Modified Higgs couplings and unitarity violation

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Prompted by the recent observation of a Higgs-like particle at the CERN Large Hadron Collider, we investigate a quantitative correlation between possible departures of the gauge and Yukawa couplings of this particle from their Standard Model expectations and the scale of unitarity violation in the processes $WW \to WW$ and $t\bar{t} \to WW$.

One of the crucial arguments for the existence of the Higgs boson in the Standard Model (SM) is that, without it, the longitudinal vector boson (V_L, where $V = W, Z$) scattering amplitudes at the tree level would uncontrollably grow with the center-of-mass energy (E). This will result in the violation of ''unitarity,'' thus implying breakdown of quantum mechanical sense of probability conservation in scattering amplitudes. In the SM, the Higgs boson possesses appropriate gauge couplings to ensure exact cancellation of the residual E^2 growth in the $V_L V_L \to V_L V_L$ scattering amplitude that survives after adding the gauge boson contributions. It has been explicitly shown in Ref. [1] how, for $E \gg M_V$, the E^2 dependence is traded in favor of the unknown m_h^2, where m_h is the Higgs boson mass. From this it was concluded that m_h should be less than about a TeV for unitarity not to be violated. An intimate relationship between unitarity and renormalizability adds a special relevance to this issue. For a renormalizable theory the tree level amplitude for $2 \to 2$ scattering should not contain any term which grows with energy [2]. In perturbative expansion of scattering amplitudes these energy growths must be canceled order by order [3]. It has been shown that the energy dependent terms in tree level amplitudes get exactly canceled if the couplings satisfy certain sets of ''unitarity sum rules'' [4]. It has also been realized that the presence of the Higgs boson is not the only option to satisfy these sum rules [5,6].

Meanwhile, a Higgs-like particle has been observed with a mass of around 125 GeV by the ATLAS and CMS collaborations of the LHC [7,8]. This is much below the upper limit coming from unitarity violation mentioned above. If this particle indeed turns out to be the SM Higgs, then the scattering amplitudes involving not only the longitudinal vector bosons but any other SM particles as external states would be well behaved for arbitrarily high energies. However, the recent observation of some excess events in the $h \to \gamma\gamma$ channel, as well as large errors associated with other decay channels, has fueled speculation that Higgs couplings to fermions and/or gauge bosons might not be exactly as predicted by the SM [9]. There is more than one way to modify the Higgs couplings. One way is to hypothesize that the WWh and the ZZh couplings are modified; more specifically, enhanced with

respect to their SM values. This would result not only in an increase in the Higgs production cross section via vector boson fusion and associated production, but also in an enhancement of the W-loop contribution to $h \to \gamma\gamma$ decay. But this would at the same time lead to excess events in the $h \to WW^*$ and $h \to ZZ^*$ channels, something which is not obvious from the data. It would also result in the violation of unitarity in longitudinal gauge boson scattering channels. This was indeed explored long back [10], however, in the absence of the LHC data there was no motivation to study the correlation between unitarity violation and the Higgs decay branching ratios at that time. If we refrain from adding any extra particle to the SM and yet attempt to account for the excess in the diphoton channel, the next natural choice would be to modify the Yukawa coupling of the top quark. As is already known, if we put the sign of the top Yukawa coupling opposite to what it is in the SM, the $h \to \gamma\gamma$ rate gets enhanced due to a constructive interference between the W-loop diagram and the top-loop diagram [9]. One of the fallouts of this sign flip is that $t\bar{t} \to V_L V_L$ scattering no longer remains unitary. In fact, as we shall show, any nontrivial admixture of CP-even and CP-odd states in the composition of the scalar particle jeopardizes the good high energy behavior of the $t\bar{t} \to V_L V_L$ amplitude even if we keep the moduli of the top Yukawa coupling and the Higgs gauge coupling to their SM values. The purpose of this paper is to explicitly demonstrate how the scales of unitarity violation in $W_L W_L \to W_L W_L$ and $t\bar{t} \to W_L W_L$ scattering processes depend on the modification parameters of the gauge and the top Yukawa couplings of the Higgs. We demonstrate what an enhanced diphoton rate may imply in this context.

In our analysis, we modify only the top Yukawa coupling, since the other Yukawa couplings are numerically much less relevant. We take

$$g_{tth} = (1-f)(\cos\delta - i\sin\delta\gamma_5)g_{tth}^{\mathrm{SM}} = (1-f)e^{-i\delta\gamma_5}g_{tth}^{\mathrm{SM}}. \tag{1a}$$

The parameter f is a measure of the overall coupling of the Higgs boson to the top quark, whereas δ is a parameter that quantifies the mixture of CP-even and CP-odd components in the Higgs boson. We also modify the gauge couplings of the Higgs boson as

$$g_{VVh} = (1 - x)g_{VVh}^{SM}, \tag{1b}$$

where V can be W or Z, as said before. We maintain equality between the WWh and ZZh couplings to respect custodial symmetry. The parameters x, f and δ are all real, and they all vanish in the SM.

We now comment on the existing experimental constraints on these modification parameters. First, it has been shown in Ref. [11] that precision electroweak measurements imply $-0.2 \leq x \leq 0.1$ at 95% C.L. for $m_h = 125$ GeV and $m_t = 173$ GeV, while from the recent LHC Higgs data analysis the 95% C.L. range has been estimated to be $-0.4 \leq x \leq 0.4$ [12,13]. Second, the allowed range of f can be extracted from recent fits of modified Higgs couplings against the LHC data. For example, for $x = 0$, the range is $-0.1 < f < 0.6$ for values of δ fixed at 0 and π [13,14]. Note that similar bounds have been obtained by the authors of Ref. [15], who considered a phase in the effective coupling due to an absorptive part in the amplitude. In this paper, we take a more conservative approach and consider a Hermitian Yukawa Lagrangian.

With the modifications prescribed in Eq. (1), one should examine unitarity constraints on scattering processes involving the top quark and the W boson. Note that we will talk about the longitudinally polarized component of the W boson only, dropping the polarization subscript L which is implicitly assumed. We have looked at the energy dependence of the elastic scattering $WW \rightarrow WW$ and the inelastic scattering $t\bar{t} \rightarrow WW$. The scattering amplitudes that we find are as follows:

$$\mathcal{A}^{WW \rightarrow WW} = 2\sqrt{2}G_F E^2 (2x - x^2)(1 + \cos\theta) + \cdots, \tag{2a}$$

$$\mathcal{A}^{t\bar{t} \rightarrow WW} = 2\sqrt{2}G_F E m_t Y(x, f, \delta) + \cdots, \tag{2b}$$

where the dots indicate subleading terms in energy which do not concern us, θ is the scattering angle, and

$$Y(x, f, \delta) = \mp[1 - (1 - x)(1 - f)e^{\mp i\delta}], \tag{3}$$

where different signs correspond to different combinations of helicities [16]. The scattering amplitude can be expanded in terms of partial waves [1]:

$$\mathcal{A}(\theta) = 16\pi \sum_{l=0}^{\infty} (2l + 1)a_l P_l(\cos\theta). \tag{4}$$

The unitarity condition $|a_0| \leq 1$ puts upper limits on the center-of-mass energy in each of these processes. These limits are as follows:

$$E \leq E_{max}^{WW} = \left(\frac{4\sqrt{2}\pi}{G_F} \frac{1}{|2x - x^2|}\right)^{\frac{1}{2}} \text{ [from } WW \rightarrow WW]; \tag{5a}$$

$$E \leq E_{max}^{tt} = \frac{4\sqrt{2}\pi}{G_F m_t} \frac{1}{|Y(x, f, \delta)|} \text{ [from } t\bar{t} \rightarrow WW]. \tag{5b}$$

Because only $\cos\delta$ appears in $|Y|$, we can take δ in the range $[0, \pi]$. Without any loss of generality, we can take

$1 - f \geq 0$ to cover the entire parameter space. In passing, let us add that the constraints from $t\bar{t} \rightarrow ZZ$ are the same in the leading order in E as that given in Eq. (5b).

We now discuss the numerical dependence of the unitarity violation scale on the nonstandard parameters expressed through our master equations given in Eq. (5). Our results are displayed in Fig. 1. The different panels correspond to different choices of δ, as indicated in the figure. For the $WW \rightarrow WW$ scattering amplitude which grows as E^2, there is contribution coming from the Higgs mediated diagram and therefore it depends on x, but there is no dependence on f and δ since the top-Higgs coupling is not involved. The latter coupling is of course relevant for the $t\bar{t} \rightarrow WW$ scattering, and the Higgs mediated graph is sensitive to all the three nonstandard parameters, i.e., x, f and δ. In all the panels the lines titled $WW \rightarrow WW$ obtained by plotting Eq. (5a) show the scale of unitarity violation as the WWh coupling departs from its SM value. The other lines mark the unitarity violation scale arising from $t\bar{t} \rightarrow WW$ and are obtained from Eq. (5b). In the limit $x = 1$, i.e., when the Higgs either does not exist or does not couple to W, unitarity is violated at a pretty low scale, $E_{max}^{WW} \approx 1.3$ TeV. As x approaches zero, E_{max}^{WW} goes up. On the other hand, the limit $f = 1$ implies that the Higgs does not couple to the top quark, so in this limit the Higgs mediated graph for $t\bar{t} \rightarrow WW$ would not exist, and hence, the unitarity violation scale arising from the above scattering would be independent of x and δ. Similar things happen in the limit $x = 1$, causing the unitarity violation scale from $t\bar{t} \rightarrow WW$ to be independent of f and δ. This is precisely the reason as to why the horizontal $f = 1$ line in all the panels meets the curvy lines for other values of f at one single point which is at $x = 1$ corresponding to $E_{max}^{tt} \approx 9$ TeV.

An important observation at this stage is the following: for $\delta \neq 0$ and $\delta \neq \pi$, the process $t\bar{t} \rightarrow WW$ is not unitary regardless of the choice of x and f. The vertical shades in the four panels restrict the values of x within the zone allowed by precision tests. One thing is quite clear that if x happens to take a value near the edge of the shade in any panel, the unitarity violation would set in for $WW \rightarrow WW$ at a scale much lower than where it would happen for $t\bar{t} \rightarrow WW$, which is easily understood from the E^2 versus E growth in the two amplitudes. But if x settles at a much smaller value, as one can see from the different panels, the unitarity violation scales from these two amplitudes get closer and at some point the hierarchy mentioned earlier is reversed.

We now consider the decay of the 125 GeV particle into two photons. Two-photon final states have a definite CP property, more specifically, a definite parity. As a result, if the initial spin-zero state is not an eigenstate of parity, the parity-even and parity-odd components will contribute incoherently. For the sake of simplicity and to provide intuitive feel for easy comparison with standard expressions, we consider the decay of a CP-even scalar state only, which amounts to taking $\delta = 0$ or π.
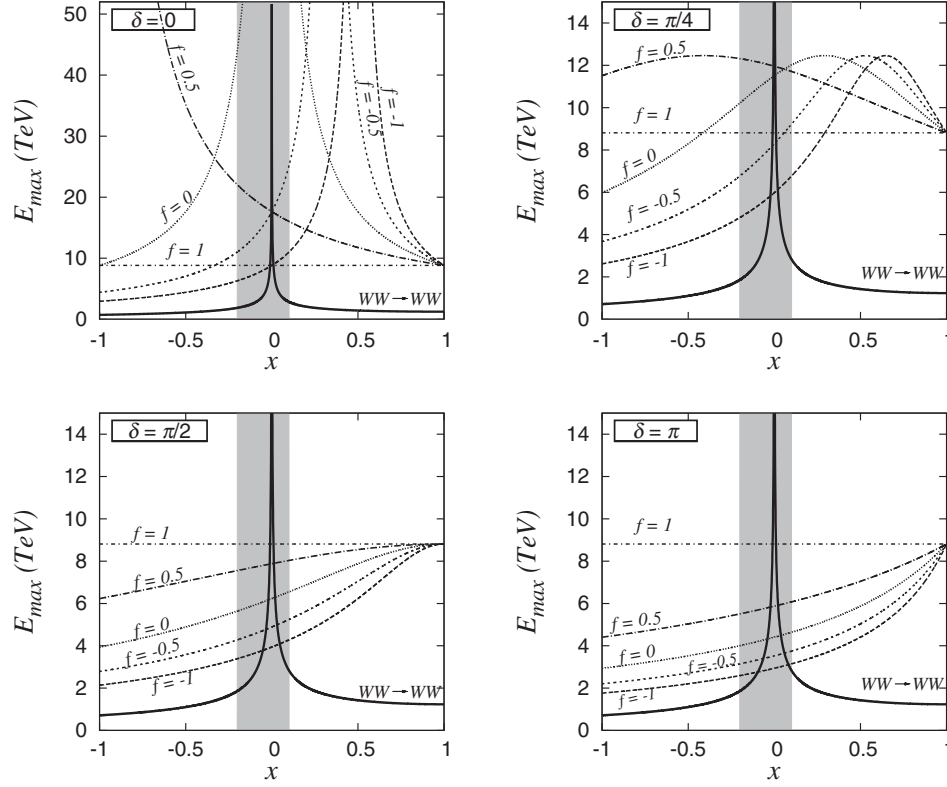
FIG. 1.   Unitarity violation scale as a function of x, for specific values of f and δ. For each panel, the scale coming from the elastic $WW \to WW$ scattering has been marked. The other lines come from $t\bar{t} \to WW$ scattering for various values of f. The vertical shaded region represents the range of x consistent with electroweak precision data. Note the different scale on the vertical axis for the plot with $\delta = 0$.

The decay $h \to \gamma\gamma$ proceeds dominantly through a W boson loop and a top loop diagram. For a CP-even h, the decay width is given by [17]

$$\Gamma(h \to \gamma\gamma) = \frac{\alpha^2 g^2}{2^{10}\pi^3} \frac{m_h^3}{M_W^2} \left| F_W + \frac{4}{3}F_t \right|^2. \quad (6)$$

For the SM, the values of F_W and F_t are given by

$$\begin{aligned} F_W^{\mathrm{SM}} &= 2 + 3\tau_W + 3\tau_W(2 - \tau_W)f(\tau_W), \\ F_t^{\mathrm{SM}} &= -2\tau_t[1 + (1 - \tau_t)f(\tau_t)], \end{aligned} \quad (7)$$

where

$$\tau_x \equiv (2m_x/m_h)^2. \quad (8)$$

For $m_h \approx 125$ GeV, $\tau_x > 1$ for both $x = W$, t. In this situation,

$$f(\tau) = [\sin^{-1}(\sqrt{1/\tau})]^2. \quad (9)$$

Using the modified Higgs couplings of Eq. (1), the expressions of the W and top loop contributions are obtained by replacing F_W^{SM} and F_t^{SM} by

$$F_W = (1 - x)F_W^{\mathrm{SM}}, \qquad F_t = (1 - f)e^{-i\delta}F_t^{\mathrm{SM}}, \quad (10)$$

where δ is either zero or π, as mentioned earlier.

We now estimate how the Higgs production cross section would be modified. For 7(8)-TeV LHC, the top loop driven gluon-gluon fusion channel contributes around 85% of the total cross section, while the associated production and the vector boson fusion together almost account for the remaining 15% [17]. The production cross section would then be modified roughly by the factor

$$\begin{aligned} \frac{\sigma(pp \to h)}{\sigma^{\mathrm{SM}}(pp \to h)} &= \frac{(1 - f)^2\sigma_{\mathrm{G}} + (1 - x)^2\sigma_{\mathrm{V}}}{\sigma_{\mathrm{G}} + \sigma_{\mathrm{V}}} \\ &\approx (1 - f)^2 85\% + (1 - x)^2 15\%. \end{aligned} \quad (11)$$

As far as the different decay channels of the Higgs are concerned, for $m_h \approx 125$ GeV, branching ratios of the SM Higgs boson are roughly as follows: 58% to $b\bar{b}$, 7% to $\tau^+\tau^-$, 3% to $c\bar{c}$, 24% to VV^* and 8% to gg [17]. We then express the modification of the total decay width by the ratio:

$$\frac{\Gamma_h}{\Gamma_h^{\mathrm{SM}}} = (58\% + 7\% + 3\%) + (1 - x)^2 24\% + (1 - f)^2 8\%. \quad (12)$$

The above expressions lead us to define

$$\mu = \frac{\sigma(pp \to h)}{\sigma^{\mathrm{SM}}(pp \to h)} \cdot \frac{\Gamma(h \to \gamma\gamma)}{\Gamma^{\mathrm{SM}}(h \to \gamma\gamma)} \cdot \frac{\Gamma_h^{\mathrm{SM}}}{\Gamma_h}. \quad (13)$$
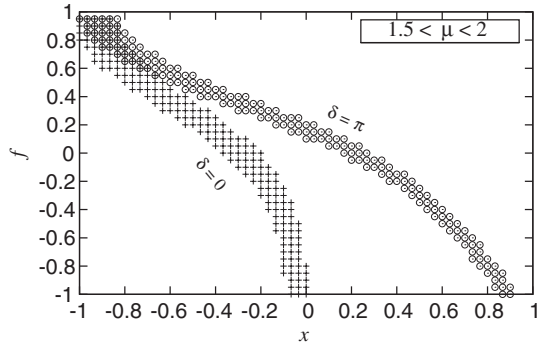
FIG. 2.   Allowed regions in the x-f plane that correspond to the diphoton enhancement ratio μ lying between 1.5 and 2, for $\delta = 0$ and $\delta = \pi$.

FIG. 3.   Unitarity violation scale plotted against diphoton enhancement. The vertical shaded region corresponds to $1.5 \leq \mu \leq 2.0$.

In Fig. 2 we have shaded different regions in the x-f plane, for the two possible choices of δ, which can account for the apparent excess of the diphoton events. Motivated by the recent LHC data, we choose μ in the range 1.5 to 2 for the sake of illustration. For $x \approx 0$ and $\delta = \pi$, we observe that

$$0.1 < f < 0.25, \tag{14}$$

which is roughly consistent with the limit quoted earlier in connection with global fits. Thus a *top-phobic Higgs*, which corresponds to $f \rightarrow 1$, is highly unlikely. We must admit though that this comparison is not entirely fair as we have modified only the top Yukawa coupling, while in the global fits all the Yukawa couplings were modified. We also admit that for the simplicity of illustration we have not taken into account the efficiency factors in the estimation of μ.

In Fig. 3, we have exhibited the correlation between the unitarity violation scale and the diphoton enhancement ratio μ. For drawing this plot, we have varied f between -1 and $+1$. Keeping in mind the relative sensitivity of the two scattering processes, we restrict x in a rather narrow range: $-0.005 < x < 0.005$. The horizontal line, appropriately labeled, corresponds to the unitarity violation scale in $WW \rightarrow WW$ scattering with $|x| = 0.005$. For smaller values of x, this line will appear at higher energy. The other curvy lines come from $t\bar{t} \rightarrow WW$ and they correspond to two different choices of δ, viz., zero and π. The thickness of the lines for $\delta = 0$ and $\delta = \pi$ come from the range of x just mentioned. For $\delta = 0$, it is hard to achieve a value of μ as large as 1.5. For $\delta = \pi$, it is possible to obtain a value of μ in the range 1.5 to 2, as can be seen by the corresponding line going through the vertical shade. The corresponding range of f, which can be read from Fig. 2, has been mentioned in Eq. (14). It is
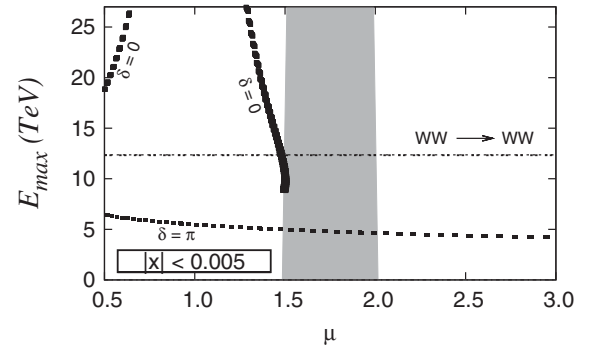
worth noting from this figure that for $\delta = \pi$, which facilitates diphoton rate enhancement, the unitarity violation scale comes down to around 5 TeV. This is true even when $x = 0$, i.e., when the gauge coupling of the Higgs boson matches the SM value and therefore the $WW \rightarrow WW$ scattering is perfectly unitary.

To summarize, even though the existence of a Higgs-like particle has been announced, precise measurements of its couplings to gauge bosons and fermions would take quite a while. The expected precision of the gauge and Yukawa couplings of the Higgs is unlikely to get better than about 25% within a year from now [18]. If the measured couplings eventually match their SM values, the theory is unitary, i.e., well behaved up to arbitrarily high energies. Otherwise, the extent of departure of the measured values of the couplings from their SM predictions would mark the scale where unknown dynamics would set in (see e.g., Ref. [19]). We have carried out a quantitative study of this scale as a function of the deviation of the Higgs couplings from their SM values through studies of the $WW \rightarrow WW$ and $t\bar{t} \rightarrow WW$ scattering processes. We have specifically focused on nonstandard effects on the gauge coupling of the Higgs and the top Yukawa coupling, as these two couplings play a crucial role in the stability of the electroweak vacuum and the perturbative unitarity of the theory. If future measurements favor Higgs couplings closer to its SM values, the expected scale of unitarity saturation would go up.

Note added.—While this work was being completed, we became aware of a similar work [20] which has addressed similar questions.

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