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Radiative corrections to the leptonic Dirac *CP*-violating phase

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Since the smallest leptonic mixing angle θ_{13} has been measured to be relatively large, it is quite promising to constrain or determine the leptonic Dirac *CP*-violating phase δ in future neutrino oscillation experiments. Given some typical values of $\delta = \pi/2$, π, and $3\pi/2$ at the low energy scale, as well as current experimental results of the other neutrino parameters, we perform a systematic study of radiative corrections to δ by using the one-loop renormalization group equations in the minimal supersymmetric standard model and the universal extra-dimensional model. It turns out that δ is rather stable against radiative corrections in both models, except for the minimal supersymmetric standard model with a very large value of $\tan\beta$. Both cases of Majorana and Dirac neutrinos are discussed. In addition, we use the preliminary indication of $\delta = (1.08^{+0.28}_{-0.31})\pi$ or $\delta = (1.67^{+0.37}_{-0.77})\pi$ from the latest global-fit analyses of data from neutrino oscillation experiments to illustrate how it will be modified by radiative corrections.

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I. INTRODUCTION

In the last two decades, our knowledge on neutrinos has been greatly improved by a number of elegant neutrino oscillation experiments [1]. Now, we are convinced that neutrinos are massive, and they can transform from one flavor to another when propagating in vacuum or in matter. The lepton flavor mixing phenomenon can be described by a 3×3 unitary matrix V, namely the leptonic mixing matrix, which is conventionally parametrized through three mixing angles θ_{12}, θ_{13} and θ_{23}, as well as three *CP*-violating phases δ, ρ and σ, viz.,

$$V = U \cdot P \equiv \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot P, \tag{1}$$

where $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$ for $ij = 12$, 13, 23. Note that $P = \mathrm{diag}(e^{i\rho}, e^{i\sigma}, 1)$ is a diagonal matrix with ρ and σ being two Majorana-type *CP*-violating phases if neutrinos are Majorana particles, while $P = \mathbf{1}$ if neutrinos are Dirac particles. Current experimental data indicate that the three leptonic mixing angles are $\theta_{12} \approx 34°$, $\theta_{13} \approx 9°$ and $\theta_{23} \approx 40°$. Two independent neutrino mass-squared differences are found to be $\Delta m^2_{21} \equiv m^2_2 - m^2_1 \approx 7.5 \times 10^{-5}\ \mathrm{eV}^2$ and $|\Delta m^2_{31}| \equiv |m^2_3 - m^2_1| \approx 2.5 \times 10^{-3}\ \mathrm{eV}^2$. The latest global-fit results of neutrino parameters are shown in Table I. However, we are still unclear whether the neutrino mass ordering is normal (i.e., $\Delta m^2_{31} > 0$) or inverted (i.e., $\Delta m^2_{31} < 0$), and the leptonic Dirac *CP*-violating phase δ remains experimentally unconstrained.

The recent results from Daya Bay [5] and RENO [6] reactor neutrino experiments have established that $\theta_{13} \approx 9°$, which is rather large. Hence, it is quite promising to determine the leptonic Dirac *CP*-violating phase δ by comparing the oscillation probabilities of neutrinos and antineutrinos in future long-baseline neutrino oscillation experiments [7]. In addition, the km^3-scale neutrino telescopes (e.g., IceCube and KM3NeT) could provide us with useful and complementary information about leptonic *CP* violation by precisely measuring the flavor composition of ultrahigh-energy astrophysical neutrinos [8]. If the Deep Core of the IceCube detector is made denser to lower the energy threshold down to a few GeV, such as the proposal PINGU [9], a large amount of atmospheric neutrino events can be collected and used to determine the neutrino mass hierarchy and perhaps the leptonic *CP*-violating phase [10]. On the other hand, a lot of neutrino mass models based on discrete flavor symmetries or phenomenological assumptions have recently been proposed to describe the observed leptonic mixing pattern, in particular a relatively large θ_{13}. Interestingly, the leptonic *CP*-violating phase δ has been predicted in some models to be rather large (e.g., $\delta > \pi/3$) or even maximal (i.e., $\delta = \pi/2$) [11,12]. In other models, leptonic *CP* violation is shown to be absent, namely $\delta = 0$ or π [13]. It is worthwhile to mention that the latest global-fit analyses of neutrino

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TABLE I. The best-fit values and 1σ ranges of the neutrino parameters from the latest global-fit analyses of neutrino oscillation experiments, where the normal neutrino mass hierarchy is assumed.

Parameter	Ref. [2]	Ref. [3]	Ref. [4]
$\sin^2\theta_{12}$	0.307 0.291–0.325	0.300 0.287–0.313	0.320 0.303–0.336
$\sin^2\theta_{13}$	0.0241 0.0216–0.0266	0.0230 0.0207–0.0253	0.0246 0.0218–0.0275
$\sin^2\theta_{23}$	0.386 0.365–0.410	0.410 0.385–0.447	0.427 0.400–0.461
$\Delta m^2_{21}/10^{-5}$ eV^2	7.54 7.32–7.80	7.50 7.32–7.69	7.62 7.43–7.81
$\Delta m^2_{31}/10^{-3}$ eV^2	2.51 2.41–2.57	2.47 2.40–2.54	2.55 2.46–2.61
δ/π	1.08 0.77–1.36	1.67 0.90–2.03	0.8 0–2.0

oscillation experiments yield $\delta = (1.08^{+0.28}_{-0.31})\pi$ [2] and $\delta = (1.67^{+0.37}_{-0.77})\pi$ [3], although the 1σ errors are still quite large.[1] Therefore, we have already obtained some preliminary information on the leptonic CP-violating phase δ from the global-fit analyses.

In this work, we are concerned with how the theoretical predictions or the observed value of δ will be modified by the radiative corrections when running from a low-energy scale to a superhigh-energy scale. This question *does* make sense if we believe that there exists at some superhigh-energy scale a unified theory for flavor mixing and CP violation in both quark and lepton sectors. Once the leptonic CP-violating phase δ is measured in future neutrino oscillation experiments, the renormalization group (RG) evolution of δ will tell us how large or small it will be at a given superhigh-energy scale. As a matter of fact, the running of leptonic mixing parameters has been extensively discussed in the literature [14,15], and more recently in Ref. [16], where the authors concentrate on the newly measured θ_{13}. Different from the previous works, we focus on δ and perform a systematic study of its running behavior in the minimal supersymmetric standard model (MSSM) and in the universal extra-dimensional model (UEDM). The motivation for such a study is twofold: (1) The leptonic CP-violating phase δ is the last fundamental parameter (except for the neutrino mass hierarchy) to be measured in the future neutrino oscillation experiments, and now both the theoretical models and the global-fit analysis can provide us with preferred values of δ at the low-energy scale. (2) The models with supersymmetry or extra spatial dimensions are the most natural extensions of the standard model (SM), which can solve the gauge hierarchy problem and offer good candidates for the dark matter.

In lack of a complete theory for neutrino mass generation, we implement the dimension-five Weinberg operator to account for tiny Majorana neutrino masses [17]. The RG running of δ in the case of Dirac neutrinos will be considered as well for comparison and completeness.

The remaining part of the present paper is organized as follows. In Sec. II, we set up the basic framework for the RG running of leptonic mixing parameters in the case of Majorana neutrinos. The renormalization group equation (RGE) of δ is derived analytically, and solved numerically. Section III is devoted to the RG running of δ in the case of Dirac neutrinos in the MSSM. We summarize our conclusions in Sec. IV. The complete set of RGEs in the SM, MSSM, and UEDM for Majorana neutrinos are collected in Appendix A, while those in the SM and MSSM for Dirac neutrinos in Appendix B.

II. RUNNING OF CP-VIOLATING PHASE: MAJORANA NEUTRINOS

First of all, we derive the RGE for the leptonic CP-violating phase δ, assuming that neutrinos are Majorana particles. Without loss of generality, we introduce the dimension-five Weinberg operator responsible for neutrino masses [17]:

$$-\mathcal{L}_\nu = \frac{1}{2}(\overline{\ell}H)\cdot\kappa\cdot(H^T\ell^C) + \text{H.c.}, \quad (2)$$

where ℓ and H stand for the lepton and Higgs doublet fields, respectively, and κ is a symmetric and complex matrix of the inverse mass dimension. After electroweak symmetry breaking, the mass matrix of three light Majorana neutrinos is given by $M_\nu = \kappa v^2$ with $v \approx 174$ GeV being the vacuum expectation value of the SM Higgs field, or by $M_\nu = \kappa(v\sin\beta)^2$ with $\tan\beta$ being the ratio of the vacuum expectation values of two Higgs

[1] The best-fit value is found to be $\delta = 0.8\pi$ for the normal mass hierarchy and $\delta = -0.03\pi$ for the inverted mass hierarchy by another global-fit group [4]. However, there is no constraint on δ within the 1σ range.

doublets in the MSSM. Note that we are working within an effective theory, and consider the running of neutrino mixing parameters below the cutoff scale Λ where new physics takes effects.

At one-loop level, the evolution of κ is governed by [14,15]

$$16\pi^2 \frac{\mathrm{d}\kappa}{\mathrm{d}t} = \alpha_\kappa + C_\kappa[(Y_l Y_l^\dagger)\kappa + \kappa(Y_l Y_l^\dagger)^T], \tag{3}$$

where $t \equiv \ln(\mu/\Lambda_{\rm EW})$ with μ being an arbitrary renormalization scale between the electroweak scale $\Lambda_{\rm EW} \approx 100$ GeV and a cutoff scale where new physics comes into play, and Y_l is the Yukawa coupling matrix of the charged leptons. The coefficients α_κ and C_κ are flavor universal, and have been explicitly given in Appendix A for the SM, the MSSM, and the UEDM. It is worth stressing that Eq. (3) takes on the same form in all the models under consideration. However, the coefficients in the RGEs may differ. We will distinguish them by adding the corresponding superscripts to these coefficients, as shown in Appendix A.

A. Analytical results

Since the RGEs of neutrino mass matrix $M_\nu = \kappa v^2$ in the SM and UEDM, or $M_\nu = \kappa(v\sin\beta)^2$ in the MSSM, are given by the same formula in Eq. (3), the evolution of neutrino mass eigenvalues and leptonic mixing parameters can be figured out in the same way. In flavor basis, where the Yukawa coupling matrix of the charged leptons is diagonal, namely $Y_l = D_l \equiv \mathrm{diag}(y_e, y_\mu, y_\tau)$, κ can be diagonalized by the leptonic mixing matrix V, namely $V^\dagger \kappa V^* = \hat{\kappa} \equiv \mathrm{diag}(\kappa_1, \kappa_2, \kappa_3)$. Generally speaking, an arbitrary 3×3 unitary matrix V' can be factorized as $V' = QUP$, where $Q = \mathrm{diag}(e^{\mathrm{i}\phi_e}, e^{\mathrm{i}\phi_\mu}, e^{\mathrm{i}\phi_\tau})$ and $P = \mathrm{diag}(e^{\mathrm{i}\rho}, e^{\mathrm{i}\sigma}, 1)$ are pure phase matrices, while the unitary matrix U consists of three mixing angles θ_{12}, θ_{13}, θ_{23} and the Dirac CP-violating phase δ [cf. Eq. (1)]. Although the phases ϕ_α (for $\alpha = e, \mu, \tau$) are unphysical and can be removed by rephasing the charged-lepton fields, we will keep them in the derivation of the RGEs for neutrino masses and leptonic mixing parameters.

Since $y_e^2 \ll y_\mu^2 \ll y_\tau^2$, we take into account the dominant contribution from the tau-lepton Yukawa coupling to the RGE of κ. Following Ref. [18], one obtains

$$16\pi^2 \frac{\mathrm{d}\kappa_i}{\mathrm{d}t} = \kappa_i(\alpha_\kappa + 2C_\kappa y_\tau^2 |U_{\tau i}|^2), \tag{4}$$

where α_κ and C_κ should bear the corresponding superscripts when Eq. (4) is applied to a specific model. Given $m_i = \kappa_i v^2$ (for $i = 1, 2, 3$), we observe that Eq. (4) determines the evolution of absolute neutrino masses. Moreover, it is straightforward to find that $U_{\alpha i}$, ρ, σ, and ϕ_α (for $\alpha = e, \mu, \tau$ and $i = 1, 2, 3$) have to fulfill the following equations:

$$\begin{aligned} \mathrm{Im}[(U^\dagger \dot{U})_{11}] + \sum_\alpha |U_{\alpha 1}|^2 \dot{\phi}_\alpha + \dot{\rho} &= 0, \\ \mathrm{Im}[(U^\dagger \dot{U})_{22}] + \sum_\alpha |U_{\alpha 2}|^2 \dot{\phi}_\alpha + \dot{\sigma} &= 0, \\ \mathrm{Im}[(U^\dagger \dot{U})_{33}] + \sum_\alpha |U_{\alpha 3}|^2 \dot{\phi}_\alpha &= 0, \end{aligned} \tag{5}$$

and

$$\begin{aligned} \mathrm{Re}[(U^\dagger \dot{U})_{12}] - \sum_\alpha I_{12}^\alpha \dot{\phi}_\alpha &= -\frac{C_\kappa y_\tau^2}{32\pi^2}\left\{\hat{\zeta}_{12}[s_{2(\rho-\sigma)} I_{12}^\tau + c_{2(\rho-\sigma)} \mathcal{R}_{12}^\tau] + \tilde{\zeta}_{12}\mathcal{R}_{12}^\tau\right\}, \\ \mathrm{Im}[(U^\dagger \dot{U})_{12}] + \sum_\alpha \mathcal{R}_{12}^\alpha \dot{\phi}_\alpha &= -\frac{C_\kappa y_\tau^2}{32\pi^2}\left\{\hat{\zeta}_{12}[s_{2(\rho-\sigma)} \mathcal{R}_{12}^\tau - c_{2(\rho-\sigma)} I_{12}^\tau] + \tilde{\zeta}_{12} I_{12}^\tau\right\}, \\ \mathrm{Re}[(U^\dagger \dot{U})_{13}] - \sum_\alpha I_{13}^\alpha \dot{\phi}_\alpha &= -\frac{C_\kappa y_\tau^2}{32\pi^2}\left\{\hat{\zeta}_{13}[s_{2\rho} I_{13}^\tau + c_{2\rho} \mathcal{R}_{13}^\tau] + \tilde{\zeta}_{13}\mathcal{R}_{13}^\tau\right\}, \\ \mathrm{Im}[(U^\dagger \dot{U})_{13}] + \sum_\alpha \mathcal{R}_{13}^\alpha \dot{\phi}_\alpha &= -\frac{C_\kappa y_\tau^2}{32\pi^2}\left\{\hat{\zeta}_{12}[s_{2\rho} \mathcal{R}_{13}^\tau - c_{2\rho} I_{13}^\tau] + \tilde{\zeta}_{13} I_{13}^\tau\right\}, \\ \mathrm{Re}[(U^\dagger \dot{U})_{23}] - \sum_\alpha I_{23}^\alpha \dot{\phi}_\alpha &= -\frac{C_\kappa y_\tau^2}{32\pi^2}\left\{\hat{\zeta}_{23}[s_{2\sigma} I_{23}^\tau + c_{2\sigma} \mathcal{R}_{23}^\tau] + \tilde{\zeta}_{23}\mathcal{R}_{23}^\tau\right\}, \\ \mathrm{Im}[(U^\dagger \dot{U})_{23}] + \sum_\alpha \mathcal{R}_{23}^\alpha \dot{\phi}_\alpha &= -\frac{C_\kappa y_\tau^2}{32\pi^2}\left\{\hat{\zeta}_{23}[s_{2\sigma} \mathcal{R}_{23}^\tau - c_{2\sigma} I_{23}^\tau] + \tilde{\zeta}_{23} I_{23}^\tau\right\}, \end{aligned} \tag{6}$$

where $\hat{\zeta}_{ij} \equiv 4\kappa_i\kappa_j/(\kappa_i^2 - \kappa_j^2)$ and $\tilde{\zeta}_{ij} \equiv 2(\kappa_i^2 + \kappa_j^2)/(\kappa_i^2 - \kappa_j^2)$ have been defined, and the overdot refers to the derivative with respect to the running parameter t. In addition, $\mathcal{R}_{ij}^\alpha \equiv \mathrm{Re}(U_{\alpha i}^* U_{\alpha j})$ and $I_{ij}^\alpha \equiv \mathrm{Im}(U_{\alpha i}^* U_{\alpha j})$. Given the standard parametrization of U in Eq. (1), the matrix elements of $U^\dagger \dot{U}$ are shown in Table II, while the coefficients $\mathcal{R}_{ij}^\alpha$ and I_{ij}^α are given in Table III. Note that Eqs. (5) and (6) form an array of differential equations linear in $\{\dot{\theta}_{12}, \dot{\theta}_{13}, \dot{\theta}_{23}, \dot{\delta}, \dot{\rho}, \dot{\sigma}, \dot{\phi}_e, \dot{\phi}_\mu, \dot{\phi}_\tau\}$, which can be explicitly solved. As a result, the RGE of δ can be approximately written as

TABLE II. Explicit expressions of $\mathrm{Re}[(U^\dagger \dot{U})_{ij}]$ and $\mathrm{Im}[(U^\dagger \dot{U})_{ij}]$ for $i \le j$ in the standard parametrization of leptonic mixing matrix.

ij	$\mathrm{Re}[(U^\dagger \dot{U})_{ij}]$	$\mathrm{Im}[(U^\dagger \dot{U})_{ij}]$
11	0	$+2s_{12}c_{12}s_{13}s_\delta \dot{\theta}_{23} + c_{12}^2 s_{13}^2 \dot{\delta}$
22	0	$-2s_{12}c_{12}s_{13}s_\delta \dot{\theta}_{23} + s_{12}^2 s_{13}^2 \dot{\delta}$
33	0	$-s_{13}^2 \dot{\delta}$
12	$\dot{\theta}_{12} + s_{13}s_\delta \dot{\theta}_{23}$	$-(c_{12}^2 - s_{12}^2) s_{13} s_\delta \dot{\theta}_{23} + s_{12}c_{12}s_{13}^2 \dot{\delta}$
13	$-s_{12}c_{13}\dot{\theta}_{23} + c_{12}c_\delta \dot{\theta}_{13} - c_{12}s_{13}c_{13}s_\delta \dot{\delta}$	$-c_{12}s_\delta \dot{\theta}_{13} - c_{12}s_{13}c_{13}c_\delta \dot{\delta}$
23	$+c_{12}c_{13}\dot{\theta}_{23} + s_{12}c_\delta \dot{\theta}_{13} - s_{12}s_{13}c_{13}s_\delta \dot{\delta}$	$-s_{12}s_\delta \dot{\theta}_{13} - s_{12}s_{13}c_{13}c_\delta \dot{\delta}$

$$\dot{\delta} \approx \frac{C_\kappa y_\tau^2}{32\pi^2}\Big\{\frac{s_{12}c_{12}s_{23}c_{23}}{s_{13}}[s_\delta(\tilde{\zeta}_{32} - \tilde{\zeta}_{31}) + (s_{(\delta+2\sigma)}\hat{\zeta}_{32} - s_{(\delta+2\rho)}\hat{\zeta}_{31})] - \hat{\zeta}_{21}s_{23}^2 s_{2(\rho-\sigma)}$$
$$- (c_{23}^2 - s_{23}^2)(s_{2\rho}s_{12}^2\hat{\zeta}_{31} + s_{2\sigma}c_{12}^2\hat{\zeta}_{32}) + c_{23}^2(s_{2(\delta+\rho)}c_{12}^2\hat{\zeta}_{31} + s_{2(\delta+\sigma)}s_{12}^2\hat{\zeta}_{32})$$
$$- \frac{s_{23}c_{23}s_{13}}{s_{12}c_{12}}[\tilde{\zeta}_{21}s_\delta - \hat{\zeta}_{21}(s_{(\delta+2\rho-2\sigma)}c_{12}^2 + s_{(\delta-2\rho+2\sigma)}s_{12}^2)] + s_{13}^2 c_{23}^2 s_{2(\rho-\sigma)}\hat{\zeta}_{21}\Big\}. \quad (7)$$

Since the last two terms in the third line of Eq. (7) are proportional to s_{13} and s_{13}^2, we have neglected the terms further suppressed by $\mathcal{O}(\Delta m_{21}^2/|\Delta m_{31}^2|)$. If neutrino masses are nearly degenerate $m_i^2 \gg |\Delta m_{31}^2| \gg \Delta m_{21}^2$, which will always be assumed in the following, we have $\hat{\zeta}_{ij} \approx \tilde{\zeta}_{ij} \approx 4m_i^2/(m_i^2 - m_j^2)$ and $\hat{\zeta}_{21} \gg |\hat{\zeta}_{32}|, |\hat{\zeta}_{31}| \gg 1$, and thus, the RG evolution of δ could be significant. To next-to-leading order, Eq. (7) approximates to

$$\dot{\delta} \approx -\frac{C_\kappa y_\tau^2}{8\pi^2}\frac{m_1^2}{\Delta m_{21}^2}\Big\{s_{23}^2 s_{2(\rho-\sigma)} + \frac{2s_{23}c_{23}}{s_{12}c_{12}s_{13}} \times \Big[s_{13}^2 c_{(\delta+\rho-\sigma)} + \frac{\Delta m_{21}^2}{\Delta m_{31}^2}s_{12}^2 c_{12}^2 c_{(\delta+\rho+\sigma)}s_{(\rho-\sigma)}\Big]\Big\}, \quad (8)$$

where we have taken m_1 as the absolute neutrino mass and ignored the difference between Δm_{31}^2 and Δm_{32}^2. Some comments are in order:

(i) In general, the evolution of δ is dominated by the leading-order term $-\hat{\zeta}_{21}s_{23}^2 s_{2(\rho-\sigma)}$ on the right-hand side of Eq. (7). At higher order, if the terms suppressed by $|\hat{\zeta}_{31}|/\hat{\zeta}_{21} = \Delta m_{21}^2/|\Delta m_{31}^2| \approx 1/30$ are taken into account, then those by $s_{13}^2 \approx 1/40$ should also be kept for consistency, since they are of the same order of magnitude, as we have done in Eq. (8). The relative error in Eq. (7) is at the level of $s_{13}|\hat{\zeta}_{31}|/\hat{\zeta}_{21} \approx 0.5\%$, given the best-fit values of θ_{13} and neutrino mass-squared differences.

(ii) It is evident from Eq. (7) that the evolution of δ is entangled with that of three mixing angles and two Majorana CP-violating phases. In particular, it depends crucially on the Majorana phases ρ and σ. It has been found that the Dirac CP-violating phase δ can be radiatively generated from ρ and σ, even if the initial value of δ is vanishing [19]. On the other hand, the RG evolution of δ becomes negligible when $\rho \approx \sigma$, while the mixing angle θ_{12} is quite sensitive to the RG effect in this case.

(iii) The RGEs of δ in the SM, the MSSM, and the UEDM are given by the same formula in Eq. (7), but with different values of the coefficient C_κ. We have $C_\kappa^{\mathrm{SM}} = -3/2$ in the SM, while $C_\kappa^{\mathrm{MSSM}} = 1$ in the MSSM and $C_\kappa^{\mathrm{UEDM}} = -3(1+s)/2$ in the UEDM, respectively. Therefore, given the same

TABLE III. The coefficients $\mathcal{R}_{ij}^\alpha$ and $\mathcal{I}_{ij}^\alpha$ for $\alpha = e, \mu, \tau$ and $ij = 12, 13, 23$ in the standard parametrization of leptonic mixing matrix.

$\mathcal{R}_{ij}^\alpha$	12	13	23
e	$s_{12}c_{12}c_{13}^2$	$c_{12}s_{13}c_{13}$	$s_{12}s_{13}c_{13}$
μ	$s_{12}c_{12}(s_{23}^2 s_{13}^2 - c_{23}^2) - (c_{12}^2 - s_{12}^2)s_{23}c_{23}s_{13}c_\delta$	$-(s_{12}c_{23} + c_{12}s_{23}s_{13}c_\delta)s_{23}c_{13}$	$+(c_{12}c_{23} - s_{12}s_{23}s_{13}c_\delta)s_{23}c_{13}$
τ	$s_{12}c_{12}(c_{23}^2 s_{13}^2 - s_{23}^2) + (c_{12}^2 - s_{12}^2)s_{23}c_{23}s_{13}c_\delta$	$+(s_{12}s_{23} - c_{12}c_{23}s_{13}c_\delta)c_{23}c_{13}$	$-(c_{12}s_{23} + s_{12}c_{23}s_{13}c_\delta)c_{23}c_{13}$

$\mathcal{I}_{ij}^\alpha$	12	13	23
e	0	0	0
μ	$+s_{23}c_{23}s_{13}s_\delta$	$c_{12}s_{23}^2 s_{13}c_{13}s_\delta$	$s_{12}s_{23}^2 s_{13}c_{13}s_\delta$
τ	$-s_{23}c_{23}s_{13}s_\delta$	$c_{12}c_{23}^2 s_{13}c_{13}s_\delta$	$s_{12}c_{23}^2 s_{13}c_{13}s_\delta$

Majorana CP-violating phases and leptonic mixing angles, the evolution of δ in the MSSM will be in the direction opposite to that in the SM and the UEDM.

Finally, we observe from Eq. (5) that the identity $\dot{\phi}_e + \dot{\phi}_\mu + \dot{\phi}_\tau + \dot{\rho} + \dot{\sigma} = 0$ holds in the standard parametrization of U. The proof is as follows. Given a general nonsingular matrix X, whose elements are functions of the running parameter t, one can prove that $\mathrm{d}[\det(X)]/\mathrm{d}t = \det(X) \cdot \mathrm{tr}[X^{-1}(\mathrm{d}X/\mathrm{d}t)]$. If we take X to be a unitary matrix U with $\det(U) = 1$ and $U^{-1} = U^\dagger$, then $\mathrm{tr}(U^\dagger \dot{U}) = 0$ can be obtained. This observation together with Eq. (5) leads to the identity $\dot{\phi}_e + \dot{\phi}_\mu + \dot{\phi}_\tau + \dot{\rho} + \dot{\sigma} = 0$. However, this identity depends on the specific parametrization of U. For instance, if $\det(U) = e^{-\mathrm{i}\phi}$ with ϕ being the Dirac CP-violating phase, then we have $\dot{\phi} = \dot{\phi}_e + \dot{\phi}_\mu + \dot{\phi}_\tau + \dot{\rho} + \dot{\sigma}$, as shown in Ref. [18].

B. Numerical results

We proceed in this subsection with the numerical solution to the RGE of the leptonic Dirac CP-violating phase δ. Since the evolution of δ in the SM is negligible even in the case of a nearly-degenerate neutrino mass spectrum, we consider only the MSSM and the UEDM. Note that no approximations to the RGE of δ will be made in our numerical calculations. Our numerical results are shown in Fig. 1, and the main points are summarized as follows.

In the MSSM, we have taken two typical values of $\tan\beta = 10$ and $\tan\beta = 30$ for illustration. In both cases, the absolute neutrino mass $m_1 = 0.1$ eV is assumed, which is consistent with the cosmological bound $m_1 + m_2 + m_3 < 1.3$ eV (95% C.L.) from the WMAP Collaboration [20]. For the initial values of δ at the electroweak scale, we have chosen $\delta = \pi/2$, π, and $3\pi/2$ as typical examples. Since the tau-lepton Yukawa coupling is given by $y_\tau^2 = m_\tau^2(1 + \tan^2\beta)/v^2$ in the MSSM, the evolution of δ should be significantly enhanced for a large value of $\tan\beta$, as shown in the upper plots of Fig. 1. For $\tan\beta = 30$, the RG running of δ is quite significant. In particular, even if $\delta = \pi$ is found at the low-energy scale, namely, there is no CP-violating effect in neutrino oscillation experiments, the maximal CP-violating phase $\delta = \pi/2$ or $3\pi/2$ can be achieved at the cutoff scale $\Lambda = 10^{14}$ GeV. In other words, one can change from the scenario with a zero CP-violating phase to that with a maximal CP-violating phase, or vice versa. For $\tan\beta = 10$, the radiative correction to δ is at most 10% even at $\Lambda = 10^{14}$ GeV.

In the UEDM, we have input two different values of the absolute neutrino mass $m_1 = 0.1$ eV and $m_1 = 0.5$ eV. As shown in the lower plots of Fig. 1, δ is rather stable against radiative corrections for $m_1 = 0.1$ eV. Even for

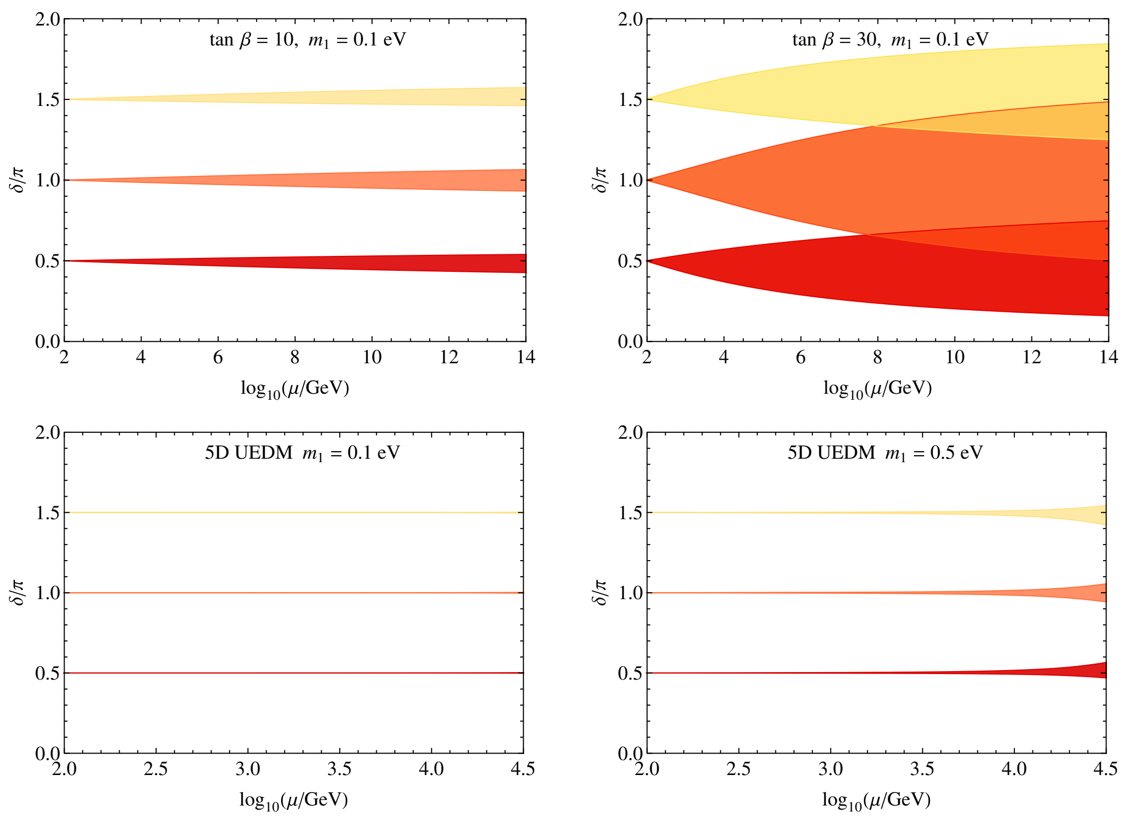

FIG. 1 (color online). Evolution of δ for Majorana neutrinos in the MSSM (upper plots) and in the UEDM (lower plots). The initial values $\delta = \pi/2$, $\delta = \pi$, and $\delta = 3\pi/2$ are assumed, while the Majorana CP-violating phases ρ and σ are marginalized. The values of θ_{12}, θ_{13}, θ_{23} and Δm_{21}^2, Δm_{31}^2 in the 1σ ranges from the global-fit analysis (for $\Delta m_{31}^2 > 0$) have been used as input [3].

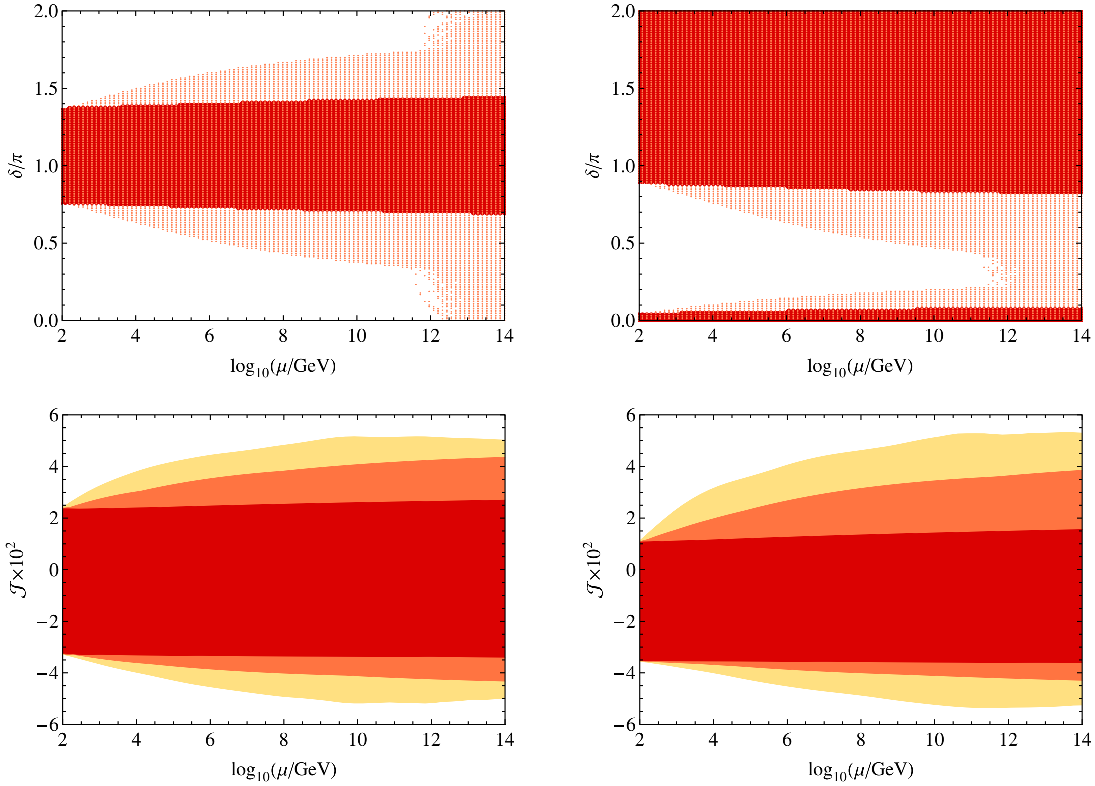

FIG. 2 (color online). Allowed values of the leptonic CP-violating phase δ (upper plots) and the Jarlskog invariant $\mathcal{J}$ (lower plots) for Majorana neutrinos at 1σ C.L. with $\tan\beta = 10$ (dark red or dark gray) and $\tan\beta = 30$ (light red or gray) in the MSSM. The result of $\mathcal{J}$ in the MSSM with $\tan\beta = 50$ is also given in the lower plots (yellow or light gray). The global-fit data from Ref. [2] are adopted for the left column, while that from Ref. [3] for the right column.

$m_1 = 0.5$ eV, which is marginally in tension with the cosmological bound, the relative change of δ at the cutoff scale $\Lambda = 3 \times 10^4$ GeV is not larger than 10%. The cutoff scale $\Lambda = 3 \times 10^4$ GeV in the UEDM has been chosen to avoid the Landau pole, where the Higgs mass is $M_H = 125$ GeV and $R^{-1} = 10$ TeV with R being the radius of the compactified extra dimension. Since the valid energy range in the UEDM is much smaller than that in the MSSM, the RG running does not develop as much. However, it should be noted that the RG running in UEDM is actually in the form of a power law, and thus can be more significant than in the SM and in the MSSM.

It should also be noted that the Majorana CP-violating phases ρ and σ have been marginalized over the range $[0, \pi)$ in our numerical results. If the specific values of ρ and σ are chosen, the variation of δ will be even smaller. Therefore, we conclude that the leptonic Dirac CP-violating phase δ is stable against radiative corrections in all the models under consideration, except for the MSSM with a large value of $\tan\beta$. In comparison, the Dirac CP-violating phase in the quark sector is stable even in the MSSM with a large value of $\tan\beta$, since the quark mass spectrum is strongly hierarchical.

Now, we turn to the RG running behavior of δ by taking the global-fit results $\delta = (1.08^{+0.28}_{-0.31})\pi$ [2] and $\delta = (1.67^{+0.37}_{-0.77})\pi$ [3] as input. Since the present uncertainty is large, we will choose the 1σ range for illustration. In the upper plots of Fig. 2, the allowed regions of δ at the superhigh-energy scale have been given in the MSSM. In the case of $\tan\beta = 30$, one can observe that δ is almost arbitrary within $[0, 2\pi)$ due to the large uncertainty of the input, so any predictions for δ from a high-energy flavor model could be made consistent with the low-energy observations by the RG running. This is true for the global-fit results from both groups [2,3]. In reality, any observable effects of CP violation should be related to the Jarlskog invariant $\mathcal{J} \equiv s_{12}c_{12}s_{23}c_{23}s_{13}c^2_{13}s_\delta$. Therefore, we also show the RG running of $\mathcal{J}$ in the MSSM for $\tan\beta = 10$, 30, 50, in the lower plots of Fig. 2. It can be observed that $\mathcal{J}$ at a superhigh-energy scale could be quite different from that at the low-energy scale, in particular for $\tan\beta = 30$ and $\tan\beta = 50$.

III. RUNNING OF CP-VIOLATING PHASE: DIRAC NEUTRINOS

The possibility for neutrinos to be Dirac particles has never been experimentally excluded. Moreover, it has been shown that the leptogenesis mechanism responsible for the matter-antimatter asymmetry in our Universe also works well in a different way for Dirac neutrinos [21]. Hence, we assume neutrinos to be Dirac particles, and give them masses through the coupling to the Higgs doublet $-\bar{\ell}_{\rm L} Y_\nu \nu_{\rm R} H + {\rm H.c.}$ with Y_ν being the neutrino Yukawa coupling matrix. It is convenient to write the RGEs of Dirac neutrino parameters as [22]

$$16\pi^2 \frac{{\rm d}\omega}{{\rm d}t} = 2\alpha_\nu \omega + C_{\nu,l}[(Y_l Y_l^\dagger)\omega + \omega(Y_l Y_l^\dagger)], \qquad (9)$$

where $\omega \equiv Y_\nu Y_\nu^\dagger$ has been defined. The RGEs of κ in the SM and the MSSM take the same form in Eq. (8), but with different coefficients α_ν and $C_{\nu,l}$, as given in Appendix B. Since the beta function for Dirac neutrino Yukawa couplings is currently not available in the UEDM, we consider only the SM and the MSSM. Similarly, as in the Majorana neutrino case, we find the RGE for the leptonic Dirac CP-violating phase δ in the case of Dirac neutrinos

$$\dot{\delta} \approx -\frac{C_{\nu,l} y_\tau^2}{16\pi^2} \frac{s_{23} c_{23} s_{13} s_\delta}{s_{12} c_{12}} \times \left[\xi_{21} + (c_{12}^2 \xi_{32} - s_{12}^2 \xi_{31}) + \frac{s_{12}^2 c_{12}^2}{s_{13}^2}(\xi_{32} - \xi_{31}) \right], \qquad (10)$$

where $\xi_{ij} \equiv (m_i^2 + m_j^2)/(m_i^2 - m_j^2)$ has been defined. The relative error in the above equation is at the level of $s_{13}(\Delta m_{21}^2/|\Delta m_{31}^2|)^2 \sim 10^{-4}$. It is worth mentioning that the last term in Eq. (10) is comparable in magnitude to the second term, since the suppression by a factor of $\Delta m_{21}^2/|\Delta m_{31}^2|$ is compensated by the enhancement from $1/s_{13}^2$. Some general comments are in order:

(i) The evolution of δ is proportional to s_δ at all orders, so δ will be kept unchanged by the RG running if $s_\delta = 0$, namely, $\delta = 0$ or $\delta = \pi$. In other words, if leptonic CP violation is absent at low energies, it will never be generated by RG running. This is quite different from the Majorana case, where δ can be radiatively generated via the nonvanishing Majorana CP-violating phases even if $\delta = 0$ or $\delta = \pi$ has been used as an initial condition.

(ii) Two qualitative differences between the SM and the MSSM should be noted. First, the tau-Yukawa coupling $y_\tau^2 = m_\tau^2(1 + \tan^2\beta)/v^2$ in the MSSM is significantly enhanced for a large value of $\tan\beta$. Hence, the RG effect is more remarkable than that in the SM. Second, the coefficient $C_{\nu,l}$ takes opposite signs in the SM and in the MSSM, indicating the evolution of δ in opposite directions in these two models.

To illustrate the RG running behavior of δ in the Dirac neutrino case, we have shown in Fig. 3 two typical examples in the MSSM. In both examples, the initial values of δ have been taken to be $\pi/2$ and $3\pi/2$, and the absolute neutrino mass is $m_1 = 0.1$ eV. The left plot is for $\tan\beta = 10$, while the right for $\tan\beta = 30$. Note that the beta function of δ is proportional to $-s_\delta$ in Eq. (9), where $C_{\nu,l} = 1$ in the MSSM. Therefore, δ increases for $\delta = 3\pi/2$, while it decreases for $\delta = \pi/2$, as the energy scale evolves towards higher energies. This feature can be clearly observed in Fig. 3. Furthermore, the variation of δ at any energy scale is quite small, compared to that in the case of Majorana neutrinos, where the arbitrary Majorana CP-violating phases play an important role in the evolution of δ. As we have already mentioned, δ will be kept unchanged if the initial values lead to $s_\delta = 0$, so the trivial cases of $\delta = 0$ and $\delta = \pi$ have not been considered.

Now, we continue with the global-fit results of δ in Refs. [2,3] as initial values. The RG running of δ in the MSSM for $\tan\beta = 10$, 30 and $\tan\beta = 50$ have been shown in the upper and middle plots of Fig. 4, respectively. As before, the absolute neutrino mass $m_1 = 0.1$ eV is assumed. In the former case, the RG running effects are insignificant, which is in accordance with the results in Fig. 3. In the latter case, however, it is interesting to note that a wide range of values $\delta \in [0.2\pi, 1.8\pi]$ cannot be reached at the superhigh-energy scale $\Lambda = 10^{14}$ GeV, no matter what initial value of δ is chosen. The reason for this behavior is that the mixing angle θ_{13} is approaching zero

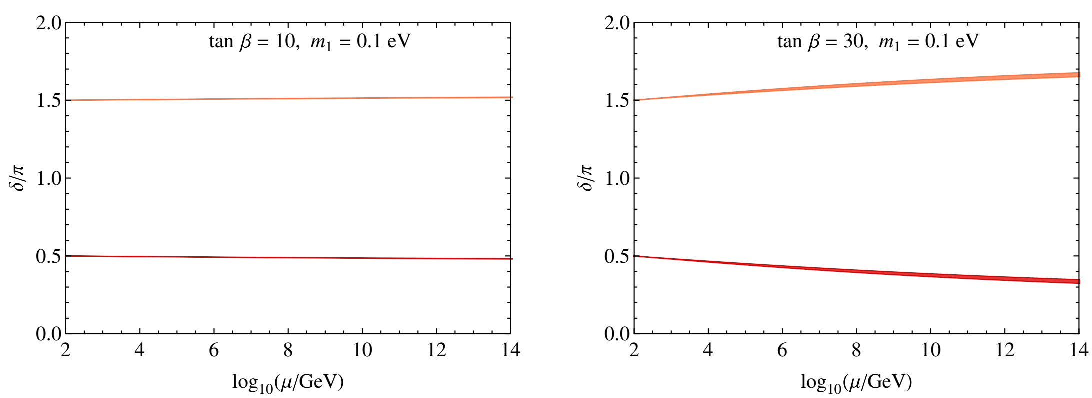

FIG. 3 (color online). Evolution of δ for Dirac neutrinos in the MSSM for $\tan\beta = 10$ (left plot) and $\tan\beta = 30$ (right plot). The initial values $\delta = \pi/2$ and $\delta = 3\pi/2$ are assumed, and the values of θ_{12}, θ_{13}, θ_{23} and Δm_{21}^2, Δm_{31}^2 in the 1σ ranges from the global-fit analysis (for $\Delta m_{31}^2 > 0$) have been used as input [3].

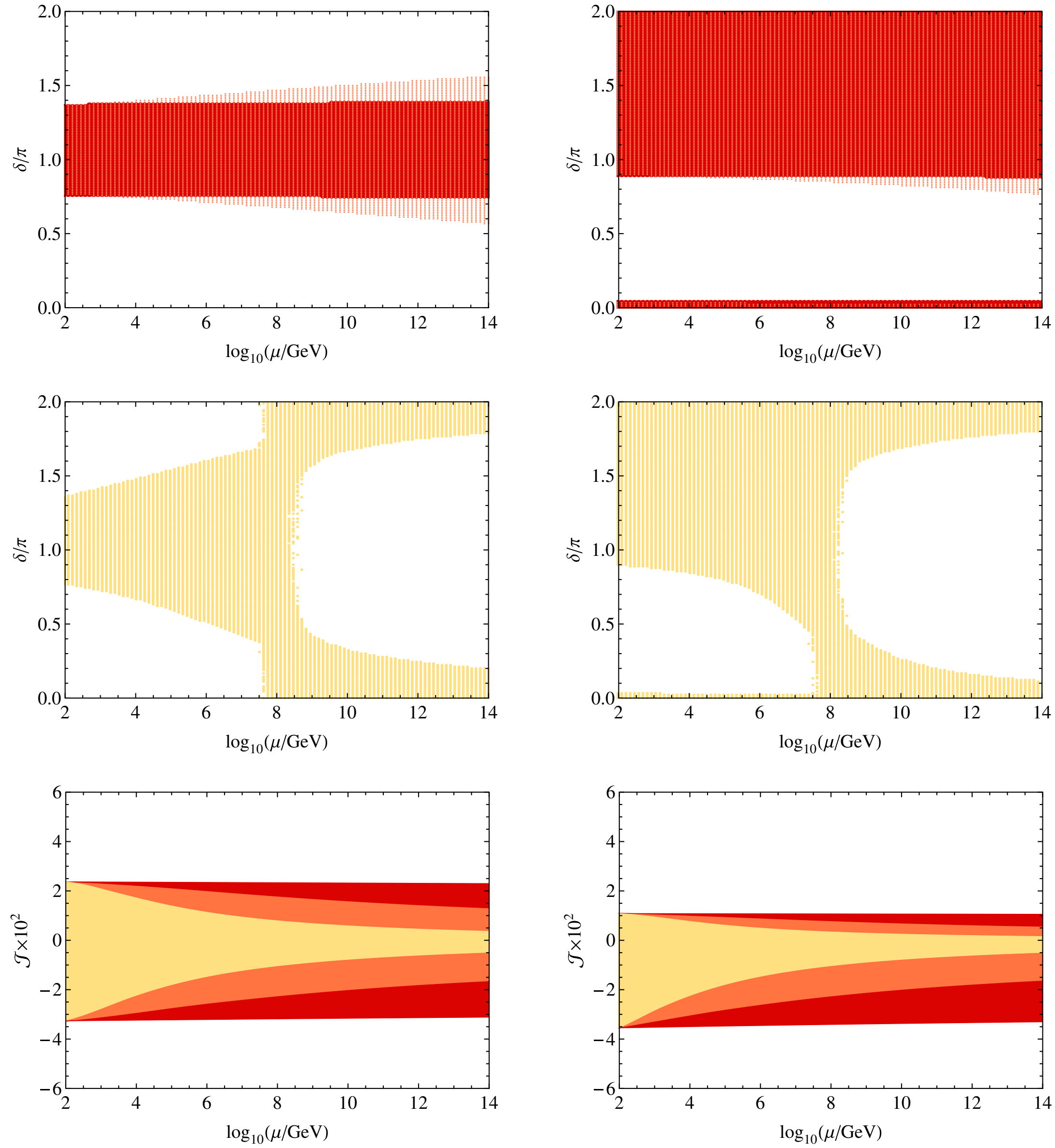

FIG. 4 (color online). Allowed values of the leptonic Dirac CP-violating phase δ (upper and middle plots) and the Jarlskog invariant $\mathcal{J}$ (lower plots) for Dirac neutrinos at 1σ C.L. with $\tan\beta = 10$ (dark red or dark gray), $\tan\beta = 30$ (light red or gray) and $\tan\beta = 50$ (yellow or light gray) in the MSSM. The absolute neutrino mass $m_1 = 0.1$ eV has been assumed. The global-fit data from Ref. [2] are adopted for the left column, while that from Ref. [3] for the right column.

around $\Lambda' = 10^8$ GeV. In the limit of an extremely small value of θ_{13}, Eq. (8) can be written as

$$\dot{\delta} \approx -\frac{y_\tau^2}{16\pi^2} s_{12} c_{12} s_{23} c_{23} s_\delta s_{13}^{-1} (\xi_{32} - \xi_{31}), \tag{11}$$

where $C_{\nu,l} = 1$ has been chosen for the MSSM. Therefore, the RG running of δ will be rapidly accelerated around $\Lambda' = 10^8$ GeV to the large-value region for $s_\delta < 0$ (i.e., $\delta > \pi$), while to the small-value region for $s_\delta > 0$ (i.e., $\delta < \pi$). This observation applies also to any initial value of δ. In fact, we have numerically checked the whole parameter region of $\delta \in [0, 2\pi)$ at low energies, and found that only $[0, 0.2\pi]$ and $[1.8\pi, 2\pi)$ can be reached at high energies. However, the exact allowed range of δ at high-energy scales really depends on the initial values of δ and three mixing angles. For $\delta = \pi$, the RG running of δ will be absent, but θ_{13} becomes negative above $\Lambda' = 10^8$ GeV, so we have to redefine $\delta \to \delta \pm \pi$ to make θ_{13} positive, leading to $\delta = 0$ or 2π at high-energy scales. In the lower plots of Fig. 4, the evolution of the Jarlskog invariant $\mathcal{J}$ is shown. Unlike the Dirac CP-violating phase δ itself, the physical observable $\mathcal{J}$ evolves smoothly over the whole range of energy scales, as it should. For $\tan\beta = 50$, the value of $|\mathcal{J}|$ can initially be as large as 2%, it becomes vanishingly small at $\Lambda = 10^{14}$ GeV. One reason for this is that δ shrinks into a small region around 0 or 2π at the high-energy scale, as indicated in the middle plots of Fig. 4. Obviously, the evolution of the three mixing angles is also relevant here.

IV. FURTHER DISCUSSIONS

In Secs. II and III, we have examined the RG running behaviors of the leptonic Dirac *CP*-violating phase δ in the cases of Majorana neutrinos and Dirac neutrinos, respectively. Now, we compare these two cases and summarize the main differences:

(i) In the Majorana case, the two Majorana *CP*-violating phases are playing a crucial role in the RG running of δ. One can start from a *CP*-conserving scenario with $\delta = 0$ or π at the low-energy scale, and end up with a *CP*-violating scenario even with $\delta = \pi/2$ or $3\pi/2$. In the Dirac case, the evolution of δ is proportional to s_δ, so the *CP* conservation at the low-energy scale definitely implies that *CP* violation is absent at a superhigh-energy scale.

(ii) The mixing angle θ_{13} could approach zero at some high-energy scale Λ' in both cases if a large value of $\tan\beta$ is assumed in the MSSM. On the other hand, there exist in the RGEs of δ some terms inversely proportional to s_{13}. Therefore, the RG running behavior of δ will be dramatically changed around Λ'. Given the global-fit values of δ within the 1σ range, it turns out that δ could be arbitrary at the high-energy scale in the Majorana case due to the marginalization over ρ and σ. In the Dirac case, δ is found to be in two narrow ranges $[0, 0.2\pi]$ or $[1.8\pi, 2\pi)$ in the MSSM with $\tan\beta = 50$.

However, if a concrete mass model for Majorana neutrinos or Dirac neutrinos is assumed, the RG running of δ may depend on the model details. In particular, when new particles or interactions come into play at some intermediate energy scale, the RGEs of the neutrino parameters are completely changed [23]. Hence, we have assumed that this is not the case in the previous discussions, at least below the cutoff scale.

As we have mentioned before, many flavor symmetry models, which are intended for describing the observed leptonic mixing angles, predict the leptonic Dirac *CP*-violating phase δ. For instance, it has been shown in Ref. [12] that $\delta \approx 2\pi/3$ (or $4\pi/3$) and $\delta \approx \pi/3$ (or $5\pi/3$) for different breaking patterns of the A_4 flavor symmetry in the type-I seesaw model, where three heavy right-handed neutrino singlets are introduced to realize the dimension-five Weinberg operator. If the vacuum alignment problem is further solved in the framework of supersymmetry, significant radiative corrections to these theoretical predictions of δ could be possible. Thus, the leptonic Dirac *CP*-violating phase to be measured in neutrino oscillation experiments is related by the RG running to the theoretical prediction at the seesaw scale. On the other hand, the *CP*-violating and out-of-equilibrium decays of the heavy right-handed neutrinos can generate the lepton number asymmetry in the early universe, which will be converted into the baryon number asymmetry via the SM sphaleron processes. In this case, the leptonic *CP* violation in neutrino oscillations can be associated with the matter-antimatter asymmetry in our Universe.

V. SUMMARY

Thanks to the recent measurements of θ_{13} in the Daya Bay and RENO experiments, the discovery of *CP* violation in neutrino oscillation experiments seems to be promising if the leptonic *CP* violation really exists and the leptonic Dirac *CP*-violating phase δ happens to be far away from 0 or π. On the other hand, we have already had a preliminary result for the leptonic *CP*-violating phase δ from the global-fit analysis of all kinds of neutrino oscillation experiments, namely $\delta = (1.08^{+0.28}_{-0.31})\pi$ [2] and $\delta = (1.67^{+0.37}_{-0.77})\pi$ [3]. Therefore, we are well motivated to study the RG running of δ from the low-energy scale to a superhigh-energy scale, where a unified model for fermion masses, flavor mixing, and *CP* violation is expected.

In the case of Majorana neutrinos, we have introduced the dimension-five Weinberg operator to account for neutrino masses. The RGE of δ has been derived analytically in great detail for the SM, the MSSM, and the UEDM, and a self-consistent approximation to it has been given as well. By a self-consistent approximation, we mean that the RGE of δ has been expanded in terms of s^2_{13} and $\Delta m^2_{21}/|\Delta m^2_{31}|$, and all the terms of the same order of magnitude should be preserved. It turns out that δ is rather stable against radiative corrections in all these models, except for the case of a large $\tan\beta$ in the MSSM (e.g., $\tan\beta = 30$ together with a nearly degenerate neutrino mass spectrum). In this case, the Majorana *CP*-violating phases play an important role in the evolution of δ such that a maximal phase $\delta = \pi/2$ or $3\pi/2$ can be radiatively generated at a superhigh-energy scale even if $\delta = \pi$ (i.e., no *CP*-violating effects in neutrino oscillation experiments) at the low-energy scale. The evolution of δ and the Jarlskog invariant $\mathcal{J}$ have been illustrated by taking the 1σ global-fit results of δ as input.

In the case of Dirac neutrinos, we have derived the RGE of δ in the SM and MSSM, and the self-consistent approximation to it has been made. Note that a nearly degenerate neutrino mass spectrum and the absolute neutrino mass $m_1 = 0.1$ eV are assumed in our analysis. The RG running effect of δ can be neglected in the SM and in the MSSM with a small $\tan\beta$ (e.g., $\tan\beta \leq 10$). However, δ can be modified by more than 30% for $\tan\beta = 30$. The evolution of δ and the Jarlskog invariant $\mathcal{J}$ have been examined by inputting the 1σ global-fit results of δ. In the case of $\tan\beta = 50$, δ in the range of $[0.2\pi, 1.8\pi]$ is found to be unreachable at $\Lambda = 10^{14}$ GeV, since the mixing angle θ_{13} approaches zero at some intermediate scale (e.g., $\Lambda' = 10^8$ GeV), which forces δ to be in a large-value region for $\delta > \pi$ or a small-value region for $\delta < \pi$. At the same time, the Jarlskog invariant $\mathcal{J}$ becomes vanishingly small at a superhigh-energy scale.

As we already know some information and will soon learn more about the leptonic Dirac CP-violating phase δ, it is thus meaningful to see how large it will be at a superhigh-energy scale. At such an energy scale, the leptonic Dirac CP-violating phase might be related to the quark Dirac CP-violating phase in a unified flavor model, or to the generation of matter-antimatter asymmetry in our Universe via the leptogenesis mechanism. In any case, the precise determination of δ in the ongoing and upcoming neutrino oscillation experiments or at a future neutrino factory will shed light on the flavor dynamics at a high-energy scale.

ACKNOWLEDGMENTS

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APPENDIX A: RGES FOR MAJORANA NEUTRINOS

1. The SM

In the SM extended with the dimension-five Weinberg operator, the RGE for κ has already been given in Eq. (3), while those for the Yukawa coupling matrices $Y_{\rm f}$ of charged fermions (i.e., $\rm f = \mathit{l}$ for charged leptons, $\rm f = u$ for up-type quarks and $\rm f = d$ for down-type quarks) can be written as

$$\begin{aligned}
16\pi^2\frac{\mathrm{d}Y_l}{\mathrm{d}t} &= [\alpha_l^{\rm SM} + C_{l,l}^{\rm SM}(Y_l Y_l^\dagger)]Y_l,\\
16\pi^2\frac{\mathrm{d}Y_{\rm u}}{\mathrm{d}t} &= [\alpha_{\rm u}^{\rm SM} + C_{\rm u,u}^{\rm SM}(Y_{\rm u} Y_{\rm u}^\dagger) + C_{\rm u,d}^{\rm SM}(Y_{\rm d} Y_{\rm d}^\dagger)]Y_{\rm u},\\
16\pi^2\frac{\mathrm{d}Y_{\rm d}}{\mathrm{d}t} &= [\alpha_{\rm d}^{\rm SM} + C_{\rm d,u}^{\rm SM}(Y_{\rm u} Y_{\rm u}^\dagger) + C_{\rm d,d}^{\rm SM}(Y_{\rm d} Y_{\rm d}^\dagger)]Y_{\rm d}.
\end{aligned} \tag{A1}$$

The relevant coefficients in Eqs. (3) and (A1) are $C_\kappa^{\rm SM} = C_{\rm u,d}^{\rm SM} = C_{\rm d,u}^{\rm SM} = -3/2$, $C_{l,l}^{\rm SM} = C_{\rm u,u}^{\rm SM} = C_{\rm d,d}^{\rm SM} = +3/2$, and

$$\begin{aligned}
\alpha_\kappa^{\rm SM} &= -3g_2^2 + \lambda + 2T_{\rm M}^{\rm SM},\\
\alpha_l^{\rm SM} &= -\frac{9}{4}g_1^2 - \frac{9}{4}g_2^2 + T_{\rm M}^{\rm SM},\\
\alpha_{\rm u}^{\rm SM} &= -\frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 + T_{\rm M}^{\rm SM},\\
\alpha_{\rm d}^{\rm SM} &= -\frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 + T_{\rm M}^{\rm SM}
\end{aligned} \tag{A2}$$

with $T_{\rm M}^{\rm SM} \equiv {\rm tr}[3(Y_{\rm u}Y_{\rm u}^\dagger) + 3(Y_{\rm d}Y_{\rm d}^\dagger) + (Y_l Y_l^\dagger)]$. The RGEs for the $SU(3)_{\rm C} \times SU(2)_{\rm L} \times U(1)_{\rm Y}$ gauge couplings g_3, g_2, and g_1 are given by

$$16\pi^2\frac{\mathrm{d}g_i}{\mathrm{d}t} = b_i^{\rm SM} g_i^3 \tag{A3}$$

with $(b_1^{\rm SM}, b_2^{\rm SM}, b_3^{\rm SM}) = (41/10, -19/6, -7)$. The quartic coupling λ of the Higgs field appears in the RGE of κ, which affects the evolution of absolute neutrino masses. It should satisfy the following RGE:

$$\begin{aligned}
16\pi^2\frac{\mathrm{d}\lambda}{\mathrm{d}t} = {}& 6\lambda^2 - 3\lambda\left(\frac{3}{5}g_1^2 + 3g_2^2\right) + \frac{3}{2}\left(\frac{9}{25}g_1^2 + \frac{6}{5}g_1^2 g_2^2 + 3g_2^2\right)\\
&+ 4\lambda T_{\rm M}^{\rm SM} - 8\,{\rm tr}[3(Y_{\rm u}Y_{\rm u}^\dagger)^2 + 3(Y_{\rm d}Y_{\rm d}^\dagger)^2 + (Y_l Y_l^\dagger)^2].
\end{aligned} \tag{A4}$$

It is worth mentioning that if the experimental uncertainties of the top quark mass M_t and the strong coupling α_s are taken into account, the SM vacuum could be stable up to the Planck scale $\Lambda_{\rm Pl} = 1.2 \times 10^{19}$ GeV [24], even for a Higgs mass $M_H = 125$ GeV indicated by the recent results of the ATLAS and CMS experiments.

2. The MSSM

In the MSSM, the RGEs in Eqs. (3) and (A1) are still applicable, but the relevant flavor-universal coefficients are as follows: $C_\kappa^{\rm MSSM} = C_{\rm u,d}^{\rm MSSM} = C_{\rm d,u}^{\rm MSSM} = 1$, $C_{l,l}^{\rm MSSM} = C_{\rm u,u}^{\rm MSSM} = C_{\rm d,d}^{\rm MSSM} = 3$, and

$$\begin{aligned}
\alpha_\kappa^{\rm MSSM} &= -\frac{6}{5}g_1^2 - 6g_2^2 + 6\,{\rm tr}(Y_{\rm u}Y_{\rm u}^\dagger),\\
\alpha_l^{\rm MSSM} &= -\frac{9}{5}g_1^2 - 3g_2^2 + {\rm tr}[3(Y_{\rm d}Y_{\rm d}^\dagger) + (Y_l Y_l^\dagger)],\\
\alpha_{\rm u}^{\rm MSSM} &= -\frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 + 36\,{\rm tr}(Y_{\rm u}Y_{\rm u}^\dagger),\\
\alpha_{\rm d}^{\rm MSSM} &= -\frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 + {\rm tr}[3(Y_{\rm d}Y_{\rm d}^\dagger) + (Y_l Y_l^\dagger)].
\end{aligned} \tag{A5}$$

The RGEs for the gauge couplings are given in Eq. (A3), but with $(b_1^{\rm MSSM}, b_2^{\rm MSSM}, b_3^{\rm MSSM}) = (33/5, 1, -3)$ in the beta functions. As we can see from the RGE of κ, the running neutrino parameters are determined by the charged-lepton Yukawa coupling matrix Y_l, especially the tau-lepton Yukawa coupling $y_\tau^2 = m_\tau^2(1 + \tan^2\beta)/v^2$, which could significantly be enhanced for a large value of $\tan\beta$. Such a unique feature can make the RG running of leptonic mixing parameters remarkable in the MSSM.

3. The UEDM

In the UEDM, all the SM fields are promoted to a higher-dimensional spacetime, so every SM particle is accompanied by a tower of Kaluza-Klein (KK) modes [25]. In the simplest UEDM with only one extra spatial dimension, which is compactified on an S^1/Z_2 orbifold with radius R, the KK parity defined as $(-1)^n$ for the nth KK mode is conserved after compactification. The mass scale of the first excited KK mode, i.e., $\mu_0 \equiv R^{-1}$, has been constrained to be larger than about 300 GeV.

If we extend the UEDM by an effective operator $(\overline{\ell}H) \cdot \hat{\kappa} \cdot (H^T \ell^C)/2$ to accommodate Majorana neutrino masses, just as in Eq. (1), then the effective Majorana neutrino mass matrix after electroweak symmetry breaking is $M_\nu = \kappa v^2$ with $\kappa = \hat{\kappa}/(\pi R)$. The RGE of κ now receives contributions from the KK modes, which are excited at the energy scale of interest. More explicitly, the RGEs for κ and the Yukawa coupling matrices of the charged fermions are also given by Eqs. (3) and (A1), but with the following coefficients [25]:

$$\begin{aligned}
\alpha_\kappa^{\rm UEDM} &= \alpha_\kappa^{\rm SM} + s\left(-\frac{1}{4}g_1^2 - \frac{11}{4}g_2^2 + \lambda + 4T_{\rm M}^{\rm SM}\right),\\
\alpha_l^{\rm UEDM} &= \alpha_l^{\rm SM} + s\left(-\frac{33}{8}g_1^2 - \frac{15}{8}g_2^2 + 2T_{\rm M}^{\rm SM}\right),\\
\alpha_{\rm u}^{\rm UEDM} &= \alpha_{\rm u}^{\rm SM} + s\left(-\frac{101}{72}g_1^2 - \frac{15}{8}g_2^2 - \frac{28}{3}g_3^2 + 2T_{\rm M}^{\rm SM}\right),\\
\alpha_{\rm d}^{\rm UEDM} &= \alpha_{\rm d}^{\rm SM} + s\left(-\frac{17}{72}g_1^2 - \frac{15}{8}g_2^2 - \frac{28}{3}g_3^2 + 2T_{\rm M}^{\rm SM}\right),
\end{aligned} \tag{A6}$$

and $C_{\rm x}^{\rm UEDM} = C_{\rm x}^{\rm SM}(1+s)$ with "x" being any relevant subscript. Note that $s \equiv \lfloor \mu/\mu_0 \rfloor$ counts the number of excited KK modes for a given energy scale μ. In addition, the coefficients in the beta functions of gauge couplings turn out to be

$$\begin{aligned}
b_1^{\rm UEDM} &= b_1^{\rm SM} + \frac{27}{2}s, \qquad b_2^{\rm UEDM} = b_2^{\rm SM} + \frac{7}{6}s,\\
b_3^{\rm UEDM} &= b_3^{\rm SM} - \frac{5}{2}s.
\end{aligned} \tag{A7}$$

Finally, the RGE for the quartic Higgs coupling λ is quite relevant in the UEDM, as in the SM case. It has been found to be [25]

$$\begin{aligned}
16\pi^2\frac{{\rm d}\lambda}{{\rm d}t} &= 6(1+s)\lambda^2 - 3(1+s)\lambda\left(\frac{3}{5}g_1^2 + 3g_2^2\right)\\
&\quad + \frac{3}{2}\left(1+\frac{4}{3}s\right)\left(\frac{9}{25}g_1^4 + \frac{6}{5}g_1^2 g_2^2 + 3g_4^2\right)\\
&\quad + 4(1+2s)\lambda T_{\rm M}^{\rm SM} - 8(1+2s){\rm tr}[3(Y_{\rm u}Y_{\rm u}^\dagger)^2\\
&\quad + 3(Y_{\rm d}Y_{\rm d}^\dagger)^2 + (Y_l Y_l^\dagger)^2].
\end{aligned} \tag{A8}$$

APPENDIX B: RGES FOR DIRAC NEUTRINOS

If the SM is extended with three right-handed neutrino singlets, then neutrinos acquire Dirac masses in the same way as the charged leptons and quarks do. At one-loop level, the RGEs of the fermion Yukawa coupling matrices read [22]

$$\begin{aligned}
16\pi^2\frac{{\rm d}Y_\nu}{{\rm d}t} &= [\alpha_\nu^{\rm SM} + C_{\nu,\nu}^{\rm SM}(Y_\nu Y_\nu^\dagger) + C_{\nu,l}^{\rm SM}(Y_l Y_l^\dagger)]Y_\nu,\\
16\pi^2\frac{{\rm d}Y_l}{{\rm d}t} &= [\alpha_l^{\rm SM} + C_{l,\nu}^{\rm SM}(Y_\nu Y_\nu^\dagger) + C_{l,l}^{\rm SM}(Y_l Y_l^\dagger)]Y_l,\\
16\pi^2\frac{{\rm d}Y_{\rm u}}{{\rm d}t} &= [\alpha_{\rm u}^{\rm SM} + C_{\rm u,u}^{\rm SM}(Y_{\rm u} Y_{\rm u}^\dagger) + C_{\rm u,d}^{\rm SM}(Y_{\rm d} Y_{\rm d}^\dagger)]Y_{\rm u},\\
16\pi^2\frac{{\rm d}Y_{\rm d}}{{\rm d}t} &= [\alpha_{\rm d}^{\rm SM} + C_{\rm d,u}^{\rm SM}(Y_{\rm u} Y_{\rm u}^\dagger) + C_{\rm d,d}^{\rm SM}(Y_{\rm d} Y_{\rm d}^\dagger)]Y_{\rm d},
\end{aligned} \tag{B1}$$

where $C_{\rm f,g}^{\rm SM} = +3/2$ (for f = g) and $-3/2$ (for f $\neq$ g), and

$$\begin{aligned}
\alpha_\nu^{\rm SM} &= -\frac{9}{20}g_1^2 - \frac{9}{4}g_2^2 + T_{\rm D}^{\rm SM},\\
\alpha_l^{\rm SM} &= -\frac{9}{4}g_1^2 - \frac{9}{4}g_2^2 + T_{\rm D}^{\rm SM},\\
\alpha_{\rm u}^{\rm SM} &= -\frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 + T_{\rm D}^{\rm SM},\\
\alpha_{\rm d}^{\rm SM} &= -\frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 + T_{\rm D}^{\rm SM}
\end{aligned} \tag{B2}$$

with $T_{\rm D}^{\rm SM} \equiv {\rm tr}[3(Y_{\rm u}Y_{\rm u}^\dagger) + 3(Y_{\rm d}Y_{\rm d}^\dagger) + (Y_\nu Y_\nu^\dagger) + (Y_l Y_l^\dagger)]$. The RGEs of fermion Yukawa coupling matrices are the same as in Eq. (B1) for the MSSM, but with different coefficients, namely $C_{\rm f,g}^{\rm MSSM} = +3$ (for f = g) and $+1$ (for f $\neq$ g), and

$$\begin{aligned}
\alpha_\nu^{\rm MSSM} &= -\frac{3}{5}g_1^2 - 3g_2^2 + {\rm tr}[3(Y_{\rm u}Y_{\rm u}^\dagger) + (Y_\nu Y_\nu^\dagger)],\\
\alpha_l^{\rm MSSM} &= -\frac{9}{5}g_1^2 - 3g_2^2 + {\rm tr}[3(Y_l Y_l^\dagger) + (Y_\nu Y_\nu^\dagger)],\\
\alpha_{\rm u}^{\rm MSSM} &= -\frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 + {\rm tr}[3(Y_{\rm u}Y_{\rm u}^\dagger) + (Y_\nu Y_\nu^\dagger)],\\
\alpha_{\rm d}^{\rm MSSM} &= -\frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 + {\rm tr}[3(Y_{\rm d}Y_{\rm d}^\dagger) + (Y_l Y_l^\dagger)].
\end{aligned} \tag{B3}$$

The RGEs of three gauge couplings g_1, g_2, and g_3 are the same as those in the case of Majorana neutrinos [see Eq. (A3)].

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