Meson dominance of hadron form factors and large-N_c phenomenology

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We discuss the pion and nucleon form factors and generalized form factors within the large-N_c approach in the spacelike region. We estimate their theoretical uncertainties through the use of the *half-width rule*, amounting to taking the half width of the resonances as the deviation of their mass parameters. The approach embodies the meson dominance of form factors and the high-energy constraints from perturbative QCD. We compare the results with the available experimental data and lattice simulations. The meson-dominance form factors are generally comparable to the available experimental data within the half-width-rule uncertainties. Our errors are comparable to the experimental uncertainties, but are smaller than the lattice errors.

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I. INTRODUCTION

Electroweak form factors provide valuable information on the internal structure of the existing composite hadrons, particularly on their Lorentz-invariant transverse densities [1] (for reviews of the nucleon form factors see, e.g., Ref. [2] and references therein). On the fundamental level, the QCD counting rules provide the high-momentum behavior of the form factors (FF) [3], which in perturbative QCD (pQCD) acquire logarithmic corrections [4]. In recent years much attention has also been paid to the so-called generalized form factors, i.e., moments of the generalized parton distributions (GPDs) (for reviews see, e.g., Refs. [5–7] and references therein), which can be directly accessed on the Euclidean lattices [8], even when no physical experiments can.

Electromagnetic form factors are by far the best understood, both theoretically and experimentally. An early and phenomenologically very successful approach to study these quantities was initiated by the vector-meson-dominance (VMD) model (for reviews see Refs. [9,10]). On a formal level, VMD is implemented in two equivalent ways. At the field-theoretic level one postulates the so-called current-field identities which state the proportionality between fields of stable mesons and the related conserved currents with identical quantum numbers. Alternately, within the framework of dispersion relations, one may saturate the matrix elements of the currents with delta-like spectral functions. However, vector mesons are unstable resonances with a finite decay width, and thus corrections to the narrow resonance approximation are expected.

There arises a natural question of *what* numerical value for the mass of the resonance one should use [11–13]. While the rigorous definition of a resonance mass m_R and width Γ_R corresponds to a pole of an analytically continued amplitude in the complex s-plane on the second Riemann sheet, $s = m_R^2 - i\Gamma_R m_R$, complex energies cannot be measured experimentally. Of course, since a resonance has a real mass distribution which generally depends on the process where the resonance is produced, a variety of possibilities arise to extract the maximum of an integrated mean value, which becomes identical and independent of the background in the narrow-resonance limit. Thus, a conservative estimate is made if the mass of a resonance is determined with an accuracy of about its half width. Following Refs. [14–18] we suggest to use the *half-width rule*: take the intrinsic uncertainty of a resonance mass as the range $m_R \pm \Gamma_R/2$ to estimate the finite width corrections. The average width-to-mass ratio listed by the Particle Data Group (PDG) was found to be $\langle \Gamma_R/m_R \rangle = 0.12(8)$, for both mesons and baryons [16,17].

From a fundamental point of view, the success of the VMD model remained a mystery until it was shown how it arises within a well-defined approximation of QCD. Indeed, in the large-N_c limit resonances become narrow [19,20] and hadronic form factors turn out to be meromorphic functions under the assumption of confinement; although in principle they contain an infinite set of resonances, they can generally be written as a sum of pole functions [21–25]. This allows us to build up an effective theory at a purely hadronic level, with no explicit reference to the underlying quark-gluon dynamics: all the specific QCD information is contained in the chiral and short-distance constraints. More generally, meson dominance of any vertex function with appropriate quantum numbers and the meromorphic property also extends to the case of

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generalized form factors. From a practical point of view, for the lowest-rank generalized form factors the only difference from the standard form factors associated to conserved currents is that while the momentum dependence remains scale-independent, the normalization undergoes the QCD evolution [26]. Thus, generalized form factors in the spacelike region provide nothing but meson masses estimated in the large-N_c limit.

It is quite remarkable that given the simplicity of the leading-N_c contribution, where only tree diagrams are needed, the $1/N_c$ corrections turn out to be extremely complicated, and despite courageous efforts [27–31] they have not been worked out completely. In the present paper we provide a simple way of estimating the size of the $1/N_c$ corrections by implementing a rather obvious idea that the mass of a resonance is determined with an accuracy of about its half width [14–18]. While this provides a large-N_c meaning to the half-width rule, it also yields rather rewarding consequences: the predicted theoretical uncertainties turn out to be comparable or larger than the corresponding experimental results, but at the same time smaller than current lattice estimates. In all considered cases we find an overall agreement between the theory and experiment.

The requirement of quark-hadron duality at large N_c involves, as a matter of principle, an infinite tower of hadronic states [24,25,32–37]. This becomes clear for two-point functions, where further asymptotic constraints between the meson, the decay amplitudes, and the meson spectra are derived.[1] A careful scrutiny of the Particle Data Table confirms, with the help of the half-width rule [17], the radial and angular momentum Regge pattern in the meson spectrum, proposed in Ref. [38]. This fact provides a phenomenological basis for large-N_c Regge-model calculations of form factors [14,39–41].

Unlike for the two-point functions, the short-distance QCD constraints for the form factors may be saturated with a finite number of states, provided that detailed pQCD information (the occurrence of the logarithmic corrections) is given up. One may take advantage of this fact by using a sufficiently large but finite number of meson states, such that the correct asymptotics is reproduced up to (slowly varying) logarithms, which allows one to impose the appropriate normalization conditions [13,23–25,42]. The implementation of the pQCD logs from the mesonic side is not at all trivial; a possible mechanism, involving infinitely many states, is suggested in Ref. [41], where it was also shown that the onset of pQCD in the pion form factor might possibly occur at "cosmologically" large momenta. We recall in this regard that almost model-independent upper and lower bounds on the spacelike form factor are established above $Q^2 \sim 7$ GeV^2 [43]. To be fair, it is not completely clear whether at present the logarithmic pQCD corrections are distinctly seen in the current experimental data. We note that approaches with an infinite number of meson resonances are considered along the holographic framework [44,45].

Usually, nucleon form factors are conveniently parameterized as dipole functions, which describe the data quite successfully in a given range of momenta. However, this can only correspond *exactly* to a sum of two simple degenerate poles with opposite residues. Actually, we will show that if the error bars on the monopole mass are taken into account, one can make the dipole overlap with a product of monopoles within the corresponding error bars provided with the half-width rule.

To summarize, our construction is based on the following assumptions:

(i) Hadronic form factors in the spacelike region are dominated by mesonic states with the relevant quantum numbers.
(ii) The high-energy behavior is given by pQCD, and the number of mesons is taken to be minimal to satisfy these conditions.
(iii) Errors in the meson-dominated form factors are estimated by means of the half-width rule, i.e., by treating resonance masses as random variables distributed with the dispersion given by the width.

The paper is organized as follows. In Sec. II we review for completeness the basics of meson dominance in the narrow-width limit for the case of the nucleon. In Sec. III we digress on the role played by the finite-width corrections, providing a large-N_c justification for the intuitively obvious half-width rule for the masses. In Secs. IV and V we carry out the analysis for several pion and nucleon form factors, respectively. Finally, in Sec. VI we draw our main conclusions.

II. MESON DOMINANCE OF FORM FACTORS

First, the implications of meson dominance on form factors will be illustrated with the nucleon form factors as an example. Quite generally, the nucleon form factors are defined as the matrix element of a given current or composite interpolating field, $J(x)$:

$$\langle N(p', s')|J(0)|N(p, s)\rangle = \bar{u}(p', s')\Gamma_J(p' - p)u(p, s), \quad (1)$$

where $u(p, s)$ and $u(p', s')$ are Dirac spinors corresponding to the initial and final four-momentum and spin states, respectively. The quantity $\Gamma_J(p' - p)$ involves the Dirac matrices. Lorentz indices are suppressed for clarity of notation. Meson dominance of the form factor corresponds to parametrizing the current with a superposition of meson fields with the same quantum numbers as the current,

$$J(x) = \sum_n f_n \Phi_n(x), \quad (2)$$

which means that the meson may decay into the vacuum through the current,

[1] In fact, the only possibility to avoid the dimension-two operators at large Q^2, not present in the conventional operator product expansion, is by assuming an infinite set of states [35,36].

$$\langle 0|J(0)|\Phi_n\rangle = f_n. \tag{3}$$

This also implies that the two-point correlator can be written as

$$\Pi_{JJ}(t) = \sum_n \frac{f_n^2}{m_n^2 - t}, \tag{4}$$

where m_n is the mass of the meson state Φ_n. On the other hand, the meson-nucleon-nucleon coupling g_n is defined via

$$\langle N(p', s')|(\partial^2 + M_n^2)\Phi_n(0)|N(p, s)\rangle = \bar{u}(p', s')g_n u(p, s) \tag{5}$$

(g_n in general involves a Dirac structure). Then

$$\langle N(p', s')|J(0)|N(p, s)\rangle = \sum_n f_n \langle N(p', s')|\Phi_n(0)|N(p, s)\rangle = \bar{u}(p')F(t)u(p), \tag{6}$$

where the form factor is

$$F(t) = \sum_n \frac{f_n g_n}{m_n^2 - t}. \tag{7}$$

It satisfies, on very general field-theoretic grounds and up to suitable subtractions, a dispersion relation:

$$F(t) = \text{c.t.} + \frac{1}{\pi}\int_{t_0}^{\infty} \frac{\text{Im}F(t')}{t' - t - i\epsilon} dt', \tag{8}$$

where "c.t." stands for counterterms and t_0 is the threshold. In the narrow-resonance approximation one has the spectral density

$$\text{Im}F(t) = \pi\sum_n c_n m_n^2 \delta(t - m_n^2), \tag{9}$$

yielding, up to subtractions, the sum of monopoles,

$$F(t) = \sum_n c_n \frac{m_n^2}{m_n^2 - t}, \tag{10}$$

corresponding to Eq. (7) with $c_n m_n^2 = f_n g_n$, which is constant, i.e., independent of t.

The asymptotic behavior of the form factors determines the number of necessary subtractions. Thus, for a form factor falling off as $\sim t^{-N}$ we have the set of conditions

$$\sum_n c_n m_n^k = 0 \qquad k = 0, \ldots N - 1, \tag{11}$$

and thus the minimum number of meson states needed to satisfy these constrains is N, whence

$$F(t) = F(0)\prod_{n=1}^{N} \frac{m_n^2}{m_n^2 - t}. \tag{12}$$

This simple ansatz already predicts the couplings in Eq. (7) "for free." One can, of course, add more resonances by multiplying Eq. (12) with the factor

$$\frac{1 - d_k t/m_k^2}{1 - t/m_k^2}, \tag{13}$$

where the unknown coefficient d_k may be determined if some couplings c_n are known from the experiment.

Quite generally, on the basis of the large-Q^2 expansion, one has [4] $(-t)^{i+1}F_i(t) \sim \log(-t/\Lambda)^{-\gamma}$, with the anomalous dimension $\gamma \sim 2$ and weakly depending on the number of flavors. Fits of the form factors hardly see any impact of these pQCD logs at the currently available momenta [46].

The corresponding radii are given by the expansion

$$\frac{F(t)}{F(0)} = 1 + \frac{t}{6}\langle r^2\rangle + \cdots \tag{14}$$

We recall that the radii are quite sensitive to chiral ($1/N_c$-suppressed) corrections and, actually, in some channels they diverge for $m_\pi \to 0$.

Both two- and three-point correlators, Eqs. (4) and (10) respectively, require in principle an infinite number of mesons. Note, however, that the sign of the residues $f_n g_n$ appearing in the form factors is arbitrary, while the sign appearing in the two-point correlator is positive. This possibly provides a quite different mechanism for cancellations, and hence for the form of how the short-distance constraints are fulfilled. In short, the two-point functions *need* infinitely many mesonic states to comply to pQCD, whereas the three-point functions, such as the form factors, can be saturated with a finite number of meson states.

It is useful to notice that we may also deduce the component of the NN potential due to the exchange of the meson states Φ_n:

$$\langle N(p_1', s_1')N(p_2', s_2')|V|N(p_1, s_1)N(p_2, s_2)\rangle = \sum_n \bar{u}(p_1', s_1')g_n u(p_1, s_1)\bar{u}(p_2', s_2')g_n u(p_2, s_2)\frac{1}{m_n^2 - t}. \tag{15}$$

Via crossing, the $\bar{N}N$ scattering amplitude can be obtained as well.

III. FINITE-WIDTH CORRECTIONS

A question of fundamental and practical importance is what mass value should one use for the meson states in the VMD expression for the form factors [11,17]. Naively, one might take the "experimental" value.[2] However, the extended VMD formula for the form factor, Eq. (10), corresponds to the large-N_c limit. As such, it is subject to the $1/N_c$ corrections which generate a corresponding mass shift. The form of these corrections can in principle be evaluated by computing meson loops within the resonance chiral perturbation theory [27–31,47]. The general structure of the correction for the form factor corresponds to the replacements

$$\frac{g_n f_n}{m_n^2 - t} \to \frac{G_n(t)f_n}{m_n^2 - t - \Sigma(t)}, \tag{16}$$

[2] This value also depends on the experimental process and may differ within the half-width rule.

where $\Sigma(t)$ is the self-energy. However, the question remains what the size of these corrections is numerically. Strictly speaking, such a question can only be answered by a lattice calculation at different values of N_c (see, e.g., Ref. [48] and references therein). Unfortunately, as we argue below, within a purely hadronic resonance theory we can only make an educated guess, since there are undetermined counterterms encoding the effects of the high-energy states that are not explicitly considered. For instance, for the case of the ρ-meson we may take into account the decay into 2π, which is a real process, but also the virtual $\bar{K}K$ excitation, etc. Our lack of an explicit knowledge on *all* excitations makes it difficult, if not impossible, to predict the mass shift reliably.

A. Mass shift and the width

To elaborate on the mass-shift effect in greater detail, let us consider the two-point function yielding the mesonic resonance propagator,

$$D(s) = \frac{1}{s - m_0^2 - \Sigma(s)}. \tag{17}$$

The mass parameter m_0 is the tree level resonance mass, which is $\mathcal{O}(N_c^0)$, whereas the self-energy, coming from meson loops, is suppressed: $\Sigma(s) = \mathcal{O}(N_c^{-1})$.

Let us consider, for instance, the self-energy correction of the scalar or vector mesons due to pion loops. Analyticity implies that the self-energy satisfies the dispersion relation[3]

$$\Sigma(s) = \text{c.t.} + \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\Sigma(s' + i0^+)}{s' - s}, \tag{18}$$

where "c.t." means suitable subtractions. The pole position, $s = s_R = m_R^2 - \mathrm{i}m_R\Gamma_R$, is given by

$$s_R - m_0^2 - \Sigma(s_R) = 0. \tag{19}$$

This is a complicated self-consistent equation, but within the $1/N_c$ expansion it can be solved perturbatively, yielding

$$s_R = m_0^2 + 2m_0\Delta m_R - i\Gamma_R m_0 + \mathcal{O}(N_c^{-2}), \tag{20}$$

where

$$\Delta m_R = \frac{1}{2m_0}\mathrm{Re}\Sigma(m_0^2), \tag{21}$$

$$\Gamma_R = -\frac{1}{m_0}\mathrm{Im}\Sigma(m_0^2). \tag{22}$$

[3]We disregard spin complications; see, e.g., Ref. [49] for details.

Note that the imaginary part is proportional to the corresponding decay width,

In general, there appears a threshold momentum dependence for the decay amplitude which is proportional to the phase space. The form reflects the spin of the resonance, such as

$$\Gamma(s) = \Gamma_R\left[\frac{\rho(s)}{\rho(m_R^2)}\right]^{2J+1}, \tag{23}$$

with $\rho(s) = \sqrt{1 - 4m_\pi^2/s} \equiv p/\sqrt{s}$, where p is the center-of-mass momentum when $m_0^2 \to s_0$. Obviously, the number of subtractions in Eq. (18) depends on the assumed off-shellness. For the previous choice of $\Gamma(s)$, which becomes a constant at large s, we need at least one subtraction, which we may choose to be, e.g., at $s = 0$, and thus in terms of the principal value integral we have

$$\Delta m_R = \frac{1}{2m_0}\left[\mathrm{Re}\Sigma(0) + \frac{1}{\pi}0 = \int_{4m_\pi^2}^{\infty} ds' \frac{m_0^2}{s'}\frac{\mathrm{Im}\Sigma(s')}{s' - m_0^2}\right]. \tag{24}$$

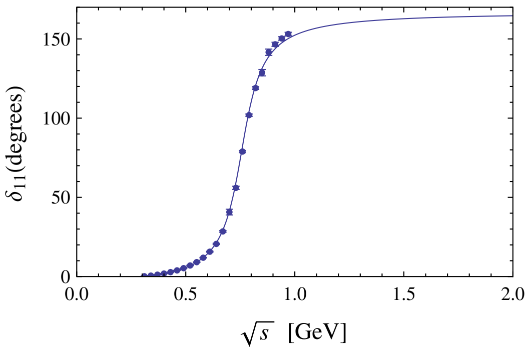
Therefore Δm_R depends on an arbitrary constant $\mathrm{Re}\Sigma(0) = \mathcal{O}(N_c^{-1})$, which cannot be determined from the dispersion integral or the lowest-order parameters and hence naively becomes independent of the width. Other momentum-dependent widths, not vanishing at high s, may introduce additional subtractions. The present discussion illustrates our statement that one cannot generically compute the mass shift in a model-independent way.[4]

This lack of predictive power within the purely hadronic theory is not surprising. However, from a microscopic point of view the meson self-energy can be understood as the coupling of the $q\bar{q}$ bound state to the meson continuum, and physical resonances turn into Feschbach resonances. The relevant scale corresponds to the string-breaking distance, defining a physical momentum scale which may be described as a transition form factor from $\bar{q}q$ states to mesonic channels. This implies that the mass shift due to closed channels is necessarily negative as it corresponds to second-order perturbation theory below the closed channels, but also that the mass shift due to the open channel scales exactly as the decay width. In the Appendix we analyze some specific models where we can see that within uncertainties a natural rough estimate of the mass shift is given by the half-width rule.

B. Finite-width effects in the spacelike region

Finite-width corrections for the pion charge form factor were pioneered by Gounaris and Sakurai [50]. They have implemented the $e^+e^- \to \pi^+\pi^-$ final-state interactions in

[4]Unless further information is available.



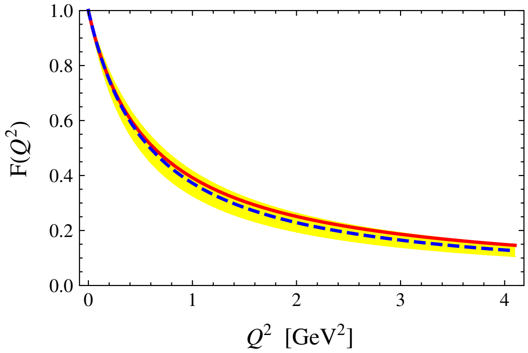


FIG. 1 (color online). The $(I, J) = (1, 1)\pi\pi$ scattering phase shift as a function of the center-of-mass energy (in GeV). We use the BW representation discussed in the text. The data are from the analysis of Ref. [86] (left panel). The pion charge form factor in the spacelike region as a function of the momentum Q^2 (in GeV^2) for the Omnes representation (solid red) is compared with the simple monopole form (dashed blue). The band corresponds to taking a monopole with the half-width rule (right panel).

the time-like region, where they are crucial. In this section we analyze the influence of widths on the spacelike region. To this end, we use Watson's theorem on final states, which can be written as[5]

$$\frac{F_V(s+\mathrm{i}0^+)}{F_V(s-\mathrm{i}0^+)} = \frac{T_{11}(s-\mathrm{i}0^+)}{T_{11}(s+\mathrm{i}0^+)} \equiv e^{2\mathrm{i}\delta_{11}(s)}, \qquad 4m_\pi^2 \le s \le 4m_K^2, \tag{25}$$

where $F_V(s)$ is the form factor, $T_{11}(s \pm \mathrm{i}0^+)$ is the $\pi\pi$ partial-wave scattering amplitude in the vector-isovector channel $(J, I) = (1, 1)$, and $\delta_{11}(s)$ is the corresponding phase shift. A well-known solution to this discontinuity equation is given in terms of the Omnes function,

$$\Omega(s) = \exp\left[\frac{s}{\pi}\int_{4m^2}^{\infty}\frac{\delta_J(s')}{s'(s-s')}\right], \tag{26}$$

which fulfills $\Omega(0) = 1$. A solution to Eq. (25) is given by just taking $F_V(s) = P(s)\Omega(s)$, with $P(s)$ being an arbitrary polynomial. Choosing $P(s) = 1$ we have

$$F_V(s) = \Omega(s). \tag{27}$$

In the case of a zero-width resonance the phase shift is $\delta_{11}(s) = \pi/2\theta(s - m_\rho^2)$, and one gets the monopole form factor,

$$F_V(s) = \frac{m_V^2}{m_V^2 - s}, \tag{28}$$

featuring VMD in its simplest version.

We use a simple Breit-Wigner (BW) parametrization for the vector-isovector $\pi\pi$-phase shift, obtained as

$$e^{2i\delta_{11}(s)} = \frac{D_V(s+i0^+)}{D_V(s-i0^+)}, \tag{29}$$

where

$$[D_V(s)]^{-1} = s - m_\rho^2 + \mathrm{i}m_\rho\Gamma_\rho\left[\frac{(s-4m_\pi^2)m_\rho^2}{(m_\rho^2-4m_\pi^2)s}\right]^{\frac{3}{2}}. \tag{30}$$

The p-wave character of the $\rho \to 2\pi$ decay can be recognized in the phase-space factor. The Omnes form factor is depicted in Fig. 1. As we can see, the finite-width correction lies within the band corresponding to the half-width rule imposed on top of the monopole form factor, considering here $m_\rho = 0.77$ GeV and $\Gamma_\rho = 0.15$ GeV.

C. The half-width rule

As we can see, the subleading $1/N_c$ corrections in the spacelike region essentially correspond to keeping the meson dominance form and to changing the parameters. By making simple calculations we have seen that a conservative bound on the mass shift is given by the half-width rule. From a spectral point of view this is quite natural when we appeal to the Källén-Lehmann representation of the resonance two-point function,

$$D(s) = \int_0^\infty d\mu^2 \frac{\rho(\mu^2)}{\mu^2 - s - i0^+}. \tag{31}$$

We may then use the probabilistic interpretation of the line shape

$$P(\mu) = Z\rho(\mu). \tag{32}$$

If we take a Breit-Wigner shape for $D(s)$ (neglecting the threshold effects), we have

[5] Another way of writing the relation, which generalizes trivially to coupled channels, is through the use of the Bethe-Salpeter equation $FT^{-1} = \Gamma V$ [51], which yields $\mathrm{Disc}F_V(s+\mathrm{i}0^+)^{-1} = \mathrm{Disc}T_{11}(s+\mathrm{i}0^+)^{-1}$. Then

$$\mathrm{Disc}F_i(s+\mathrm{i}0^+)^{-1} = \mathrm{Disc}\sum_j T_{ij}(s+\mathrm{i}0^+)^{-1}.$$

Above the $\bar{K}K$ production threshold Watson's theorem requires considering also the kaon form factor and the corresponding extension to coupled channels, i.e., the mixing with $\pi\pi \to \bar{K}K$ transitions.

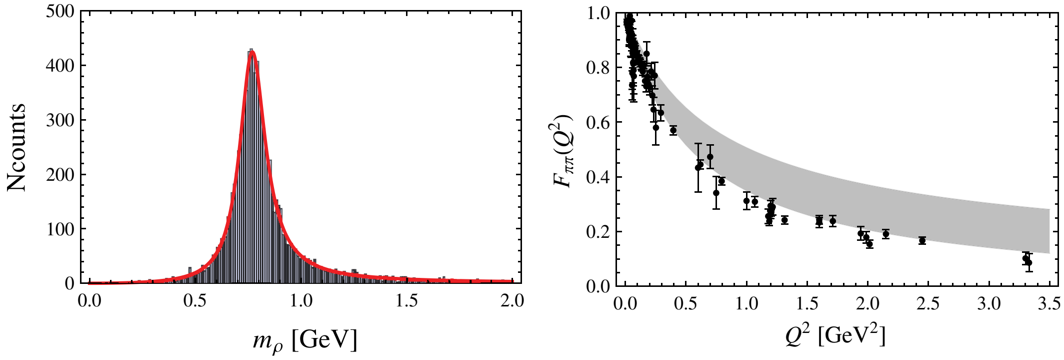


FIG. 2 (color online). Sampling of the ρ-meson mass according to the BW spectral distribution. We sample $N = 10^4$ values and bin them with $\Delta m = 20$ MeV (left panel). The monopole form factor $F_{\pi\pi}(Q^2) = m_\rho^2/(m_\rho^2 + Q^2)$ sampled with the previous distribution is compared to the experimental data [87–97] (right panel).

$$P_{\rm BW}(\mu) = \frac{1}{\pi}\frac{2\Gamma\mu^2}{(\mu^2 - M^2)^2 + \Gamma^2\mu^2}, \quad (33)$$

which is normalized to unity, $\int d\mu P(\mu) = 1$.

The random implementation for a given distribution is obtained in a standard way by inverting the relation expressing the coordinate independence of probabilities,

$$P(\mu)d\mu = dz, \quad (34)$$

with $z \in U[0, 1]$ being a uniformly distributed variable.[6] The result for a BW shape in the case of the ρ-meson for $N = 10^4$ samples is shown in Fig. 2. The idea amounts to treating the resonance mass as a random variable and to propagating its effect in all observables. Of course, different shapes produce somewhat different confidence levels. For definiteness, we will take Gaussians which have shorter tails and are symmetric around the resonance value.

In Fig. 2 we plot the monopole form factor

$$F_V(Q^2) = \frac{m_\rho^2}{m_\rho^2 + Q^2} \quad (35)$$

according to the BW distribution of the mass.

IV. PION FORM FACTORS

A. Electromagnetic form factor

The charge form factor of the pion is given by

$$\langle\pi^+(p')|J_\mu^{\rm em}(0)|\pi^+(p)\rangle = (p'^\mu + p^\mu)F_V(q^2), \quad (36)$$

with $q = p' - p$ and the electromagnetic current $J_\mu^{\rm em}(x) = \sum_{q=u,d,s,\ldots} e_q\bar{q}(x)\gamma_\mu q(x)$, where e_q denotes the quark charges in units of the elementary charge. The charge normalization requires

$$F_V(0) = 1. \quad (37)$$

Actually, in the spacelike region, where $t = -Q^2$, $F(t)$ is real and at large Q^2 values the pQCD methods can be applied, yielding asymptotically [52–57]

$$F_V(-Q^2) = \frac{16\pi f_\pi^2\alpha(Q^2)}{Q^2}\left[1 + 6.58\frac{\alpha(Q^2)}{\pi} + \cdots\right], \quad Q^2 \gg m^2, \quad (38)$$

with $f_\pi = 92.3$ MeV denoting the pion weak decay constant, and m standing for the lowest vector-meson mass. If we ignore the slowly varying logarithm, we get $F_V(t) = \mathcal{O}(t^{-1})$, and in the large-$N_c$ limit one has

$$F_V(t) = \sum_{V=\rho,\rho',\ldots} c_V\frac{m_V^2}{m_V^2 - t}, \quad (39)$$

where $\sum_V c_V = 1$ and $c_V = g_{V\pi\pi}F_V/m_V$.

The simplest formula fulfilling this constraint and Eq. (37) is the VMD solution

$$F_V(t) = \frac{m_\rho^2}{m_\rho^2 - t}, \quad (40)$$

whence $g_{\rho\pi\pi}f_\rho = m_\rho$. The $\rho - \gamma$ coupling is given by

$$\Gamma(\rho \to e^+e^-) = \frac{4\pi\alpha^2}{3}\frac{F_\rho^2}{m_\rho}, \quad (41)$$

whereas the $\rho \to \pi\pi$ decay is

$$\Gamma(\rho \to \pi\pi) = \frac{g_{\rho\pi\pi}^2 m_\rho}{48\pi}. \quad (42)$$

Using the PDG numbers one gets for $m_\rho = 0.77$ GeV the values $f_\rho = 0.156$ GeV and $g_{\rho\pi\pi} = 6$.

One of the important features of the pion form factor is that the radius has large chiral corrections; thus, we may improve the phenomenology by adding one extra meson, ρ'. Then

[6] For a Lorentz distribution, $P(\mu) = Z\mu/((\mu^2 - M_R^2)^2 + \Gamma_R^2 M_R^2)$, the relation is given by $\mu^2 = M_R^2 + M_R\Gamma_R\tan(\pi z/2)$, with $z \in U[0, 1]$. This distribution must be cut and normalized when negative μ^2 values are generated.

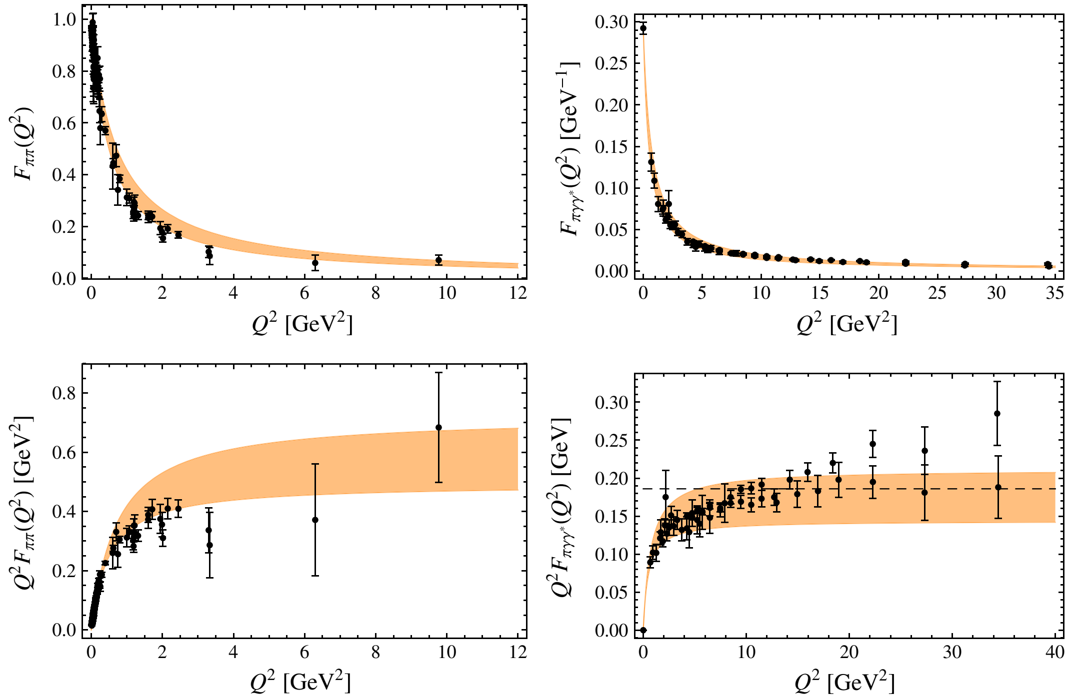


FIG. 3 (color online). Top row: The pion charge form factor with the half-width-rule compared with the experimental data [87–97] (left) and the pion-photon transition form factor compared with the experimental data [61–64] (right). Bottom row: The pion charge form factor multiplied by Q^2 (left) and the pion-photon transition form factor multiplied by Q^2 (right); the horizontal dashed line represents the asymptotic value $2f_\pi$.

$$F_V(t) = (1-c)\frac{m_\rho^2}{m_\rho^2 - t} + c\frac{m_{\rho'}^2}{m_{\rho'}^2 - t}, \tag{43}$$

such that

$$\frac{1}{6}\langle r^2\rangle = (1-c)\frac{1}{m_\rho^2} + c\frac{1}{m_{\rho'}^2}. \tag{44}$$

Thus by imposing the physical value of $\langle r^2\rangle$ we get the value of c for any m_ρ and $m_{\rho'}$. Taking again the PDG values for those quantities [$m_\rho = 0.77549(34)$ GeV, $m'_\rho = 1.465(25)$ GeV, $\langle r^2\rangle = (0.672(8)\ \mathrm{fm})^2$] we obtain $c = -0.227(39)$. This result is shown in Fig. 3.

One could also carry out the analysis the other way around, starting from Eqs. (40) and (43) with all of the constants treated as free parameters to be determined by a fit to the experimental data. It was shown in Refs. [11,37] that by this method one can retain a precise value for $\langle r^2\rangle$, even though the masses do not have their precise physical values. In the large-N_c limit, when the functions become meromorphic, this fitting procedure is mathematically safe thanks to the convergence theorems from the Padé theory [37]. In this framework, Fig. 3 of Ref. [11] shows the half-width rule as a good estimation of the systematic error done on the determination of the poles when fitting the spacelike data.

B. Axial form factor

The axial form factor of the pion is intimately related to the pion radiative decay $\pi^\pm \to l^\pm \gamma\nu$ (with l standing for e or μ) and its hadronic contribution. The decay proceeds via ordinary inner bremsstrahlung from the weak decay $\pi^\pm \to l^\pm\nu$ accompanied by the photon radiated from the external charged particles, and the structure-dependent interaction (SD) between the photon and the virtual hadronic states, with contributions of both vector and axial-vector form factors.

For the transition $\pi^+ \to \gamma\nu_e e^+$ [58], the structure-dependent amplitude is given by

$$M_{\mathrm{SD}} = ieG_F\cos\theta_c \bar{u}_\nu\gamma_\mu(1-\gamma_5)v_e\epsilon_\nu^* M^{\mu\nu}/\sqrt{2}m_\pi, \tag{45}$$

where ϵ^μ is the polarization vector of the photon, G_F is the weak interaction coupling constant, and θ_c is the Cabibbo angle. The hadronic contribution is enclosed in the amplitude $M^{\mu\nu}$ and reads

$$M^{\mu\nu} = F_V(t)\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta - iF_A(t)[q^\nu(q^\mu + p^\mu) - g^{\mu\nu}q\cdot(q+p)], \tag{46}$$

with $F_{V,A}(t)$ denoting the vector and axial-vector form factors, respectively. By assuming axial-meson dominance, we have

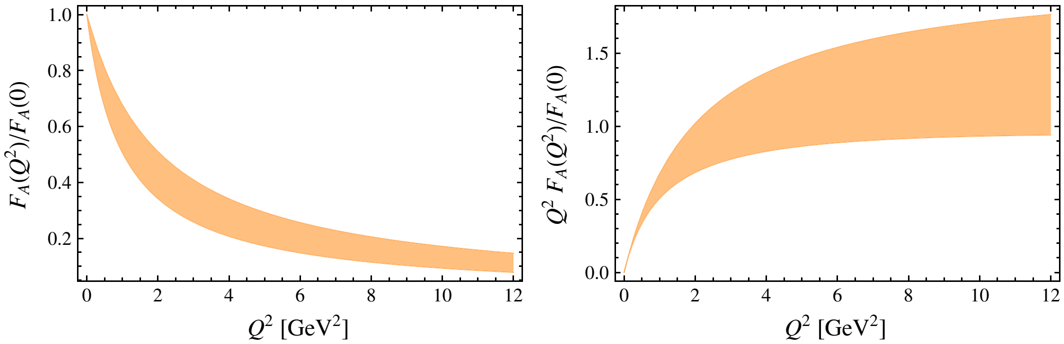


FIG. 4 (color online). Axial form factor in the spacelike region $t < 0$.

$$F_A(t) = F_A(0)\frac{M_A^2}{M_A^2 - t}, \tag{47}$$

with $F_A(0) = 0.0119(1)$ [59]. The result obtained with the half-width rule is presented in Fig. 4.

C. Transition form factor

The pion-photon transition form factor $\pi^0 \to \gamma\gamma^*$ has been subjected to vigorous discussion in recent years. Firstly, its value at the origin is fixed by the chiral anomaly,

$$F(0) = \frac{1}{4\pi^2 f_\pi}, \tag{48}$$

while its asymptotic behavior is given by

$$F(Q^2) \to \frac{6 f_\pi}{N_c Q^2} + \cdots \tag{49}$$

A simple model fulfilling both conditions is

$$F(Q^2) = \frac{1}{4\pi^2 f_\pi}\frac{m_\rho^2}{m_\rho^2 + Q^2}, \tag{50}$$

provided one has the relation

$$m_\rho^2 = \frac{24\pi^2 f_\pi^2}{N_c}, \tag{51}$$

which gives $m_\rho = 823$ MeV for $f_\pi = 92.6$ MeV or $m_\rho = 770$ MeV for $f_\pi = 86.6$ MeV in the chiral limit. The result is shown in Fig. 3.

If we include two resonances [60], ρ and ρ', we get, after imposing the anomaly and large-Q^2 behavior,

$$F(Q^2) = \frac{1}{4\pi^2 f_\pi}\frac{m_\rho^2 m_{\rho'}^2 + 24 f_\pi^2 \pi^2 Q^2/N_c}{(m_\rho^2 + Q^2)(m_{\rho'}^2 + Q^2)}. \tag{52}$$

The result is shown in Fig. 5, using $m_\rho = 0.775$ GeV, $m'_\rho = 1.465$ GeV, $\Gamma_\rho = 0.150$ GeV, and $\Gamma'_\rho = 0.400$ GeV.

One could even go beyond this approximation by including a third resonance (the ρ''). This introduces a new parameter that can be fixed by the derivative of the form factor at the origin, the parameter a_π [13]:

$$F(Q^2) = \frac{1}{4\pi^2 f_\pi}\frac{m_\rho^2 m_{\rho'}^2 m_{\rho''}^2 + bQ^2 + 24 f_\pi^2 \pi^2 Q^4/N_c}{(m_\rho^2 + Q^2)(m_{\rho'}^2 + Q^2)(m_{\rho''}^2 + Q^2)}, \tag{53}$$

where the parameter b can be obtained through a matching procedure to the low-energy expansion of $F(Q^2)$, i.e., $b = m_\rho^2 m_{\rho'}^2 m_{\rho''}^2 (\frac{a_\pi}{m_\pi^2} + \frac{1}{m_\rho^2} + \frac{1}{m_{\rho'}^2} + \frac{1}{m_{\rho''}^2})$. Given $m_\pi = 0.135$ GeV, m_ρ, m'_ρ, $m''_\rho = 1.720(20)$ GeV, and $a_\pi = 0.032(4)$, we obtain $b = 5.82(18)$.

Figure 3 of Ref. [13] shows how the half-width rule provides a good estimate of the systematic error on the determination of poles of rational approximants, such as Eqs. (52) and (54), when fitting to the spacelike data [61–64].

D. Gravitational form factor

The gravitational quark form factors of the pion [65], Θ_1 and Θ_2, are defined through the matrix element of the quark part of the energy-momentum tensor in the one-pion state:

$$\langle \pi^b(p')|\Theta^{\mu\nu}(0)|\pi^a(p)\rangle = \frac{1}{2}\delta^{ab}[(g^{\mu\nu}q^2 - q^\mu q^\nu)\Theta_1(q^2) + 4P^\mu P^\nu \Theta_2(q^2)], \tag{54}$$

where $P = \frac{1}{2}(p' + p)$, $q = p' - p$, and a, b are the isospin indices. The gravitational form factors satisfy the low-energy

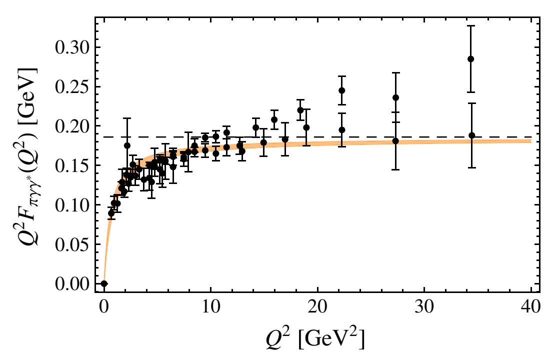


FIG. 5 (color online). Band: the pion-photon transition form factor of Eq. (52). Points: various experimental data [61–64]. The horizontal line represents the theoretic asymptotic value of $2f_\pi$.

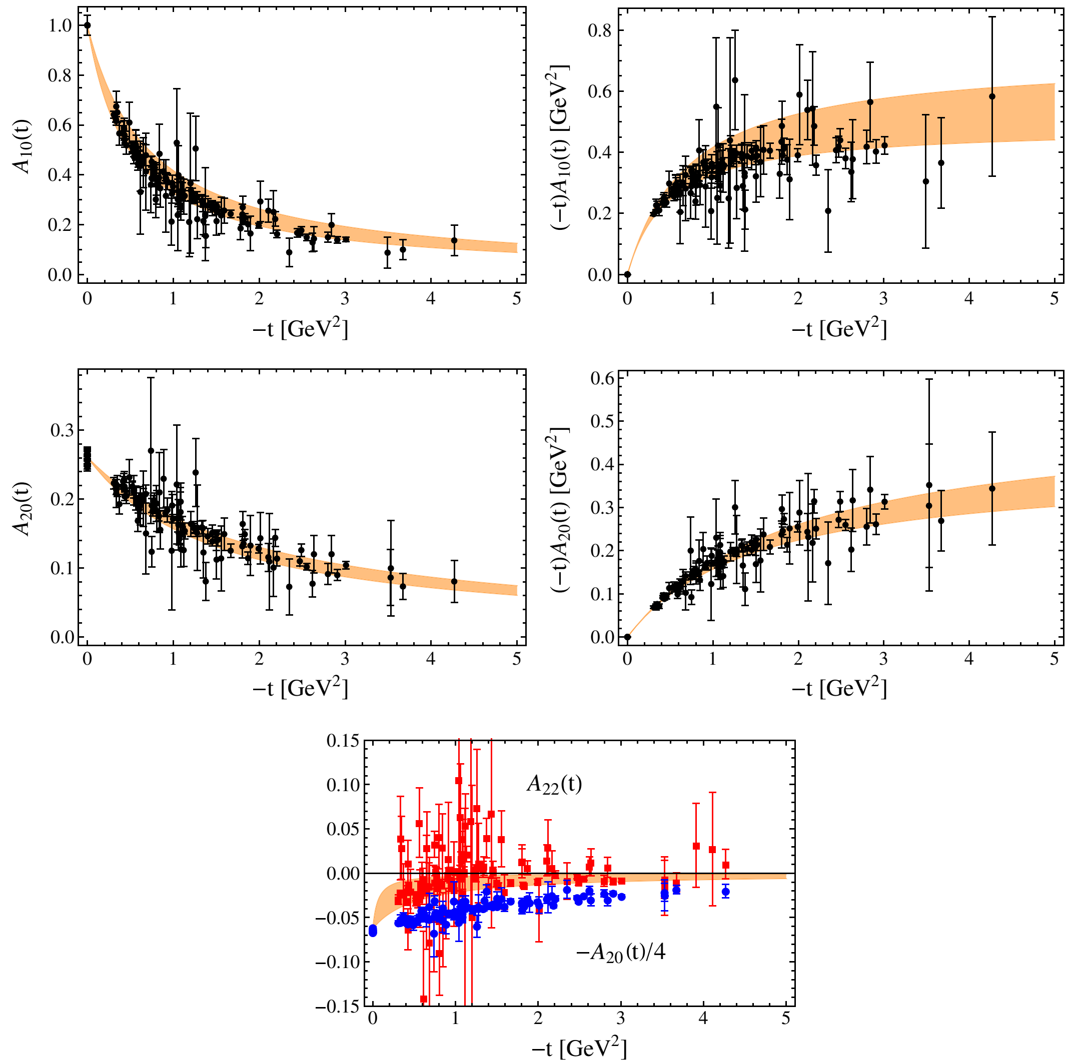


FIG. 6 (color online). Electromagnetic form factor $A_{10}(t)$, the corresponding $(-t)A_{10}(t)$, and the quark part of the gravitational form factor $A_{20}(t)$ and $(-t)A_{20}(t)$, compared to the lattice data from Ref. [66] (top and middle row, respectively). The spin-0 gravitational form factor of the pion, $A_{22}(t)$, from the lattice calculation of Refs. [66,67] is extrapolated to the physical pion mass. Red squares correspond to $A_{22}(t)$ and blue circles to $-A_{20}(t)/4$ (bottom row).

theorem, $\Theta_1(0) - \Theta_2(0) = \mathcal{O}(m_\pi^2)$ [65]. The trace part is dominated by scalar states, while the traceless component is dominated by spin-2 tensor mesons.

Following the standard notation for the moments of the pion GPDs [26], we introduce

$$A_{20}(t) = \frac{1}{2}\Theta_1(t), \qquad A_{22}(t) = -\frac{1}{2}\Theta_2(t), \tag{55}$$

where the symbols $\Theta_i(t)$ denote the quark parts of the gravitational form factors of Eq. (54). We also consider the moment

$$A_{10}(t) = F_V(t), \tag{56}$$

with $F_V(t)$ denoting the electromagnetic form factor described in Sec. IVA.

The results of monopole representations with the half-width rule are shown in Fig. 6, where we consider

$$A_{10}(t) = \frac{m_\rho^2}{m_\rho^2 - t}, \qquad A_{20}(t) = A_{20}(0)\frac{m_{f_2}^2}{m_{f_2}^2 - t}, \tag{57}$$

with $m_{f_2} = 1.320$ GeV, $\Gamma_{f_2} = 0.185$ GeV, and $A_{20}(0) = 0.261$. We also show in Fig. 6 the low-energy theorem that relates $A_{20}(t)$ with the other moment, $A_{22}(t)$. The relation $A_{22}(0) = -\frac{1}{4}A_{20}(0)$ is compared to the lattice data of Refs. [66,67].

V. NUCLEON FORM FACTORS

A. Electromagnetic form factors

In the nonstrange sector the electromagnetic current is given by

$$J^\mu_{\rm em}(x) = \frac{1}{2}J^\mu_B(x) + J^{\mu 3}_V(x), \tag{58}$$

where $J_B^\mu(x)$ is the baryon current and $J_V^{\mu 3}(x)$ is the third component of the isospin current:

$$J_B^\mu(x) = \bar{q}(x)\gamma^\mu q(x), \qquad J_V^{\mu a}(x) = \bar{q}(x)\gamma^\mu \frac{\tau^a}{2} q(x). \quad (59)$$

The matrix elements of these currents are

$$\begin{aligned}\langle N(p')|J_B^\mu(0)|N(p)\rangle &= \bar{u}(p')\Big[\gamma^\mu F_1^{I=0}(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M_N}F_2^{I=0}(q^2)\Big]u(p),\\ \langle N(p')|J_V^{\mu a}(0)|N(p)\rangle &= \bar{u}(p')\frac{\tau^a}{2}\Big[\gamma^\mu F_1^{I=1}(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M_N}F_2^{I=1}(q^2)\Big]u(p), \end{aligned} \quad (60)$$

where $q = p' - p$, and F_1 and F_2 are the Dirac and Pauli form factors, respectively. The relation to the proton and neutron form factors is

$$F_i^p = (F_i^{I=0} + F_i^{I=1}), \qquad F_i^n = (F_i^{I=0} - F_i^{I=1}), \quad (61)$$

where

$$F_1^p(0) = 1, \qquad F_1^n(0) = 0, \quad (62)$$

$$F_2^p(0) = \kappa_p, \qquad F_2^n(0) = \kappa_n. \quad (63)$$

The quantities $\kappa_p = 1.793$ and $\kappa_n = -1.913$ are the anomalous proton and neutron magnetic moments, respectively. The electric and magnetic Sachs form factors are defined as

$$\begin{aligned} G_E^p(q^2) &= F_1^p(q^2) + \frac{q^2}{4M_N^2}F_2^p(q^2),\\ G_E^n(q^2) &= F_1^n(q^2) + \frac{q^2}{4M_N^2}F_2^n(q^2),\\ G_M^p(q^2) &= F_1^p(q^2) + F_2^p(q^2),\\ G_M^n(q^2) &= F_1^n(q^2) + F_2^n(q^2). \end{aligned} \quad (64)$$

The normalization conditions become

$$G_E^p(0) = 1, \qquad G_E^n(0) = 0, \quad (65)$$

$$G_M^p(0) = \mu_p, \qquad G_M^n(0) = \mu_n, \quad (66)$$

where $\mu_p = 2.79\mu_N$ and $\mu_n = -1.91\mu_N$, with $\mu_N = e/(2M_N)$ denoting the nuclear magneton.

The asymptotic behavior for $t \to -\infty$ is given by [3]

$$t^{i+1}F_i(t) \to [\log(-t/\Lambda^2)]^{-\gamma}, \qquad (i = 1, 2), \quad (67)$$

where the anomalous dimension $\gamma \sim 2$ is weakly dependent on the number of flavors. As mentioned before, such a slowly changing log behavior cannot be reproduced with a finite number of resonances, and thus we assume it to be constant. At the leading order in the large-N_c expansion the form factors read

$$\begin{aligned} F_1^{I=0}(t) &= \sum_V \frac{g_{\omega NN} f_{\omega\gamma}}{m_\omega^2 - t}, \qquad F_2^{I=0}(t) = \sum_V \frac{f_{\omega NN} f_{\omega\gamma}}{m_\omega^2 - t},\\ F_1^{I=1}(t) &= \sum_V \frac{g_{\rho NN} f_{\rho\gamma}}{m_\rho^2 - t}, \qquad F_2^{I=1}(t) = \sum_V \frac{f_{\rho NN} f_{\rho\gamma}}{m_\rho^2 - t}. \end{aligned} \quad (68)$$

We define the strong isoscalar and isovector vertices

$$\begin{aligned}\langle N(p')|\omega^\mu|N(p)\rangle &= \bar{u}(p')\Big[g_{\omega NN}\gamma^\mu + f_{\omega NN}\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\Big]u(p),\\ \langle N(p')|\rho_a^\mu|N(p)\rangle &= \bar{u}(p')\frac{\tau_a}{2}\Big[g_{\rho NN}\gamma^\mu + f_{\rho NN}\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\Big]u(p). \end{aligned} \quad (69)$$

According to Eq. (67), the minimum number of resonances is two and three for the Dirac and Pauli form factors, respectively. We use the normalization conditions at the origin, the asymptotic conditions, and fix the vector and tensor couplings to the lowest-lying resonance, $g_{\omega NN}$, $f_{\omega NN}$, $g_{\rho NN}$, $f_{\rho NN}$ in the isoscalar and isovector channels. Our illustrative goal here is to predict the form factors without attempting a detailed fit to the data.

For the case of mesons the $1/N_c$ counting provides strict bookkeeping, with the leading diagrams taking the form of trees. For the baryon sector this is not the case. The reason is that in some channels the meson-baryon coupling scales as $\sqrt{N_c}$ (for instance, the pion-nucleon coupling). This leads to enhancement, such that the coupling of the current via a meson loop acquires the same N_c scaling as the tree-level coupling (see Refs. [68,69] and references therein). In view of this, the meson-dominance approach is heuristic, assuming more than the $1/N_c$ expansion. We will show, however, that this approach, and in particular the half-width rule, work quite well for the nucleon form factors.

After imposing all the conditions on the form factors in Eq. (68), we get

$$\begin{aligned} F_1^{I=0}(t) &= \frac{1}{2}\frac{1 - c_0 t/m_{\omega''}^2}{(1 - t/m_\omega^2)(1 - t/m_{\omega'}^2)(1 - t/m_{\omega''}^2)},\\ F_1^{I=1}(t) &= \frac{1}{2}\frac{1 - c_1 t/m_{\rho''}^2}{(1 - t/m_\rho^2)(1 - t/m_{\rho'}^2)(1 - t/m_{\rho''}^2)},\\ F_2^{I=0}(t) &= \frac{1}{2}\frac{1}{(1 - t/m_\omega^2)(1 - t/m_{\omega'}^2)(1 - t/m_{\omega''}^2)},\\ F_2^{I=1}(t) &= \frac{1}{2}\frac{1}{(1 - t/m_\rho^2)(1 - t/m_{\rho'}^2)(1 - t/m_{\rho''}^2)}, \end{aligned} \quad (70)$$

where the constants c_0 and c_1 are determined from the values of $g_{\omega NN}$ and $g_{\rho NN}$, respectively, as

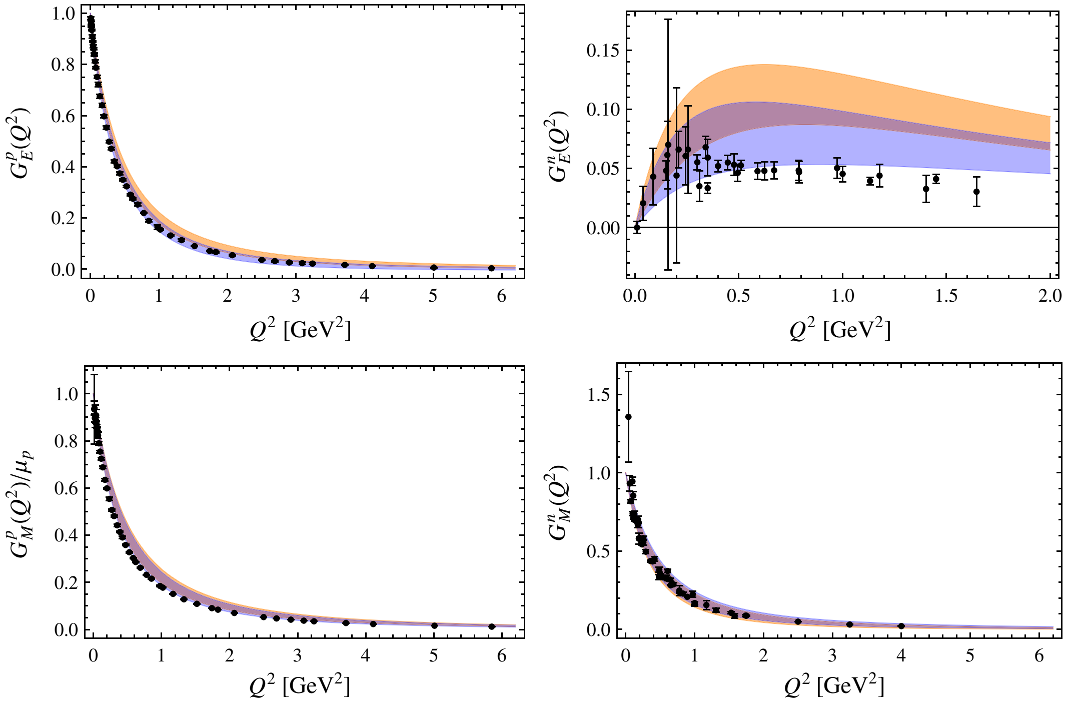


FIG. 7 (color online). The electromagnetic nucleon form factors compared to the data for the proton [98] and neutron [99] (and references therein). Orange (lighter) band: $g_{\omega NN} = 9$. Blue (darker) band: $g_{\omega NN} = 12$.

$$\frac{g_{\omega NN} f_{\omega\gamma}}{m_\omega^2} = \frac{1}{2}\frac{1 - c_0 m_\omega^2/m_{\omega''}^2}{(1 - m_\omega^2/m_{\omega'}^2)(1 - m_\omega^2/m_{\omega''}^2)},$$
$$\frac{g_{\rho NN} f_{\rho\gamma}}{m_\rho^2} = \frac{1}{2}\frac{1 - c_0 m_\rho^2/m_{\rho''}^2}{(1 - m_\rho^2/m_{\rho'}^2)(1 - m_\rho^2/m_{\rho''}^2)}. \tag{71}$$

Thus, with the electromagnetic decay widths for the vector mesons, $F_\rho = 0.149$ GeV, we find [Eq. (41)] $\Gamma(\rho \to e^+ e^-) = 4\pi\alpha^2 F_\rho^2/m_\rho/3$, which is numerically equal to 6.4 keV.

Detailed fits implementing VMD [70] require large Okubo-Zweig-Iizuka violations as well as a huge departure from the flavor SU(3) symmetry (see also Ref. [71]). In particular, the value of $g_{\omega NN} \gg g_{\omega NN}^{\mathrm{SU(3)}}$ [46,72]. However, the simple Kelly parametrization [73] provides a successful fit in the spacelike region as a rational function with the correct large-momentum behavior.

The result of varying the masses according to the half-width rule is presented in Fig. 7. We consider the SU(3) case where $g_{\omega NN}/g_{\rho NN} = f_{\omega NN}/f_{\rho NN} = 3$, as well as a 30% violation for the ratio (the orange and blue bands in Fig. 7, respectively). We recall that in the meson-exchange models [74] or in dispersive analyses of the nucleon form factors [46,72] even much larger symmetry breaking is needed to comply with the phenomenology.

B. Axial and pseudoscalar form factors

The axial matrix element of the nucleon is defined by (for a recent review see, e.g., Ref. [75] and references therein)

$$\langle N(p')|J_A^{\mu a}(0)|N(p)\rangle = \bar{u}(p')\frac{\tau^a}{2}\gamma_5\Big[\gamma_\mu G_A(q^2) + \frac{q^\mu}{2M_N}G_P(q^2)\Big]u(p), \tag{72}$$

where $G_A(q^2)$ and $G_P(q^2)$ are the axial and the induced pseudoscalar form factors, respectively. The QCD axial current is

$$J_A^{\mu a}(x) = \bar{q}(x)\gamma^\mu \frac{\tau^a}{2} q(x), \tag{73}$$

while the current-field identity relating it to the axial-meson field $A^{\mu\nu}$ and the pseudoscalar field P is

$$J_A^{\mu a}(x) = \sum f_A \partial_\nu A^{a\mu\nu}(x) + \sum f_P \partial^\mu P^a(x). \tag{74}$$

Therefore, we have

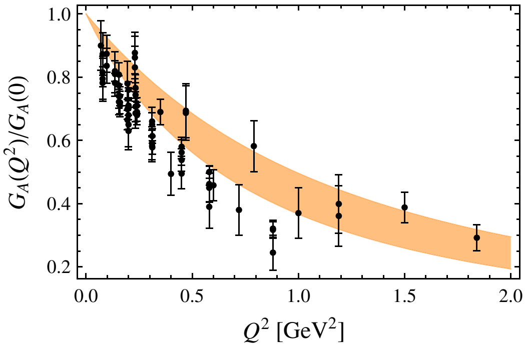
$$G_A(t) = g_A + \sum_A \frac{f_A g_{ANN} t}{M_A^2 - t},$$
$$G_P(t) = -\sum_A \frac{4M_N^2 f_A g_{ANN}}{M_A^2 - t} + \sum_P \frac{4M_N F_P g_{PNN}}{M_P^2 - t}. \tag{75}$$

We use here the extended partially conserved axial-vector current form [76], which for the on-shell mesons reads

$$\partial_\mu J_A^{\mu a}(x) = \sum f_P M_P^2 P^a(x), \tag{76}$$

yielding the following relation among the form factors:

$$2M_N G_A(t) + \frac{t}{2M_N} G_P(t) = \sum_P \frac{2M_P^2 F_P}{M_P^2 - t} g_{PNN}. \tag{77}$$



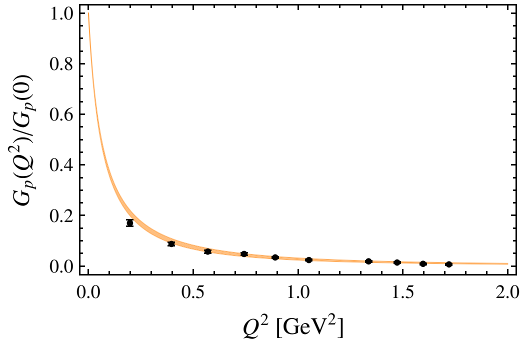


FIG. 8 (color online). The isovector-axial nucleon form factors using two axial masses (left panel). The isovector-pseudoscalar nucleon form factor using two axial masses and the lightest pion considered in Ref. [100] (right panel).

The pseudoscalar-nucleon coupling is defined by

$$\langle p'|(\partial^2 + M_P^2)P_n^a(x)|p\rangle = g_{PNN}\bar{u}(p')i\gamma_5\tau^a u(p). \quad (78)$$

From here we get the (extended) Goldberger-Treiman relation

$$M_N g_A = \sum_P F_P g_{PNN} = f_\pi g_{\pi NN} + f_{\pi'} g_{\pi' NN} + \cdots \quad (79)$$

The high-energy behavior of the weak form factors in QCD was discussed many years ago [77,78]. At high Q^2, one has for the isovector [79] and isoscalar [80] the asymptotic behavior

$$Q^4 G_A(-Q^2) \to \text{const.} \quad (80)$$

We also have the sum rules

$$g_A = \sum_A f_A g_{ANN}, \qquad 0 = \sum_A f_A g_{ANN} M_A^2. \quad (81)$$

It is noteworthy that most determinations of the axial form factor proceed via a dipole fit,

$$G_A(t) = \frac{g_A}{(1 - t/\Lambda_A^2)^2}, \quad (82)$$

suggesting a $1/t^2$ falloff at large t. The values of the parameter are $\Lambda_A = 1.026(21)$ GeV or $\Lambda_A = 1.069(16)$ GeV, depending on the process [75]. In the literature Λ_A is denoted and called the *axial mass* (see, e.g., Ref. [75]). This is not our axial meson mass, since Eq. (82), although phenomenologically successful, cannot be justified from a field-theoretic point of view and is in contradiction with the large-N_c-motivated parametrization.

The minimum meson-dominance ansatz compatible with low- and high-energy constraints reads

$$G_A(t) = g_A \frac{m_{a_1}^2 m_{a_1'}^2}{(m_{a_1}^2 - t)(m_{a_1'}^2 - t)},$$
$$G_P(t) = G_A(t)\frac{G_P(0)}{m_p^2 - t}. \quad (83)$$

By applying the half-width rule to this parametrization, i.e., using $m_{a_1} = 1.230$ GeV, $m_{a_1'} = 1.647$, $\Gamma_{a_1} = 0.425$ GeV, and $\Gamma_{a_1'} = 0.254$ GeV, we get the results depicted in Fig. 8. As we can see, the results are in reasonable agreement with the data. Actually, the two axial mesons are incorporated as a product of monopoles, but since they have an overlapping spectrum, the net effect is essentially a dipole form factor with an average mass which is somewhat larger than the usual dipole cutoff.

C. Gravitational form factors

The discussion of the nucleon gravitational form factors follows closely the pion case with suitable changes. For the nucleon case, the quark contributions to these form factors have been determined by the QCDSF Collaboration [81] and the LHPC Collaboration [82].

The decomposition, corresponding to the energy-momentum tensor matrix elements taken between nucleon states, reads

$$\langle p'|\Theta_{\mu\nu}^q|p\rangle = \bar{u}(p')\Big[A_{20}^q(t)\frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} + B_{20}^q(t)\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{4M_N} + C_{20}^q(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{M_N}\Big]u(p), \quad (84)$$

where $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ (the Bjorken-Drell notation), the scalar functions are moments of the GPDs, the momentum transfer is denoted as $\Delta = p' - p$, and the average nucleon momentum is $P = (p' + p)/2$. Taking the trace and applying the Gordon identity, $2M_N\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p')\times(i\sigma^{\mu\rho}\Delta_\rho + 2P^\mu)u(p)$, as well as the Dirac equation, $(\not{p} - M_N)u(p) = 0$ and $\bar{u}(p')(\not{p}' - M_N) = 0$, we obtain the following expression for the spin-0 gravitational form factor of the nucleon:

$$\Theta_N^q(t) = M_N\Big[A_{20}^q(t) + \frac{t}{4M_M^2}B_{20}^q(t) - \frac{3t}{M_N^2}C_{20}^q(t)\Big], \quad (85)$$

whereas the spin-2 (normalized) component becomes

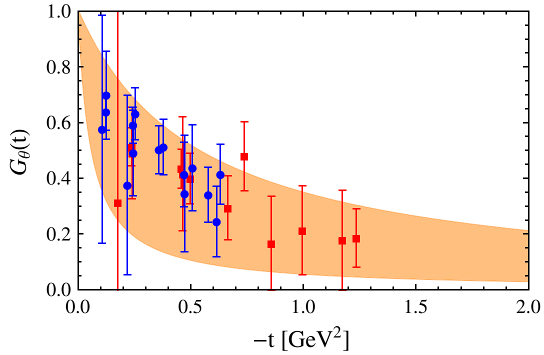


FIG. 9 (color online). Spin-0 gravitational form factor of the nucleon, $G_\theta(t)$, obtained from the lattice simulations of Ref. [82] at the pion masses $m_\pi = 352.3$ MeV (blue circles) and $m_\pi = 356.6$ MeV (red squares).

$$F_T^q(t) = \frac{A_{20}^q(t)}{A_{20}^q(0)}. \tag{86}$$

The trace is dominated by 0^{++} scalar states. For the monopole representation of the gravitational form factor,

$$G_\theta(t) = \frac{m_{f_0}^2}{m_{f_0}^2 - t}, \tag{87}$$

depicted in Fig. 9, we use the updated values of the sigma meson properties from the latest edition of the PDG tables [59], i.e., $m_\sigma = 475$ MeV and $\Gamma_\sigma = 550$ MeV.

The traceless part of the energy-momentum tensor corresponds to a spin-2 isoscalar gravitational form factor which naturally couples to the f_2 meson within a tensor-dominance approximation. For the nucleon this FF has been determined by two lattice groups in Refs. [81,82]. Actually, in Ref. [81] a dipole fit describes the data successfully, namely

$$F_T(t) = \frac{1}{(1 - t/\Lambda_T^2)^2}, \tag{88}$$

with $\Lambda_T = 1.1(2)$ GeV, if a linear extrapolation in m_π to the physical point is assumed. Assuming an asymptotic falloff for the form factor, such that $F_T(t) = \mathcal{O}(t^{-2})$, we just take a sum of two monopoles that reduces to

$$F_T(t) = \frac{m_{f_2}^2}{m_{f_2}^2 - t}\frac{m_{f_2'}^2}{m_{f_2'}^2 - t}. \tag{89}$$

The PDG tables quote $m_{f_2} = 1.320$ GeV and $\Gamma_{f_2} = 0.185$ GeV, and for the first excited state gives $m_{f_2'} = 1.525$ GeV and $\Gamma_{f_2'} = 0.073$ GeV.[7]

[7]This is somewhat different from the results of Ref. [83], with a width about four times larger for the $f_2(1565)$ meson. In any case, for the spin-2 form factor the difference becomes rather irrelevant in the region below 1 GeV.

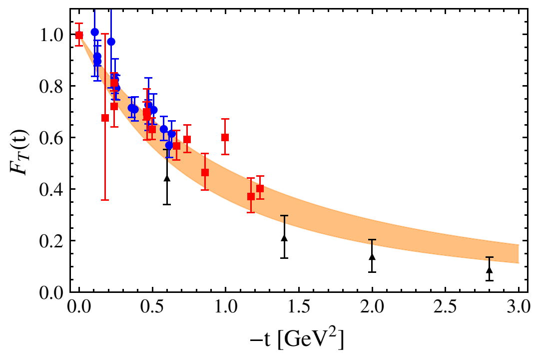


FIG. 10 (color online). Spin-2 isoscalar gravitational form factor of the nucleon, $F_T(t)$, obtained from the lattice simulations of Ref. [82] at the pion masses $m_\pi = 352.3$ MeV and $m_\pi = 356.6$ MeV (blue circles and red squares, respectively), together with the results of Ref. [82] linearly extrapolated to the physical pion mass (black triangles).

As we can see from Fig. 10, similarly to the case of the axial nucleon FF, the dipole FF with an uncertainty essentially corresponds to two monopoles after the half-width rule has been implemented.

VI. CONCLUSIONS

In the present work we have taken advantage of the well-known fact that in the large-N_c limit of QCD the generalized hadronic form factors, probing bilinear $\bar{q}q$ operators with given J^{PC} quantum numbers, feature generalized meson dominance of $\bar{q}q$ states with the same quantum numbers. They assume the monopole form

$$\langle A(p')|J(0)|\langle B(p)\rangle \sim \sum_n \frac{c_n^{AB} m_n^2}{m_n^2 - t}, \tag{90}$$

where m_n are the meson masses and c_n^{AB} are suitable couplings. Thus generalized form factors at some finite momentum transfer essentially measure the masses of the lowest-lying mesons.

The goal of this paper was to present a comprehensive analysis of the pion and nucleon form factors, providing a theoretical uncertainty following from the half-width rule. We have incorporated the following:

(i) The correct asymptotic power-like behavior of form factors (short-distance constraints).
(ii) Low-momentum normalization constraints.
(iii) The minimum number of mesons with the relevant quantum numbers in each channel.
(iv) Theoretical error estimates based on the half-width rule.

Given the approximate nature of the underlying large-N_c expansion, we should not expect perfect agreement with data. Rather, the addressed question is how we can estimate the accuracy of the large-N_c expansion for hadronic form factors, which are basic experimental quantities. In the present paper we provide arguments in favor of the simple rule, where a rough estimate is given by varying the resonance mass within its width range. As we have seen, this provides a surprisingly close answer to the data, which fall within the bands produced with the half-width rule. While this presumably is a conservative assumption, it still provides a cheap estimate based on an independent source of information.

Extension to other baryons and mesons is straightforward, as well as to the transition matrix elements. One useful application of the meson-dominance scheme is the *a priori* determination of generalized form factors, just based on the PDG tables, for which abundant data have begun being produced on the lattice. Our calculations show that improving on the uncertainty predictions based of the half-width rule may require highly refined lattice studies beyond the present accuracy.

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APPENDIX: CUTOFF DEPENDENCE OF THE MASS SHIFT

In this appendix we analyze the mass shift of the lowest-lying scalar and vector resonances due to the most relevant decays, $S \to \pi\pi$ and $V \to \pi\pi$. With a chiral derivative coupling, one obtains [75]

$$\begin{aligned} \Gamma_S(s) &= \frac{3m_S s}{16\pi f_\pi^4}\rho_S\left[c_d + (c_m - c_d)\frac{2m_\pi^2}{m_S^2}\right]^2, \\ \Gamma_V(s) &= \frac{G_V^2 m_V s}{48\pi f_\pi^4}\rho_V^3, \end{aligned} \tag{A1}$$

where $\rho_R = \rho(m_R^2) = \sqrt{1 - 4m_\pi^2/m_R^2}$. Note the additional s factors appearing in the widths. By using the dispersion relation for the self-energy, Eq. (18), we can see that the real part of the integral is quartically divergent and thus three subtractions are needed. This is equivalent to fixing the mass and the width. On the contrary, the imaginary part is finite and provides the width at the physical value. This argument shows that we need more input information, and until then have we no predictive power. Within a large-N_c environment [84] we use BW and not pole reference subtraction values for both mesons in this study.

In order to get an estimate, we assume some form of the hadronic form factor for the vertex $\rho\pi\pi$. Out of ignorance, we consider different cutoff functions,

$$G_{\rho\pi\pi}(q^2) = \left(\frac{q^2 + \Lambda^2}{s + \Lambda^2}\right)^n, \tag{A2}$$

which preserve the imaginary part, $G_{\rho\pi\pi}(m_\rho^2)$, and hence the width. Here $n = 1$ corresponds to a monopole, $n = 2$ corresponds to a dipole, and the limit $n \to \infty$ corresponds to a sharp cutoff. We naturally expect Λ to be in the range above m_ρ and around $m_{\rho'}$, which corresponds to ignoring all states above these energies. With these conditions, and assuming a small correction, we get a mass shift

$$\Delta m_\rho = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds \frac{G_{\rho\pi\pi}(s)\Gamma_{\rho\pi\pi}(s)}{s(s - m_\rho^2)}. \tag{A3}$$

The results are depicted in Fig. 11 as a function of the cutoff scale. As expected, the overall order of magnitude is quite compatible with the half-width rule.

In a theory with a finite number of resonances there is, of course, an implicit high-energy cutoff corresponding to the next (not included) meson. Assuming a vanishing contribution of the higher-energy states, one may make a rough estimate of the mass shift. For instance, for the ρ-meson one has [85] a shift of about half the width. Of course, this may be partly a numerical coincidence, but it illustrates the point that parametrically the mass shift and the width scale in a similar way.

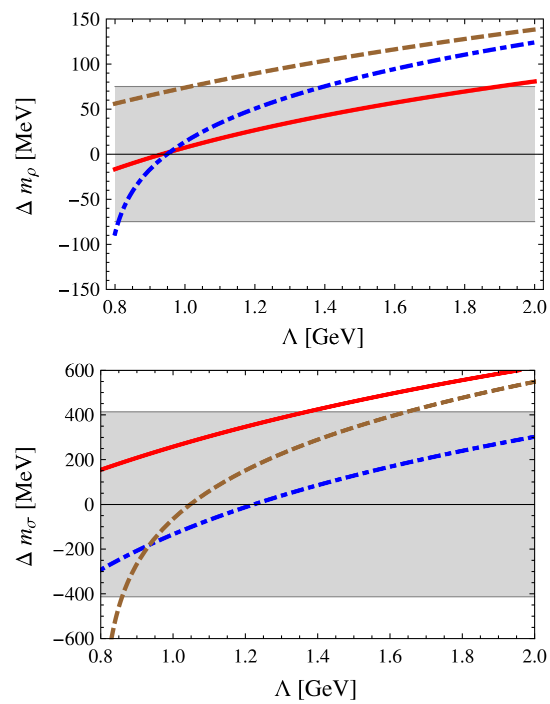

FIG. 11 (color online). Mass shifts Δm (in GeV) of the ρ-meson (top panel) and the σ-meson (bottom panel), due to pion loops, plotted as functions of the cutoff Λ (in GeV) for several cutoff functions: monopole (dashed brown), dipole (solid red), and sharp cutoff (dot-dashed blue).

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