_B^2}\right)^2}, \tag{3}$$

where a, b, and $F(0)$ are the three parameters and their values are listed in Table I.

B. Evaluation of FSI effects

Now let us turn to evaluate the long-distance effects at hadron level. For the effective vertices, the flavor-SU(4) symmetry is assumed. The coupling of pseudoscalar and vector mesons is

$$\mathcal{L}_0 = \mathrm{Tr}(\partial_\mu \phi^\dagger \partial^\mu \phi) - \frac{1}{2}\mathrm{Tr}(F^\dagger_{\mu\nu}F^{\mu\nu}), \tag{4}$$

where $F^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$, and ϕ and V represent the 4×4 pseudoscalar and vector meson matrices in SU(4), respectively,

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta + \frac{\eta_c}{\sqrt{12}} & D_s^- \\ D^0 & D^+ & D_s^+ & -\frac{3\eta_c}{\sqrt{12}}, \end{pmatrix}, \tag{5}$$

$$V = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega'}{\sqrt{6}} + \frac{J/\psi}{\sqrt{12}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega'}{\sqrt{6}} + \frac{J/\psi}{\sqrt{12}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & -\sqrt{\frac{2}{3}}\omega' + \frac{J/\psi}{\sqrt{12}} & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & -\frac{3J/\psi}{\sqrt{12}}. \end{pmatrix}. \tag{6}$$

For the gauge invariance, covariant derivatives replace the regular ones:

$$\partial_\mu\phi \to \mathcal{D}_\mu\phi = \partial_\mu\phi - \frac{ig}{2}[V_\mu, \phi], \qquad F_{\mu\nu} \to \partial_\mu V_\nu - \partial_\nu V_\mu - \frac{ig}{2}[V_\mu, V_\nu]. \tag{7}$$

Now we are ready to write down the relevant terms in the pseudoscalar-vector coupling:

$$\mathcal{L} = \mathcal{L}_0 + ig\,\mathrm{Tr}(\partial^\mu\phi[\phi, V^\nu]) + g'\epsilon_{\alpha\beta\mu\nu}\partial^\alpha V^\beta \partial^\mu V^\nu \phi + \cdots, \tag{8}$$

where the third term on the right-hand side of Eq. (8) is involved additionally as a vector-vector-pseudoscalar coupling [20]. In the process that $D_s^{(*)+}D_s^{(*)-}$ rescatter into K^+K^-, the corresponding Lagrangian is

$$\begin{aligned} \mathcal{L}_{KD_sD^*} &= ig_{KD_sD^*}\{D_\mu^{-*}[\partial^\mu K^+ D_s^- - K^+\partial^\mu D_s^-] + \bar{D}^{0*}[K^-\partial^\mu D_s^+ - \partial^\mu K^- D_s^+]\}, \\ \mathcal{L}_{KD_s^*D} &= ig_{KD_s^*D}\{D_{s\mu}^{-*}[K^+\partial^\mu D^0 - \partial^\mu K^+ D^0] + D_{s\mu}^{+*}[\partial^\mu K^-\bar{D}^0 - K^-\partial^\mu \bar{D}^0]\}, \\ \mathcal{L}_{KD_s^*D^*} &= g_{KD_s^*D^*}\epsilon^{\alpha\beta\mu\nu}[\partial_\alpha \bar{D}^{0*}_\beta \partial_\mu D_{s\nu}^{+*}K^- + \partial_\alpha D_{s\beta}^{-*}\partial_\mu D_\nu^{0*}K^+], \end{aligned} \tag{9}$$

and the effective vertices for πDD^* and πD^*D^* are written as [14,21]

$$\mathcal{L}_{\pi DD^*} = \frac{i}{2}g_{\pi DD^*}(\bar{D}\tau_i D^{*\mu}\partial_\mu\pi_i - \partial^\mu\bar{D}\tau_i D^*_\mu \pi_i - \mathrm{H.c.}), \qquad \mathcal{L}_{\pi D^*D^*} = -g_{\pi D^*D^*}\varepsilon^{\mu\nu\alpha\beta}\partial^\mu \bar{D}^*_\nu \pi \partial_\alpha D^{*\beta}. \tag{10}$$

TABLE II. The values of the coupling constants involved in our work.

g	Value	g	Value
$g_{D^*D\psi}$	4.2 GeV^{-1} [21], $\frac{7.7}{m_D}$ GeV^{-1} [14]	$g_{DD\psi}$	7.9 [21], 8.0 [22]
$g_{\pi DD^*}$	8.8 [14]	$g_{\pi D^*D^*}$	8.9 GeV^{-1} [21], 9.1 GeV^{-1} [23]

In Eq. (9), the values of the coupling constants should be obtained by fitting the experiment data. However, it is noticed that in previous literature those coupling constants are still not well fixed yet (see Table II). So, in this work, we use an average value for each coupling constant, and the uncertainty is considered as a systematical error, which might be attributed to our input parameters. For example, there are no available data for determining the coupling constants $g_{KD_sD^*}$ and $g_{KD_s^*D}$; we are going to fix them based on the SU(4) symmetry, which tells us that $g_{KD_sD^*} = g_{KD_s^*D} = \sqrt{2}g_{\rho\pi\pi} = \sqrt{3}g_{\psi DD}/2$, and by fitting the data, $g_{\rho\pi\pi} = 8.8$ and $g_{\psi DD} = 7.9$ are set (see Table II). We may have two different values for $g_{KD_sD^*}$ and $g_{KD_s^*D}$: 6.0 and 9.4, but by our strategy we adopt an average value for $g_{KD_sD^*}$ and $g_{KD_s^*D}$ as $g_{KD_sD^*} = g_{KD_s^*D} = 7.7 \pm 1.7$. We use the same method to get the values for other relevant coupling constants as long as there are no data to directly fix them and retain the errors. Then we have

$$\begin{aligned} g_{KD_sD^*} &= g_{KD_s^*D} = \sqrt{2}g_{\rho\pi\pi} = \frac{\sqrt{3}}{2}g_{\psi DD} = 7.7 \pm 0.9, \\ g_{KD_s^*D^*} &= \frac{1}{\sqrt{3}}g_{\psi D^*D} = (2.40 \pm 0.02)\,\mathrm{GeV}^{-1}, \\ g_{\pi D^*D^*} &= (9.0 \pm 0.1)\,\mathrm{GeV}^{-1}, \quad g_{\pi DD^*} = 8.8. \end{aligned} \tag{11}$$

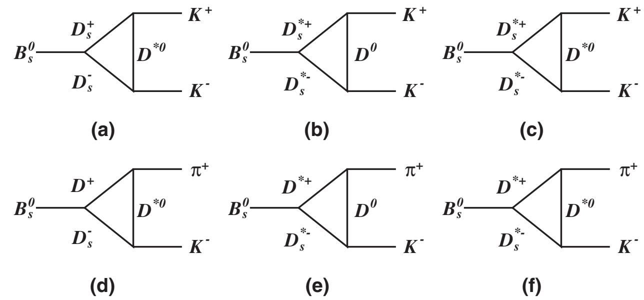

FIG. 1. The QCD one-particle-exchange process in $D_s^{(*)+}D_s^{(*)-} \to K^+K^-$.

The authors of Ref. [24] gave a simple relation between $g_{\pi D^*D^*}$ and $g_{\pi DD^*}$ as $g_{\pi D^*D^*} = 1.4g_{\pi DD^*}$, which can also be used to determine $g_{\pi DD^*}$ or $g_{\pi D^*D^*}$ from each other.

To evaluate the FSI effects, we calculate the absorptive part (see above arguments) of the one-particle-exchange triangle diagrams. Figure 1 shows the diagrams of $B_s \to D_s^{(*)+}D_s^{(*)-} \to K^+K^-$ by exchanging $D^{0(*)}$. For calculating the absorptive part of the triangle, with the Cutkosky cutting rule [25,26], the related amplitudes are

$$\begin{aligned} &\mathcal{A}_1[B_s(p) \to D_s^+(p_1)D_s^-(p_2) \to K^+(p_3)K^-(p_4)] \\ &\quad = \frac{1}{2}\int d\tilde{p}_1 d\tilde{p}_2 (2\pi)^4 \delta(p - p_1 - p_2)\mathcal{A}[B_s \to D_s^+(p_1)D_s^-(p_2)](-g_{KD_sD^*})[-i(p_1 + p_3)_\mu](-g_{KD_sD^*})[-i(p_2 + p_4)_\nu] \\ &\qquad \times \left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_{D^{0*}}^2}\right]\frac{i}{q^2 - m_{D^{0*}}^2}\mathcal{F}^2(q^2, m_{D^{0*}}^2), \end{aligned} \tag{12a}$$

$$\begin{aligned} &\mathcal{A}_2[B_s(p) \to D_s^*(p_1)D_s^*(p_2) \to K^+(p_3)K^-(p_4)] \\ &\quad = \frac{1}{2}\int d\tilde{p}_1 d\tilde{p}_2 (2\pi)^4 \delta(p - p_1 - p_2)\mathcal{A}[B_s \to D_s^{+*}(p_1)D_s^{-*}(p_2)](-g_{KD_s^*D})[-i(q + p_3)_\xi](-g_{KD_s^*D})[-i(q - p_4)_\sigma] \\ &\qquad \times \left[-g_{\mu\xi} + \frac{p_{1\mu}p_{1\xi}}{m_{D_s^*}^2}\right]\left[-g_{\nu\sigma} + \frac{p_{2\nu}p_{2\sigma}}{m_{D_s^*}^2}\right]\frac{i}{q^2 - m_{D^0}^2}\mathcal{F}^2(q^2, m_{D^0}^2), \end{aligned} \tag{12b}$$

$$\begin{aligned} &\mathcal{A}_3[B_s(p) \to D_s^{*+}(p_1)D_s^{*-}(p_2) \to K^+(p_3)K^-(p_4)] \\ &\quad = \frac{1}{2}\int d\tilde{p}_1 d\tilde{p}_2 (2\pi)^4 \delta(p - p_1 - p_2)\mathcal{A}[B_s \to D_s^{+*}(p_1)D_s^{-*}(p_2)](ig_{KD_s^*D^*})(-ip_{1\sigma})(-iq_\alpha)\epsilon^{\sigma\omega\alpha\beta}(ig_{KD_s^*D^*}) \\ &\qquad \times (-ip_{2\xi})(iq_\kappa)\epsilon^{\xi\lambda\kappa\rho}\left[-g_{\beta\rho} + \frac{q_\beta q_\rho}{m_{D^{0*}}^2}\right]\left[-g_{\mu\omega} + \frac{p_{1\mu}p_{1\omega}}{m_{D_s^*}^2}\right]\left[-g_{\nu\lambda} + \frac{p_{2\nu}p_{2\lambda}}{m_{D_s^*}^2}\right]\frac{i}{q^2 - m_{D^{0*}}^2}\mathcal{F}^2(q^2, m_{D^{*0}}^2); \end{aligned} \tag{12c}$$

instead, for the decay channel $B_s \to D^{(*)+}D_s^{(*)-} \to \pi^+K^-$, we have

$$\begin{aligned} &\mathcal{A}_1[B_s(p) \to D^+(p_1)D_s^-(p_2) \to \pi^+(p_3)K^-(p_4)] \\ &\quad = \frac{1}{2}\int d\tilde{p}_1 d\tilde{p}_2 (2\pi)^4 \delta(p - p_1 - p_2)\mathcal{A}[B_s \to D^+(p_1)D_s^-(p_2)](-g_{\pi DD^*})[-i(p_1 + p_3)_\mu](-g_{KD_sD^*}) \\ &\qquad \times [-i(p_2 + p_4)_\nu]\left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_{D^{0*}}^2}\right]\frac{i}{q^2 - m_{D^{0*}}^2}\mathcal{F}^2(q^2, m_{D^{0*}}^2), \end{aligned} \tag{12d}$$

$$\mathcal{A}_2[B_s(p) \to D^{*+}(p_1) D_s^{*-}(p_2) \to \pi^+(p_3) K^-(p_4)] = \frac{1}{2}\int d\tilde{p}_1 d\tilde{p}_2 (2\pi)^4 \delta(p - p_1 - p_2) \mathcal{A}[B_s \to D^{+*}(p_1) D_s^{-*}(p_2)](-g_{\pi DD^*})[-i(q+p_3)_\xi](-g_{KD_s^*D})[-i(q-p_4)_\sigma] \times \bigg[-g_{\mu\xi} + \frac{p_{1\mu}p_{1\xi}}{m_{D^*}^2}\bigg]\bigg[-g_{\nu\sigma} + \frac{p_{2\nu}p_{2\sigma}}{m_{D_s^*}^2}\bigg]\frac{i}{q^2 - m_{D^0}^2}\mathcal{F}^2(q^2, m_{D^0}^2), \tag{12e}$$

$$\mathcal{A}_3[B_s(p) \to D^{*+}(p_1) D_s^{*-}(p_2) \to \pi^+(p_3) K^-(p_4)] = \frac{1}{2}\int d\tilde{p}_1 d\tilde{p}_2 (2\pi)^4 \delta(p - p_1 - p_2) \mathcal{A}[B_s \to D^{+*}(p_1) D_s^{-*}(p_2)](ig_{\pi D^*D^*})(-ip_{1\sigma})(-iq_\alpha)\epsilon^{\sigma\omega\alpha\beta}(ig_{KD_s^*D^*}) \times (-ip_{2\xi})(iq_\kappa)\epsilon^{\xi\lambda\kappa\rho}\bigg[-g_{\beta\rho} + \frac{q_\beta q_\rho}{m_{D^{0*}}^2}\bigg]\bigg[-g_{\mu\omega} + \frac{p_{1\mu}p_{1\omega}}{m_{D^*}^2}\bigg]\bigg[-g_{\nu\lambda} + \frac{p_{2\nu}p_{2\lambda}}{m_{D_s^*}^2}\bigg]\frac{i}{q^2 - m_{D^{0*}}^2}\mathcal{F}^2(q^2, m_{D^{*0}}^2), \tag{12f}$$

where $d\tilde{p} = dp^3/((2\pi)^3 2E)$, q is the momentum of the exchanged $D^{0(*)}$ meson, and a dipole form factor $\mathcal{F}(q^2, m^2) = (\Lambda^2 - m^2)^2/(\Lambda^2 - q^2)^2$ where $\Lambda = m + \beta\Lambda_{\rm QCD}$[1] is introduced to compensate its off-shell effect [21].

III. NUMERICAL RESULT

With the amplitudes given in Eq. (12), we can easily calculate the decay width of the sequential processes $B_s \to D_s^{(*)} D_s^{(*)} \to K^+K^-$. We take the relevant parameters from PDG [3] as $m_{B_s} = 5.366$ GeV, $m_{D_s} = 1.968$ GeV, $m_{D_s^*} = 2.112$ GeV, $m_{D^0} = 1.864$ GeV, $m_{D^{0*}} = 2.007$ GeV, $m_{D^\pm} = 1.869$ GeV, $m_{D^{*\pm}} = 2.01$ GeV; and the value of the weak decay constants, such as $f_\pi f_K f_D f_{D_s}$, can be found in Ref. [27]; $a_1 = 1.14$ is set in our numerical computations [13]. The total amplitudes are

$$|\mathcal{A}(B_s \to K^+K^-)| = |\mathcal{A}^{\rm direct}(B_s \to K^+K^-) + \mathcal{A}_1(B_s \to D_s^+ D_s^- \to K^+K^-) + \mathcal{A}_2(B_s \to D_s^{*+} D_s^{*-} \to K^+K^-) + \mathcal{A}_3(B_s \to D_s^{*+} D_s^{*-} \to K^+K^-)| = (4.1 \pm 0.8) \times 10^{-8}, \tag{13}$$

$$|\mathcal{A}(B_s \to \pi^+K^-)| = |\mathcal{A}^{\rm direct}(B_s \to \pi^+K^-) + \mathcal{A}_1(B_s \to D^+ D_s^- \to \pi^+K^-) + \mathcal{A}_2(B_s \to D^{*+} D_s^{*-} \to \pi^+K^-) + \mathcal{A}_3(B_s \to D^{*+} D_s^{*-} \to \pi^+K^-)| = 30.4^{+0.2}_{-0.1} \times 10^{-9}, \tag{14}$$

where $A^{\rm direct}$ $(B_s \to K^+K^-)$ is the penguin contribution [7]. So, the branching ratios are

$$\mathcal{B}(B_s \to K^+K^-) = \frac{\Gamma_{B_s \to K^+K^-}}{\Gamma_{\rm tot}} = \frac{1}{32\pi^2\Gamma_{\rm tot}}\frac{|p_K|}{m_{B_s}^2}\int |\mathcal{A}_{\rm tot}(B_s \to KK)|^2 d\Omega = 13.6^{+6.3}_{-5.1} \times 10^{-6},$$
$$\mathcal{B}(B_s \to \pi^+K^-) = \frac{\Gamma_{B_s \to \pi K}}{\Gamma_{\rm tot}} = \frac{1}{32\pi^2\Gamma_{\rm tot}}\frac{|p_K|}{m_{B_s}^2}\int |\mathcal{A}_{\rm tot}(B_s \to K\pi)|^2 d\Omega = 7.8^{+0.1}_{-0.1} \times 10^{-6}, \tag{15}$$

where the errors are systematical, originating from the uncertainty of the coupling constants.

In Table III we list the theoretical predictions on $\mathcal{B}(B_s \to K^+K^-)$ in various approaches. It is noted that most of the predicted cental values are lower than the newly measured value [7–10] as long as the contribution from FSI is not included. We add the contributions from FSI to that calculated in pQCD (NLO); then one can find that the resultant central value is consistent with the data [28]: $\mathcal{B}(B_s \to K^+K^-) = (3.3 \pm 0.9) \times 10^{-5}$ within 1σ.

The deviations of our theoretical prediction from the data might come from the loophole in our calculation. As indicated above, we only consider the absorptive part of the hadronic triangle. Indeed the dispersive part may also make substantial contributions [29]. As suggested in the literature, the dispersive contribution should be smaller than the absorptive one (it is consistent with the general principle of the quantum field theory), or at most has the same magnitude as that of the absorptive part. If the contribution of the dispersive part is indeed of the same order as the absorptive part, then taking it into account, we may have a result that is even closer to the data.

In general, even though we cannot precisely reproduce the experimental data, we can confirm ourselves that the

[1]Here, m denotes the mass of the exchanged meson and β is a phenomenological parameter which is set to 1 in our calculation. $\Lambda_{\rm QCD}$ is taken as 220 MeV.

TABLE III. Various approaches and predictions on $\mathcal{B}(B_s \to K^+K^-)$ (in units of 10^{-6}), and the ratio $\mathcal{R} = \mathcal{B}(B_s \to \pi^+K^-)/\mathcal{B}(B_s \to K^+K^-)$.

	QCDF [7]	pQCD (LO) [8]	pQCD (NLO) [9]	SCET [10]	FSI	FSI + pQCD (NLO)	Experiment [4,5]
$\mathcal{B}$	$25.2^{+12.7+12.5}_{-7.2-9.1}$	$13.6^{+8.6}_{-5.2}$	$15.6^{+5.1}_{-3.9}$	$18.2 \pm 6.7 \pm 1.1 \pm 0.5$	$13.6^{+6.3}_{-5.1}$	$29.2^{+8.1}_{-6.5}$	33 ± 9
$\mathcal{R}$[a]	0.194	0.360	0.314	0.269	0.360	0.167	0.148

[a]Since the main contribution for the transition $B_s \to \pi^+K^-$ comes from the tree diagram and all predictions made in various models on this channel are close to each other, we use the data $\mathcal{B}(B_s \to \pi^+K^-) = 4.9 \times 10^{-6}$ as input in our calculations.

FSI is important and non-negligible for understanding the $\mathcal{B}(B_s \to K^+K^-) > \mathcal{B}(B_s \to \pi^+K^-)$ conflict.

IV. CONCLUSION AND DISCUSSION

As expected, the LHCb is extensively aiming on the study of the B physics, especially to look for some "anomalies" in experiments, which need high statistics and precise measurements. For B_s's charmless nonleptonic two-body decay, the early Monte Carlo studies show that nearly 37 K $B_s \to K^+K^-$ signals will be seen at 2 fb^{-1} integrated luminosity [30]. On the other hand, 314 ± 27 $B_s \to \pi^+K^-$ signals were observed based on the 2010 data with its integrated luminosity 0.35 fb^{-1} [31]. We believe that the statistics of the B_s decay into K^+K^- and π^+K^- is sufficient to draw a definite decision about their branching ratios. The latest result reported by the LHCb Collaboration shows that the branching ratios have been measured as [32]

$$\frac{\mathcal{B}(H_b \to F)}{\mathcal{B}(H'_b \to F')} = \frac{f_{H'_b}}{f_{H_b}} \frac{N(H_b \to F)}{N(H'_b \to F')} \frac{\epsilon_{\text{rec}}(H'_b \to F')}{\epsilon_{\text{rec}}(H_b \to F)} \frac{\epsilon_{\text{PID}}(F')}{\epsilon_{\text{PID}}(F)}, \tag{16}$$

where the $f_{H_b^{(\prime)}}$ is the possibility of b quark hadronizing into hadron H, N is the observed number of signals for certain decay modes, ϵ_{rec} is the efficiency of the reconstruction excluding the particle identification (PID) cuts, and ϵ_{PID} is just the efficiency of PID cuts.

It is noted that in Table III, the experimental data are taken from Refs. [4,5], but the 2012 data of PDG indicate that the branching ratio of $B_s \to K^+K^-$ is $(2.64 \pm 0.28) \times 10^{-5}$ [28], which is smaller than the data in Refs. [4,5]. Moreover, the LHCb Collaboration reports that with the 0.37 fb^{-1} 2011 data, the branching ratios of $B_s \to K^+K^-$ and $B_s \to \pi^+K^-$ are experimentally determined as

$$\begin{aligned} \mathcal{B}(B_s \to K^+K^-) &= (23.0 \pm 0.7 \pm 2.3) \times 10^{-6}, \\ \mathcal{B}(B_s \to \pi^+K^-) &= (5.4 \pm 0.4 \pm 0.6) \times 10^{-6}, \end{aligned} \tag{17}$$

where the former uncertainties are statistical and the later one is systematical, which include the uncertainties of PID calibration, final state radiation with soft gamma, signal shape used for fitting, and the impact of the background: the additional three-body background,[2] the combinatorial background, and the cross-feed background.[3] Since $B_s \to K^+K^-$ and $B_s \to D_s^{(*)+}\bar{D}_s^{(*)-} \to K^+K^-$ have the same final states, it means we cannot single out the contributions of FSI by simply measuring the cross section in experiments. Even though our theoretical prediction on $\mathcal{B}(B_s \to K^+K^-)$ presented in Table III is slightly above the value of the LHCb measurements, it is still consistent with the data within the experimental error tolerance. Therefore we are expecting more precise measurements in the future.

Cheng and Chua suggested that the decay $B_s \to K^+K^-$ is dominated by the penguin diagram, which does not suffer from large CKM suppression [7]. In that scenario the conflict $\mathcal{B}(B_s \to K^+K^-) > \mathcal{B}(B_s \to \pi^+K^-)$ could be partly explained. In our work, we show that the FSI definitely makes an important contribution because $B_s \to D_s^{(*)}\bar{D}_s^{(*)}$ are dominant decay modes of B_s as confirmed by the data and the scattering $D_s^{(*)+}D_s^{(*)-} \to K^+K^-$ is allowed by all the symmetry requirements. Therefore, we might consider that the penguin contribution and the FSI effects are in parallel to contribute to the decay $B_s \to K^+K^-$.[4] Actually, there are several phenomenological parameters in the model for evaluating the FSI effects, which were obtained by fitting the earlier data of heavy flavor processes. One thing can be sure, and that is that the FSI should exist and contribute to the process $B_s \to K^+K^-$, but how significant it would be is determined by both the requirement of experimental measurements and the theoretical estimate. If the data on $\mathcal{B}(B_s \to K^+K^-)$ are indeed going down, the FSI effects for $B_s \to K^+K^-$ would be less significant; by that situation, one can further restrict the involved phenomenological parameters at this energy region. On another aspect, the errors of the measurements are still too large to make a definite conclusion yet, so we are expecting more precise measurements not only from LHCb, but also the future super-B factory.

Definitely, the FSI effects also play roles in other similar decay channels, such as $B^\pm \to K^\pm\omega$. Therefore, careful studies on such modes with the same theoretical framework

[2]Miss or misidentify the pion or kaon in final state.

[3]The uncertainties from the distribution of signal in data and simulation.

[4]In this work we do not consider the penguin contribution, but only that of the FSI effects to the decay. We acknowledge the possible contribution from the penguin diagram as well, further study involving both of them and moreover their interference will be made in our later works.

would be helpful for more accurately evaluating the FSI effects. The PDG indicates that $\mathcal{B}(B^\pm \to K^\pm\omega)/\mathcal{B}(B^\pm \to \pi^\pm\omega) = (6.7 \times 10^{-6})/(6.9 \times 10^{-6})$, so one would expect that $\mathcal{B}(B \to K\omega)$ is also enhanced by the FSI. Moreover, a measurement on the polarization of ω might be helpful to gain more information about the role of the FSI mechanism, and it will be studied in our coming work.

From the experimental aspect, as the FSI is a scattering fully governed by the strong interaction, its proper time is too short to be measured in the LHCb detector. Although the LHCb can well reconstruct the events $B_s \to K^+(\pi^+)K^-$ and identify the π^+K^- purely with the PID cuts [33], it is impossible to distinguish between the direct decays $B_s \to K^+(\pi^+)K^-$ and the sequential ones. Considering the good ability of LHCb for tracking charged particles, we believe that $B^\pm \to K(\pi)^\pm\omega$ is another good channel to study FSI. The decay rates of $\mathcal{B}(B \to \bar{D}^0 D^{(*)+})$ have been well measured as $\sim 1.8 \pm 0.2 \times 10^{-2}$, and we can make Monte Carlo simulations on the rescattering of $\bar{D}^0 D_s^{(*)+}$ into $K(\pi)^+\omega$ to complete the theoretical scenario. The abnormal ratio of the two channels $\mathcal{B}(B^\pm \to K^\pm\omega)/\mathcal{B}(B^\pm \to \pi^\pm\omega) = (6.7 \times 10^{-6})/(6.9 \times 10^{-6})$ [28] can also be explained as the FSI contribution as well.[5] With the $B^\pm \to DK(\pi)^\pm$ measurement [34] and considering the efficiency, about 0.8 K $B^+ \to K(\pi)^+\omega$ can been seen in the 2011 database and it implies that the FSI contribution to the polarization of the vector meson ω might be distinguished from that of the direct decay. Comparing more accurate data with our theoretical calculation, one can expect to gain more knowledge on the FSI effects, for example, how to determine the parameters in the dipole form factor etc.

One object can be recommended for pining down the role of the FSI effects. As is well known, the direct CP violation is proportional to $\sin(\alpha_1 - \alpha_2) \cdot \sin(\phi_1 - \phi_2)$, where the subscripts 1 and 2 refer to two different routes and α, ϕ are the weak and strong phases, respectively. Therefore, if a direct CP violation exists in B_s decays, there must at least be two different routes to the final states which possess different weak and strong phases. In fact, the penguin and tree diagrams have different strong and weak phases. The weak phase of the tree diagram is coming from $V_{ub}V_{us}^*$ and does not posses a strong phase. Instead, the weak phase of the penguin is from $V_{cb}V_{cs}^*$ and the strong phase is due to the existence of the absorptive part of the loop. In our scenario, the sequential processes $B_s(\bar{B}_s) \to D_s^{(*)}\bar{D}_s^{(*)} \to K^+K^-$, which have the same weak phase $V_{cb}V_{cs}^*$, but a different strong phase from the penguin. Because of the extra contribution, the amplitude would become

$$M = M^{\rm tree}e^{i\alpha_1} + M^{\rm penguin}e^{i(\alpha_2+\phi_1)} + M^{\rm seq}e^{i(\alpha_2+\phi_2)},$$

where the last term is the FSI contribution.

Thus, the interference between the tree diagram with a sum of the penguin and sequential processes would be different from the situation where the tree diagram only interferes with the penguin. Therefore, by measuring the CP violation of $\mathcal{B}(B_s \to K^+K^-) - \mathcal{B}(\bar{B}_s \to K^+K^-)$, one can distinguish between the penguin contributions and the FSI effects. But since the experimental errors and theoretical uncertainties are not well controlled so far, we cannot make a more definite prediction on the CP violation yet; we would wait for more precise data to be available and then continue our theoretical computations.

Moreover, according to the above arguments, there does not exist a tree diagram for $B_s \to K^0\bar{K}^0$. Consequently, the direct transition $B_s \to K^0\bar{K}^0$ is uniquely determined by the penguin diagram and FSI effects. Because the tree contribution to $B_s \to K^+K^-$ is very small, the rate of $B_s \to K^0\bar{K}^0$ must be close to that of $B_s \to K^+K^-$. However, the interference between penguin (neglecting the Cabibbo-suppressed penguin diagrams where t quark and u quark are intermediate agents) and FSI does not lead to a direct CP violation at all, because they have the same weak phase. This can be confirmed by future experiments.

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[5] $\mathcal{B}(B \to K(\pi)\omega) = 6.7(6.9) \times 10^{-6}$, at the same time, the Cabibbo suppression determines that the branching ratio of the $K\omega$ final state should be smaller.

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