

**Chameleon Brans-Dicke cosmology**

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We consider a generalized Brans-Dicke model in which the scalar field has a potential function and is also allowed to couple nonminimally with the matter sector. We assume a power-law form for the potential and the coupling functions as the inputs of the model and show that acceleration of the Universe can be realized for a constrained range of exponents of the potential function. We also argue that this accelerating phase is consistent with a large and positive Brans-Dicke parameter. In our analysis, the potential plays a more important role with respect to the coupling function in the dynamics of the Universe as the latter does not contribute to any of the relations characterizing evolution of the scale factor of the Universe and the scalar field. However, we will show that the coupling function is closely related to magnitude and direction of the energy transfer between matter and the scale field. We use this fact and some thermodynamic aspects of the model to put some constraints on the coupling function. In particular, we argue that the second law of thermodynamics constrains direction of the overall energy transfer.

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**I. INTRODUCTION**

Cosmological observations on the expansion history of the Universe can be interpreted as evidence either for existence of some exotic matter components or for modification of the gravitational theory. In the first route of interpretation one can take a mysterious cosmic fluid with sufficiently large and negative pressure, dubbed dark energy. In the second route, however, one attributes the accelerating expansion to a modification of general relativity. Such modified gravity models can be obtained in different ways. For instance, one can replace the Ricci scalar in the Einstein-Hilbert action by some functions  $f(R)$  (for a review see, e.g., Ref. [1] and references therein), or by considering a scalar partner for the metric tensor for describing geometry of spacetime, the so-called scalar-tensor gravity. The prototype of the latter approach is Brans-Dicke (BD) theory [2] whose original motivation was the search for a theory containing Mach's principle. As the simplest and best-studied generalization of general relativity, it is natural to think about the BD scalar field as a possible candidate for producing cosmic acceleration without invoking auxiliary fields or exotic matter systems. In fact, there have been many attempts to show that the BD model can potentially explain the cosmic acceleration. It is shown that this theory can actually produce a nondecelerating expansion for low negative values of the BD parameter  $\omega$  [3]. Unfortunately, this conflicts with the lower bound imposed on this parameter by solar system experiments [4]. Due to this difficulty, some authors propose modifications of the BD model such as introducing some potential functions for the scalar field [5], or considering a field-dependent BD parameter [6] without resolving the problem. In a general scalar-tensor theory there is a nonminimal

coupling between the scalar field and Ricci scalar while the former minimally couples with the matter sector. In other terms, there is no an explicit coupling between the scalar field and matter systems in Jordan frame representation. In BD theory, in its original form, the motivation for such a minimal coupling was to keep the theory in accord with the weak equivalence principle [2]. However, there has been recently a tendency in the literature to go a step further and consider a nonminimal coupling between the scalar field and matter systems as well by introducing an arbitrary function of the scalar field as a coupling function. In these models, the scalar field is regarded as a chameleon field as it can be heavy enough in the environment of the laboratory tests so that the local gravity constraints are satisfied. Meanwhile, it can be light enough in the low-density cosmological environment to be considered as a candidate for dark energy. Such a chameleon-matter coupling was first introduced in the BD model to achieve accelerating expansion of the Universe for sufficiently large BD parameter [7]. It is shown that even though the absolute value of  $\omega$  is enlarged due to such a nonminimal coupling, its negative sign makes this theory remain inconsistent with observations. Later works apply nonminimal coupling to general scalar-tensor theories. For instance, Ref. [8] considers stability analysis and possibility of phantom crossing with an assumption that potential of the scalar field and the coupling function have power-law forms, and bouncing solutions and some cosmological tests are investigated in Ref. [9]. In the present work, we will show that within a class of solutions of the field equations  $\omega$  can take positive and large values. We also discuss some thermodynamic aspects of the model and show that the overall energy transfer should be into the matter system if the second law of thermodynamics is to be fulfilled. We use this fact and some recent observations to tightly constrain the exponent of the coupling function.

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## II. THE MODEL

We consider the action functional

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left\{ \phi R - \frac{\omega}{g} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - 2V(\phi) + 2f(\phi)L_m \right\}, \quad (1)$$

where  $R$  is the Ricci scalar,  $\phi$  is the BD scalar field, and  $V(\phi)$  and  $f(\phi)$  are some analytic functions. Here the matter Lagrangian density, denoted by  $L_m$ , is coupled with  $\phi$  via the function  $f(\phi)$ . This allows a nonminimal coupling between the matter system and  $\phi$ ; thus the latter appears as a chameleon field. Taking  $f(\phi) = 1$ , we return to the BD action with a potential function  $V(\phi)$ .

Varying the action with respect to the metric  $g_{\mu\nu}$  and  $\phi$  yields the field equations, given by

$$\phi G_{\mu\nu} = T_{\mu\nu}^\phi + f(\phi)T_{\mu\nu}^m, \quad (2)$$

$$(2\omega + 3)\square\phi + 2(2V - V'\phi) = T^m f - 2f'\phi L_m, \quad (3)$$

where prime indicates differentiation with respect to  $\phi$ ,  $T^m = g^{\mu\nu}T_{\mu\nu}^m$ , and

$$T_{\mu\nu}^\phi = \frac{\omega}{\phi} \left( \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla_\alpha \phi \nabla^\alpha \phi \right) + (\nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \square\phi) - V(\phi)g_{\mu\nu}, \quad (4)$$

$$T_{\mu\nu}^m = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}. \quad (5)$$

Due to the explicit coupling of the matter system with  $\phi$ , the stress tensor  $T_{\mu\nu}^m$  is not divergence free. This can be seen by applying the Bianchi identities  $\nabla^\mu G_{\mu\nu} = 0$  to (2), which leads to

$$\nabla^\mu T_{\mu\nu}^m = (L_m g_{\mu\nu} - T_{\mu\nu}^m) \nabla^\mu \ln f. \quad (6)$$

As it is clear from (6), details of the energy exchange between matter and  $\phi$  depend on the explicit form of the matter Lagrangian density  $L_m$ . Here we consider a perfect fluid energy-momentum tensor as a matter system with energy density  $\rho_m$  and pressure  $p_m$ .

There are different choices for the perfect fluid Lagrangian density; all of them lead to the same energy-momentum tensor and field equations in the context of general relativity [10,11]. The two Lagrangian densities that have been widely used in the literature are  $L_m = p_m$  and  $L_m = -\rho_m$  [12–14]. For a perfect fluid that does not couple explicitly to  $\phi$  [i.e., for  $f(\phi) = 1$ ], the two Lagrangian densities  $L_m = p_m$  and  $L_m = -\rho_m$  are perfectly equivalent, as discussed in Refs. [13,14]. However, in the model presented here the expression of  $L_m$  enters explicitly the field equations and all results strongly depend on the choice of  $L_m$ . In fact, it is shown that there is a strong debate about equivalency of different

expressions attributed to the Lagrangian density of a coupled perfect fluid [15].

Here we consider  $L_m = p_m$ . In this case, although fluid elements follow geodesics of the background metric and there is no additional force, the matter is still nonconserved [16]:

$$\dot{\rho}_m + 3H(\gamma + 1)\rho_m = -(\gamma + 1)\rho_m \frac{\dot{f}}{f}. \quad (7)$$

This has the solution

$$\rho_m = \rho_0 a^{-3(\gamma+1)} f^{-(\gamma+1)} \quad (8)$$

with  $\rho_0$  being an integration constant. This indicates that evolution of matter density strongly depends on the coupling function. When  $f = 1$ , (8) reduces to the standard evolution law for the matter energy density. However, as we have already stated, the coupling function in this framework does not affect a matter system with  $\gamma = -1$  (a cosmological constant) as it is clear from (8).

## III. A COSMOLOGICAL SETTING

Now we apply the above framework to a homogeneous and isotropic Universe described by the Friedmann-Robertson-Walker metric. In a spatially flat case, Eqs. (2) and (3) become

$$3\frac{\dot{a}^2}{a^2} = \frac{f}{\phi} \rho_m + \frac{1}{2} \omega \frac{\dot{\phi}^2}{\phi^2} - 3H \frac{\dot{\phi}}{\phi} + \frac{V}{\phi}, \quad (9)$$

$$3\frac{\ddot{a}}{a} = -\frac{3\rho_m}{\phi(2\omega + 3)} \left[ \gamma\phi f' + f \left( \omega \left( \gamma + \frac{1}{3} \right) + 1 \right) \right] - \omega \frac{\dot{\phi}^2}{\phi^2} + 3H \frac{\dot{\phi}}{\phi} + \frac{1}{(2\omega + 3)} \left[ 3V' + (2\omega - 3) \frac{V}{\phi} \right], \quad (10)$$

$$(2\omega + 3)(\ddot{\phi} + 3H\dot{\phi}) - 2(2V - \phi V') = (1 - 3\gamma)f\rho_m + 2\gamma\phi f' \rho_m. \quad (11)$$

To proceed further, we choose  $V(\phi) = V_0 \phi^{l_1}$  and  $f(\phi) = f_0 \phi^{l_2}$  in which  $l_1$  and  $l_2$  are constant parameters and  $V_0$  and  $f_0$  are positive quantities. We assume power-law forms for evolution of the scale factor and the scalar field  $a(t) = a_0 t^n$  and  $\phi(t) = \phi_0 t^m$ . Inserting these ansatz functions together with (8) into (9)–(11) gives the parameters  $n$  and  $m$  in terms of  $l_1$  and  $l_2$ . Since the left-hand side of (9)–(11) falls as  $t^{-2}$ , we arrive at the following relationships:

$$m(l_1 - 1) = -2, \quad (12)$$

$$m(l_2 - 1) - (3n + ml_2)(\gamma + 1) = -2. \quad (13)$$

Combining (12) and (13) gives

$$n = \frac{2(\gamma l_2 + l_1)}{3(\gamma + 1)(l_1 - 1)}. \quad (14)$$

For  $\gamma \neq 0$ , the conditions for the Universe to be expanding ( $n > 0$ ) and accelerating ( $n > 1$ ) are the following:<sup>1</sup>

$$\begin{aligned} n > 0 &\rightarrow l_2 > -l_1/\gamma & l_1 > 1, \\ l_2 < -l_1/\gamma & & l_1 < 1, \end{aligned} \quad (15)$$

$$\begin{aligned} n > 1 &\rightarrow l_2 > \frac{3}{2}(l_1 - 1) + \frac{1}{2\gamma}(l_1 - 3) & l_1 > 1, \\ l_2 < \frac{3}{2}(l_1 - 1) + \frac{1}{2\gamma}(l_1 - 3) & & l_1 < 1. \end{aligned} \quad (16)$$

As an illustration, let us consider the potential  $V(\phi) = V_0\phi^2$  in a radiation-dominated Universe. In this case,  $\gamma = 1/3$  and expansion of the Universe imply  $l_2 > -6$ . On the other hand, the success of big bang nucleosynthesis gives us strong evidence of the radiation-dominated decelerated phase which leads to  $l_2 < 0$ . Thus a scalar field with a quadratic potential which couples with radiation requires that  $-6 < l_2 < 0$ . Note that the relation (12) indicates that for  $l_1 > 1$  the scalar field is a decreasing function of time ( $m < 0$ ). This implies that the coupling function  $f \propto t^{m l_2}$  increases with time or the matter-chameleon coupling gets stronger as the Universe expands.

This feature is changed in a matter-dominated Universe. For  $\gamma = 0$ , the expression (14) reduces to

$$n = \frac{2l_1}{3(l_1 - 1)}. \quad (17)$$

This relation indicates that  $n > 1$  cannot be realized if  $l_1 < 0$ . On the other hand, when  $l_1 > 0$  the accelerating expansion of the Universe requires that  $1 < l_1 < 3$ . The parameter  $l_2$  does not enter the above condition and it seems that the scalar field effectively decouples from the matter system in a matter-dominated Universe. Exploring the relations (12) and (13) reveals that  $l_2$  also disappears from this relation when we set  $\gamma = 0$ .

As it is evident, the above conditions do not put any constraint on the BD parameter  $\omega$ . To obtain such a constraint, one should consider the condition for which the right-hand side of (10) becomes positive. For  $\gamma = 0$ , the condition is

$$\begin{aligned} V_0\phi_0^{l_1-1} \left[ \frac{2\omega + 3(l_1 - 1)}{2\omega + 3} \right] - \frac{\rho_0 a_0^{-3}(\omega + 3)}{\phi_0(2\omega + 3)} - m^2\omega \\ - \frac{6n}{l_1 - 1} > 0. \end{aligned} \quad (18)$$

At this point, attention is focused on the question of whether it is possible to have accelerated expansion for a large positive  $\omega$ . To answer the question, let us consider the above expression for  $\omega \gg 1$ ,

<sup>1</sup>We assume that the matter system satisfies the weak energy condition  $\gamma + 1 > 0$ .

$$V_0\phi_0^{l_1-1} - \frac{\rho_0 a_0^{-3}}{2\phi_0} - \frac{6n}{l_1 - 1} > m^2\omega. \quad (19)$$

If the left-hand side effectively takes a negative sign then the inequality cannot be satisfied for a positive BD parameter. To make an estimation, we first note that  $\phi_0^{-1} \sim G \sim M_p^{-2}$  and  $\rho_0 a_0^{-3} \sim M$  with  $G$ ,  $M_p$ , and  $M$  being respectively the gravitational constant, the Planck mass, and the total mass content in the Universe. Second, it is an observational fact that matter density of the Universe is of the same order of the critical density  $\rho_c = 3H^2 M_p^2 / 8\pi$  with  $H$  being the Hubble parameter. This leads to a relation between  $M$  and  $H$  such that  $M/M_p^2 \sim R$ , of which  $R \sim H^{-1}$  is the Hubble radius [17]. Substituting these results into (19) gives

$$M_p^{2(l_1-1)} \left( V_0 - \frac{R^{l_1}}{M^{l_1-1}} \right) - \frac{6n}{l_1 - 1} > m^2\omega. \quad (20)$$

Using  $R \sim 10^{26}$  (meter) and  $M \sim 10^{96}$  (meter)<sup>-1,2</sup> one can see that the term containing  $V_0$  is a leading term on the left-hand side of (20) unless  $l_1$  takes values near the lower bound, namely,  $l_1 \approx 1$ . In other words, when  $l_1$  is not close to unity one can write  $\omega < M_p^{2(l_1-1)} V_0 / m^2$  which indicates that the BD parameter can take positive large values. Note that there is no need for fine-tuning of the constant  $V_0$  since the upper bound given by the latter relation is sufficiently large due to the appearance of  $M_p$ . It can be easily checked that for  $l_1 \approx 1$ ,  $R^{l_1}/M^{l_1-1} \approx R \sim 10^{26}$  (meter) and the left-hand side of (20) take a negative sign. In this case, the inequality cannot be satisfied for  $\omega > 0$ . It is interesting to note that the coupling function, or the parameter  $l_2$ , does not play any role in the above argument and it is only the potential function that is relevant. As we have already seen, this is also the case (for  $\gamma = 0$ ) when one enters the arguments concerning the expansion of the Universe in the matter-dominated era. Despite the irrelevant role of the coupling function in the latter arguments, it affects the dynamics of the matter energy density. The evolution law (8) can be rewritten as

$$\rho_m \propto a^{-3(\gamma+1)+\varepsilon}, \quad (21)$$

where

$$\varepsilon = -\frac{m}{n} l_2(\gamma + 1). \quad (22)$$

In fact, the parameter  $\varepsilon$  measures the modification of the matter expansion rate due to its interaction with the scalar field. The relation (21) states that when  $\varepsilon > 0$ , matter is created and energy is constantly injecting into the matter so that the latter will dilute more slowly compared to its standard evolution  $\rho_m \propto a^{-3(\gamma+1)}$ . Similarly, when  $\varepsilon < 0$  the reverse is true, namely, that matter is annihilated and the direction of the energy transfer is outside of the matter

<sup>2</sup>We use the unit system in which  $\hbar = c = 1$ .

system so that the rate of the dilution is faster than the standard one. We may use (13) to write (22) in the form

$$n = \frac{2l_2(\gamma + 1)}{3l_2(\gamma + 1)^2 - \varepsilon(\gamma l_2 + 1)}. \quad (23)$$

When  $m = 0$ ,  $\phi$  takes a constant configuration which is given by  $\phi_0 \sim G^{-1}$  and the model (1) reduces to Einstein gravity with a cosmological term. This case corresponds to  $\varepsilon = 0$  and the solution (23) reduces to  $n = 2/3(\gamma + 1)$ , the case in the standard cosmology. For  $\gamma = 0$ , the expression (23) takes the form

$$n = \frac{2}{3 - \frac{\varepsilon}{l_2}}. \quad (24)$$

One can use this relation to constrain the parameters  $\varepsilon$  and  $l_2$ . We first note that from the modified matter expansion rate (21) we should expect  $|\varepsilon| \ll 1$  since so far there has been no report from observations about an anomalous expansion rate. In other terms, if there exists any deviation from the standard evolution of matter density it must be very small. Taking  $\varepsilon$  as an independent parameter, one can compare the modified expansion rate with observations and perform data fitting to estimate  $|\varepsilon|$ . In fact, it is shown that cosmological observations are consistent with  $|\varepsilon| < 0.1$  [18–20]. This result puts an upper bound on absolute value of the parameter  $l_2$ . The condition for accelerating expansion inferred by (24) is  $\varepsilon/l_2 > 1$  which leads to  $|l_2| < 0.1$ .

On the other hand, the expression (24) implies that the parameters  $\varepsilon$  and  $l_2$  should have the same sign in order that  $n > 1$ . Thus direction of the energy transfer between the matter system and the scalar field is characterized either by  $\varepsilon$  or  $l_2$ . Following [19], we argue that  $\varepsilon > 0$  as required by the second law of thermodynamics. To do this, we should investigate some thermodynamic features of the matter-chameleon coupling described by (1). A thermodynamic description of a perfect fluid matter system requires the knowledge of the particle flux  $N^\alpha = nu^\alpha$  and the entropy flux  $S^\alpha = n\sigma u^\alpha$  where  $n = N/a^3$  and  $\sigma = S/N$  are, respectively, the concentration and the specific entropy (per particle) of the created or annihilated particles. Since the energy density of matter is given by  $\rho_m = nM$  with  $M$  being the mass of each particle, the appearance of the extra term in the energy balance equation (7) means that this extra change of  $\rho_m$  can be attributed to a change of  $n$  or  $M$ . Here we assume that the mass of each matter particle remains constant and the extra term in the energy balance equation only leads to a change of the number density  $n$ . In this case, Eq. (7) can be written as ( $\gamma = 0$ )

$$\dot{n} + 3Hn = n\Gamma, \quad (25)$$

where

$$\Gamma \equiv -\frac{\dot{f}}{f} = \varepsilon \frac{\dot{a}}{a} \quad (26)$$

is the decay rate. We also assume that the overall energy transfer is adiabatic processes in which matter particles are continuously created or annihilated while the specific entropy per particle remains constant during the whole processes ( $\dot{\sigma} = 0$ ) [21]. This means that

$$\frac{\dot{S}}{S} = \frac{\dot{N}}{N} = \Gamma. \quad (27)$$

From  $n \propto a^{-3+\varepsilon}$ , we can see that the total number of particles scales as  $N \propto a^\varepsilon$ , and the second law of thermodynamics  $\dot{S} \geq 0$  implies that  $\varepsilon \geq 0$ . This conclusion can also be drawn by (26) since  $\Gamma \geq 0$  requires that  $\varepsilon \geq 0$  in an expanding Universe. Hence, the chameleon scalar field should suffer energy reduction and the matter system should gain energy during expansion of the Universe if the second law of thermodynamics is to be fulfilled.

#### IV. CONCLUSIONS

In this work we have studied some features of the generalized BD model in which the scalar field is allowed to couple nonminimally with the matter sector. The matter expansion law and the geodesic equation have modified due to this nonminimal coupling. The modification depends on the choice of  $L_m$ , Lagrangian density of the matter system. For instance, in our choice the matter conservation equation is modified while there is no extra force in the geodesic equation.

We have found power-law solutions for the scale factor and the scalar field. In this class of solutions, when  $\gamma = 0$  the accelerating expansion of the Universe can be realized for  $1 < l_1 < 3$ . Our analysis also indicates that this accelerating phase is consistent with a positive and large BD parameter.

The fact that the parameter  $l_2$  does not contribute to the condition for cosmic speedup in the case of  $\gamma = 0$  may be regarded as a signal for an irrelevant role of the function  $f(\phi)$  in the matter-dominated era. However, it is evident from (8) that  $f(\phi)$  is important in the evolution of matter density. We have reformulated the matter expansion law as  $\rho_m \propto a^{-3(\gamma+1)+\varepsilon}$  where the parameter  $\varepsilon$  characterizes both magnitude and direction of the energy transfer. We have argued that such a reformulation has two important consequences. First, since the magnitude of  $\varepsilon$  can be fixed by observation (as it is recently suggested that  $|\varepsilon| < 0.1$ ) accelerating expansion which requires that  $l_2 < \varepsilon$  sets an upper bound on the absolute value of  $l_2$ . Second, the sign of  $\varepsilon$  is restricted by the second law of thermodynamics to assume only positive values so that the direction of energy transfer is into the matter system. This constrains the exponent of the coupling function to take values within the range  $0 < l_2 < 0.1$ .

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