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Exotic $QQ\bar{q}\,\bar{q}$, $QQ\bar{q}\,\bar{s}$, and $QQ\bar{s}\,\bar{s}$ states

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After constructing the possible $J^P = 0^-, 0^+, 1^-$, and 1^+ $QQ\bar{q}\,\bar{q}$ tetraquark interpolating currents in a systematic way, we investigate the two-point correlation functions and extract the corresponding masses with the QCD sum rule approach. We study the $QQ\bar{q}\,\bar{q}$, $QQ\bar{q}\,\bar{s}$, and $QQ\bar{s}\,\bar{s}$ systems with various isospins $I = 0, 1/2, 1$. Our numerical analysis indicates that the masses of doubly bottomed tetraquark states are below the threshold of the two-bottom mesons, two-bottom baryons, and one doubly bottomed baryon plus one antinucleon. Very probably these doubly bottomed tetraquark states are stable.

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I. INTRODUCTION

In the past decade, many charmonium or charmonium-like states were observed in the *B* factories, some of which do not fit in the conventional quark model and are considered as the candidates of the exotic states such as molecular states, tetraquark states, hybrid mesons, baryonium states, etc. For experimental reviews of these new states, see Refs. [1–5].

Hadronic molecular states are loosely bound states composed of a pair of mesons. They are probably bound by the long-range, color-singlet pion exchange. These interesting states generally lie very close to the open-charm/bottom threshold. Some near-threshold charmoniumlike states such as X(3872) and Z(4430) are very good candidates of the molecular states composed of a pair of charmed and anticharmed mesons. In fact, such a possibility was studied extensively in Refs. [6–12].

Tetraquarks are composed of four quarks. They are bound by colored force between quarks. They decay through rearrangement. Some are charged. Some carry strangeness. There are many states within the same multiplet. The low-lying scalar mesons below 1 GeV have been considered good candidates of the tetraquark states. Some of the recently observed charmoniumlike states were also suggested to be candidates of the hidden-charm tetraquark states [13–19]. We have studied the possible pseudoscalar, scalar, vector, and axial-vector hidden-charm/bottom tetraquark states systematically in the framework of the QCD sum rule [16–19].

Besides the hidden-charm/bottom $Q\bar{Q}q\bar{q}$-type tetraquark states, the doubly-charmed/bottomed tetraquark states $QQ\bar{q}\,\bar{q}$ are also very interesting, where Q denotes a heavy quark (beauty or charm) and q denotes one light quark (up, down, strange). Such a four-quark configuration is allowed in QCD. In QED we have the hydrogen atom. One light electron circles around the proton. When two electrons are shared by two protons, a hydrogen molecule is formed. In QCD we have the heavy meson. One light antiquark circles around the heavy quark. If the two light antiquarks were shared by the two heavy quarks in the doubly charmed/bottomed tetraquark system, we would have the QCD analogue of the hydrogen molecule!

On the other hand, if the heavy quark QQ pair is spatially close, it would act as a pointlike antiheavy quark color source $\bar{Q}$ and pick up two light quarks $\bar{q}\,\bar{q}$ to form the bound state $QQ\bar{q}\,\bar{q}$. The existence and stability of such systems have been studied in many different models, such as the MIT bag model [20], chiral quark model [21,22], constituent quark model [23–27], and chiral perturbation theory [28]. Some other useful references are [29–38]. However, their existence and stability are model dependent up to now. More theoretical investigations will be helpful in the clarification of the situation.

In this work, we will discuss the $QQ\bar{q}\,\bar{q}$ systems using the method of QCD sum rule. We first construct the $QQ\bar{q}\,\bar{q}$ currents with $J^P = 0^-, 0^+, 1^-$, and 1^+ in a systematic way. These currents have no definite *C* parity due to their special flavor structures. The isospins of these currents can be $I = 0, 1/2, 1$ with specific quark contents. With the independent currents, we investigate the two-point correlation functions and spectral densities. After performing the QCD sum rule analysis, we extract the masses of the possible $0^-, 0^+, 1^-, 1^+$ $QQ\bar{q}\,\bar{q}$, $QQ\bar{q}\,\bar{s}$, and $QQ\bar{s}\,\bar{s}$ states.

The paper is organized as follows. In Sec. II, we construct the $QQ\bar{q}\,\bar{q}$ currents with $J^P = 0^-, 0^+, 1^-$, and 1^+. In Sec. III, we calculate the correlation functions with the operator product expansion (OPE) method and extract the spectral densities. The results are collected in the Appendix. In Sec. V, we perform the numerical analysis

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and extract the masses of these tetraquark states. We discuss the possible decay patterns of these doubly charmed/bottomed tetraquark states in Sec. V. The last section is a brief summary.

II. TETRAQUARK INTERPOLATING CURRENTS

In this section, we construct the diquark-antidiquark currents with $J^P = 0^-$, 0^+, 1^-, and 1^+ using the same technique in our previous works [16,17]. One can construct the tetraquark current with the qq basis or $\bar{q}q$ basis, $(qq)(\bar{q}\bar{q})$ or $(\bar{q}q)(\bar{q}q)$. However, they can be related by the Fierz transformation. In this work, we consider the first set. Considering the Lorentz structures, there are five independent diquark fields without derivatives: $q_a^T C q_b$, $q_a^T C\gamma_5 q_b$, $q_a^T C\gamma_\mu q_b$, $q_a^T C\gamma_\mu\gamma_5 q_b$, and $q_a^T C\sigma_{\mu\nu} q_b$, where a, b are the color indices. Since $q_a^T C\sigma_{\mu\nu}\gamma_5 q_b$ and $q_a^T C\sigma_{\mu\nu} q_b$ carry different parity, we consider both operators although they are equivalent. These diquark bilinear can be in the symmetric **6** representation or antisymmetric $\bar{\mathbf{3}}$ representation in the color and flavor $SU(3)$ space. Considering the Lorentz structures, we list the properties of these diquark operators in Table I.

Being composed of two quark (antiquark) fields, the diquark (antidiquark) fields should satisfy Fermi statistics. As shown in Table I, the flavor and color structures are entangled for every diquark operator. For example, the flavor and color structures of the scalar diquark operator $q_a^T C\gamma_5 q_b$ are either $(\mathbf{6_f}, \mathbf{6_c})$ or $(\bar{\mathbf{3}}_\mathbf{f}, \bar{\mathbf{3}}_\mathbf{c})$. For $QQ\bar{q}\bar{q}$ systems, the heavy quark pair has the symmetric flavor structure $\mathbf{6_f}$. Its flavor and color structures are then fixed as $(\mathbf{6_f}, \mathbf{6_c})$. To construct the color singlet $QQ\bar{q}\bar{q}$ currents, the heavy quark pair QQ and the light antiquark pair $\bar{q}\bar{q}$ should have the same color structures.

According to Ref. [16], there are ten color singlet $QQ\bar{q}\bar{q}$ currents with $J^P = 0^-$:

$$
\begin{aligned}
S_\pm &= Q_a^T C Q_b(\bar{q}_a\gamma_5 C\bar{q}_b^T \pm \bar{q}_b\gamma_5 C\bar{q}_a^T),\\
V_\pm &= Q_a^T C\gamma_5 Q_b(\bar{q}_a C\bar{q}_b^T \pm \bar{q}_b C\bar{q}_a^T),\\
T_\pm &= Q_a^T C\sigma_{\mu\nu} Q_b(\bar{q}_a\sigma^{\mu\nu}\gamma_5 C\bar{q}_b^T \pm \bar{q}_b\sigma^{\mu\nu}\gamma_5 C\bar{q}_a^T),\\
A_\pm &= Q_a^T C\gamma_\mu Q_b(\bar{q}_a\gamma^\mu\gamma_5 C\bar{q}_b^T \pm \bar{q}_b\gamma^\mu\gamma_5 C\bar{q}_a^T),\\
P_\pm &= Q_a^T C\gamma_\mu\gamma_5 Q_b(\bar{q}_a\gamma^\mu C\bar{q}_b^T \pm \bar{q}_b\gamma^\mu C\bar{q}_a^T),
\end{aligned}
\tag{1}
$$

where "+" denotes the symmetric color structure $[\mathbf{6_c}]_{QQ} \otimes [\bar{\mathbf{6}}_\mathbf{c}]_{\bar{q}\bar{q}}$ and "−" denotes the antisymmetric color structure $[\bar{\mathbf{3}}_\mathbf{c}]_{QQ} \otimes [\mathbf{3_c}]_{\bar{q}\bar{q}}$. Due to the symmetry constraint, it's enough to keep one light diquark piece only in the bracket of Eq. (1) within the calculation. We keep two terms in Eq. (1) to illustrate the color symmetry explicitly.

By considering the symmetric flavor structure for heavy quark pair QQ, only the currents which satisfy the Pauli principle survive. For the pseudoscalar currents in Eq. (1), S_+, V_+, T_-, A_-, and P_+ survive and all the other currents vanish. According to Table I, the $QQ\bar{q}\bar{q}$ ($q = u, d$) operators S_+, V_+, T_- are isovector currents and A_-, P_+ are isoscalar currents. Finally, we obtain the following $QQ\bar{q}\bar{q}$ interpolating currents with $J^P = 0^-$, 0^+, 1^-, and 1^+:

(i) The tetraquark interpolating currents with $J^P = 0^-$ are

$$
\begin{aligned}
\eta_1 &= Q_a^T C Q_b(\bar{q}_a\gamma_5 C\bar{q}_b^T + \bar{q}_b\gamma_5 C\bar{q}_a^T),\\
\eta_2 &= Q_a^T C\gamma_5 Q_b(\bar{q}_a C\bar{q}_b^T + \bar{q}_b C\bar{q}_a^T),\\
\eta_3 &= Q_a^T C\sigma_{\mu\nu} Q_b(\bar{q}_a\sigma^{\mu\nu}\gamma_5 C\bar{q}_b^T - \bar{q}_b\sigma^{\mu\nu}\gamma_5 C\bar{q}_a^T),\\
\eta_4 &= Q_a^T C\gamma_\mu Q_b(\bar{q}_a\gamma^\mu\gamma_5 C\bar{q}_b^T - \bar{q}_b\gamma^\mu\gamma_5 C\bar{q}_a^T),\\
\eta_5 &= Q_a^T C\gamma_\mu\gamma_5 Q_b(\bar{q}_a\gamma^\mu C\bar{q}_b^T + \bar{q}_b\gamma^\mu C\bar{q}_a^T),
\end{aligned}
\tag{2}
$$

in which η_1, η_2, η_3 are isovector currents with $[\bar{\mathbf{6}}_\mathbf{f}]_{\bar{q}\bar{q}}$ and $I = 1$, η_4, η_5 are isoscalar currents with $[\mathbf{3_f}]_{\bar{q}\bar{q}}$ and $I = 0$.

(ii) The tetraquark interpolating currents with $J^P = 0^+$ are

$$
\begin{aligned}
\eta_1 &= Q_a^T C Q_b(\bar{q}_a C\bar{q}_b^T + \bar{q}_b C\bar{q}_b^T),\\
\eta_2 &= Q_a^T C\gamma_5 Q_b(\bar{q}_a\gamma_5 C\bar{q}_b^T + \bar{q}_b\gamma_5 C\bar{q}_b^T),\\
\eta_3 &= Q_a^T C\gamma_\mu Q_b(\bar{q}_a\gamma^\mu C\bar{q}_b^T - \bar{q}_b\gamma^\mu C\bar{q}_b^T),\\
\eta_4 &= Q_a^T C\gamma_\mu\gamma_5 Q_b(\bar{q}_a\gamma^\mu\gamma_5 C\bar{q}_b^T + \bar{q}_b\gamma^\mu\gamma_5 C\bar{q}_b^T),\\
\eta_5 &= Q_a^T C\sigma_{\mu\nu} Q_b(\bar{q}_a\sigma^{\mu\nu} C\bar{q}_b^T - \bar{q}_b\sigma^{\mu\nu} C\bar{q}_a^T),
\end{aligned}
\tag{3}
$$

and all the scalar interpolating currents are isovector currents with $[\bar{\mathbf{6}}_\mathbf{f}]_{\bar{q}\bar{q}}$ and $I = 1$.

TABLE I. Properties of the diquark operators.

$q\Gamma q$	J^P	States	(Flavor, color)
$q_a^T C\gamma_5 q_b$	0^+	$^1S_0$	$(\mathbf{6_f}, \mathbf{6_c})$, $(\bar{\mathbf{3}}_\mathbf{f}, \bar{\mathbf{3}}_\mathbf{c})$
$q_a^T C q_b$	0^-	$^3P_0$	$(\mathbf{6_f}, \mathbf{6_c})$, $(\bar{\mathbf{3}}_\mathbf{f}, \bar{\mathbf{3}}_\mathbf{c})$
$q_a^T C\gamma_\mu\gamma_5 q_b$	1^-	$^3P_1$	$(\mathbf{6_f}, \mathbf{6_c})$, $(\bar{\mathbf{3}}_\mathbf{f}, \bar{\mathbf{3}}_\mathbf{c})$
$q_a^T C\gamma_\mu q_b$	1^+	$^3S_1$	$(\mathbf{6_f}, \bar{\mathbf{3}}_\mathbf{c})$, $(\bar{\mathbf{3}}_\mathbf{f}, \mathbf{6_c})$
$q_a^T C\sigma_{\mu\nu} q_b$	1^-, for $\mu, \nu = 1, 2, 3$ 1^+, for $\mu = 0, \nu = 1, 2, 3$	$^1P_1$ $^3S_1$	$(\mathbf{6_f}, \bar{\mathbf{3}}_\mathbf{c})$, $(\bar{\mathbf{3}}_\mathbf{f}, \mathbf{6_c})$
$q_a^T C\sigma_{\mu\nu}\gamma_5 q_b$	1^+, for $\mu, \nu = 1, 2, 3$ 1^-, for $\mu = 0, \nu = 1, 2, 3$	$^3S_1$ $^1P_1$	$(\mathbf{6_f}, \bar{\mathbf{3}}_\mathbf{c})$, $(\bar{\mathbf{3}}_\mathbf{f}, \mathbf{6_c})$

TABLE II. The quark contents and isospins of the tetraquark currents.

Quark content	$[\bar{q}\,\bar{q}]_{\mathbf{f}}$	I	$J^P = 0^-$	$J^P = 0^+$	$J^P = 1^-$	$J^P = 1^+$
$QQ\bar{q}\,\bar{q}$	$\bar{\mathbf{6}}_{\mathbf{f}}$	1	η_1, η_2, η_3	$\eta_1, \eta_2, \eta_3, \eta_4, \eta_5$	$\eta_1, \eta_2, \eta_3, \eta_4$	$\eta_1, \eta_2, \eta_3, \eta_4$
$QQ\bar{s}\,\bar{s}$	$\bar{\mathbf{6}}_{\mathbf{f}}$	0	η_1, η_2, η_3	$\eta_1, \eta_2, \eta_3, \eta_4, \eta_5$	$\eta_1, \eta_2, \eta_3, \eta_4$	$\eta_1, \eta_2, \eta_3, \eta_4$
$QQ\bar{q}\,\bar{s}$	$\bar{\mathbf{6}}_{\mathbf{f}}$	1/2	η_1, η_2, η_3	$\eta_1, \eta_2, \eta_3, \eta_4, \eta_5$	$\eta_1, \eta_2, \eta_3, \eta_4$	$\eta_1, \eta_2, \eta_3, \eta_4$
	$\mathbf{3}_{\mathbf{f}}$		η_4, η_5		$\eta_5, \eta_6, \eta_7, \eta_8$	$\eta_5, \eta_6, \eta_7, \eta_8$
$QQ\bar{u}\,\bar{d}$	$\mathbf{3}_{\mathbf{f}}$	0	η_4, η_5	...	$\eta_5, \eta_6, \eta_7, \eta_8$	$\eta_5, \eta_6, \eta_7, \eta_8$

(iii) The tetraquark interpolating currents with $J^P = 1^-$ are

$$\begin{aligned} &\eta_1 = Q_a^T C\gamma_\mu\gamma_5 Q_b(\bar{q}_a\gamma_5 C\bar{q}_b^T + \bar{q}_b\gamma_5 C\bar{q}_a^T), \qquad \eta_2 = Q_a^T C\gamma_5 Q_b(\bar{q}_a\gamma_\mu\gamma_5 C\bar{q}_b^T + \bar{q}_b\gamma_\mu\gamma_5 C\bar{q}_a^T),\\ &\eta_3 = Q_a^T C\sigma_{\mu\nu} Q_b(\bar{q}_a\gamma^\nu C\bar{q}_b^T - \bar{q}_b\gamma^\nu C\bar{q}_a^T), \qquad \eta_4 = Q_a^T C\gamma^\nu Q_b(\bar{q}_a\sigma_{\mu\nu} C\bar{q}_b^T - \bar{q}_b\sigma_{\mu\nu} C\bar{q}_a^T),\\ &\eta_5 = Q_a^T C\gamma_\mu Q_b(\bar{q}_a C\bar{q}_b^T - \bar{q}_b C\bar{q}_a^T), \qquad \eta_6 = Q_a^T C Q_b(\bar{q}_a\gamma_\mu C\bar{q}_b^T + \bar{q}_b\gamma_\mu C\bar{q}_a^T),\\ &\eta_7 = Q_a^T C\sigma_{\mu\nu}\gamma_5 Q_b(\bar{q}_a\gamma^\nu\gamma_5 C\bar{q}_b^T - \bar{q}_b\gamma^\nu\gamma_5 C\bar{q}_a^T), \qquad \eta_8 = Q_a^T C\gamma^\nu\gamma_5 Q_b(\bar{q}_a\sigma_{\mu\nu}\gamma_5 C\bar{q}_b^T + \bar{q}_b\sigma_{\mu\nu}\gamma_5 C\bar{q}_a^T) \end{aligned} \tag{4}$$

in which η_1, η_2, η_3, η_4 are isovector currents with $[\bar{\mathbf{6}}_{\mathbf{f}}]_{\bar{q}\,\bar{q}}$ and $I = 1$, η_5, η_6, η_7, η_8 are isoscalar currents with $[\mathbf{3}_{\mathbf{f}}]_{\bar{q}\,\bar{q}}$ and $I = 0$.

(iv) The tetraquark interpolating currents with $J^P = 1^+$ are

$$\begin{aligned} &\eta_1 = Q_a^T C\gamma_\mu\gamma_5 Q_b(\bar{q}_a C\bar{q}_b^T + \bar{q}_b C\bar{q}_a^T), \qquad \eta_2 = Q_a^T C Q_b(\bar{q}_a\gamma_\mu\gamma_5 C\bar{q}_b^T + \bar{q}_b\gamma_\mu\gamma_5 C\bar{q}_a^T),\\ &\eta_3 = Q_a^T C\sigma_{\mu\nu}\gamma_5 Q_b(\bar{q}_a\gamma^\nu C\bar{q}_b^T - \bar{q}_b\gamma^\nu C\bar{q}_a^T), \qquad \eta_4 = Q_a^T C\gamma^\nu Q_b(\bar{q}_a\sigma_{\mu\nu}\gamma_5 C\bar{q}_b^T - \bar{q}_b\sigma_{\mu\nu}\gamma_5 C\bar{q}_a^T),\\ &\eta_5 = Q_a^T C\gamma_\mu Q_b(\bar{q}_a\gamma_5 C\bar{q}_b^T - \bar{q}_b\gamma_5 C\bar{q}_a^T), \qquad \eta_6 = Q_a^T C\gamma_5 Q_b(\bar{q}_a\gamma_\mu C\bar{q}_b^T + \bar{q}_b\gamma_\mu C\bar{q}_a^T),\\ &\eta_7 = Q_a^T C\sigma_{\mu\nu} Q_b(\bar{q}_a\gamma^\nu\gamma_5 C\bar{q}_b^T - \bar{q}_b\gamma^\nu\gamma_5 C\bar{q}_a^T), \qquad \eta_8 = Q_a^T C\gamma^\nu\gamma_5 Q_b(\bar{q}_a\sigma_{\mu\nu} C\bar{q}_b^T + \bar{q}_b\sigma_{\mu\nu} C\bar{q}_a^T), \end{aligned} \tag{5}$$

in which η_1, η_2, η_3, η_4 are isovector currents with $[\bar{\mathbf{6}}_{\mathbf{f}}]_{\bar{q}\,\bar{q}}$ and $I = 1$, η_5, η_6, η_7, η_8 are isoscalar currents with $[\mathbf{3}_{\mathbf{f}}]_{\bar{q}\,\bar{q}}$ and $I = 0$.

For the isovector ($I = 1$) currents, we do not differentiate the up and down quarks in our analysis and denote them by q. However, they should be differentiated for the isoscalar ($I = 0$) currents because the flavor structures of the light anti-diquark $\bar{q}\,\bar{q}$ are antisymmetric. The quark contents are $QQ\bar{u}\,\bar{d}$ for these currents. For the $QQ\bar{s}\,\bar{s}$ systems, only the currents with $[\bar{\mathbf{6}}_{\mathbf{f}}]_{\bar{q}\,\bar{q}}$ flavor structures survive. The isospins for these systems are $I = 0$. We will also discuss the $QQ\bar{q}\,\bar{s}$ systems ($I = 1/2$) by using all the currents in Eqs. (2)–(5). We pick up the interpolating currents with different quark contents in Table II. To calculate the two-point correlation functions, the Wick contractions of the currents for $QQ\bar{u}\,\bar{d}$ and $QQ\bar{q}\,\bar{s}$ systems are different from those for the $QQ\bar{q}\,\bar{q}$ and $QQ\bar{s}\,\bar{s}$ systems.

III. QCD SUM RULE

In QCD sum rule [39–41], we consider the two-point correlation functions of the interpolation currents. For the scalar and pseudoscalar currents, the two-point correlation functions read

$$\Pi(q^2) \equiv i\int d^4x e^{iqx}\langle 0|T\eta(x)\eta^\dagger(0)|0\rangle, \tag{6}$$

where $\eta(x)$ is the corresponding interpolating current. The two-point correlation functions of the vector and axial-vector currents are

$$\begin{aligned} \Pi_{\mu\nu}(q^2) &= i\int d^4x e^{iqx}\langle 0|T\eta_\mu(x)\eta_\nu^\dagger(0)|0\rangle\\ &= \Pi(q^2)\Big(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\Big) + \Pi_0(q^2)\frac{q_\mu q_\nu}{q^2}. \end{aligned} \tag{7}$$

There are two parts of $\Pi_{\mu\nu}$ with different Lorentz structures because η_μ is not a conserved current. $\Pi(q^2)$ is related to the vector and axial-vector meson, while Π_0 is the scalar and pseudoscalar current polarization function. At the hadron level, we express the correlation function in the form of the dispersion relation with spectral function,

$$\Pi(q^2) = (q^2)^N\int_{4(m_q+m_Q)^2}^\infty \frac{\rho(s)}{s^N(s-q^2-i\varepsilon)}ds + \sum_{n=0}^{N-1} b_n(q^2)^n, \tag{8}$$

where

$$\begin{aligned} \rho(s) &\equiv \sum_n \delta(s-m_n^2)\langle 0|\eta|n\rangle\langle n|\eta^\dagger|0\rangle\\ &= f_X^2\delta(s-m_X^2) + \text{continuum}, \end{aligned} \tag{9}$$

where m_X is the mass of the resonance X and f_X is the decay constant of the meson,

$$\langle 0|\eta|X\rangle = f_X, \qquad \langle 0|\eta_\mu|X\rangle = f_X\epsilon^X_\mu, \tag{10}$$

where ϵ^X_μ is the polarization vector of X ($\epsilon^X \cdot q = 0$).

One can calculate the correlation functions at the quark-gluon level via the OPE method. Using the same technique as in Refs. [13,16–18,42,43], we calculate the Wilson coefficients while the light quark propagator and heavy quark propagator are adopted as

$$\begin{aligned} iS_q^{ab} &= \frac{i\delta^{ab}}{2\pi^2 x^4}\hat{x} + \frac{i}{32\pi^2}\frac{\lambda^n_{ab}}{2}g_s G^n_{\mu\nu}\frac{1}{x^2}(\sigma^{\mu\nu}\hat{x} + \hat{x}\sigma^{\mu\nu}) \\ &\quad - \frac{\delta^{ab}}{12}\langle\bar{q}q\rangle + \frac{\delta^{ab}x^2}{192}\langle g_s\bar{q}\sigma\cdot Gq\rangle \\ &\quad - \frac{m_q\delta^{ab}}{4\pi^2 x^2} + \frac{i\delta^{ab}m_q\langle\bar{q}q\rangle}{48}\hat{x}, \\ iS_Q^{ab} &= \frac{i\delta^{ab}}{\hat{p} - m_Q} + \frac{i}{4}g_s\frac{\lambda^n_{ab}}{2}G^n_{\mu\nu} \\ &\quad \times \frac{\sigma^{\mu\nu}(\hat{p} + m_Q) + (\hat{p} + m_Q)\sigma^{\mu\nu}}{(p^2 - m_Q^2)^2} \\ &\quad + \frac{i\delta^{ab}}{12}\langle g_s^2 GG\rangle m_Q \frac{p^2 + m_Q\hat{p}}{(p^2 - m_Q^2)^4}, \end{aligned} \tag{11}$$

where $\hat{x} = \gamma_\mu x^\mu$, $\hat{p} = \gamma_\mu p^\mu$, $\langle\bar{q}g_s\sigma\cdot Gq\rangle = \langle g_s\bar{q}\sigma^{\mu\nu}G_{\mu\nu}q\rangle$, $\langle g_s^2 GG\rangle = \langle g_s^2 G_{\mu\nu}G^{\mu\nu}\rangle$. The spectral density is obtained with: $\rho(s) = \frac{1}{\pi}\,\mathrm{Im}\Pi(q^2)$.

In order to suppress the higher-state contributions and remove the subtraction terms in Eq. (8), we perform the Borel transformation to the correlation function,

$$L_{M_B}\Pi(p^2) = \lim_{\substack{-p^2,n\to\infty \\ -p^2/n = M_B^2}} \frac{1}{n!}(-p^2)^{n+1}\left(\frac{d}{dp^2}\right)^n \Pi(p^2). \tag{12}$$

After performing the Borel transformation and equating the two representations of the correlation function with the quark-hadron duality, we obtain

$$\Pi(M_B^2) = f_X^2 e^{-m_X^2/M_B^2} = \int_{4(m_q+m_Q)^2}^{s_0} ds e^{-s/M_B^2}\rho(s), \tag{13}$$

where s_0 is the threshold parameter, and M_B is the Borel parameter. We can extract the meson mass m_X,

$$m_X^2 = \frac{\int_{4(m_q+m_Q)^2}^{s_0} ds e^{-s/M_B^2} s\rho^{\mathrm{OPE}}(s)}{\int_{4(m_q+m_Q)^2}^{s_0} ds e^{-s/M_B^2}\rho^{\mathrm{OPE}}(s)}. \tag{14}$$

For all the tetraquark currents in Eqs. (2)–(5), we collect the spectral densities $\rho(s)$ in the Appendix, respectively. We neglect the three-gluon condensate $\langle g_s^3 fGGG\rangle$ because their contribution is negligible.

IV. NUMERICAL ANALYSIS

In the QCD sum rule analysis, we use the following values of the parameters [39,44–47] in the chiral limit ($m_u = m_d = 0$):

$$\begin{aligned} m_s(1\ \mathrm{GeV}) &= 125 \pm 20\ \mathrm{MeV}, \\ m_c(m_c) &= (1.23 \pm 0.09)\ \mathrm{GeV}, \\ m_b(m_b) &= (4.2 \pm 0.07)\ \mathrm{GeV}, \\ \langle\bar{q}q\rangle &= -(0.23 \pm 0.03)^3\ \mathrm{GeV}^3, \\ \langle\bar{s}s\rangle &= (0.8 \pm 0.1)\langle\bar{q}q\rangle, \\ \langle\bar{q}g_s\sigma\cdot Gq\rangle &= -M_0^2\langle\bar{q}q\rangle, \\ M_0^2 &= (0.8 \pm 0.2)\ \mathrm{GeV}, \\ \langle g_s^2 GG\rangle &= (0.48 \pm 0.14)\ \mathrm{GeV}^4. \end{aligned} \tag{15}$$

The Borel mass M_B and the threshold value s_0 are two pivotal parameters. Requiring the convergence of the OPE leads to the lower bound $M^2_{B\,\mathrm{min}}$ of the Borel parameter. In the present work, we require that the most important condensate contribution be less than one fourth of the perturbative term. We require that the pole contribution be larger than 30%, which determines the upper bound $M^2_{B\,\mathrm{max}}$ of the Borel parameter. The pole contribution (PC) is defined as

$$\mathrm{PC} = \frac{\int_0^{s_0} ds e^{-s/M_B^2}\rho(s)}{\int_0^{\infty} ds e^{-s/M_B^2}\rho(s)}, \tag{16}$$

which depends on both the Borel mass M_B^2 and the threshold value s_0. s_0 is chosen around the region where the variation of m_X with M_B is minimum in the Borel working region. For a genuine hadron state, the extracted mass from the sum rule analysis is expected to be stable with the reasonable variation of the Borel parameter M_B^2 and threshold s_0.

For all the isovector $cc\bar{q}\,\bar{q}$ and isoscalar $cc\bar{u}\,\bar{d}$ systems, the most important nonperturbative corrections come from the four-quark condensate $\langle\bar{q}q\rangle^2$. Both the quark condensate $\langle\bar{q}q\rangle$ and the quark-gluon mixed condensate $\langle\bar{q}g_s\sigma\cdot Gq\rangle$ vanish when we let $m_u = m_d = m_q = 0$. For the $(I, J^P) = (1, 0^-)$ $cc\bar{q}\,\bar{q}$ system, only the interpolating currents η_1 and η_3 lead to a stable mass sum rule after performing the QCD sum rule analysis. In Fig. 1, we show the mass curves of the extracted hadron mass m_X with M_B^2 and s_0 for the current η_3 with $(I, J^P) = (1, 0^-)$. The variation of m_X with the Borel mass M_B is very weak around $s_0 \sim 23\ \mathrm{GeV}^2$. For η_2, the stability of the mass curves is much worse and m_X grows monotonically with s_0 and M_B. The situation is very similar for the $(I, J^P) = (0, 0^-)$ $cc\bar{s}\,\bar{s}$ systems. Now we keep the m_s related terms in the spectral densities. These terms are very important corrections for the OPE series. The dominant nonperturbative contribution is the quark condensate $\langle\bar{s}s\rangle$ for η_2^s and η_3^s. We show the variations of m_{X^s} with the Borel mass M_B^2 and threshold parameter s_0 for the current η_3^s in Fig. 2.

With the parameters in Eq. (15), we list the working region of the Borel parameters, threshold value s_0, the extracted masses for the currents with $J^P = 0^-$ in Table III. The pole contribution and the masses are

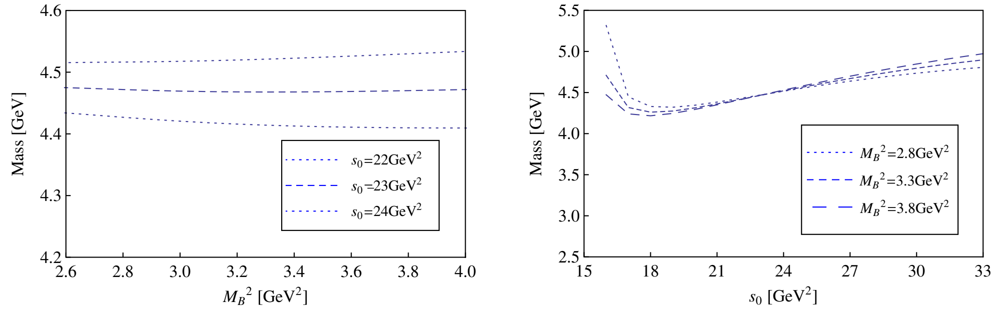

FIG. 1 (color online). The variation of m_X with M_B and s_0 for the current η_3 with $(I, J^P) = (1, 0^-)$ for $cc\bar{q}\,\bar{q}$ system.

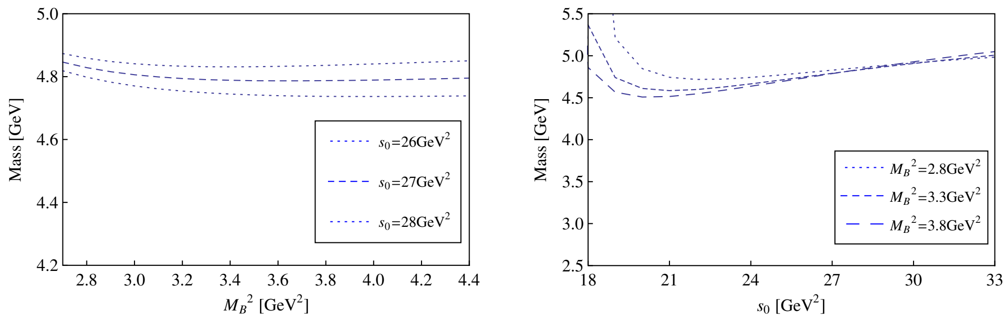

FIG. 2 (color online). The variation of m_{X^s} with M_B and s_0 for the current η_3^s with $(I, J^P) = (0, 0^-)$ for $cc\bar{s}\,\bar{s}$ system.

extracted using the corresponding threshold values s_0 and Borel parameters M_B^2 listed in the table. For the isovector currents η_1, η_2, and η_3, one can also investigate the $QQ\bar{u}\,\bar{d}$ systems besides the $QQ\bar{u}\,\bar{u}$ and $QQ\bar{d}\,\bar{d}$ systems. As mentioned in Sec. II, the Wick contractions of the currents for the $QQ\bar{u}\,\bar{d}$ systems are different from those for the $QQ\bar{d}\,\bar{d}$ and $QQ\bar{u}\,\bar{u}$ systems. However, the correlation functions are the same in the chiral limit ($m_u = m_d = m_q = 0$) except for a constant coefficient. We denote them as $QQ\bar{q}\,\bar{q}$ when we discuss the isovector systems. We take into account the uncertainty of the values of the threshold parameter and variation of the Borel mass to obtain the errors. The other

TABLE III. The numerical results for the doubly charmed/bottomed $QQ\bar{q}\,\bar{q}$, $QQ\bar{q}\,\bar{s}$, and $QQ\bar{s}\,\bar{s}$ systems with $J^P = 0^-$.

	Current	s_0 (GeV^2)	$[M^2_{B\,\min}, M^2_{B\,\max}]$ (GeV^2)	M_B^2 (GeV^2)	m_X (GeV)	PC (%)	f_X (GeV^5)	Open charm/bottom threshold (GeV)
$cc\bar{q}\,\bar{q}$	η_1	24	3.0–3.9	3.4	4.43 ± 0.12	41.2	0.0674	
	η_3	23	2.6–3.6	3.1	4.47 ± 0.12	42.6	0.312	3.872
$cc\bar{u}\,\bar{d}$	η_4	22	2.7–3.4	3.0	4.43 ± 0.13	38.4	0.0870	
	η_5	23	2.5–3.7	3.2	4.41 ± 0.14	41.5	0.106	
$cc\bar{q}\,\bar{s}$	η_1	24	2.9–3.8	3.4	4.45 ± 0.16	40.1	0.0489	
	η_2	24	2.7–3.7	3.4	4.68 ± 0.12	43.1	0.106	
	η_3	26	3.0–4.4	3.8	4.71 ± 0.14	40.6	0.245	3.975
	η_4	25	2.6–4.1	3.4	4.64 ± 0.13	44.9	0.136	
	η_5	24	2.6–4.1	3.4	4.50 ± 0.16	45.9	0.124	
$cc\bar{s}\,\bar{s}$	η_1	25	2.8–4.0	3.4	4.46 ± 0.13	44.3	0.0731	4.081
	η_3	27	2.7–4.3	3.4	4.79 ± 0.17	47.7	0.558	
$bb\bar{q}\,\bar{q}$	η_1	125	7.0–9.6	8.0	10.6 ± 0.3	48.6	0.207	
	η_3	120	6.8–9.4	8.0	10.5 ± 0.3	43.7	1.60	10.60
$bb\bar{u}\,\bar{d}$	η_5	115	7.0–8.1	7.5	10.3 ± 0.2	36.5	0.367	
$bb\bar{q}\,\bar{s}$	η_1	124	7.2–9.6	8.5	10.6 ± 0.2	40.8	0.188	
	η_3	120	7.2–9.1	8.0	10.6 ± 0.2	40.2	0.853	10.69
	η_5	115	7.0–7.9	7.5	10.4 ± 0.2	33.8	0.378	
$bb\bar{s}\,\bar{s}$	η_1	125	6.7–9.6	8.0	10.6 ± 0.3	47.9	0.286	10.78
	η_3	120	6.6–8.9	8.0	10.6 ± 0.3	38.0	1.74	

TABLE IV. The numerical results for the doubly charmed/bottomed $QQ\bar{q}\,\bar{q}$, $QQ\bar{q}\,\bar{s}$, and $QQ\bar{s}\,\bar{s}$ systems with $J^P = 0^+$.

	Current	s_0 (GeV^2)	$[M^2_{B\,\mathrm{min}}, M^2_{B\,\mathrm{max}}]$ (GeV^2)	M_B^2 (GeV^2)	m_X (GeV)	PC (%)	f_X (GeV^5)	Open charm/bottom threshold (GeV)
$cc\bar{q}\,\bar{s}$	η_2	22	2.8–3.6	3.2	4.16 ± 0.14	39.0	0.0548	3.833
	η_3	20	2.6–3.4	3.0	4.02 ± 0.18	39.3	0.0561	
$cc\bar{s}\,\bar{s}$	η_1	28	3.2–4.1	3.4	5.05 ± 0.15	43.3	0.136	3.937
	η_2	22	2.6–3.8	3.2	4.27 ± 0.11	43.2	0.0933	
$bb\bar{q}\,\bar{q}$	η_2	120	7.0–9.8	8.2	10.3 ± 0.3	48.2	0.590	
	η_3	115	6.9–9.0	8.0	10.2 ± 0.3	40.3	0.539	10.56
	η_5	115	6.7–8.8	8.0	10.2 ± 0.3	39.4	1.10	
$bb\bar{q}\,\bar{s}$	η_3	115	6.5–8.8	8.0	10.2 ± 0.3	40.3	0.398	
	η_4	115	5.8–8.6	7.2	10.2 ± 0.3	45.6	0.337	10.65
	η_5	120	6.2–9.8	8.0	10.3 ± 0.3	49.3	0.806	
$bb\bar{s}\,\bar{s}$	η_1	130	7.5–9.8	8.5	11.0 ± 0.2	41.4	0.391	
	η_2	120	6.4–9.8	8.0	10.4 ± 0.3	49.7	0.632	
	η_3	115	6.3–9.0	8.0	10.2 ± 0.3	40.5	0.560	10.73
	η_4	120	6.2–8.4	8.0	10.4 ± 0.3	41.9	0.486	
	η_5	115	6.2–8.8	8.0	10.2 ± 0.3	38.9	1.14	

possible error sources are the truncation of the OPE series and the uncertainty of the quark masses. The uncertainty of the condensate values are not included.

Replacing m_c with m_b in the correlation functions and repeating the same analysis procedures, we obtain the results of the doubly bottomed systems. We also collect the numerical results of the $bb\bar{q}\,\bar{q}$, $bb\bar{q}\,\bar{s}$, and $bb\bar{s}\,\bar{s}$ systems with $J^P = 0^-$ in Table III.

The derivation and analysis of the QCD sum rules of the $J^P = 0^+$, 1^-, 1^+ $QQ\bar{q}\,\bar{q}$, $QQ\bar{q}\,\bar{s}$, and $QQ\bar{s}\,\bar{s}$ systems are very similar to the $J^P = 0^-$ case. We omit details and collect the numerical results in Tables IV, V, and VI, for the $J^P = 0^+$, 1^- and 1^+ systems, respectively.

There are no stable sum rules for the 0^+ and 1^+ $cc\bar{q}\,\bar{q}$ and $cc\bar{u}\,\bar{d}$ systems. These tetraquarks probably do not

TABLE V. The numerical results for the doubly-charmed/bottomed $QQ\bar{q}\,\bar{q}$, $QQ\bar{q}\,\bar{s}$, and $QQ\bar{s}\,\bar{s}$ systems with $J^P = 1^-$.

	Current	s_0 (GeV^2)	$[M^2_{B\,\mathrm{min}}, M^2_{B\,\mathrm{max}}]$ (GeV^2)	M_B^2 (GeV^2)	m_X (GeV)	PC (%)	f_X (GeV^5)	Open charm/bottom threshold (GeV)
$cc\bar{q}\,\bar{q}$	η_1	23	3.0–3.6	3.3	4.35 ± 0.14	38.6	0.0490	
$cc\bar{u}\,\bar{d}$	η_6	23	3.1–3.7	3.4	4.34 ± 0.16	37.9	0.0395	3.730
	η_7	22	2.6–3.4	3.0	4.41 ± 0.12	39.4	0.0690	
	η_8	23	2.6–3.5	3.0	4.42 ± 0.14	41.1	0.0940	
$cc\bar{q}\,\bar{s}$	η_1	23	2.7–3.6	3.2	4.37 ± 0.17	39.1	0.0357	
	η_2	24	2.9–3.8	3.2	4.59 ± 0.13	43.0	0.0838	
	η_6	23	2.9–3.7	3.4	4.35 ± 0.16	37.7	0.0353	3.833
	η_7	24	2.4–3.9	3.4	4.58 ± 0.14	39.8	0.105	
	η_8	24	2.4–3.9	3.4	4.52 ± 0.13	41.1	0.114	
$cc\bar{s}\,\bar{s}$	η_1	24	2.8–3.7	3.3	4.47 ± 0.13	40.7	0.0603	
	η_3	23	2.5–3.5	3.0	4.47 ± 0.14	40.9	0.101	3.937
	η_4	26	2.8–4.2	3.3	4.74 ± 0.17	49.1	0.196	
$bb\bar{q}\,\bar{q}$	η_1	125	7.0–9.6	8.0	10.6 ± 0.3	47.8	0.229	
$bb\bar{u}\,\bar{d}$	η_6	120	7.2–8.9	8.0	10.4 ± 0.2	40.5	0.142	10.56
	η_8	120	8.2–9.4	8.8	10.5 ± 0.2	35.9	0.492	
$bb\bar{q}\,\bar{s}$	η_1	120	7.2–8.8	8.0	10.5 ± 0.2	37.9	0.124	
	η_6	120	7.2–8.9	8.0	10.4 ± 0.2	40.6	0.145	10.65
	η_8	120	7.6–9.3	8.4	10.5 ± 0.2	37.8	0.491	
$bb\bar{s}\,\bar{s}$	η_1	125	6.6–9.6	8.0	10.6 ± 0.3	47.1	0.240	
	η_3	120	6.7–9.0	8.0	10.5 ± 0.3	40.1	0.490	10.73
	η_4	120	6.8–8.9	8.0	10.6 ± 0.3	38.8	0.655	

TABLE VI. The numerical results for the doubly charmed/bottomed $QQ\bar{q}\,\bar{q}$, $QQ\bar{q}\,\bar{s}$, and $QQ\bar{s}\,\bar{s}$ systems with $J^P = 1^+$.

	Current	s_0 (GeV^2)	$[M^2_{B\,\min}, M^2_{B\,\max}]$ (GeV^2)	M_B^2 (GeV^2)	m_X (GeV)	PC (%)	f_X (GeV^5)	Open charm/bottom threshold (GeV)
$cc\bar{q}\,\bar{s}$	η_1	28	3.0–4.2	3.6	4.96 ± 0.11	42.1	0.0801	3.975
	η_2	27	3.1–4.0	3.6	4.87 ± 0.11	38.5	0.0726	
	η_3	21	2.4–3.4	2.8	4.12 ± 0.17	47.5	0.0571	
	η_4	21	2.5–3.4	2.8	4.13 ± 0.16	47.9	0.0574	
	η_5	21	2.8–3.7	3.2	4.12 ± 0.16	41.7	0.0378	
	η_6	21	3.0–3.7	3.2	4.17 ± 0.12	41.5	0.0718	
	η_7	21	2.2–3.3	2.8	4.15 ± 0.17	42.9	0.0465	
$cc\bar{s}\,\bar{s}$	η_1	29	3.2–4.5	3.8	5.03 ± 0.13	42.5	0.138	4.081
	η_2	30	3.2–4.6	3.8	5.12 ± 0.14	45.9	0.150	
	η_3	21	2.2–3.4	2.8	4.17 ± 0.16	45.4	0.0838	
	η_4	21	2.2–3.4	2.8	4.19 ± 0.16	45.7	0.0849	
$bb\bar{q}\,\bar{q}$	η_3	115	6.5–8.8	7.8	10.2 ± 0.3	41.4	0.459	10.604
	η_4	115	6.8–8.8	7.8	10.2 ± 0.3	41.7	0.454	
$bb\bar{u}\,\bar{d}$	η_5	115	7.0–9.0	8.0	10.2 ± 0.3	42.8	0.215	
	η_6	115	7.0–9.2	8.0	10.2 ± 0.3	42.0	0.304	
	η_7	115	6.5–8.6	7.6	10.2 ± 0.3	43.2	0.241	
	η_8	115	6.8–8.8	7.6	10.2 ± 0.3	41.7	0.343	
$bb\bar{q}\,\bar{s}$	η_1	125	6.9–8.6	7.6	10.7 ± 0.3	42.1	0.155	10.691
	η_2	125	6.9–8.8	7.6	10.7 ± 0.4	44.5	0.170	
	η_3	120	6.2–9.8	8.0	10.4 ± 0.3	48.9	0.452	
	η_4	120	6.5–9.8	8.0	10.4 ± 0.3	49.3	0.446	
	η_5	120	6.6–9.8	8.0	10.3 ± 0.3	52.3	0.298	
	η_6	120	6.6–9.8	8.0	10.3 ± 0.4	52.1	0.418	
	η_7	120	6.2–9.6	8.0	10.4 ± 0.3	48.1	0.342	
	η_8	120	5.8–9.6	8.0	10.4 ± 0.3	46.3	0.491	
$bb\bar{s}\,\bar{s}$	η_1	130	7.0–9.7	8.5	11.0 ± 0.3	40.8	0.336	10.781
	η_2	130	7.2–9.9	8.5	10.9 ± 0.3	42.9	0.370	
	η_3	120	6.2–9.8	8.0	10.4 ± 0.3	48.1	0.657	
	η_4	120	6.2–9.8	8.0	10.4 ± 0.3	48.5	0.651	

exist. The doubly bottomed systems are more stable as the heavy quark mass increases.

In the $J^P = 1^+$ $cc\bar{s}\,\bar{s}$ system, the extracted mass is about 5.03–5.12 GeV from the currents η_1^s and η_2^s. In contrast, the mass extracted from currents η_3^s and η_4^s is around 4.17 GeV, which is much lower than that from η_1^s and η_2^s. The same situations occur in the $J^P = 1^+$ $cc\bar{q}\,\bar{s}$, $bb\bar{q}\,\bar{s}$ and $bb\bar{s}\,\bar{s}$ cases. According to Table I, the diquark fields $Q_a^T C\gamma_\mu\gamma_5 Q_b$ and $Q_a^T C Q_b$ are P-wave operators while $Q_a^T C\gamma_\mu Q_b$ and $Q_a^T C\sigma_{\mu\nu}\gamma_5 Q_b$ are S-wave operators. So the interpolating currents η_1^s and η_2^s contain two P-wave operators, whereas η_3^s and η_4^s contain two S-wave operators. In other words, the extracted masses from the currents η_3^s and η_4^s correspond to the ground state mass of the $J^P = 1^+$ $cc\bar{s}\,\bar{s}$ system while the masses of η_1^s and η_2^s correspond to the orbitally excited state. That is the underlying mechanism which renders the extracted mass from η_1^s and η_2^s is much higher than that from η_3^s and η_4^s. The similar situation occurs in the current η_1^s with $J^P = 0^+$ for the $cc\bar{s}\,\bar{s}$ and $bb\bar{s}\,\bar{s}$ systems.

There is another intuitive way to understand the difference of the various interpolating currents. According to textbook knowledge about the quark model, the interaction between the quark pair for the symmetric color structure $[\mathbf{6_c}]_{QQ} \otimes [\bar{\mathbf{6}}_\mathbf{c}]_{\bar{q}\,\bar{q}}$ is repulsive, whereas the interaction for the antisymmetric color structure $[\bar{\mathbf{3}}_\mathbf{c}]_{QQ} \otimes [\mathbf{3_c}]_{\bar{q}\,\bar{q}}$ is attractive. Thus, the currents with the symmetric color structure will result in higher mass than those with the antisymmetric color structure. There were similar observations of the effect of the color structure on the tetraquark spectrum in Refs. [22,27].

V. DECAY PATTERNS OF THE $QQ\bar{q}\,\bar{q}$ AND $QQ\bar{s}\,\bar{s}$ STATES

In this section, we study the decay patterns of the possible doubly charmed/bottomed states. From the numerical results of Tables III, IV, V, and VI, the extracted masses of the $cc\bar{q}\,\bar{q}$, $cc\bar{q}\,\bar{s}$, and $cc\bar{s}\,\bar{s}$ doubly charmed states are above the $D^{(*)(+/0)}_{(0/1)}D^{(*)(+/0)}_{(0/1)}$, $D^{(*)+}_{(0/1)}D^{(*)+}_{s(0/1)}$, and

TABLE VII. The possible two-meson decay modes of the $cc\bar{q}\bar{q}$ and $cc\bar{s}\bar{s}$ states.

J^P	S-wave	P-wave
0^-	$D^0+D_0^*(2400)^0, D^++D_0^*(2400)^0, D^0+D_{s0}^*(2317)^+,$ $D^++D_{s0}^*(2317)^+, D_0^*(2400)^0+D_s^+,$ $D^*(2007)^0+D_{s1}^+, D_1(2420)^0+D_s^*(2112)^+,$ $D_{s0}^*(2317)^++D_s^+,\ldots$	$D^0+D^*(2007)^0, D^++D^*(2007)^0, D^0+D^*(2010)^+,$ $D^++D^*(2010)^+, D^*(2007)^0+D^*(2007)^0,$ $D^*(2010)^++D^*(2010)^+, D^*(2007)^0+D^*(2010)^+,$ $D^*(2007)^0+D_s^+, D^*(2010)^++D_s^+, D^++D_s^*(2112)^+,$ $D_s^++D_s^*(2112)^+, D_s^*(2112)^++D_s^*(2112)^+,\ldots$
0^+	$D^++D_s^+, D^0+D_s^+, D_s^++D_s^+, D_s^{*+}+D_s^{*+},$ $D_{s0}^{*+}+D_{s0}^{*+}, D_{s1}^+(2460)+D_{s1}^+(2460),\ldots$	$D_{s0}^*(2317)^++D_s^*(2112)^+, D_s^++D_{s1}(2460)^+,$ $D_s^++D_{s1}(2536)^+,\ldots$
1^-	$D^0+D_1^0, D^++D_1^0, D^{*0}+D_{s0}^{*+},$ $D^{*+}+D_{s0}^{*+}, D_s^++D_{s1}^+,\ldots$	$D^0+D^0, D^++D^+, D^0+D^+, D^{*0}+D^{*0}, D^{*+}+D^{*+},$ $D^0+D^{*0}, D^0+D^{*+}, D^++D^{*0}, D^++D^{*+}, D^++D_s^{*+},$ $D^0+D_s^{*+}, D^{*0}+D_s^+, D^{*+}+D_s^+, D_s^++D_s^+, D_s^{*+}+D_s^{*+},\ldots$
1^+	$D^++D_s^{*+}, D^0+D_s^{*+}, D^{*0}+D_s^+, D^{*+}+D_s^+,$ $D_s^++D_s^{*+}, D_{s0}^{*+}+D_{s1}^+, D_{s0}^{*+}+D_{s1}^+, D_{s1}^++D_{s1}^+,\ldots$	$D^0+D_{s0}^{*+}, D^++D_{s0}^{*+}, D_0^{*0}+D_s^+, D^{*0}+D_{s1}^+, D^{*+}+D_{s1}^+,$ $D_1^0+D_s^{*+}, D_s^++D_{s0}^{*+}, D_{s0}^{*+}+D_s^{*+}, D_s^++D_{s1}^+, D_s^++D_{s1}^+,\ldots$

$D_{s(0/1)}^{(*)+}D_{s(0/1)}^{(*)+}$ thresholds. The possible decay modes of the $cc\bar{q}\bar{q}$, $cc\bar{q}\bar{s}$, and $cc\bar{s}\bar{s}$ states with different quantum numbers are listed in Table VII. Both the S-wave and P-wave decay patterns are allowed. These $cc\bar{q}\bar{q}$, $cc\bar{q}\bar{s}$, and $cc\bar{s}\bar{s}$ tetraquarks will decay easily through rearrangement or the so-called fall-apart mechanism. They are very broad resonances. It may be difficult to observe them experimentally.

The situation is very different for the doubly bottomed states. As we emphasize in the previous section, the η_1^s current with $J^P=0^+$ and the η_1^s, η_2^s currents with $J^P=1^+$ explore the excited tetraquark states because of their special diquark structure. We focus on the doubly bottomed ground states. The extracted masses of these states as shown in Tables III, IV, V, and VI are lower than the open bottom thresholds $\bar{B}^0\bar{B}^0=10.56$ GeV and $\bar{B}_s^0\bar{B}_s^0=10.73$ GeV. These doubly bottomed states cannot decay into the two $b\bar{q}$ or $b\bar{s}$ mesons, which implies the existence of the doubly bottomed bound states $bb\bar{q}\bar{q}$ and $bb\bar{s}\bar{s}$. This observation is consistent with the conclusions of Refs. [20,21,28].

Except for the two-meson decay modes, these doubly charmed/bottomed tetraquark states may also decay into the two-baryon final states: a pair of bottom and antibottom baryons or one doubly bottomed baryon plus one antinucleon. The lightest bottom baryon is Λ_b. Its mass is 5.62 GeV. In other words, these doubly bottomed tetraquark states do not decay into a pair of bottom and antibottom baryons.

TABLE VIII. The possible two-baryon decay modes of the $cc\bar{q}\bar{q}$ and $cc\bar{s}\bar{s}$ states.

J^P	S-wave	P-wave
0^-		$\bar{N}+\Xi_{cc}$
0^+	$\bar{N}+\Omega_{cc}$	
1^-		$\ldots$
1^+	$\bar{N}+\Omega_{cc}$	

The mass of the doubly charmed baryons discovered by the SELEX Collaboration is $m_{\Xi_{cc}}=3519\pm1$ MeV. For the other doubly charmed/bottomed baryons, their masses have been estimated in the quark model, $m_{\Omega_{cc}}=3.8$ GeV, $m_{\Xi_{bb}}=10.2$ GeV [32,48]. The possible two-baryon decay patterns of the doubly charmed tetraquark states are listed in Table VIII. In contrast, the doubly bottomed $bb\bar{q}\bar{q}$ states cannot decay into $\bar{N}\Xi_{bb}$ because their masses in Tables III, IV, V, and VI are below $m_{\bar{N}}+m_{\Xi_{bb}}$. The above analysis also supports the existence of the doubly bottomed tetraquark states.

VI. SUMMARY

In order to explore the possible $QQ\bar{q}\bar{q}$, $QQ\bar{q}\bar{s}$, and $QQ\bar{s}\bar{s}$ states with $J^P=0^-$, 0^+, 1^-, and 1^+, we have constructed the possible tetraquark interpolating operators without derivatives in a systematic way. Because of Fermi statistics, the wave functions of (QQ) and $\bar{q}\bar{q}$ should be antisymmetric (color $\times$ flavor $\times$ orbital $\times$ spin). We obtain 26 color-singlet interpolating currents with these quantum numbers, in which 16 currents possess symmetric light quark flavor structure $[\bar{\mathbf{6}}_{\mathbf{f}}]_{\bar{q}\bar{q}}$ and the other 10 currents belong to $[\mathbf{3}_{\mathbf{f}}]_{\bar{q}\bar{q}}$. The properties of these currents, such as the isospins, the flavor structures, and the J^P quantum numbers, are summarized in Table II.

Then we make the operator product expansion and extract the spectral densities for every interpolating current. Because of the special Lorentz structures of the currents, the quark condensate $\langle\bar{q}q\rangle$ and $\langle\bar{q}g_s\sigma\cdot Gq\rangle$ vanishes for $QQ\bar{q}\bar{q}$ systems. For the $QQ\bar{q}\bar{s}$ and $QQ\bar{s}\bar{s}$ systems, we keep the m_s-related terms in the spectral densities. These terms give important contributions to the correlation functions. Now the most important corrections come from the quark condensate and the four-quark condensate.

In the working range of the Borel parameter, only 0^- and 1^- $cc\bar{q}\bar{q}$ systems give a stable mass sum rule. The masses of the possible 0^- and 1^- $cc\bar{q}\bar{q}$ states are

4.45 ± 0.12 GeV and 4.35 ± 0.14 GeV. There does not exist a stable mass sum rule for the 0^+ and 1^+ $cc\bar{q}\bar{q}$ systems. The QCD sum rules of the tetraquark systems become more stable as the quark mass increases. According to our analysis, stable QCD sum rules exist for the following channels: $cc\bar{q}\bar{s}$, $cc\bar{s}\bar{s}$, $bb\bar{q}\bar{q}$, $bb\bar{q}\bar{s}$, and $bb\bar{s}\bar{s}$.

Unfortunately, the doubly charmed tetraquark states are found to lie above the two-meson threshold. These states will decay very rapidly through the fall-apart mechanism. Very probably they may be too broad to be identified as a resonance experimentally. In contrast, it's very interesting to note that the masses of the doubly bottomed tetraquark states are below the open bottom thresholds and the $\bar{N} + \Omega_{bb}$ threshold. In other words, the tetraquark states $bb\bar{q}\bar{q}$, $bb\bar{q}\bar{s}$, and $bb\bar{s}\bar{s}$ are stable. Once produced, they decay via electromagnetic and weak interactions only. The $bb\bar{q}\bar{q}$, $bb\bar{q}\bar{s}$, and $bb\bar{s}\bar{s}$ states may be searched for at facilities such as LHCb and RHIC in the future, where plenty of heavy quarks are produced [49].

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APPENDIX: THE SPECTRAL DENSITIES

In this appendix, we list the spectral densities of the tetraquark interpolating currents with different quantum numbers, respectively. The spectral densities read

$$\rho^{\rm OPE}(s) = \rho^{\rm pert}(s) + \rho^{\langle\bar{q}q\rangle}(s) + \rho^{\langle GG\rangle}(s) + \rho^{\langle\bar{q}Gq\rangle}(s) + \rho^{\langle\bar{q}q\rangle^2}(s). \quad \text{(A1)}$$

The Borel transformation of the correlation functions reads

$$\Pi(M_B^2) = \int_{4(m_Q+m_q)^2}^{\infty} ds\rho^{\rm OPE}(s)e^{-s/M_B^2} + \Pi^{\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma\cdot Gq\rangle}(M_B^2). \quad \text{(A2)}$$

For the interpolating currents with symmetric light quark flavor structure $[\bar{\mathbf{6}}_{\mathbf{f}}]_{\bar{q}\bar{q}}$, there are two types of Wick contraction when we calculate the two-point correlations for the $QQ\bar{q}\bar{q}$ and $QQ\bar{q}\bar{s}$ systems, as mentioned in Sec. II and IV. We list the spectral densities for both of them. For the currents with antisymmetric light quark flavor structure $[\mathbf{3}_{\mathbf{f}}]_{\bar{q}\bar{q}}$, we just list the spectral densities for the $QQ\bar{q}\bar{s}$ systems while keeping the m_s-related terms. The spectral densities for the $QQ\bar{s}\bar{s}$ and $QQ\bar{u}\bar{d}$ systems can be obtained from the expressions of the $QQ\bar{q}\bar{q}$ and $QQ\bar{q}\bar{s}$ systems, respectively, by replacing the corresponding parameters.

1. The spectral densities for the currents in the $QQ\bar{q}\bar{q}$ systems

For the interpolating currents with $J^P = 0^-$,

$$\rho_1^{\rm pert}(s) = \int_{\alpha_{\rm min}}^{\alpha_{\rm max}} d\alpha \int_{\beta_{\rm min}}^{\beta_{\rm max}} d\beta \frac{(1-\alpha-\beta)(m^2(\alpha+\beta-2)+\alpha\beta s)(\alpha\beta s - m^2(\alpha+\beta))^3}{64\pi^6\alpha^3\beta^3},$$

$$\rho_1^{\langle\bar{q}q\rangle}(s) = -m_q\langle\bar{q}q\rangle \int_{\alpha_{\rm min}}^{\alpha_{\rm max}} d\alpha \int_{\beta_{\rm min}}^{\beta_{\rm max}} d\beta \frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(\alpha+\beta+1)-2\alpha\beta s)}{2\pi^4\alpha\beta},$$

$$\rho_1^{\langle GG\rangle}(s) = \langle g_s^2GG\rangle \int_{\alpha_{\rm min}}^{\alpha_{\rm max}} d\alpha \int_{\beta_{\rm min}}^{\beta_{\rm max}} d\beta \bigg\{\frac{m^2(1-\alpha-\beta)^2(2m^2(\alpha+\beta)+m^2-3\alpha\beta s)}{192\pi^6\alpha^3} + (m^2(\alpha+\beta)-\alpha\beta s)\bigg[\frac{(1-\alpha-\beta)^2(2\alpha^2\beta s - m^2(\alpha(\alpha+\beta+2)-8\beta))}{512\pi^6\alpha^3\beta^2} - \frac{m^2(\alpha+\beta+1)-2\alpha\beta s}{256\pi^6\alpha\beta}\bigg]\bigg\},$$

$$\rho_1^{\langle\bar{q}Gq\rangle}(s) = -\frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle(s-4m^2)}{8\pi^4}\sqrt{1-\frac{4m^2}{s}},$$

$$\rho_1^{\langle\bar{q}q\rangle^2}(s) = \frac{\langle\bar{q}q\rangle^2(s-4m^2)}{3\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_1^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) = \frac{\langle\bar{q}q\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{3\pi^2}\int_0^1 dx\bigg\{\frac{m^4(2x-1)}{M_B^2(1-x)x^2} + \frac{m^2}{x(1-x)} + M_B^2\bigg\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \quad \text{(A3)}$$

$$\rho_2^{\rm pert}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)^3(m^2(3\alpha+3\beta-2)-\alpha\beta s)}{64\pi^6\alpha^3\beta^3},$$

$$\rho_2^{\langle\bar{q}q\rangle}(s) = m_q\langle\bar{q}q\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{3(m^2(\alpha+\beta-1)-2\alpha\beta s)(m^2(\alpha+\beta)-\alpha\beta s)}{2\pi^4\alpha\beta},$$

$$\rho_2^{\langle GG\rangle}(s) = \langle g_s^2GG\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \left\{\frac{m^2(1-\alpha-\beta)^2(m^2(2\alpha+2\beta-1)+m^2-3\alpha\beta s)}{192\pi^6\alpha^3} + (m^2(\alpha+\beta)-\alpha\beta s) \times\left[\frac{(1-\alpha-\beta)^2(2\alpha^2\beta s-m^2(\alpha(\alpha+\beta-2)+8\beta))}{512\pi^6\alpha^3\beta^2} - \frac{m^2(\alpha+\beta-1)-2\alpha\beta s}{256\pi^6\alpha\beta}\right]\right\},$$

$$\rho_2^{\langle\bar{q}Gq\rangle}(s) = \frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle s}{8\pi^4}\sqrt{1-\frac{4m^2}{s}}, \qquad \rho_2^{\langle\bar{q}q\rangle^2}(s) = -\frac{\langle\bar{q}q\rangle^2 s}{3\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_2^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) = -\frac{\langle\bar{q}q\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{3\pi^2}\int_0^1 dx\left\{\frac{m^4}{M_B^2(1-x)x^2} + \frac{m^2}{x(1-x)} + M_B^2\right\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \tag{A4}$$

$$\rho_3^{\rm pert}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{3(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)^4}{16\pi^6\alpha^3\beta^3},$$

$$\rho_3^{\langle\bar{q}q\rangle}(s) = m_q\langle\bar{q}q\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{6(m^2(\alpha+\beta-2)-2\alpha\beta s)(m^2(\alpha+\beta)-\alpha\beta s)}{\pi^4\alpha\beta},$$

$$\rho_3^{\langle GG\rangle}(s) = \langle g_s^2GG\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \left\{\frac{m^2(1-\alpha-\beta)^2(2m^2(\alpha+\beta)-3\alpha\beta s)}{16\pi^6\alpha^3} - \frac{(\alpha(\alpha+12\beta-2)+\beta(9\beta-10)+1)(m^2(\alpha+\beta)-2\alpha\beta s)(m^2(\alpha+\beta)-\alpha\beta s)}{64\pi^6\alpha^2\beta^2}\right\},$$

$$\rho_3^{\langle\bar{q}Gq\rangle}(s) = \frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle}{\pi^4}\left\{3m^2\sqrt{1-\frac{4m^2}{s}} - \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{2m^2}{\alpha}\right\}, \qquad \rho_3^{\langle\bar{q}q\rangle^2}(s) = -\frac{8\langle\bar{q}q\rangle^2 m^2}{\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_3^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) = -\frac{4m^2\langle\bar{q}q\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{3\pi^2}\int_0^1 dx\left\{\frac{3m^2-2xM_B^2}{x^2M_B^2}\right\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \tag{A5}$$

For the interpolating currents with $J^P = 0^+$,

$$\rho_1^{\rm pert}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1-\alpha-\beta)(m^2(\alpha+\beta-2)+\alpha\beta s)(\alpha\beta s-m^2(\alpha+\beta))^3}{64\pi^6\alpha^3\beta^3},$$

$$\rho_1^{\langle\bar{q}q\rangle}(s) = 3m_q\langle\bar{q}q\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(\alpha+\beta+1)-2\alpha\beta s)}{2\pi^4\alpha\beta},$$

$$\rho_1^{\langle GG\rangle}(s) = \langle g_s^2GG\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \left\{\frac{m^2(1-\alpha-\beta)^2(2m^2(\alpha+\beta)+m^2-3\alpha\beta s)}{192\pi^6\alpha^3} + (m^2(\alpha+\beta)-\alpha\beta s) \times\left[\frac{(1-\alpha-\beta)^2(2\alpha^2\beta s-m^2(\alpha(\alpha+\beta+2)-8\beta))}{512\pi^6\alpha^3\beta^2} - \frac{m^2(\alpha+\beta+1)-2\alpha\beta s}{256\pi^6\alpha\beta}\right]\right\},$$

$$\rho_1^{\langle\bar{q}Gq\rangle}(s) = \frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle(s-4m^2)}{8\pi^4}\sqrt{1-\frac{4m^2}{s}}, \qquad \rho_1^{\langle\bar{q}q\rangle^2}(s) = \frac{\langle\bar{q}q\rangle^2(s-4m^2)}{3\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_1^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) = -\frac{\langle\bar{q}q\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{3\pi^2}\int_0^1 dx\left\{\frac{m^4(2x-1)}{M_B^2(1-x)x^2} + \frac{m^2}{x(1-x)} + M_B^2\right\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \tag{A6}$$

$$\rho_2^{\rm pert}(s)=\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)^3(m^2(3\alpha+3\beta-2)-\alpha\beta s)}{64\pi^6\alpha^3\beta^3},$$

$$\rho_2^{\langle\bar{q}q\rangle}(s)=-m_q\langle\bar{q}q\rangle\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\frac{(m^2(\alpha+\beta-1)-2\alpha\beta s)(m^2(\alpha+\beta)-\alpha\beta s)}{2\pi^4\alpha\beta},$$

$$\rho_2^{\langle GG\rangle}(s)=\langle g_s^2GG\rangle\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\bigg\{\frac{m^2(1-\alpha-\beta)^2(m^2(2\alpha+2\beta-1)+m^2-3\alpha\beta s)}{192\pi^6\alpha^3}+(m^2(\alpha+\beta)-\alpha\beta s)\times\bigg[\frac{(1-\alpha-\beta)^2(2\alpha^2\beta s-m^2(\alpha(\alpha+\beta-2)+8\beta))}{512\pi^6\alpha^3\beta^2}-\frac{m^2(\alpha+\beta-1)-2\alpha\beta s}{256\pi^6\alpha\beta}\bigg]\bigg\},$$

$$\rho_2^{\langle\bar{q}Gq\rangle}(s)=-\frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle s}{8\pi^4}\sqrt{1-\frac{4m^2}{s}},\qquad \rho_2^{\langle\bar{q}q\rangle^2}(s)=\frac{\langle\bar{q}q\rangle^2 s}{3\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_2^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2)=\frac{\langle\bar{q}q\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{3\pi^2}\int_0^1dx\bigg\{\frac{m^4}{M_B^2(1-x)x^2}+\frac{m^2}{x(1-x)}+M_B^2\bigg\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \tag{A7}$$

$$\rho_3^{\rm pert}(s)=\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\frac{(1-\alpha-\beta)(m^2(2\alpha+2\beta-1)-\alpha\beta s)(m^2(\alpha+\beta)-\alpha\beta s)}{32\pi^6\alpha^3\beta^3},$$

$$\rho_3^{\langle\bar{q}q\rangle}(s)=m_q\langle\bar{q}q\rangle\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\frac{3m^2(m^2(\alpha+\beta)-\alpha\beta s)}{2\pi^4\alpha\beta},$$

$$\rho_3^{\langle GG\rangle}(s)=\langle g_s^2GG\rangle\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\bigg\{\frac{m^2(1-\alpha-\beta)^2(m^2(4\alpha\beta-3\alpha+4\beta^2-4\beta)+3\alpha(1-2\beta)\beta s)}{192\pi^6\alpha^3\beta}+(m^2(\alpha+\beta)-\alpha\beta s)\bigg[\frac{(1-\alpha-\beta)(m^2(\alpha+\beta-1)-2\alpha\beta s)}{64\pi^6\alpha^2\beta}+\frac{m^2(m^2(\alpha+\beta)-\alpha\beta s)}{128\pi^6\alpha\beta}\bigg]\bigg\},$$

$$\rho_3^{\langle\bar{q}Gq\rangle}(s)=-\frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle}{8\pi^4}\bigg\{(2m^2+s)\sqrt{1-\frac{4m^2}{s}}-\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\frac{2m^2(\alpha+\beta-1)-3\alpha\beta s}{\alpha}\bigg\},$$

$$\rho_3^{\langle\bar{q}q\rangle^2}(s)=\frac{\langle\bar{q}q\rangle^2(2m^2+s)}{3\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_3^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2)=\frac{\langle\bar{q}q\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{6\pi^2}\int_0^1dx\bigg\{\frac{2m^4(2-x)}{M_B^2(1-x)x^2}+\frac{m^2}{1-x}+2M_B^2(1-x)\bigg\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \tag{A8}$$

$$\rho_4^{\rm pert}(s)=\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\frac{(1-\alpha-\beta)(m^2-\alpha\beta s)(m^2(\alpha+\beta)-\alpha\beta s)^3}{16\pi^6\alpha^3\beta^3},$$

$$\rho_4^{\langle\bar{q}q\rangle}(s)=m_q\langle\bar{q}q\rangle\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(4\alpha+4\beta+5)-8\alpha\beta s)}{\pi^4\alpha\beta},$$

$$\rho_4^{\langle GG\rangle}(s)=\langle g_s^2GG\rangle\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\bigg\{\frac{m^2(1-\alpha-\beta)^2(m^2(4\alpha\beta+3\alpha+4\beta^2+4\beta)-3\alpha\beta(2\beta+1)s)}{96\pi^6\alpha^2\beta}+(m^2(\alpha+\beta)-\alpha\beta s)\bigg[\frac{5(1-\alpha-\beta)(m^2(\alpha+\beta+1)-2\alpha\beta s)}{64\pi^6\alpha^2\beta}+\frac{m^2}{128\pi^6\alpha\beta}\bigg]\bigg\},$$

$$\rho_4^{\langle\bar{q}Gq\rangle}(s)=\frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle}{4\pi^4}\bigg\{(s-6m^2)\sqrt{1-\frac{4m^2}{s}}+\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\frac{10m^2(\alpha+\beta+1)-15\alpha\beta s}{2\alpha}\bigg\},$$

$$\rho_4^{\langle\bar{q}q\rangle^2}(s)=-\frac{2\langle\bar{q}q\rangle^2(s-6m^2)}{3\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_4^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2)=\frac{\langle\bar{q}q\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{6\pi^2}\int_0^1dx\bigg\{\frac{4m^4(2-3x)}{M_B^2(1-x)x^2}+\frac{m^2(15x-14)}{x(1-x)}-2M_B^2\frac{5x^2-7x+2}{1-x}\bigg\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \tag{A9}$$

$$\rho_5^{\rm pert}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{3(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)^4}{16\pi^6\alpha^3\beta^3},$$

$$\rho_5^{\langle\bar{q}q\rangle}(s) = m_q\langle\bar{q}q\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{6(m^2(\alpha+\beta+2)-2\alpha\beta s)(m^2(\alpha+\beta)-\alpha\beta s)}{\pi^4\alpha\beta},$$

$$\rho_5^{\langle GG\rangle}(s) = \langle g_s^2 GG\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \bigg\{\frac{m^2(1-\alpha-\beta)^2(2m^2(\alpha+\beta)-3\alpha\beta s)}{16\pi^6\alpha^3} - \frac{(\alpha(\alpha+12\beta-2)+\beta(9\beta-10)+1)(m^2(\alpha+\beta)-2\alpha\beta s)(m^2(\alpha+\beta)-\alpha\beta s)}{64\pi^6\alpha^2\beta^2}\bigg\},$$

$$\rho_5^{\langle\bar{q}Gq\rangle}(s) = -\frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle}{\pi^4}\left\{3m^2\sqrt{1-\frac{4m^2}{s}} - \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{2m^2}{\alpha}\right\},$$

$$\rho_5^{\langle\bar{q}q\rangle^2}(s) = \frac{8\langle\bar{q}q\rangle^2 m^2}{\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_5^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) = \frac{4m^2\langle\bar{q}q\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{3\pi^2}\int_0^1 dx\bigg\{\frac{3m^2-2xM_B^2}{x^2M_B^2}\bigg\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \tag{A10}$$

For the interpolating currents with $J^P = 1^-$,

$$\rho_1^{\rm pert}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \bigg\{\frac{(1-\alpha-\beta)^2(\alpha\beta s-m^2(\alpha+\beta-4))(m^2(\alpha+\beta)-\alpha\beta s)^3}{128\pi^6\alpha^3\beta^3} + \frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)^4}{64\pi^6\alpha^3\beta^3}\bigg\},$$

$$\rho_1^{\langle\bar{q}q\rangle}(s) = -m_q\langle\bar{q}q\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(\alpha+\beta+2)-3\alpha\beta s)}{4\pi^4\alpha\beta},$$

$$\rho_1^{\langle GG\rangle}(s) = \langle g_s^2 GG\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \bigg\{\frac{(m^2(\alpha+\beta+2)-\alpha\beta s)(\alpha\beta s-m^2(\alpha+\beta))}{512\pi^6\alpha\beta} + (1-\alpha-\beta)^2 \times \bigg[\frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(3\alpha^2+3\alpha\beta-16\beta^3+48\beta)-5\alpha^2\beta s)}{3072\pi^6\alpha^3\beta^2} + \frac{m^2(2m^2(\alpha+\beta)+m^2-3\alpha\beta s)}{192\pi^6\alpha^3}\bigg]\bigg\},$$

$$\rho_1^{\langle\bar{q}Gq\rangle}(s) = -\frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle(s-4m^2)}{12\pi^4}\sqrt{1-\frac{4m^2}{s}},$$

$$\rho_1^{\langle\bar{q}q\rangle^2}(s) = \frac{2\langle\bar{q}q\rangle^2(s-4m^2)}{9\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_1^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) = \frac{\langle\bar{q}q\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{3\pi^2}\int_0^1 dx\bigg\{\frac{m^4(2x-1)}{M_B^2x^2(1-x)} + \frac{m^2(2-x)}{1-x} + M_B^2 x\bigg\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \tag{A11}$$

$$\rho_2^{\rm pert}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)^3(m^2(7(\alpha+\beta)(\alpha+\beta+1)-8)-3\alpha\beta s(\alpha+\beta+1))}{384\pi^6\alpha^3\beta^3},$$

$$\rho_2^{\langle\bar{q}q\rangle}(s) = m_q\langle\bar{q}q\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{3(m^2(\alpha+\beta)-\alpha\beta s)(m^2(\alpha+\beta-1)-2\alpha\beta s)}{2\pi^4\alpha\beta} + \frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)(5\alpha\beta s-m^2(3\alpha+3\beta-2))}{4\pi^4\alpha\beta},$$

$$\rho_2^{\langle GG\rangle}(s) = \langle g_s^2 GG\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \Big\{ \frac{(1-\alpha-\beta)^3(m^2(\alpha+\beta)-\alpha\beta s)(5\alpha^2\beta s - m^2(3\alpha^2+3\alpha\beta-4\alpha+16\beta))}{3072\pi^6\alpha^3\beta^2}$$
$$- \frac{(1-\alpha-\beta)^2(m^2(\alpha+\beta)-\alpha\beta s)(m^2(\alpha^2+\alpha\beta-2\alpha+8\beta))}{512\pi^6\alpha^3\beta^2}$$
$$+ \frac{m^2(1-\alpha-\beta)^2(m^2(3\alpha^2+6\alpha\beta+2\alpha+3\beta^2+2\beta-2)-\alpha\beta s(4\alpha+4\beta+5))}{576\pi^6\alpha^3}$$
$$+ \frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)(m^2(3\alpha+3\beta-2)-5\alpha\beta s)}{1536\pi^6\alpha\beta}$$
$$+ \frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(\alpha+\beta-1)-2\alpha\beta s)}{768\pi^6\alpha\beta}\Big\},$$

$$\rho_2^{\langle\bar{q}Gq\rangle}(s) = \frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle s}{8\pi^4}\sqrt{1-\frac{4m^2}{s}},$$

$$\rho_2^{\langle\bar{q}q\rangle^2}(s) = -\frac{2\langle\bar{q}q\rangle^2 s}{3\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_2^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) = -\frac{\langle\bar{q}q\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{3\pi^2}\int_0^1 dx\Big\{\frac{m^4}{M_B^2x^2(1-x)} + \frac{m^2}{(1-x)x} + M_B^2\Big\} e^{-\frac{m^2}{M_B^2(1-x)x}}. \quad \text{(A12)}$$

$$\rho_3^{\mathrm{pert}}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \Big\{ \frac{(1-\alpha-\beta)^3(m^2(\alpha+\beta)-\alpha\beta s)(m^2(3\alpha+3\beta+1)-7\alpha\beta s)}{192\pi^6\alpha^3\beta^3}$$
$$+ \frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)(\alpha\beta s(\alpha+\beta-7)-m^2(\alpha^2+2\alpha\beta-3\alpha+\beta^3-3\beta-4))}{256\pi^6\alpha^3\beta^3}\Big\},$$

$$\rho_3^{\langle\bar{q}q\rangle}(s) = m_q\langle\bar{q}q\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \Big\{ \frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)(m^2(15\alpha+15\beta+2)-25\alpha\beta s)}{8\pi^4\alpha\beta}$$
$$+ \frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(3\alpha+3\beta-5)-4\alpha\beta s)}{4\pi^4\alpha\beta}\Big\},$$

$$\rho_3^{\langle GG\rangle}(s) = \langle g_s^2 GG\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \Big\{ \frac{m^2(1-\alpha-\beta)^3(m^2(15\alpha+15\beta+1)-20\alpha\beta s)}{1152\pi^6\alpha^3}$$
$$+ \frac{(1-\alpha-\beta)^3(m^2(\alpha+\beta)-\alpha\beta s)(25\alpha^2\beta s - m^2(15\alpha^2+15\alpha\beta+4\alpha-24\beta))}{9126\pi^6\alpha^3\beta^2}$$
$$+ \frac{(1-\alpha-\beta)^2(m^2(\alpha+\beta)^2-\alpha\beta s)(m^2(15\alpha^2\beta-3\alpha^2+15\alpha\beta^2+\alpha\beta-2\alpha+12\beta)+\alpha^2(6-25\beta)\beta s)}{1536\pi^6\alpha^3\beta^2}$$
$$+ \frac{(\alpha+\beta-1)(m^2(\alpha+\beta)-\alpha\beta s)(\alpha(25\alpha+14)\beta s - m^2(15\alpha^2+\alpha(15\beta+8)+6\beta+8))}{1536\pi^6\alpha^2\beta}$$
$$+ \frac{m^2(\alpha+\beta-1)^2(m^2(6\alpha+6\beta+1)-9\alpha\beta s)}{384\pi^6\alpha^3} + \frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(3\alpha+3\beta-5)-4\alpha\beta s)}{768\pi^6\alpha\beta}\Big\},$$

$$\rho_3^{\langle\bar{q}Gq\rangle}(s) = \frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle s}{8\pi^4}\left\{\frac{16m^2-s}{6}\sqrt{1-\frac{4m^2}{s}} + \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{\alpha\beta s - 3m^2}{3\alpha}\right\},$$

$$\rho_3^{\langle\bar{q}q\rangle^2}(s) = \frac{\langle\bar{q}q\rangle^2(s-16m^2)}{18\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_3^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) = \frac{\langle\bar{q}q\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{18\pi^2}\int_0^1 dx\Big\{\frac{m^4(12x-9)}{M_B^2x^2(1-x)} - \frac{2m^2(3x-4)}{(1-x)} - \frac{3M_B^2(2x^2-3x+1)}{1-x}\Big\} e^{-\frac{m^2}{M_B^2(1-x)x}}. \quad \text{(A13)}$$

$$
\begin{aligned}
\rho_4^{\mathrm{pert}}(s) &= \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \bigg\{ \frac{(1-\alpha-\beta)^3 (m^2(\alpha+\beta)-\alpha\beta s)^3 (m^2(3\alpha+3\beta+2)-7\alpha\beta s)}{192\pi^6\alpha^3\beta^3}, \\
\rho_4^{\langle\bar{q}q\rangle}(s) &= m_q\langle\bar{q}q\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \bigg\{ \frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)(5\alpha\beta s-m^2(3\alpha+3\beta+1))}{2\pi^4\alpha\beta} \\
&\quad + \frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(13\alpha+13\beta-14)-23\alpha\beta s)}{8\pi^4\alpha\beta} \bigg\}, \\
\rho_4^{\langle GG\rangle}(s) &= \langle g_s^2 GG\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \bigg\{ \frac{(1-\alpha-\beta)^3 (m^2(\alpha+\beta)-\alpha\beta s)(5\alpha^2\beta s-3m^2(\alpha^2+\alpha\beta+2\beta))}{1152\pi^6\alpha^3\beta^2} \\
&\quad + \frac{(1-\alpha-\beta)^2 (m^2(\alpha+\beta)-\alpha\beta s)(5\alpha^2\beta(6\beta+1)s-m^2(18\alpha^2\beta+3\alpha^2+18\alpha\beta^2+19\alpha\beta+8\beta^2-24\beta))}{3072\pi^6\alpha^3\beta^2} \\
&\quad - \frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)(m^2(-6\alpha^2+3\alpha-6\alpha\beta+5\beta-4)+\alpha(10\alpha-9)\beta s)}{768\pi^6\alpha^2\beta} \\
&\quad + \frac{(m^2\alpha+\beta)-\alpha\beta s)(m^2(7\alpha+7\beta-2)-13\alpha\beta s)}{1536\pi^6\alpha\beta} - \frac{(1-\alpha-\beta)^2 m^2}{1152\pi^6\alpha^3} \\
&\quad \times [2(1-\alpha-\beta)(m^2(6\alpha+6\beta+1)-8\alpha\beta s)+3(m^2(6\alpha+6\beta-1)-9\alpha\beta s)] \bigg\}, \\
\rho_4^{\langle\bar{q}Gq\rangle}(s) &= \frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle s}{12\pi^4} \bigg\{ (2m^2+s)\sqrt{1-\frac{4m^2}{s}} + \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{3m^2(\alpha+\beta-1)-4\alpha\beta s}{2\alpha} \bigg\}, \\
\rho_4^{\langle\bar{q}q\rangle^2}(s) &= \frac{2\langle\bar{q}q\rangle^2(2m^2+s)}{9\pi^2}\sqrt{1-\frac{4m^2}{s}}, \\
\Pi_4^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) &= \frac{\langle\bar{q}q\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{18\pi^2} \int_0^1 dx \bigg\{ \frac{m^4(6x-9)}{M_B^2 x^2(1-x)} - \frac{m^2(3x^2-4x+3)}{x(1-x)} - 6M_B^2(1-x) \bigg\} e^{-\frac{m^2}{M_B^2(1-x)x}}.
\end{aligned}
\tag{A14}
$$

For the interpolating currents with $J^P = 1^+$,

$$
\begin{aligned}
\rho_1^{\mathrm{pert}}(s) &= \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \bigg\{ \frac{(1-\alpha-\beta)^2(\alpha\beta s-m^2(\alpha+\beta-4))(m^2(\alpha+\beta)-\alpha\beta s)^3}{128\pi^6\alpha^3\beta^3} \\
&\quad + \frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)^4}{64\pi^6\alpha^3\beta^3} \bigg\}, \\
\rho_1^{\langle\bar{q}q\rangle}(s) &= 3m_q\langle\bar{q}q\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(\alpha+\beta+2)-3\alpha\beta s)}{4\pi^4\alpha\beta}, \\
\rho_1^{\langle GG\rangle}(s) &= \langle g_s^2 GG\rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \bigg\{ \frac{(m^2(\alpha+\beta+2)-\alpha\beta s)(\alpha\beta s-m^2(\alpha+\beta))}{512\pi^6\alpha\beta} + (1-\alpha-\beta)^2 \\
&\quad \times \bigg[\frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(3\alpha^2+3\alpha\beta-16\beta^3+48\beta)-5\alpha^2\beta s)}{3072\pi^6\alpha^3\beta^2} + \frac{m^2(2m^2(\alpha+\beta)+m^2-3\alpha\beta s)}{192\pi^6\alpha^3}, \\
\rho_1^{\langle\bar{q}Gq\rangle}(s) &= \frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle(s-4m^2)}{12\pi^4}\sqrt{1-\frac{4m^2}{s}}, \\
\rho_1^{\langle\bar{q}q\rangle^2}(s) &= -\frac{2\langle\bar{q}q\rangle^2(s-4m^2)}{9\pi^2}\sqrt{1-\frac{4m^2}{s}}, \\
\Pi_1^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) &= -\frac{\langle\bar{q}q\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{3\pi^2} \int_0^1 dx \bigg\{ \frac{m^4(2x-1)}{M_B^2 x^2(1-x)} + \frac{m^2(2-x)}{1-x} + M_B^2 x \bigg\} e^{-\frac{m^2}{M_B^2(1-x)x}}.
\end{aligned}
\tag{A15}
$$

$$\rho_2^{\text{pert}}(s)=\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\frac{(1-\alpha-\beta)(\alpha\beta s-m^2(\alpha+\beta))(m^2(\alpha+\beta+1)(\alpha+\beta)-8)+3\alpha\beta s(\alpha+\beta+1))}{384\pi^6\alpha^3\beta^3},$$

$$\rho_2^{\langle\bar{q}q\rangle}(s)=m_q\langle\bar{q}q\rangle\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\bigg\{\frac{(1-\alpha\beta s)(m^2(\alpha+\beta)-\alpha\beta s)(5\alpha\beta s-m^2(3\alpha+3\beta+2))}{4\pi^4\alpha\beta}$$
$$+\frac{3(m^2(\alpha+\beta)-\alpha\beta s)(m^2(\alpha+\beta+1)-2\alpha\beta s)}{2\pi^4\alpha\beta},$$

$$\rho_2^{\langle GG\rangle}(s)=\langle g_s^2GG\rangle\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\bigg\{\frac{m^2(\alpha+\beta-1)^2(m^2(3\alpha^2+6\alpha\beta+4\alpha+3\beta^2+4\beta-1)-\alpha\beta s(4\alpha+4\beta+5))}{576\pi^6\alpha^3}$$
$$+(m^2(\alpha+\beta)-\alpha\beta s)\bigg[\frac{(1-\alpha-\beta)^3(\alpha-4\beta)}{768\pi^6\alpha^3\beta^2}-\frac{(1-\alpha-\beta)^2(m^2(\alpha^2+\alpha\beta+2\alpha-8\beta)-2\alpha\beta s)}{512\pi^6\alpha^3\beta^2}$$
$$+\frac{(1-\alpha-\beta)(m^2(3\alpha+3\beta+2)-5\alpha\beta s)}{1536\pi^6\alpha\beta}-\frac{m^2(\alpha+\beta+1)-2\alpha\beta s}{768\pi^6\alpha\beta}\bigg]\bigg\},$$

$$\rho_2^{\langle\bar{q}Gq\rangle}(s)=\frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle(s-4m^2)}{8\pi^4}\sqrt{1-\frac{4m^2}{s}},$$

$$\rho_2^{\langle\bar{q}q\rangle^2}(s)=-\frac{\langle\bar{q}q\rangle^2(s-4m^2)}{3\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_2^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2)=\frac{\langle\bar{q}q\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{3\pi^2}\int_0^1 dx\bigg\{\frac{m^4(1-2x)}{M_B^2x^2(1-x)}-\frac{m^2}{x(1-x)}-M_B^2\bigg\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \quad \text{(A16)}$$

$$\rho_3^{\text{pert}}(s)=\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)\bigg[\frac{(7-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)}{256\pi^6\alpha^3\beta^3}$$
$$+\frac{(1-\alpha-\beta)(m^2(3\alpha^2+6\alpha\beta-4\alpha+3\beta^2-4\beta-4)-7(\alpha+\beta-1)\alpha\beta s)}{192\pi^6\alpha^3\beta^3}\bigg],$$

$$\rho_3^{\langle\bar{q}q\rangle}(s)=m_q\langle\bar{q}q\rangle\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\frac{m^2(\alpha+\beta)-\alpha\beta s}{8\pi^4\alpha\beta}[2(m^2(3\alpha+3\beta+5)-4\alpha\beta s)$$
$$-(1-\alpha-\beta)(m^2(-15\alpha-15\beta+2)+25\alpha\beta s)],$$

$$\rho_3^{\langle GG\rangle}(s)=\langle g_s^2GG\rangle\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\bigg\{(m^2(\alpha+\beta)-\alpha\beta s)\bigg[\frac{(1-\alpha-\beta)^3(25\alpha^2\beta s-m^2(15\alpha^2+15\alpha\beta-4\alpha+24\beta))}{9216\pi^6\alpha^3\beta^2}$$
$$+\frac{(1-\alpha-\beta)(m^2(15\alpha^2\beta-3\alpha^2+15\alpha\beta^2-7\alpha\beta+2\alpha-12\beta)+\alpha^2(6-25\beta)\beta s)}{1536\pi^6\alpha^3\beta^2}$$
$$+\frac{(1-\alpha-\beta)(m^2(3\alpha^2+3\alpha\beta+4\alpha+6\beta-8)-\alpha^2(5\alpha+14\beta)\beta s)}{1536\pi^6\alpha^2\beta}+\frac{m^2(3\alpha+3\beta+5)-4\alpha\beta s}{768\pi^6\alpha\beta}\bigg]$$
$$+\frac{m^2(\alpha+\beta-1)^2(\alpha\beta s(20\alpha+20\beta-47)-m^2(15\alpha^2+\alpha(30\beta-34)+15\beta^2-34\beta+4))}{1152\pi^6\alpha^3}\bigg\},$$

$$\rho_3^{\langle\bar{q}Gq\rangle}(s)=\frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle s}{48\pi^4}\bigg\{(16m^2-s)\sqrt{1-\frac{4m^2}{s}}+\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\frac{6m^2+2\alpha\beta s}{\alpha}\bigg\},$$

$$\rho_3^{\langle\bar{q}q\rangle^2}(s)=\frac{\langle\bar{q}q\rangle^2(s+20m^2)}{18\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_3^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2)=\frac{\langle\bar{q}q\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{18\pi^2}\int_0^1 dx\bigg\{\frac{m^4(9-6x)}{M_B^2x^2(1-x)}-\frac{2m^2(3x^2-7x+3)}{x(1-x)}-\frac{3M_B^2(2x^2-3x+1)}{1-x}\bigg\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \quad \text{(A17)}$$

$$\rho_4^{\rm pert}(s)=\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\bigg\{\frac{(1-\alpha-\beta)^3(m^2(\alpha+\beta)-\alpha\beta s)^3(m^2(3\alpha+3\beta+2)-7\alpha\beta s)}{192\pi^6\alpha^3\beta^3},$$

$$\rho_4^{\langle\bar qq\rangle}(s)=-m_q\langle\bar qq\rangle\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{max}}d\beta\bigg\{\frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)(5\alpha\beta s-m^2(3\alpha+3\beta+1))}{2\pi^4\alpha\beta}$$
$$-\frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(\alpha+\beta+10)-3\alpha\beta s)}{8\pi^4\alpha\beta}\bigg\},$$

$$\rho_4^{\langle GG\rangle}(s)=\langle g_s^2GG\rangle\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\bigg\{\frac{(1-\alpha-\beta)^3(m^2(\alpha+\beta)-\alpha\beta s)(5\alpha^2\beta s-3m^2(\alpha^2+\alpha\beta+2\beta))}{1152\pi^6\alpha^3\beta^2}$$
$$+\frac{(1-\alpha-\beta)^2(m^2(\alpha+\beta)-\alpha\beta s)(5\alpha^2\beta(6\beta+1)s-m^2(18\alpha^2\beta+3\alpha^2+18\alpha\beta^2+19\alpha\beta+8\beta^2-24\beta))}{3072\pi^6\alpha^3\beta^2}$$
$$-\frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)(m^2(-6\alpha^2+3\alpha-6\alpha\beta+5\beta-4)+\alpha(10\alpha-9)\beta s)}{768\pi^6\alpha^2\beta}$$
$$+\frac{(m^2\alpha+\beta)-\alpha\beta s)(m^2(7\alpha+7\beta-2)-13\alpha\beta s)}{1536\pi^6\alpha\beta}-\frac{(1-\alpha-\beta)^2m^2}{1152\pi^6\alpha^3}$$
$$\times[2(1-\alpha-\beta)(m^2(6\alpha+6\beta+1)-8\alpha\beta s)+3(m^2(6\alpha+6\beta-1)-9\alpha\beta s)]\bigg\},$$

$$\rho_4^{\langle\bar qGq\rangle}(s)=-\frac{m_q\langle\bar q\sigma\cdot Gq\rangle s}{12\pi^4}\bigg\{(2m^2+s)\sqrt{1-\frac{4m^2}{s}}+\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\frac{3m^2(\alpha+\beta-1)-4\alpha\beta s)}{2\alpha}\bigg\},$$

$$\rho_4^{\langle\bar qq\rangle^2}(s)=\frac{2\langle\bar qq\rangle^2(2m^2+s)}{9\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_4^{\langle\bar qq\rangle\langle\bar qGq\rangle}(M_B^2)=\frac{\langle\bar qq\rangle\langle\bar q\sigma\cdot Gq\rangle}{18\pi^2}\int_0^1dx\bigg\{\frac{m^4(9-6x)}{M_B^2x^2(1-x)}+\frac{m^2(3x^2-4x+3)}{x(1-x)}+6M_B^2(1-x)\bigg\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \qquad \text{(A18)}$$

2. The spectral densities for the currents in the $QQ\bar q\bar s$ systems

For the interpolating currents with $J^P=0^-$,

$$\rho_1^{\rm pert}(s)=\frac{1}{2^7\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\frac{(1-\alpha-\beta)[(2-\alpha-\beta)m^2-\alpha\beta s][(\alpha+\beta)m^2-\alpha\beta s]^3}{\alpha^3\beta^3},$$

$$\rho_1^{\langle\bar qq\rangle}(s)=-\frac{m_s(2\langle\bar qq\rangle-\langle\bar ss\rangle)}{8\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\frac{[(1+\alpha+\beta)m^2-2\alpha\beta s][(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta},$$

$$\rho_1^{\langle GG\rangle}(s)=\frac{\langle g_s^2GG\rangle}{3\times2^{10}\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\bigg\{\frac{8(1-\alpha-\beta)^2m^2}{\alpha^3}\bigg[\frac{[(3\alpha+4\beta)m^2-3\alpha\beta s]}{\beta}+[(2\alpha+2\beta)m^2-3\alpha\beta s]\bigg]$$
$$-\frac{3[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\bigg[\frac{(1-\alpha-\beta)^2[(2+\alpha+\beta)m^2-2\alpha\beta s]}{\alpha\beta}+2[(1+\alpha+\beta)m^2-2\alpha\beta s]\bigg]\bigg\},$$

$$\rho_1^{\langle\bar qGq\rangle}(s)=\frac{m_s\langle\bar q\sigma\cdot Gq\rangle(4m^2-s)}{32\pi^4}\sqrt{1-4m^2/s},$$

$$\rho_1^{\langle\bar qq\rangle^2}(s)=-\frac{\langle\bar qq\rangle\langle\bar ss\rangle(4m^2-s)}{6\pi^2}\sqrt{1-4m_c^2/s},$$

$$\Pi_1^{\langle\bar qGq\rangle\langle\bar qq\rangle}(M_B^2)=\frac{\langle\bar q\sigma\cdot Gq\rangle\langle\bar ss\rangle+\langle\bar s\sigma\cdot Gs\rangle\langle\bar qq\rangle}{12\pi^2}\int_0^1d\alpha\bigg\{\frac{m^4(2\alpha-1)}{M_B^2\alpha^2(1-\alpha)}+\frac{m^2}{\alpha(1-\alpha)}+M_B^2\bigg\}e^{-\frac{m^2}{\alpha(1-\alpha)M_B^2}}. \qquad \text{(A19)}$$

$$\rho_2^{\mathrm{pert}}(s)=\frac{1}{2^7\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\frac{(1-\alpha-\beta)[(3\alpha+3\beta-2)m^2-\alpha\beta s][(\alpha+\beta)m^2-\alpha\beta s]^3}{\alpha^3\beta^3},$$

$$\rho_2^{\langle\bar{q}q\rangle}(s)=\frac{m_s(2\langle\bar{q}q\rangle+\langle\bar{s}s\rangle)}{8\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\frac{[(\alpha+\beta-1)m^2-2\alpha\beta s][(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta},$$

$$\rho_2^{\langle GG\rangle}(s)=-\frac{\langle g_s^2GG\rangle}{3\times2^{10}\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\left\{\frac{8(1-\alpha-\beta)^2m^2}{\alpha^3}\left[\frac{[(3\alpha+4\beta)m^2-3\alpha\beta s]}{\beta}-[(2\alpha+2\beta)m^2-3\alpha\beta s]\right]\right.$$
$$\left.+\frac{3[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\left[\frac{(1-\alpha-\beta)^2[(\alpha+\beta-2)m^2-2\alpha\beta s]}{\alpha\beta}+2[(\alpha+\beta-1)m^2-2\alpha\beta s]\right]\right\},$$

$$\rho_2^{\langle\bar{q}Gq\rangle}(s)=\frac{m_s\langle\bar{q}\sigma\cdot Gq\rangle s}{32\pi^4}\sqrt{1-4m^2/s},\qquad\rho_2^{\langle\bar{q}q\rangle^2}(s)=-\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle s}{6\pi^2}\sqrt{1-4m_c^2/s},$$

$$\Pi_2^{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle}(M_B^2)=-\frac{\langle\bar{q}\sigma\cdot Gq\rangle\langle\bar{s}s\rangle+\langle\bar{s}\sigma\cdot Gs\rangle\langle\bar{q}q\rangle}{12\pi^2}\int_0^1d\alpha\left\{\frac{m^4}{M_B^2\alpha^2(1-\alpha)}+\frac{m^2}{\alpha(1-\alpha)}+M_B^2\right\}e^{-\frac{m^2}{\alpha(1-\alpha)M_B^2}}. \tag{A20}$$

$$\rho_3^{\mathrm{pert}}(s)=\frac{3}{2^6\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\frac{(1-\alpha-\beta)[(\alpha+\beta)m^2-\alpha\beta s]^4}{\alpha^3\beta^3},$$

$$\rho_3^{\langle\bar{q}q\rangle}(s)=\frac{3m_s}{4\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\frac{[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}[[(\alpha+\beta)m^2-2\alpha\beta s]\langle\bar{s}s\rangle-2m^2\langle\bar{q}q\rangle],$$

$$\rho_3^{\langle GG\rangle}(s)=\frac{\langle g_s^2GG\rangle}{2^7\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\left\{\frac{4(1-\alpha-\beta)^2[2(\alpha+\beta)m^2-3\alpha\beta s]m^2}{\alpha^3}\right.$$
$$\left.+\frac{[(\alpha+\beta)m^2-2\alpha\beta s][(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\left[\frac{(1-\alpha-\beta)(\alpha+9\beta-1)}{\alpha\beta}-2\right]\right\},$$

$$\rho_3^{\langle\bar{q}Gq\rangle}(s)=\frac{3m_s\langle\bar{q}\sigma\cdot Gq\rangle m^2}{8\pi^4}\sqrt{1-4m^2/s}+\frac{m_s\langle\bar{q}\sigma\cdot Gq\rangle m^2}{2\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\frac{1}{\alpha},$$

$$\rho_3^{\langle\bar{q}q\rangle^2}(s)=-\frac{2\langle\bar{q}q\rangle\langle\bar{s}s\rangle m^2}{\pi^2}\sqrt{1-4m_c^2/s},$$

$$\Pi_3^{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle}(M_B^2)=\frac{\langle\bar{q}\sigma\cdot Gq\rangle\langle\bar{s}s\rangle+\langle\bar{s}\sigma\cdot Gs\rangle\langle\bar{q}q\rangle}{6\pi^2}\int_0^1d\alpha\left\{\frac{2m^2}{3\alpha}-\frac{m^4}{2M_B^2\alpha^2}\right\}e^{-\frac{m^2}{\alpha(1-\alpha)M_B^2}}. \tag{A21}$$

$$\rho_4^{\mathrm{pert}}(s)=\frac{1}{2^6\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\frac{(1-\alpha-\beta)[(2\alpha+2\beta-1)m^2-\alpha\beta s][(\alpha+\beta)m^2-\alpha\beta s]^3}{\alpha^3\beta^3},$$

$$\rho_4^{\langle\bar{q}q\rangle}(s)=\frac{m_s}{8\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\frac{[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\{2(\langle\bar{q}q\rangle+\langle\bar{s}s\rangle)[(\alpha+\beta)m^2-2\alpha\beta s]-(4\langle\bar{q}q\rangle+\langle\bar{s}s\rangle)m^2\},$$

$$\rho_4^{\langle GG\rangle}(s)=-\frac{\langle g_s^2GG\rangle}{3\times2^7\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\left\{\frac{(1-\alpha-\beta)^2m^2}{\alpha^3}\left[\frac{3[(\alpha+\beta)m^2-\alpha\beta s]}{\beta}-[(4\alpha+4\beta-1)m^2-6\alpha\beta s]\right]\right.$$
$$\left.-\frac{[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\left[\frac{3(1-\alpha-\beta)[(\alpha+\beta-1)m^2-2\alpha\beta s]}{\alpha}+\frac{3m^2}{2}\right]\right\},$$

$$\rho_4^{\langle\bar{q}Gq\rangle}(s)=\frac{m_s\langle\bar{q}\sigma\cdot Gq\rangle(2m^2+s)}{32\pi^4}\sqrt{1-4m^2/s}-\frac{m_s\langle\bar{q}\sigma\cdot Gq\rangle}{32\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\frac{[2(\alpha+\beta-1)m^2-3\alpha\beta s]}{\alpha},$$

$$\rho_4^{\langle\bar{q}q\rangle^2}(s)=-\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle(2m^2+s)}{6\pi^2}\sqrt{1-4m_c^2/s},$$

$$\Pi_4^{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle}(M_B^2)=-\frac{\langle\bar{q}\sigma\cdot Gq\rangle\langle\bar{s}s\rangle+\langle\bar{s}\sigma\cdot Gs\rangle\langle\bar{q}q\rangle}{12\pi^2}\int_0^1d\alpha\left\{\frac{m^4(2-\alpha)}{M_B^2\alpha^2(1-\alpha)}-\frac{m^2(1-3\alpha)}{2\alpha(1-\alpha)}+(1-\alpha)M_B^2\right\}e^{-\frac{m^2}{\alpha(1-\alpha)M_B^2}}. \tag{A22}$$

$$\rho_5^{\rm pert}(s)=\frac{1}{2^5\pi^6}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\frac{(1-\alpha-\beta)(m^2-\alpha\beta s)[(\alpha+\beta)m^2-\alpha\beta s]^3}{\alpha^3\beta^3},$$

$$\rho_5^{\langle\bar qq\rangle}(s)=-\frac{m_s}{4\pi^4}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\frac{[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\{2(\langle\bar qq\rangle-\langle\bar ss\rangle)[(\alpha+\beta)m^2-2\alpha\beta s]+(4\langle\bar qq\rangle-\langle\bar ss\rangle)m^2\},$$

$$\rho_5^{\langle GG\rangle}(s)=\frac{\langle g_s^2GG\rangle}{3\times2^8\pi^6}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\bigg\{\frac{4(1-\alpha-\beta)^2m^2}{\alpha^3}\bigg[\frac{3[(\alpha+\beta)m^2-\alpha\beta s]}{\beta}+[(4\alpha+4\beta+1)m^2-6\alpha\beta s]\bigg]$$
$$+\frac{3[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\bigg[\frac{10(1-\alpha-\beta)[(\alpha+\beta+1)m^2-2\alpha\beta s]}{\alpha}+m^2\bigg]\bigg\},$$

$$\rho_5^{\langle\bar qGq\rangle}(s)=\frac{m_s\langle\bar q\sigma\cdot Gq\rangle(6m^2-s)}{16\pi^4}\sqrt{1-4m^2/s}+\frac{5m_s\langle\bar q\sigma\cdot Gq\rangle}{32\pi^4}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\frac{[2(\alpha+\beta+1)m^2-3\alpha\beta s]}{\alpha},$$

$$\rho_5^{\langle\bar qq\rangle^2}(s)=-\frac{\langle\bar qq\rangle\langle\bar ss\rangle(6m^2-s)}{3\pi^2}\sqrt{1-4m_c^2/s},$$

$$\Pi_5^{\langle\bar qGq\rangle\langle\bar qq\rangle}(M_B^2)=\frac{\langle\bar q\sigma\cdot Gq\rangle\langle\bar ss\rangle+\langle\bar s\sigma\cdot Gs\rangle\langle\bar qq\rangle}{24\pi^2}\int_0^1d\alpha\bigg\{\frac{m^4(12\alpha-8)}{M_B^2\alpha^2(1-\alpha)}+\frac{m^2(9-5\alpha)}{\alpha(1-\alpha)}+(4-10\alpha)M_B^2\bigg\}e^{-\frac{m^2}{\alpha(1-\alpha)M_B^2}}. \tag{A23}$$

For the interpolating currents with $J^P=0^+$,

$$\rho_1^{\rm pert}(s)=\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\frac{(1-\alpha-\beta)(m^2(\alpha+\beta-2)+\alpha\beta s)(\alpha\beta s-m^2(\alpha+\beta))^3}{128\pi^6\alpha^3\beta^3},$$

$$\rho_1^{\langle\bar qq\rangle}(s)=m_q(2\langle\bar qq\rangle+\langle\bar ss\rangle)\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(\alpha+\beta+1)-2\alpha\beta s)}{8\pi^4\alpha\beta},$$

$$\rho_1^{\langle GG\rangle}(s)=\frac{\langle g_s^2GG\rangle}{2}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\bigg\{\frac{m^2(1-\alpha-\beta)^2(2m^2(\alpha+\beta)+m^2-3\alpha\beta s)}{192\pi^6\alpha^3}+(m^2(\alpha+\beta)-\alpha\beta s)$$
$$\times\bigg[\frac{(1-\alpha-\beta)^2(2\alpha^2\beta s-m^2(\alpha(\alpha+\beta+2)-8\beta))}{512\pi^6\alpha^3\beta^2}-\frac{m^2(\alpha+\beta+1)-2\alpha\beta s}{256\pi^6\alpha\beta}\bigg]\bigg\},$$

$$\rho_1^{\langle\bar qGq\rangle}(s)=\frac{m_q(4\langle\bar q\sigma\cdot Gq\rangle+\langle\bar s\sigma\cdot Gs\rangle)(s-4m^2)}{128\pi^4}\sqrt{1-\frac{4m^2}{s}},\qquad\rho_1^{\langle\bar qq\rangle^2}(s)=\frac{\langle\bar qq\rangle\langle\bar ss\rangle(s-4m^2)}{6\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_1^{\langle\bar qq\rangle\langle\bar qGq\rangle}(M_B^2)=-\frac{\langle\bar qq\rangle\langle\bar s\sigma\cdot Gs\rangle+\langle\bar ss\rangle\langle\bar q\sigma\cdot Gq\rangle}{12\pi^2}\int_0^1dx\bigg\{\frac{m^4(2x-1)}{M_B^2(1-x)x^2}+\frac{m^2}{x(1-x)}+M_B^2\bigg\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \tag{A24}$$

$$\rho_2^{\rm pert}(s)=\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)^3(m^2(3\alpha+3\beta-2)-\alpha\beta s)}{128\pi^6\alpha^3\beta^3},$$

$$\rho_2^{\langle\bar qq\rangle}(s)=-m_q(2\langle\bar qq\rangle-\langle\bar ss\rangle)\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\frac{(m^2(\alpha+\beta-1)-2\alpha\beta s)(m^2(\alpha+\beta)-\alpha\beta s)}{8\pi^4\alpha\beta},$$

$$\rho_2^{\langle GG\rangle}(s)=\frac{\langle g_s^2GG\rangle}{2}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}d\beta\bigg\{\frac{m^2(1-\alpha-\beta)^2(m^2(2\alpha+2\beta-1)+m^2-3\alpha\beta s)}{192\pi^6\alpha^3}+(m^2(\alpha+\beta)-\alpha\beta s)$$
$$\times\bigg[\frac{(1-\alpha-\beta)^2(2\alpha^2\beta s-m^2(\alpha(\alpha+\beta-2)+8\beta))}{512\pi^6\alpha^3\beta^2}-\frac{m^2(\alpha+\beta-1)-2\alpha\beta s}{256\pi^6\alpha\beta}\bigg]\bigg\},$$

$$\rho_2^{\langle\bar qGq\rangle}(s)=-\frac{m_q(4\langle\bar q\sigma\cdot Gq\rangle-\langle\bar s\sigma\cdot Gs\rangle)s}{128\pi^4}\sqrt{1-\frac{4m^2}{s}},\qquad\rho_2^{\langle\bar qq\rangle^2}(s)=\frac{\langle\bar qq\rangle\langle\bar ss\rangle s}{6\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_2^{\langle\bar qq\rangle\langle\bar qGq\rangle}(M_B^2)=\frac{\langle\bar qq\rangle\langle\bar s\sigma\cdot Gs\rangle+\langle\bar ss\rangle\langle\bar q\sigma\cdot Gq\rangle}{12\pi^2}\int_0^1dx\bigg\{\frac{m^4}{M_B^2(1-x)x^2}+\frac{m^2}{x(1-x)}+M_B^2\bigg\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \tag{A25}$$

$$\rho_3^{\rm pert}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1-\alpha-\beta)(m^2(2\alpha+2\beta-1)-\alpha\beta s)(m^2(\alpha+\beta)-\alpha\beta s)}{64\pi^6\alpha^3\beta^3},$$

$$\rho_3^{\langle\bar{q}q\rangle}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{m_q m^2(m^2(\alpha+\beta)-\alpha\beta s)}{8\pi^4\alpha\beta}(\langle\bar{q}_2q_2\rangle(m^2(2\alpha+2\beta-1)-4\alpha\beta s) - 2\langle\bar{q}_1q_1\rangle(m^2(\alpha+\beta-2)-2\alpha\beta s),$$

$$\rho_3^{\langle GG\rangle}(s) = \frac{\langle g_s^2GG\rangle}{2}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \left\{\frac{m^2(1-\alpha-\beta)^2(m^2(4\alpha\beta-3\alpha+4\beta^2-4\beta)+3\alpha(1-2\beta)\beta s)}{192\pi^6\alpha^3\beta} + (m^2(\alpha+\beta)-\alpha\beta s)\left[\frac{(1-\alpha-\beta)(m^2(\alpha+\beta-1)-2\alpha\beta s)}{64\pi^6\alpha^2\beta} + \frac{m^2(m^2(\alpha+\beta)-\alpha\beta s)}{128\pi^6\alpha\beta}\right]\right\},$$

$$\rho_3^{\langle\bar{q}Gq\rangle}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{m_q(\langle\bar{s}\sigma\cdot Gs\rangle(m^2(2\alpha+2\beta-1)-3\alpha\beta s)+\langle\bar{q}\sigma\cdot Gq\rangle(3\alpha\beta s-2m^2(\alpha+\beta-1)))}{32\pi^4\alpha} + \frac{m_q(m^2\langle\bar{s}\sigma\cdot Gs\rangle-\langle\bar{q}\sigma\cdot Gq\rangle(2m^2+s))}{32\pi^4},$$

$$\rho_3^{\langle\bar{q}q\rangle^2}(s) = \frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle(2m^2+s)}{6\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_3^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) = \frac{\langle\bar{q}q\rangle\langle\bar{s}\sigma\cdot Gs\rangle+\langle\bar{s}s\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{24\pi^2}\int_0^1 dx\left\{\frac{2m^4(2-x)}{M_B^2(1-x)x^2}+\frac{m^2}{1-x}+2M_B^2(1-x)\right\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \quad \text{(A26)}$$

$$\rho_4^{\rm pert}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1-\alpha-\beta)(m^2-\alpha\beta s)(m^2(\alpha+\beta)-\alpha\beta s)^3}{32\pi^6\alpha^3\beta^3},$$

$$\rho_4^{\langle\bar{q}q\rangle}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{m_q m^2(m^2(\alpha+\beta)-\alpha\beta s)}{4\pi^4\alpha\beta}(\langle\bar{q}_2q_2\rangle(m^2(2\alpha+2\beta+1)-4\alpha\beta s) + 2\langle\bar{q}_1q_1\rangle(m^2(\alpha+\beta+2)-2\alpha\beta s),$$

$$\rho_4^{\langle GG\rangle}(s) = \frac{\langle g_s^2GG\rangle}{2}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \left\{\frac{m^2(1-\alpha-\beta)^2(m^2(4\alpha\beta+3\alpha+4\beta^2+4\beta)-3\alpha\beta(2\beta+1)s)}{96\pi^6\alpha^2\beta} + (m^2(\alpha+\beta)-\alpha\beta s)\left[\frac{5(1-\alpha-\beta)(m^2(\alpha+\beta+1)-2\alpha\beta s)}{64\pi^6\alpha^2\beta} + \frac{m^2}{128\pi^6\alpha\beta}\right]\right\},$$

$$\rho_4^{\langle\bar{q}Gq\rangle}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{5m_q(\langle\bar{s}\sigma\cdot Gs\rangle(m^2(2\alpha+2\beta+1)-3\alpha\beta s)+\langle\bar{q}\sigma\cdot Gq\rangle(2m^2(\alpha+\beta+1)-3\alpha\beta s))}{32\pi^4\alpha} + \frac{m_q(m^2\langle\bar{s}\sigma\cdot Gs\rangle+\langle\bar{q}\sigma\cdot Gq\rangle(s-6m^2))}{32\pi^4}\sqrt{1-\frac{4m^2}{s}},$$

$$\rho_4^{\langle\bar{q}q\rangle^2}(s) = -\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle(s-6m^2)}{3\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_4^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) = \frac{\langle\bar{q}q\rangle\langle\bar{s}\sigma\cdot Gs\rangle+\langle\bar{s}s\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{24\pi^2}\int_0^1 dx\left\{\frac{4m^4(2-3x)}{M_B^2(1-x)x^2}+\frac{m^2(15x-14)}{x(1-x)} - 2M_B^2\frac{5x^2-7x+2}{1-x}\right\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \quad \text{(A27)}$$

$$\rho_5^{\mathrm{pert}}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{3(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)^4}{32\pi^6\alpha^3\beta^3},$$

$$\rho_5^{\langle\bar{q}q\rangle}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{3m_q(m^2(\alpha+\beta)-\alpha\beta s)((m^2(\alpha+\beta)-2\alpha\beta s)\langle\bar{s}s\rangle + 2m^2\langle\bar{q}q\rangle)}{2\pi^4\alpha\beta},$$

$$\rho_5^{\langle GG\rangle}(s) = \frac{\langle g_s^2 GG\rangle}{2}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \Big\{\frac{m^2(1-\alpha-\beta)^2(2m^2(\alpha+\beta)-3\alpha\beta s)}{16\pi^6\alpha^3} - \frac{(\alpha(\alpha+12\beta-2)+\beta(9\beta-10)+1)(m^2(\alpha+\beta)-2\alpha\beta s)(m^2(\alpha+\beta)-\alpha\beta s)}{64\pi^6\alpha^2\beta^2}\Big\},$$

$$\rho_5^{\langle\bar{q}Gq\rangle}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{m_q(2m^2(\alpha+\beta)-3\alpha\beta s)\langle\bar{s}\sigma\cdot Gs\rangle + 2m^2\langle\bar{q}\sigma\cdot Gq\rangle}{4\pi^4\alpha} \times \frac{m_q\langle\bar{s}\sigma\cdot Gs\rangle(s-2m^2) - 12m_q m^2\langle\bar{q}\sigma\cdot Gq\rangle}{16\pi^4}\sqrt{1-\frac{4m^2}{s}},$$

$$\rho_5^{\langle\bar{q}q\rangle^2}(s) = \frac{4\langle\bar{q}q\rangle\langle\bar{s}s\rangle m^2}{\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_5^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) = \frac{m^2(\langle\bar{q}q\rangle\langle\bar{s}\sigma\cdot Gs\rangle + \langle\bar{s}s\rangle\langle\bar{q}\sigma\cdot Gq\rangle)}{3\pi^2}\int_0^1 dx\Big\{\frac{3m^2-2xM_B^2}{x^2M_B^2}\Big\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \tag{A28}$$

For the interpolating currents with $J^P = 1^-$,

$$\rho_1^{\mathrm{pert}}(s) = \frac{1}{2^8\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1-\alpha-\beta)[(\alpha+\beta)m^2-\alpha\beta s]^3}{\alpha^3\beta^3}\{(1+\alpha+\beta)[(\alpha+\beta)m^2-\alpha\beta s] + 4(1-\alpha-\beta)m^2\},$$

$$\rho_1^{\langle\bar{q}q\rangle}(s) = -\frac{m_s(2\langle\bar{q}q\rangle - \langle\bar{s}s\rangle)}{16\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{[(2+\alpha+\beta)m^2-3\alpha\beta s][(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta},$$

$$\rho_1^{\langle GG\rangle}(s) = \frac{\langle g_s^2GG\rangle}{3\times 2^{11}\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \Big\{\frac{16(1-\alpha-\beta)^2m^2}{\alpha^3}\Big[\frac{[(3\alpha+4\beta)m^2-3\alpha\beta s]}{\beta} + [(\alpha+\beta)m^2-2\alpha\beta s]\Big] + \frac{[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\Big[\frac{(1-\alpha-\beta)^2[3(\alpha+\beta)m^2-5\alpha\beta s]}{\alpha\beta} - 6[(2+\alpha+\beta)m^2-3\alpha\beta s]\Big]\Big\},$$

$$\rho_1^{\langle\bar{q}Gq\rangle}(s) = \frac{m_s\langle\bar{q}\sigma\cdot Gq\rangle(4m^2-s)}{48\pi^4}\sqrt{1-4m^2/s},$$

$$\rho_1^{\langle\bar{q}q\rangle^2}(s) = -\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle(4m^2-s)}{9\pi^2}\sqrt{1-4m_c^2/s},$$

$$\Pi_1^{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle}(M_B^2) = \frac{\langle\bar{q}\sigma\cdot Gq\rangle\langle\bar{s}s\rangle + \langle\bar{s}\sigma\cdot Gs\rangle\langle\bar{q}q\rangle}{12\pi^2}\int_0^1 d\alpha\Big\{\frac{m^4(2\alpha-1)}{M_B^2\alpha^2(1-\alpha)} + \frac{m^2(2-\alpha)}{(1-\alpha)} + M_B^2\alpha\Big\}e^{-\frac{m^2}{\alpha(1-\alpha)M_B^2}}. \tag{A29}$$

$$\rho_2^{\mathrm{pert}}(s) = \frac{1}{3\times 2^8\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1-\alpha-\beta)[(\alpha+\beta)m^2-\alpha\beta s]^3}{\alpha^3\beta^3} \times\{3(1+\alpha+\beta)[(\alpha+\beta)m^2-\alpha\beta s] - 4(1-\alpha-\beta)(2+\alpha+\beta)m^2\},$$

$$\rho_2^{\langle\bar{q}q\rangle}(s) = -\frac{m_s}{16\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\{[(\alpha+\beta)m^2+3\alpha\beta s]\langle\bar{s}s\rangle - 4[(\alpha+\beta-1)m^2-2\alpha\beta s]\langle\bar{q}q\rangle\},$$

$$\rho_2^{\langle GG\rangle}(s) = -\frac{\langle g_s^2 GG\rangle}{3^2\times 2^{11}\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \bigg\{\frac{16(1-\alpha-\beta)^2 m^2}{\alpha^3}\bigg[\frac{[(3\alpha+4\beta)m^2-3\alpha\beta s](2+\alpha+\beta)}{\beta} - 3[(\alpha+\beta)m^2-2\alpha\beta s]\bigg] - \frac{3[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\bigg[\frac{(1-\alpha-\beta)^2[(\alpha+\beta+8)m^2+9\alpha\beta s]}{\alpha\beta} + 2[(5\alpha+5\beta-4)m^2-5\alpha\beta s]\bigg]\bigg\},$$

$$\rho_2^{\langle\bar{q}Gq\rangle}(s) = \frac{m_s\langle\bar{q}\sigma\cdot Gq\rangle s}{32\pi^4}\sqrt{1-4m^2/s},$$

$$\rho_2^{\langle\bar{q}q\rangle^2}(s) = -\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle s}{6\pi^2}\sqrt{1-4m_c^2/s},$$

$$\Pi_2^{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle}(M_B^2) = -\frac{\langle\bar{q}\sigma\cdot Gq\rangle\langle\bar{s}s\rangle + \langle\bar{s}\sigma\cdot Gs\rangle\langle\bar{q}q\rangle}{12\pi^2}\int_0^1 d\alpha\bigg\{\frac{m^4}{M_B^2\alpha^2(1-\alpha)} + \frac{m^2}{(1-\alpha)} + M_B^2\bigg\}e^{-\frac{m^2}{\alpha(1-\alpha)M_B^2}}. \quad \text{(A30)}$$

$$\rho_3^{\text{pert}}(s) = -\frac{1}{3\times 2^9\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1-\alpha-\beta)[(\alpha+\beta)m^2-\alpha\beta s]^3}{\alpha^3\beta^3} \times\{9(1+\alpha+\beta)[(\alpha+\beta)m^2-\alpha\beta s] + 4(1-\alpha-\beta)(4-\alpha-\beta)m^2\},$$

$$\rho_3^{\langle\bar{q}q\rangle}(s) = -\frac{m_s}{32\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\{[(\alpha+\beta+4)m^2-9\alpha\beta s]\langle\bar{s}s\rangle - 4(3m^2-\alpha\beta s)\langle\bar{q}q\rangle\},$$

$$\rho_3^{\langle GG\rangle}(s) = \frac{\langle g_s^2 GG\rangle}{3^2\times 2^{11}\pi^6}\int_{\alpha_{min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \bigg\{\frac{8(1-\alpha-\beta)^2 m^2}{\alpha^3}\bigg[\frac{[(3\alpha+4\beta)m^2-3\alpha\beta s](\alpha+\beta-4)}{\beta} - 9[(\alpha+\beta)m^2-2\alpha\beta s]\bigg] + \frac{[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\bigg[\frac{(1-\alpha-\beta)^2[(5\alpha+5\beta+16)m^2-27\alpha\beta s]}{\alpha\beta} - \frac{72(1-\alpha-\beta)[(\alpha+\beta+1)m^2-2\alpha\beta s]}{\alpha} - 6[(\alpha+\beta-8)m^2-5\alpha\beta s]\bigg]\bigg\},$$

$$\rho_3^{\langle\bar{q}Gq\rangle}(s) = \frac{m_s\langle\bar{q}\sigma\cdot Gq\rangle(16m^2-s)}{192\pi^4}\sqrt{1-4m^2/s} + \frac{m_s\langle\bar{q}\sigma\cdot Gq\rangle}{96\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{min}}^{\beta_{\max}} d\beta \frac{(3m^2-\alpha\beta s)}{\alpha},$$

$$\rho_3^{\langle\bar{q}q\rangle^2}(s) = -\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle(16m^2-s)}{36\pi^2}\sqrt{1-4m_c^2/s},$$

$$\Pi_3^{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle}(M_B^2) = \frac{\langle\bar{q}\sigma\cdot Gq\rangle\langle\bar{s}s\rangle + \langle\bar{s}\sigma\cdot Gs\rangle\langle\bar{q}q\rangle}{72\pi^2}\int_0^1 d\alpha\bigg\{\frac{3(3-4\alpha)m^4}{M_B^2\alpha^2(1-\alpha)} - \frac{3m^2}{\alpha(1-\alpha)} + \frac{m^2(4-6\alpha)}{(1-\alpha)} + (3-6\alpha)M_B^2\bigg\}e^{-\frac{m^2}{\alpha(1-\alpha)M_B^2}}. \quad \text{(A31)}$$

$$\rho_4^{\text{pert}}(s) = -\frac{1}{3\times 2^9\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1-\alpha-\beta)[(\alpha+\beta)m^2-\alpha\beta s]^3}{\alpha^3\beta^3}\{9(1+\alpha+\beta)[(\alpha+\beta)m^2-\alpha\beta s] - 4(1-\alpha-\beta)(1+2\alpha+2\beta)m^2\},$$

$$\rho_4^{\langle\bar{q}q\rangle}(s) = \frac{m_s}{16\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\{[(2\alpha+2\beta-1)m^2+3\alpha\beta s]\langle\bar{s}s\rangle - [3(\alpha+\beta-2)m^2-5\alpha\beta s]\langle\bar{q}q\rangle\},$$

$$\rho_4^{\langle GG\rangle}(s) = \frac{\langle g_s^2 GG\rangle}{3^2\times 2^{11}\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha\int_{\beta_{\min}}^{\beta_{max}} d\beta\Big\{\frac{8(1-\alpha-\beta)^2 m^2}{\alpha^3}\Big[\frac{3[(3\alpha+4\beta)m^2-3\alpha\beta s]}{\beta} + 2(1-\alpha-\beta)[3(\alpha+\beta)m^2-4\alpha\beta s]-9[(\alpha+\beta)m^2-2\alpha\beta s]\Big]-\frac{3[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\times\Big[\frac{(1-\alpha-\beta)^2[7(\alpha+\beta)m^2-13\alpha\beta s]}{\alpha\beta}+\frac{16(1-\alpha-\beta)[(\alpha+\beta-3)m^2-3\alpha\beta s]}{\alpha} + 2[(19\alpha+19\beta-2)m^2-33\alpha\beta s]\Big]\Big\},$$

$$\rho_4^{\langle\bar q Gq\rangle}(s) = \frac{m_s\langle\bar q\sigma\cdot Gq\rangle(2m^2+s)}{48\pi^4}\sqrt{1-4m^2/s}-\frac{m_s\langle\bar q\sigma\cdot Gq\rangle}{96\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha\int_{\beta_{\min}}^{\beta_{\max}} d\beta\frac{[3(\alpha+\beta-1)m^2-4\alpha\beta s]}{\alpha},$$

$$\rho_4^{\langle\bar q q\rangle^2}(s) = -\frac{\langle\bar q q\rangle\langle\bar s s\rangle(2m^2+s)}{9\pi^2}\sqrt{1-4m_c^2/s},$$

$$\Pi_4^{\langle\bar q Gq\rangle\langle\bar q q\rangle}(M_B^2) = \frac{\langle\bar q\sigma\cdot Gq\rangle\langle\bar s s\rangle+\langle\bar s\sigma\cdot Gs\rangle\langle\bar q q\rangle}{72\pi^2}\int_0^1 d\alpha\Big\{\frac{3(3-2\alpha)m^4}{M_B^2\alpha^2(1-\alpha)}+\frac{(2+\alpha)m^2}{\alpha(1-\alpha)}-3m^2+6(1-\alpha)M_B^2\Big\}e^{-\frac{m^2}{\alpha(1-\alpha)M_B^2}}. \tag{A32}$$

$$\rho_5^{\rm pert}(s) = \frac{1}{2^9\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha\int_{\beta_{\min}}^{\beta_{\max}} d\beta\frac{(1-\alpha-\beta)[(\alpha+\beta)m^2-\alpha\beta s]^3}{\alpha^3\beta^3}\{(1+\alpha+\beta)[(\alpha+\beta)m^2-\alpha\beta s] - 4(1-\alpha-\beta)m^2\},$$

$$\rho_5^{\langle\bar q q\rangle}(s) = \frac{m_s(2\langle\bar q q\rangle+\langle\bar s s\rangle)}{32\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha\int_{\beta_{\min}}^{\beta_{\max}} d\beta\frac{[(\alpha+\beta-2)m^2-3\alpha\beta s][(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta},$$

$$\rho_5^{\langle GG\rangle}(s) = -\frac{\langle g_s^2 GG\rangle}{3\times 2^{11}\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha\int_{\beta_{\min}}^{\beta_{\max}} d\beta\Big\{\frac{8(1-\alpha-\beta)^2m^2}{\alpha^3}\Big[\frac{3[(\alpha+\beta)m^2-\alpha\beta s]}{\beta}-[(\alpha+\beta-1)m^2-2\alpha\beta s]\Big] + \frac{[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\Big[\frac{(1-\alpha-\beta)^2[3(\alpha+\beta)m^2-5\alpha\beta s]}{\alpha\beta}-6[(\alpha+\beta-2)m^2-3\alpha\beta s]\Big]\Big\},$$

$$\rho_5^{\langle\bar q Gq\rangle}(s) = \frac{m_s\langle\bar q\sigma\cdot Gq\rangle(2m^2+s)}{96\pi^4}\sqrt{1-4m^2/s},\qquad \rho_5^{\langle\bar q q\rangle^2}(s) = -\frac{\langle\bar q q\rangle\langle\bar s s\rangle(2m^2+s)}{18\pi^2}\sqrt{1-4m_c^2/s},$$

$$\Pi_5^{\langle\bar q Gq\rangle\langle\bar q q\rangle}(M_B^2) = -\frac{\langle\bar q\sigma\cdot Gq\rangle\langle\bar s s\rangle+\langle\bar s\sigma\cdot Gs\rangle\langle\bar q q\rangle}{24\pi^2}\int_0^1 d\alpha\Big\{\frac{m^4}{M_B^2\alpha^2(1-\alpha)}+\frac{m^2(2-\alpha)}{(1-\alpha)}+M_B^2\alpha\Big\}e^{-\frac{m^2}{\alpha(1-\alpha)M_B^2}}. \tag{A33}$$

$$\rho_6^{\rm pert}(s) = -\frac{1}{3\times 2^8\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha\int_{\beta_{\min}}^{\beta_{\max}} d\beta\frac{(1-\alpha-\beta)[(\alpha+\beta)m^2-\alpha\beta s]^3}{\alpha^3\beta^3}\times\{3(1+\alpha+\beta)[(\alpha+\beta)m^2-\alpha\beta s]+4(1-\alpha-\beta)(2+\alpha+\beta)m^2\},$$

$$\rho_6^{\langle\bar q q\rangle}(s) = -\frac{m_s}{16\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha\int_{\beta_{\min}}^{\beta_{\max}} d\beta\frac{[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\{3[(\alpha+\beta)m^2-\alpha\beta s]\langle\bar s s\rangle-4[(\alpha+\beta+1)m^2-2\alpha\beta s]\langle\bar q q\rangle\},$$

$$\rho_6^{\langle GG\rangle}(s) = -\frac{\langle g_s^2 GG\rangle}{3^2\times 2^{11}\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha\int_{\beta_{\min}}^{\beta_{\max}} d\beta\Big\{\frac{16(1-\alpha-\beta)^2m^2}{\alpha^3}\Big[\frac{3(2+\alpha+\beta)[(\alpha+\beta)m^2-\alpha\beta s]}{\beta} + 2[(2\alpha+2\beta+1)m^2-3\alpha\beta s]\Big]-\frac{3[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\Big[\frac{(1-\alpha-\beta)^2[(7\alpha+7\beta+8)m^2-9\alpha\beta s]}{\alpha\beta} - 2[(\alpha+\beta+4)m^2-5\alpha\beta s]\Big]\Big\},$$

$$\rho_6^{\langle\bar q Gq\rangle}(s) = \frac{m_s\langle\bar q\sigma\cdot Gq\rangle(4m^2-s)}{32\pi^4}\sqrt{1-4m^2/s},\quad \rho_6^{\langle\bar q q\rangle^2}(s) = -\frac{\langle\bar q q\rangle\langle\bar s s\rangle(4m^2-s)}{6\pi^2}\sqrt{1-4m_c^2/s},$$

$$\Pi_6^{\langle\bar q Gq\rangle\langle\bar q q\rangle}(M_B^2) = -\frac{\langle\bar q\sigma\cdot Gq\rangle\langle\bar s s\rangle+\langle\bar s\sigma\cdot Gs\rangle\langle\bar q q\rangle}{12\pi^2}\int_0^1 d\alpha\Big\{\frac{m^4(1-2\alpha)}{M_B^2\alpha^2(1-\alpha)}-\frac{m^2}{\alpha(1-\alpha)}-M_B^2\Big\}e^{-\frac{m^2}{\alpha(1-\alpha)M_B^2}}. \tag{A34}$$

$$\begin{aligned}
\rho_7^{\mathrm{pert}}(s) &= -\frac{1}{3\times 2^9\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1-\alpha-\beta)[(\alpha+\beta)m^2-\alpha\beta s]^3}{\alpha^3\beta^3} \\
&\quad\times\{9(1+\alpha+\beta)[(\alpha+\beta)m^2-\alpha\beta s]-4(1-\alpha-\beta)(4-\alpha-\beta)m^2\}, \\
\rho_7^{\langle\bar{q}q\rangle}(s) &= -\frac{m_s}{32\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\{[(5\alpha+5\beta-4)m^2-9\alpha\beta s]\langle\bar{s}s\rangle-4(3m^2+\alpha\beta s)\langle\bar{q}q\rangle\}, \\
\rho_7^{\langle GG\rangle}(s) &= \frac{\langle g_s^2GG\rangle}{3^2\times 2^{11}\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \bigg\{\frac{8(1-\alpha-\beta)^2m^2}{\alpha^3}\bigg[\frac{3(4-\alpha-\beta)[(\alpha+\beta)m^2-\alpha\beta s]}{\beta} \\
&\quad-[(10\alpha+10\beta-4)m^2-18\alpha\beta s]\bigg]+\frac{[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\bigg[\frac{(1-\alpha-\beta)^2[(13\alpha+13\beta-16)m^2-27\alpha\beta s]}{\alpha\beta} \\
&\quad-\frac{24(1-\alpha-\beta)[(5\alpha+5\beta-3)m^2-6\alpha\beta s]}{\alpha}-6[(5\alpha+5\beta+8)m^2-5\alpha\beta s]\bigg]\bigg\}, \\
\rho_7^{\langle\bar{q}Gq\rangle}(s) &= \frac{m_s\langle\bar{q}\sigma\cdot Gq\rangle(20m^2+s)}{192\pi^4}\sqrt{1-4m^2/s}+\frac{m_s\langle\bar{q}\sigma\cdot Gq\rangle}{96\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(3m^2+\alpha\beta s)}{\alpha}, \\
\rho_7^{\langle\bar{q}q\rangle^2}(s) &= -\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle(20m^2+s)}{36\pi^2}\sqrt{1-4m_c^2/s}, \\
\Pi_7^{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle}(M_B^2) &= -\frac{\langle\bar{q}\sigma\cdot Gq\rangle\langle\bar{s}s\rangle+\langle\bar{s}\sigma\cdot Gs\rangle\langle\bar{q}q\rangle}{72\pi^2}\int_0^1 d\alpha\bigg\{\frac{3(3-2\alpha)m^4}{M_B^2\alpha^2(1-\alpha)}-\frac{(7-10\alpha)m^2}{\alpha(1-\alpha)}+6m^2+(3-6\alpha)M_B^2\bigg\}e^{-\frac{m^2}{\alpha(1-\alpha)M_B^2}}.
\end{aligned} \tag{A35}$$

$$\begin{aligned}
\rho_8^{\mathrm{pert}}(s) &= -\frac{1}{3\times 2^8\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1-\alpha-\beta)[(\alpha+\beta)m^2-\alpha\beta s]^3}{\alpha^3\beta^3}\{9(1+\alpha+\beta)[(\alpha+\beta)m^2-\alpha\beta s] \\
&\quad+4(1-\alpha-\beta)(1+2\alpha+2\beta)m^2\}, \\
\rho_8^{\langle\bar{q}q\rangle}(s) &= -\frac{m_s}{16\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\{[(7\alpha+7\beta-2)m^2-9\alpha\beta s]\langle\bar{s}s\rangle \\
&\quad-2[3(\alpha+\beta+2)m^2-5\alpha\beta s]\langle\bar{q}q\rangle\}, \\
\rho_8^{\langle GG\rangle}(s) &= -\frac{\langle g_s^2GG\rangle}{3^2\times 2^{11}\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \bigg\{\frac{16(1-\alpha-\beta)^2m^2}{\alpha^3}\bigg[\frac{3(1+2\alpha+2\beta)[(\alpha+\beta)m^2-\alpha\beta s]}{\beta} \\
&\quad+[(11\alpha+11\beta+1)m^2-18\alpha\beta s]\bigg]-\frac{3[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha\beta}\bigg[\frac{(1-\alpha-\beta)^2[3(\alpha+\beta)m^2-5\alpha\beta s]}{\alpha\beta} \\
&\quad-\frac{240(1-\alpha-\beta)[(\alpha+\beta)m^2-\alpha\beta s]}{\alpha}-2[(7\alpha+7\beta-2)m^2-9\alpha\beta s]\bigg]\bigg\}, \\
\rho_8^{\langle\bar{q}Gq\rangle}(s) &= \frac{m_s\langle\bar{q}\sigma\cdot Gq\rangle(7m^2-s)}{24\pi^4}\sqrt{1-4m^2/s}+\frac{5m_s\langle\bar{q}\sigma\cdot Gq\rangle}{96\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \\
&\quad\times\int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{[3(\alpha+\beta+1)m^2-4\alpha\beta s]}{\alpha}, \\
\rho_8^{\langle\bar{q}q\rangle^2}(s) &= -\frac{2\langle\bar{q}q\rangle\langle\bar{s}s\rangle(7m^2-s)}{9\pi^2}\sqrt{1-4m_c^2/s}, \\
\Pi_8^{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle}(M_B^2) &= -\frac{\langle\bar{q}\sigma\cdot Gq\rangle\langle\bar{s}s\rangle+\langle\bar{s}\sigma\cdot Gs\rangle\langle\bar{q}q\rangle}{72\pi^2}\int_0^1 d\alpha\bigg\{\frac{6(3-4\alpha)m^4}{M_B^2\alpha^2(1-\alpha)}+\frac{2(2\alpha-11)m^2}{\alpha(1-\alpha)} \\
&\quad-\frac{6(\alpha-3)m^2}{(1-\alpha)}+(21\alpha-12)M_B^2\bigg\}e^{-\frac{m^2}{\alpha(1-\alpha)M_B^2}}.
\end{aligned} \tag{A36}$$

For the interpolating currents with $J^P = 1^+$,

$$\rho_1^{\rm pert}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \bigg\{ \frac{(1-\alpha-\beta)^2(\alpha\beta s - m^2(\alpha+\beta-4))(m^2(\alpha+\beta)-\alpha\beta s)^3}{256\pi^6\alpha^3\beta^3} + \frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)^4}{128\pi^6\alpha^3\beta^3} \bigg\},$$

$$\rho_1^{\langle\bar{q}q\rangle}(s) = m_q(\langle\bar{q}q\rangle + \langle\bar{s}s\rangle) \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(\alpha+\beta+2)-3\alpha\beta s)}{16\pi^4\alpha\beta},$$

$$\rho_1^{\langle GG\rangle}(s) = \frac{\langle g_s^2 GG\rangle}{2} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \bigg\{ \frac{(m^2(\alpha+\beta+2)-\alpha\beta s)(\alpha\beta s - m^2(\alpha+\beta))}{512\pi^6\alpha\beta} + (1-\alpha-\beta)^2 \times \bigg[\frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(3\alpha^2+3\alpha\beta-16\beta^3+48\beta)-5\alpha^2\beta s)}{3072\pi^6\alpha^3\beta^2} + \frac{m^2(2m^2(\alpha+\beta)+m^2-3\alpha\beta s)}{192\pi^6\alpha^3},$$

$$\rho_1^{\langle\bar{q}Gq\rangle}(s) = \frac{m_q(4\langle\bar{q}\sigma\cdot Gq\rangle + \langle\bar{s}\sigma\cdot Gs\rangle)(s-4m^2)}{192\pi^4}\sqrt{1-\frac{4m^2}{s}}, \qquad \rho_1^{\langle\bar{q}q\rangle^2}(s) = -\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle(s-4m^2)}{9\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_1^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) = -\frac{\langle\bar{q}q\rangle\langle\bar{s}\sigma\cdot Gs\rangle + \langle\bar{s}s\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{12\pi^2}\int_0^1 dx \bigg\{ \frac{m^4(2x-1)}{M_B^2 x^2(1-x)} + \frac{m^2(2-x)}{1-x} + M_B^2 x \bigg\} e^{-\frac{m^2}{M_B^2(1-x)x}}. \tag{A37}$$

$$\rho_2^{\rm pert}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1-\alpha-\beta)(\alpha\beta s - m^2(\alpha+\beta))(m^2(\alpha+\beta+1)(\alpha+\beta)-8)+3\alpha\beta s(\alpha+\beta+1))}{768\pi^6\alpha^3\beta^3},$$

$$\rho_1^{\langle\bar{q}q\rangle}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \bigg\{ \frac{m_q\langle\bar{q}q\rangle(m^2(\alpha+\beta)-\alpha\beta s)(m^2(\alpha+\beta+1)-2\alpha\beta s)}{4\pi^4\alpha\beta} + \frac{m_q\langle\bar{s}s\rangle(m^2(\alpha+\beta)(3\alpha+3\beta+1)-(5\alpha+5\beta-1)\alpha\beta s)}{16\pi^4\alpha\beta},$$

$$\rho_2^{\langle GG\rangle}(s) = \frac{\langle g_s^2 GG\rangle}{2} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \bigg\{ \frac{m^2(\alpha+\beta-1)^2(m^2(3\alpha^2+6\alpha\beta+4\alpha+3\beta^2+4\beta-1)-\alpha\beta s(4\alpha+4\beta+5))}{576\pi^6\alpha^3} + (m^2(\alpha+\beta)-\alpha\beta s)\bigg[\frac{(1-\alpha-\beta)^3(\alpha-4\beta)}{768\pi^6\alpha^3\beta^2} - \frac{(1-\alpha-\beta)^2(m^2(\alpha^2+\alpha\beta+2\alpha-8\beta)-2\alpha\beta s)}{512\pi^6\alpha^3\beta^2} + \frac{(1-\alpha-\beta)(m^2(3\alpha+3\beta+2)-5\alpha\beta s)}{1536\pi^6\alpha\beta} - \frac{m^2(\alpha+\beta+1)-2\alpha\beta s}{768\pi^6\alpha\beta} \bigg]\bigg\},$$

$$\rho_2^{\langle\bar{q}Gq\rangle}(s) = \frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle(s-4m^2)}{32\pi^4}\sqrt{1-\frac{4m^2}{s}}, \qquad \rho_2^{\langle\bar{q}q\rangle^2}(s) = -\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle(s-4m^2)}{6\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_2^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) = \frac{\langle\bar{q}q\rangle\langle\bar{s}\sigma\cdot Gs\rangle + \langle\bar{s}s\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{12\pi^2}\int_0^1 dx \bigg\{ \frac{m^4(1-2x)}{M_B^2 x^2(1-x)} - \frac{m^2}{x(1-x)} - M_B^2 \bigg\} e^{-\frac{m^2}{M_B^2(1-x)x}}. \tag{A38}$$

$$\rho_3^{\rm pert}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta (1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)\bigg[\frac{(7-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)}{512\pi^6\alpha^3\beta^3} + \frac{(1-\alpha-\beta)(m^2(3\alpha^2+6\alpha\beta-4\alpha+3\beta^2-4\beta-4)-7(\alpha+\beta-1)\alpha\beta s)}{384\pi^6\alpha^3\beta^3} \bigg],$$

$$\rho_3^{\langle\bar{q}q\rangle}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \bigg\{ \frac{m_q\langle\bar{q}q\rangle(3m^2+\alpha\beta s)(m^2(\alpha+\beta)-\alpha\beta)}{8\pi^4\alpha\beta} + \frac{m_q\langle\bar{s}s\rangle(m^2(\alpha+\beta)-\alpha\beta s)(\alpha\beta s(25\alpha+25\beta-37)-m^2((\alpha+\beta)(15(\alpha+\beta)-23)+4))}{32\pi^4\alpha\beta} \bigg\},$$

$$\rho_3^{\langle GG\rangle}(s)=\frac{\langle g_s^2GG\rangle}{2}\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\{(m^2(\alpha+\beta)-\alpha\beta s)\Big[\frac{(1-\alpha-\beta)^3(25\alpha^2\beta s-m^2(15\alpha^2+15\alpha\beta-4\alpha+24\beta))}{9216\pi^6\alpha^3\beta^2}$$
$$+\frac{(1-\alpha-\beta)(m^2(15\alpha^2\beta-3\alpha^2+15\alpha\beta^2-7\alpha\beta+2\alpha-12\beta)+\alpha^2(6-25\beta)\beta s)}{1536\pi^6\alpha^3\beta^2}$$
$$+\frac{(1-\alpha-\beta)(m^2(3\alpha^2+3\alpha\beta+4\alpha+6\beta-8)-\alpha^2(5\alpha+14\beta)\beta s)}{1536\pi^6\alpha^2\beta}+\frac{m^2(3\alpha+3\beta+5)-4\alpha\beta s}{768\pi^6\alpha\beta}\Big]$$
$$+\frac{m^2(\alpha+\beta-1)^2(\alpha\beta s(20\alpha+20\beta-47)-m^2(15\alpha^2+\alpha(30\beta-34)+15\beta^2-34\beta+4))}{1152\pi^6\alpha^3}\Big\},$$
$$\rho_3^{\langle\bar{q}Gq\rangle}(s)=\Big\{\int\!\!\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle(3m^2+\alpha\beta s)}{96\pi^4\alpha}$$
$$-\frac{m_q\langle\bar{s}\sigma\cdot Gs\rangle(m^2(72\alpha^2+117\alpha\beta+45\beta^2-56\alpha-56\beta+8)-3\alpha\beta s(32\alpha+20\beta-25))}{576\pi^4\alpha}\Big\}$$
$$-\frac{6m_q\langle\bar{q}\sigma\cdot Gq\rangle(20m^2+s)+\langle\bar{s}\sigma\cdot Gs\rangle(7s-52m^2)}{1152\pi^4}\sqrt{1-\frac{4m^2}{2}},$$
$$\rho_3^{\langle\bar{q}q\rangle^2}(s)=\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle(s+20m^2)}{36\pi^2}\sqrt{1-\frac{4m^2}{s}},$$
$$\Pi_3^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(s)=\frac{\langle\bar{q}q\rangle\langle\bar{s}\sigma\cdot Gs\rangle+\langle\bar{s}s\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{72\pi^2}\int_0^1dx\Big\{\frac{m^4(9-6x)}{M_B^2x^2(1-x)}-\frac{2m^2(3x^2-7x+3)}{x(1-x)}-\frac{3M_B^2(2x^2-3x+1)}{1-x}\Big\}e^{-\frac{m^2}{M_B^2(1-x)x}}.\tag{A39}$$

$$\rho_4^{\mathrm{pert}}(s)=\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\Big\{\frac{(1-\alpha-\beta)^3(m^2(\alpha+\beta)-\alpha\beta s)^3(m^2(3\alpha+3\beta+2)-7\alpha\beta s)}{384\pi^6\alpha^3\beta^3},$$
$$\rho_4^{\langle\bar{q}q\rangle}(s)=\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\Big\{\frac{m_q\langle\bar{q}_1q_1\rangle(m^2(\alpha+\beta)-\alpha\beta s)(5\alpha\beta s-3m^2(\alpha+\beta-2))}{16\pi^4\alpha\beta}$$
$$+\frac{m_q\langle\bar{s}s\rangle(m^2(\alpha+\beta)-\alpha\beta s)(m^2(-12\alpha^2-24\alpha\beta-12\beta^2+15\alpha+15\beta+2)+\alpha\beta s(20\alpha+20\beta-33))}{32\pi^4\alpha\beta}\Big\},$$
$$\rho_3^{\langle GG\rangle}(s)=\frac{\langle g_s^2GG\rangle}{2}\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\Big\{\frac{(1-\alpha-\beta)^3(m^2(\alpha+\beta)-\alpha\beta s)(5\alpha^2\beta s-3m^2(\alpha^2+\alpha\beta+2\beta))}{1152\pi^6\alpha^3\beta^2}$$
$$+\frac{(1-\alpha-\beta)^2(m^2(\alpha+\beta)-\alpha\beta s)(5\alpha^2\beta(6\beta+1)s-m^2(18\alpha^2\beta+3\alpha^2+18\alpha\beta^2+19\alpha\beta+8\beta^2-24\beta))}{3072\pi^6\alpha^3\beta^2}$$
$$-\frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)(m^2(-6\alpha^2+3\alpha-6\alpha\beta+5\beta-4)+\alpha(10\alpha-9)\beta s)}{768\pi^6\alpha^2\beta}$$
$$+\frac{(m^2\alpha+\beta)-\alpha\beta s)(m^2(7\alpha+7\beta-2)-13\alpha\beta s)}{1536\pi^6\alpha\beta}-\frac{(1-\alpha-\beta)^2m^2}{1152\pi^6\alpha^3}$$
$$\times[2(1-\alpha-\beta)(m^2(6\alpha+6\beta+1)-8\alpha\beta s)+3(m^2(6\alpha+6\beta-1)-9\alpha\beta s)]\Big\},$$
$$\rho_4^{\langle\bar{q}Gq\rangle}(s)=\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\Big\{\frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle(4\alpha\beta s-3m^2(\alpha+\beta-1))}{96\pi^4\alpha}$$
$$\times\frac{m_q\langle\bar{s}\sigma\cdot Gs\rangle(m^2(9\alpha^2-18\alpha\beta-27\beta^2+38\alpha+38\beta-2)-3\alpha\beta s(4\alpha-12\beta+19))}{576\pi^4\alpha}\Big\}$$
$$+\frac{m_q(12\langle\bar{q}\sigma\cdot Gq\rangle(2m^2+s)-\langle\bar{s}\sigma\cdot Gs\rangle(5s-8m^2))}{576\pi^4}\sqrt{1-\frac{4m^2}{s}},$$

$$\rho_4^{\langle\bar{q}q\rangle^2}(s) = \frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle(2m^2+s)}{9\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_4^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(s) = \frac{\langle\bar{q}q\rangle\langle\bar{s}\sigma\cdot Gs\rangle+\langle\bar{s}s\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{72\pi^2}\int_0^1 dx\left\{\frac{m^4(9-6x)}{M_B^2x^2(1-x)}+\frac{m^2(3x^2-4x+3)}{x(1-x)}+6M_B^2(1-x)\right\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \quad \text{(A40)}$$

$$\rho_5^{\rm pert}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)^3(m^2((\alpha+\beta)(\alpha+\beta+5)-4)-\alpha\beta s(\alpha+\beta+1))}{512\pi^6\alpha^3\beta^3},$$

$$\rho_5^{\langle\bar{q}q\rangle}(s) = 3m_q(\langle\bar{s}s\rangle-2\langle\bar{q}q\rangle)\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(\alpha+\beta-2)-3\alpha\beta s)}{32\pi^4\alpha\beta},$$

$$\rho_5^{\langle GG\rangle}(s) = \langle g_s^2GG\rangle\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta\{(m^2(\alpha+\beta)-\alpha\beta s)\left[-\frac{(\alpha^2+\alpha(8\beta-2)+(\beta-1)^2)}{6144\pi^6\alpha^2\beta^2}\right.$$
$$\left.+\frac{6\alpha^2\beta(m^2(\alpha+\beta-1)-2\alpha\beta s)-(\alpha-\beta-1)(m^2(\alpha^2+\alpha\beta+4\beta(\beta+3))-2\alpha^2\beta s)}{3072\pi^6\alpha^3}\right]$$
$$\left.+\frac{m^2(1-\alpha-\beta)^2(m^2(2\alpha+2\beta-1)-3\alpha\beta s)}{768\pi^6\alpha^3}\right\},$$

$$\rho_5^{\langle\bar{q}Gq\rangle}(s) = -\frac{m_q(\langle\bar{q}\sigma\cdot Gq\rangle+\langle\bar{s}\sigma\cdot Gs\rangle)(s+m^2)}{192\pi^4}\sqrt{1-\frac{4m^2}{s}}, \qquad \rho_5^{\langle\bar{q}q\rangle^2}(s) = \frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle(s+2m^2)}{18\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_5^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) = \frac{\langle\bar{q}q\rangle\langle\bar{s}\sigma\cdot Gs\rangle+\langle\bar{s}s\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{24\pi^2}\int_0^1 dx\left\{\frac{m^4}{M_B^2x^2(1-x)}+\frac{m^2(2-x)}{1-x}+M_B^2x\right\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \quad \text{(A41)}$$

$$\rho_6^{\rm pert}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)^3(m^2(7(\alpha+\beta)(\alpha+\beta+1)-8)-3\alpha\beta s(\alpha+\beta+1))}{768\pi^6\alpha^3\beta^3},$$

$$\rho_6^{\langle\bar{q}q\rangle}(s) = \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta(m^2(\alpha+\beta)-\alpha\beta s)\left\{\frac{m_q\langle\bar{q}q\rangle(2\alpha\beta s-m^2(\alpha+\beta-1))}{4\pi^4\alpha\beta}\right.$$
$$\left.+\frac{m_q\langle\bar{s}s\rangle(3m^2(\alpha^2+\alpha(2\beta-1)+\beta(\beta 01))+\alpha\beta s(-5\alpha-5\beta+1))}{16\pi^4\alpha\beta}\right\},$$

$$\rho_6^{\langle GG\rangle}(s) = \frac{\langle g_s^2GG\rangle}{2}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta\left\{\frac{(1-\alpha-\beta)^3(m^2(\alpha+\beta)-\alpha\beta s)(5\alpha^2\beta s-m^2(3\alpha^2+3\alpha\beta-4\alpha+16\beta))}{3072\pi^6\alpha^3\beta^2}\right.$$
$$-\frac{(1-\alpha-\beta)^2(m^2(\alpha+\beta)-\alpha\beta s)(m^2(\alpha^2+\alpha\beta-2\alpha+8\beta))}{512\pi^6\alpha^3\beta^2}$$
$$+\frac{m^2(1-\alpha-\beta)^2(m^2(3\alpha^2+6\alpha\beta+2\alpha+3\beta^2+2\beta-2)-\alpha\beta s(4\alpha+4\beta+5))}{576\pi^6\alpha^3}$$
$$+\frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)(m^2(3\alpha+3\beta-2)-5\alpha\beta s)}{1536\pi^6\alpha\beta}$$
$$\left.+\frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(\alpha+\beta-1)-2\alpha\beta s)}{768\pi^6\alpha\beta}\right\},$$

$$\rho_6^{\langle\bar{q}Gq\rangle}(s) = -\frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle s}{32\pi^4}\sqrt{1-\frac{4m^2}{s}}, \qquad \rho_6^{\langle\bar{q}q\rangle^2}(s) = \frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle s}{6\pi^2}\sqrt{1-\frac{4m^2}{s}},$$

$$\Pi_6^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2) = -\frac{\langle\bar{q}q\rangle\langle\bar{s}\sigma\cdot Gs\rangle+\langle\bar{s}s\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{12\pi^2}\int_0^1 dx\left\{\frac{m^4}{M_B^2x^2(1-x)}+\frac{m^2}{(1-x)x}+M_B^2\right\}e^{-\frac{m^2}{M_B^2(1-x)x}}. \quad \text{(A42)}$$

$$\rho_7^{\mathrm{pert}}(s)=\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\bigg\{\frac{(1-\alpha-\beta)^3(m^2(\alpha+\beta)-\alpha\beta s)(m^2(3\alpha+3\beta+1)-7\alpha\beta s)}{384\pi^6\alpha^3\beta^3}$$
$$+\frac{(1-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)(\alpha\beta s(\alpha+\beta-7)-m^2(\alpha^2+2\alpha\beta-3\alpha+\beta^3-3\beta-4))}{512\pi^6\alpha^3\beta^3}\bigg\},$$
$$\rho_7^{\langle\bar{q}q\rangle}(s)=\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta(m^2(\alpha+\beta)-\alpha\beta s)\bigg\{\frac{m_q\langle\bar{q}q\rangle(3m^2-\alpha\beta s)}{8\pi^4\alpha\beta}$$
$$\times\frac{m_q\langle\bar{s}s\rangle(m^2(-15\alpha^2+\alpha(19-30\beta)-15\beta^2+19\beta+4)+\alpha\beta s(25\alpha+25\beta-37))}{32\pi^4\alpha\beta}\bigg\},$$
$$\rho_7^{\langle GG\rangle}(s)=\frac{\langle g_s^2GG\rangle}{2}\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\bigg\{\frac{m^2(1-\alpha-\beta)^3(m^2(15\alpha+15\beta+1)-20\alpha\beta s)}{1152\pi^6\alpha^3}$$
$$+\frac{(1-\alpha-\beta)^3(m^2(\alpha+\beta)-\alpha\beta s)(25\alpha^2\beta s-m^2(15\alpha^2+15\alpha\beta+4\alpha-24\beta))}{9126\pi^6\alpha^3\beta^2}$$
$$+\frac{(1-\alpha-\beta)^2(m^2(\alpha+\beta)^2-\alpha\beta s)(m^2(15\alpha^2\beta-3\alpha^2+15\alpha\beta^2+\alpha\beta-2\alpha+12\beta)+\alpha^2(6-25\beta)\beta s)}{1536\pi^6\alpha^3\beta^2}$$
$$+\frac{(\alpha+\beta-1)(m^2(\alpha+\beta)-\alpha\beta s)(\alpha(25\alpha+14)\beta s-m^2(15\alpha^2+\alpha(15\beta+8)+6\beta+8))}{1536\pi^6\alpha^2\beta}$$
$$+\frac{m^2(\alpha+\beta-1)^2(m^2(6\alpha+6\beta+1)-9\alpha\beta s)}{384\pi^6\alpha^3}+\frac{(m^2(\alpha+\beta)-\alpha\beta s)(m^2(3\alpha+3\beta-5)-4\alpha\beta s)}{768\pi^6\alpha\beta}\bigg\},$$
$$\rho_7^{\langle\bar{q}Gq\rangle}(s)=\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\bigg\{\frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle(3m^2-\alpha\beta s)}{96\pi^4\alpha}$$
$$+\frac{m_q\langle\bar{s}\sigma\cdot Gs\rangle m^2(-72\alpha^2-13\alpha(9\beta-4)-45\beta^2+52\beta+8)+3\alpha\beta s(32\alpha+20\beta-25)}{576\pi^4\alpha}\bigg\}$$
$$-\frac{m_q\langle\bar{q}\sigma\cdot Gq\rangle(96m^2-6s)+m_q\langle\bar{s}\sigma\cdot Gs\rangle(8m^2+8s)}{1152\pi^4}\sqrt{1-\frac{4m^2}{s}},$$
$$\rho_7^{\langle\bar{q}q\rangle^2}(s)=-\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle(s-16m^2)}{36\pi^2}\sqrt{1-\frac{4m^2}{s}},$$
$$\Pi_7^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(M_B^2)=\frac{\langle\bar{q}q\rangle\langle\bar{s}\sigma\cdot Gs\rangle+\langle\bar{s}s\rangle\langle\bar{q}\sigma\cdot Gq\rangle}{72\pi^2}\int_0^1dx\bigg\{\frac{m^4(12x-9)}{M_B^2x^2(1-x)}-\frac{2m^2(3x-4)}{(1-x)}-\frac{3M_B^2(2x^2-3x+1)}{1-x}\bigg\}e^{-\frac{m^2}{M_B^2(1-x)x}}.\tag{A43}$$

$$\rho_8^{\mathrm{pert}}(s)=\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\bigg\{\frac{(1-\alpha-\beta)(7-\alpha-\beta)(m^2(\alpha+\beta)-\alpha\beta s)}{256\pi^6\alpha^3\beta^3}$$
$$+\frac{m^2(-3\alpha^2+\alpha(5-6\beta)-3\beta^2+5\beta+1)+7\alpha\beta s(\alpha+\beta-1)}{192\pi^6\alpha^3\beta^3}(1-\alpha-\beta)^2(m^2(\alpha+\beta)-\alpha\beta s)\bigg\},$$
$$\rho_8^{\langle\bar{q}q\rangle}(s)=\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta(m^2(\alpha+\beta)-\alpha\beta s)\bigg\{\frac{m_q\langle\bar{q}q\rangle(3m^2(\alpha+\beta+2)-5\alpha\beta s)}{8\pi^4\alpha\beta}$$
$$+\frac{m_q\langle\bar{s}s\rangle(\alpha\beta s(20\alpha+20\beta-33)-m^2(12\alpha^2+\alpha(24\beta-23)+12\beta^2-23\beta+2))}{16\pi^4\alpha\beta}\bigg\},$$

$$\rho_8^{\langle GG\rangle}(s)=\langle g_s^2GG\rangle\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\Big\{-\frac{(\alpha^2+\alpha(20\beta-2)+21\beta^2-22\beta+1)(\alpha\beta s-m^2(\alpha+\beta))^2}{6144\pi^6\alpha^2\beta^2}$$
$$+(m^2(\alpha+\beta)-\alpha\beta s)\Big[\frac{(1-\alpha-\beta)^3(5\alpha\beta s-3m^2(\alpha+\beta))}{2304\pi^6\alpha^2\beta^2}$$
$$-\frac{(\alpha+\beta-1)^2(m^2(\alpha^2(1-45\beta)-5\alpha\beta(9\beta-5)-8\beta(3\beta+1))+\alpha^2\beta(75\beta-2)s)}{3072\pi^6\alpha^3\beta^2}$$
$$-\frac{m^2(6\alpha^3+3\alpha^2(4\beta+3)+\alpha(6\beta^2+29\beta+1)+20(\beta^2-1))-2\alpha\beta s(5\alpha^2+\alpha(5\beta+12)+20(\beta-1))}{1536\pi^6\alpha^2\beta}\Big]$$
$$+\frac{m^2(\alpha+\beta-1)^2(m^2(-12\alpha^2-8\alpha(3\beta-4)-12\beta^2+32\beta+1)+\alpha\beta s(16\alpha+16\beta-43))}{1152\pi^6\alpha^3}\Big\},$$
$$\rho_8^{\langle\bar qGq\rangle}(s)=\int_{\alpha_{\min}}^{\alpha_{\max}}d\alpha\int_{\beta_{\min}}^{\beta_{\max}}d\beta\Big\{\frac{5m_q\langle\bar q\sigma\cdot Gq\rangle(3m^2(\alpha+\beta-1)-4\alpha\beta s)}{96\pi^4\alpha}$$
$$+\frac{m_q\langle\bar s\sigma\cdot Gs\rangle(m^2(-171\alpha^2+\alpha(230-306\beta)+5(-27\beta^2+46\beta+2))+3\alpha\beta s(76\alpha+60\beta-95))}{576\pi^4\alpha}\Big\}$$
$$-\frac{24m_q\langle\bar q\sigma\cdot Gq\rangle(7m^2-s)-m_q\langle\bar s\sigma\cdot Gs\rangle(14m^2-5s)}{576\pi^4}\sqrt{1-\frac{4m^2}{s}},$$
$$\rho_8^{\langle\bar qq\rangle^2}(s)=\frac{2\langle\bar qq\rangle\langle\bar ss\rangle(7m^2-s)}{9\pi^2}\sqrt{1-\frac{4m^2}{s}},$$
$$\Pi_8^{\langle\bar qq\rangle\langle\bar qGq\rangle}(M_B^2)=\frac{\langle\bar qq\rangle\langle\bar s\sigma\cdot Gs\rangle+\langle\bar ss\rangle\langle\bar q\sigma\cdot Gq\rangle}{72\pi^2}\int_0^1dx\Big\{\frac{6m^4(3-4x)}{M_B^2x^2(1-x)}-\frac{m^2(6x^2-32x+27)}{x(1-x)}-\frac{3M_B^2(7x^2-11x+4)}{1-x}\Big\}e^{-\frac{m^2}{M_B^2(1-x)x}},\tag{A44}$$

where m is the heavy quark Q mass m_Q. Some other notations are

$$\alpha_{\max}=\frac{1+\sqrt{1-4m^2/s}}{2}\qquad\alpha_{\min}=\frac{1-\sqrt{1-4m^2/s}}{2}\qquad\beta_{\max}=1-\alpha\qquad\beta_{\min}=\frac{\alpha m^2}{\alpha s-m^2}.\tag{A45}$$

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