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Composite inflation from super Yang-Mills theory, orientifold, and one-flavor QCD

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Recent investigations have shown that inflation can be driven by four-dimensional strongly interacting theories nonminimally coupled to gravity. We explore this paradigm further by considering composite inflation driven by orientifold field theories. The advantage of using these theories resides in the fact that at large number of colors they feature certain super Yang-Mills properties. In particular, we can use for inflation the bosonic part of the Veneziano-Yankielowicz effective theory. Furthermore, we include the $1/N$ as well as fermion mass corrections at the effective Lagrangian level allowing us to explore the effects of these corrections on the inflationary slow-roll parameters. Additionally, the orientifold field theory with fermionic matter transforming according to the two-index antisymmetric representation for three colors is QCD. Therefore, this model can be interpreted as a new nonminimally coupled QCD theory of inflation. The scale of composite inflation, for all the models presented here, is of the order of 10^{16} GeV. Unitarity studies of the inflaton scattering suggest that the cutoff of the model is at the Planck scale.

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I. INTRODUCTION

Little is known about the mechanism underlying the inflationary physics [1–3] postulated to occur soon after the birth of our Universe. The simplest models of inflation make use of elementary scalar fields. However, the fundamental constituents of space-time are spinors. Scalars can be built out of the fundamental spinors but not vice versa. It is, therefore, interesting to investigate whether the inflaton field can emerge as a composite state of a new strongly interacting gauge theory [4,5]. Holographic models of composite inflation are also being currently investigated [6,7].

To investigate this class of inflationary models one may use low-energy effective theories constrained by the global symmetries of the theory, as well as conformal symmetries. There is an interesting class of models featuring only fermionic and gluonic degrees of freedom for which one can use also supersymmetric relations [8,9] to further constrain the low-energy effective theory. These are gauge theories with fermionic matter transforming according to the two-index representation of the underlying $SU(N)$ gauge dynamics. These theories can be connected to $\mathcal{N} = 1$ super Yang-Mills (SYM) at a large number of colors and are also known as orientifold theories.[1] The field content of the theories is reported in the Table I.

The name of *orientifold field theory* is borrowed from string-theory terminology. In fact, these theories were shown to live on a brane configuration of type 0A string theory [12,13] which consists of NS5 branes, D4 branes, and an *orientifold* plane. The gauge groups in the parent and daughter theories are the same, and so are the gauge couplings.

In Ref. [14], the effective Lagrangians for orientifold theories were constructed in terms of the relevant low-lying color-singlet states. The effective Lagrangians of this type have a long history [15–28] and are known to concisely encode nonperturbative aspects of strongly coupled theories, such as the vacuum structure and symmetries, both exact and anomalous.[2]

We will start by summarizing the low-energy effective Lagrangians [14] and then couple them nonminimally to gravity. Since the bosonic sector of the nonsupersymmetric orientifold field theories at large N maps into the one of SYM, we identify first the gluino-ball state in SYM with the inflaton. We then explore the consequences on the inflationary dynamics. We will then include the $1/N$ corrections as well as the small gluino mass corrections. We investigate the inflationary parameters and check the consistency of our results against the slow-roll conditions and inflaton-inflaton scattering. We discover that the inflationary dynamics to lowest order in $1/N$ is insensitive to corrections coming

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[1] For matter transforming according to the two-index antisymmetric representation it was recognized some time ago [10] that these theories can be viewed as a large N generalization of QCD different from the 't Hooft large N. Yet, another distinct large N generalization of QCD was introduced in Ref. [11].

[2] Among recent developments in this direction was the demonstration of how the information on the center of the SU(N) gauge group (i.e., Z_N) is efficiently transferred to the hadronic states [29]. This demonstration led to a deeper understanding of the deconfining phase transition [30] in pure Yang-Mills theory. When quarks were added, either in the fundamental or in the adjoint representations of the gauge group, a link between the chiral and deconfining phase transitions was uncovered [31].

TABLE I. The fermion sector of the orientifold theories. ψ and $\tilde{\psi}$ are two Weyl fermions, while G_μ stands for the gauge bosons. In the left (right) parts of the table the fermions are in the two-index symmetric (antisymmetric) representation of the gauge group SU(N). $U_V(1)$ is the conserved global symmetry while the $U_A(1)$ symmetry is lost at the quantum level due to the chiral anomaly.

	$SU(N)$	$U_V(1)$	$U_A(1)$
$\psi_{\{ij\}}$	$\Box\Box$	1	1
$\tilde{\psi}^{\{ij\}}$	$\overline{\Box\Box}$	-1	1
G_μ	Adj	0	0

	$SU(N)$	$U_V(1)$	$U_A(1)$
$\psi_{[ij]}$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	1	1
$\tilde{\psi}^{[ij]}$	$\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$	-1	1
G_μ	Adj	0	0

from the axial anomaly sector of the orientifold field theories. However, it does depend on the corrections to the vacuum energy and the gluino mass term. The orientifold field theory with two-index antisymmetric matter and three colors is QCD with one flavor. Therefore, one can interpret the model as a new nonminimally coupled one-flavor QCD inflationary model. For all the models the compositeness scale is shown to be around 10^{16} GeV and the unitarity constraint from inflaton scattering is safely around the Planck scale.

II. NONMINIMAL SUPER YANG-MILLS INFLATION

Before considering the coupling to gravity it is instructive to briefly review the construction of the SYM effective Lagrangian while setting up the notation.

A. Super Yang-Mills effective action and setup

The Lagrangian of SU(N) supersymmetric gluodynamics is[3]

$$\mathcal{L} = \frac{1}{4g^2}\int d^2\theta\,\mathrm{Tr}W^2 + \mathrm{H.c.} = -\frac{1}{4g^2}G^a_{\mu\nu}G^{a\mu\nu} + \frac{1}{2g^2}D^aD^a + \frac{i}{g^2}\lambda^a\sigma^\mu\mathcal{D}_\mu\bar{\lambda}^a, \quad (1)$$

where g is the gauge coupling, $G^a_{\mu\nu}$ is the gauge field strength, λ^a is the gluino field, and D^a the auxiliary field. The vacuum angle is set to zero and

$$\mathrm{Tr}W^2 \equiv \frac{1}{2}W^{a,\alpha}W^a_\alpha = -\frac{1}{2}\lambda^{a,\alpha}\lambda^a_\alpha. \quad (2)$$

The effective Lagrangian in supersymmetric gluodynamics was constructed by Veneziano and Yankielowicz (VY) [23]. In terms of the composite color-singlet chiral superfield S,

$$S = \frac{3}{32\pi^2N}\mathrm{Tr}W^2, \quad (3)$$

it reads

$$\mathcal{L}_{\mathrm{VY}} = \frac{9N^2}{4\alpha}\int d^2\theta d^2\bar{\theta}(SS^\dagger)^{\frac{1}{3}} + \frac{N}{3}\int d^2\theta\left\{S\ln\left(\frac{S}{\Lambda^3}\right)^N - NS\right\} + \mathrm{H.c.}, \quad (4)$$

where Λ is an invariant scale of the theory. The factor N^2 is singled out in the Kähler term to make the parameter α scale as N^0; see Eq. (9) below. The standard definition of the fundamental scale parameter is [32]

$$\Lambda_{\mathrm{st}} = \mu\left(\frac{16\pi^2}{\beta_0 g^2(\mu)}\right)^{\beta_1/\beta_0^2}\exp\left(-\frac{8\pi^2}{\beta_0 g^2(\mu)}\right), \quad (5)$$

which for the SYM theory is exact [33] and reduces to

$$\Lambda^3_{\mathrm{SUSY\ YM}} = \mu^3\left(\frac{16\pi^2}{3Ng^2(\mu)}\right)\exp\left(-\frac{8\pi^2}{Ng^2(\mu)}\right). \quad (6)$$

The exact value of the gluino condensate is due to the holomorphic property of SYM theory and in terms of $\Lambda^3_{\mathrm{SUSY\ YM}}$ reads [34,35]

$$\langle S\rangle = \frac{9}{32\pi^2}\Lambda^3_{\mathrm{SUSY\ YM}}. \quad (7)$$

Comparing with Eq. (4) one deduces that

$$\Lambda^3 = \frac{9}{32\pi^2}\Lambda^3_{\mathrm{SUSY\ YM}} \quad (8)$$

is N independent. The gluino condensate scales as N as it should. To determine the normalization of the constant α, we require the mass of the physical excitations to be N independent,

$$\alpha \sim N^0. \quad (9)$$

Indeed, the common mass of the bosonic and fermionic components of S is $M = 2\alpha\Lambda/3$. The chiral superfield S at the component level has the standard decomposition $S(y) = \varphi(y) + \sqrt{2}\theta\Sigma(y) + \theta^2F(y)$, where y^μ is the chiral coordinate, $y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$, and

[3]The Grassmann integration is defined in such a way that $\int\theta^2d^2\theta = 2$.

$$\begin{aligned} \varphi &= \frac{-3\lambda^{a,\alpha}\lambda^a_\alpha}{64\pi^2 N}, \\ \sqrt{2}\Sigma &= \frac{3}{64\pi^2 N}[G^a_{\alpha\beta}\lambda^{a,\beta} + 2iD^a\lambda^a_\alpha], \\ F &= \frac{3}{64\pi^2 N}\bigg[-\frac{1}{2}G^a_{\mu\nu}G^{a\mu\nu} + \frac{i}{2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} + \text{f.t.}\bigg], \end{aligned} \tag{10}$$

where f.t. stands for (irrelevant) fermion terms.

The complex field φ describes the scalar and pseudoscalar gluino balls while Σ is their fermionic composite partner and the F field is an auxiliary field. To construct the low-energy effective potential one uses the axial and trace anomalies. These are

$$\partial^\mu J_\mu = \frac{N}{16\pi^2}G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}, \qquad J_\mu = -\frac{1}{g^2}\lambda^a\sigma_\mu\bar{\lambda}^a, \tag{11}$$

and

$$\vartheta^\mu_\mu = -\frac{3N}{32\pi^2}G^a_{\mu\nu}G^{a,\mu\nu}, \tag{12}$$

where J_μ is the chiral current and $\vartheta^{\mu\nu}$ is the standard symmetric energy-momentum tensor.

In SYM theory these two anomalies belong to the same supermultiplet [36] and, hence, the coefficients are the same (up to a trivial 3/2 factor due to normalizations). In the orientifold theory, the coefficients of the chiral and scale anomalies coincide only at $N = \infty$; the subleading terms are different.

Summarizing, the component bosonic form of the VY Lagrangian is

$$\begin{aligned} \mathcal{L}_{\rm SYM} &= \frac{N^2}{\alpha}(\varphi\bar{\varphi})^{-\frac{2}{3}}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\bar{\varphi} - V_{\rm SYM}, \\ V_{\rm SYM} &= \frac{4\alpha N^2}{9}(\varphi\bar{\varphi})^{\frac{2}{3}}\ln\Big(\frac{\varphi}{\Lambda^3}\Big)\ln\Big(\frac{\bar{\varphi}}{\Lambda^3}\Big), \end{aligned} \tag{13}$$

with α as the constant. We consider this effective action to be the large N limit of orientifold field theories and neglect all the fermionic degrees of freedom which are supposed to decouple in this limit.

B. Super Yang-Mills nonminimally coupled to gravity

The next step is to take the scalar component part of the superglueball action and couple it nonminimally to gravity. The action in the Jordan frame reads

$$\begin{aligned} \mathcal{S}^{\rm J}_{\rm SYM} = \int d^4x\sqrt{-g}\bigg[&-\frac{\mathcal{M}^2 + N^2\xi(\varphi\bar{\varphi})^{\frac{1}{3}}}{2}g^{\mu\nu}R_{\mu\nu} \\ &+ \frac{N^2}{\alpha}(\varphi\bar{\varphi})^{-\frac{2}{3}}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\bar{\varphi} - V_{\rm SGI}\bigg]. \end{aligned} \tag{14}$$

We focus on the modulus of φ which we shall continue to call φ. The gravity-composite dynamics model is diagonalized by imposing a conformal transformation,

$$g_{\mu\nu} \to \Omega^2 g_{\mu\nu}, \qquad \Omega^2 = \frac{\mathcal{M}^2 + N^2\xi\varphi^{\frac{2}{3}}}{M_{\rm P}^2}. \tag{15}$$

We then land in the Einstein frame and the action reads

$$\begin{aligned} \mathcal{S}^{\rm E}_{\rm SYM} = \int d^4x\sqrt{-g}\bigg[&-\frac{M_{\rm P}^2}{2}g^{\mu\nu}R_{\mu\nu} \\ &+ \frac{N^2}{\alpha}\Omega^{-2}\Big(1 + \alpha f\frac{N^2\xi^2}{3M_{\rm P}^2}\Omega^{-2}\varphi^{\frac{2}{3}}\Big)g^{\mu\nu}\varphi^{-\frac{4}{3}}\partial_\mu\varphi\partial_\nu\varphi \\ &- \Omega^{-4}V_{\rm SYM}(\varphi)\bigg], \end{aligned} \tag{16}$$

where $f = 1(0)$ is the metric (Palatini) formulation and

$$V_{\rm SYM}(\varphi) = \frac{4\alpha N^2}{9}\varphi^{\frac{4}{3}}\Big(\ln\Big(\frac{\varphi}{\Lambda^3}\Big)\Big)^2. \tag{17}$$

We now introduce a canonically normalized field χ related to φ via

$$\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\chi(\varphi)\partial_\nu\chi(\varphi) = \frac{1}{2}\Big(\frac{d\chi}{d\varphi}\Big)^2\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi, \tag{18}$$

with

$$\frac{1}{2}\Big(\frac{d\chi}{d\varphi}\Big)^2 = \frac{N^2}{\alpha}\Omega^{-2}\Big(1 + \alpha f\frac{N^2\xi^2}{3M_{\rm P}^2}\Omega^{-2}\varphi^{\frac{2}{3}}\Big)\varphi^{-\frac{4}{3}}. \tag{19}$$

In terms of the canonically normalized field we have

$$\begin{aligned} \mathcal{S}^{\rm E}_{\rm SYM} = \int d^4x\sqrt{-g}\bigg[&-\frac{1}{2}M_{\rm P}^2 g^{\mu\nu}R_{\mu\nu} \\ &+ \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - U(\chi)\bigg], \end{aligned} \tag{20}$$

with

$$U(\chi) \equiv \Omega^{-4}V_{\rm SYM}(\varphi). \tag{21}$$

C. Slow-roll parameters for nonminimally coupled super Yang-Mills

We will analyze the dynamics in the Einstein frame and, therefore, define the slow-roll parameters in terms of U and χ,

$$\begin{aligned} \epsilon &= \frac{M_{\rm P}^2}{2}\Big(\frac{dU/d\chi}{U}\Big)^2, \qquad \eta = M_{\rm P}^2\Big(\frac{d^2U/d\chi^2}{U}\Big), \\ \mathcal{N} &= \frac{1}{M_{\rm P}^2}\int_{\chi_{\rm end}}^{\chi_{\rm Ini}}\frac{U}{dU/d\chi}d\chi. \end{aligned} \tag{22}$$

We consider here the large field regime, i.e.,

$$\varphi^{\frac{2}{3}} \gg \frac{\mathcal{M}^2}{N^2\xi}. \tag{23}$$

In this limit ϵ becomes, in the φ variable,

$$\epsilon_{\rm SYM} \simeq \frac{1}{(\ln(\frac{\varphi}{\Lambda^3}))^2((\alpha\xi)^{-1} + f\cdot\frac{1}{3})}. \tag{24}$$

Inflation ends when $\epsilon_{\rm SYM} = 1$, such that

$$\frac{\varphi_{\rm end}^{\rm SYM}}{\Lambda^3} = \exp\left(\frac{1}{\sqrt{((\alpha\xi)^{-1} + f \cdot \frac{1}{3})}}\right). \quad (25)$$

In the large field limit the number of e-folding is

$$\mathcal{N} \simeq \frac{1}{2}\left[\left((\alpha\xi)^{-1} + f \cdot \frac{1}{3}\right)\left(\ln\left(\frac{\varphi}{\Lambda^3}\right)\right)^2\right]_{\varphi_{\rm end}}^{\varphi_{\rm ini}}. \quad (26)$$

A simple way to determine the value of $\varphi_{\rm ini}$ associated to when inflation starts is to require a minimal number of e-foldings compatible with a successful inflation, i.e., $\mathcal{N} = 60$. This leads to

$$\frac{\varphi_{\rm ini}^{\rm SYM}}{\Lambda^3} \simeq \exp\left(\sqrt{\frac{121}{(\alpha\xi)^{-1} + f \cdot \frac{1}{3}}}\right). \quad (27)$$

Further relevant information can be extracted using the WMAP [37] normalization condition,

$$\frac{U_{\rm ini}}{\epsilon_{\rm ini}^{\rm SYM}} = (0.0276 M_{\rm P})^4. \quad (28)$$

The label *ini* signifies that this expression has to be evaluated at the beginning of the inflationary period. This condition helps estimating the magnitude of the nonminimal coupling. We deduce

$$U_{\rm ini} \simeq \frac{4\alpha M_{\rm P}^4}{9N^2\xi^2}\left(\ln\left(\frac{\varphi_{\rm ini}^{\rm SYM}}{\Lambda^3}\right)\right)^2 \simeq \frac{4\alpha M_{\rm P}^4}{9N^2\xi^2}\left(\frac{121}{(\alpha\xi)^{-1} + f \cdot \frac{1}{3}}\right), \quad (29)$$

and

$$\epsilon_{\rm ini}^{\rm SYM} \simeq \frac{1}{\left(\ln\left(\frac{\varphi_{\rm ini}^{\rm SYM}}{\Lambda^3}\right)\right)^2\left((\alpha\xi)^{-1} + f \cdot \frac{1}{3}\right)} \simeq \frac{1}{\left(\frac{121}{(\alpha\xi)^{-1} + f \cdot \frac{1}{3}}\right)\left((\alpha\xi)^{-1} + f \cdot \frac{1}{3}\right)} = 0.0083. \quad (30)$$

We can, therefore, determine the magnitude of the nonminimal coupling which depends, in principle, on whether we use the Palatini ($f = 0$) or the metric formulation ($f = 1$). In the case of the Palatini formulation, we have

$$N^2\xi \simeq 1.1 \times 10^{10}\alpha^2 \quad \text{Palatini}. \quad (31)$$

The situation for the metric case turns out to be subtle because of the interplay between the structure of the nonminimal coupling to gravity and the large N counting. In the limit in which $(\alpha\xi)^{-1} \ll \frac{1}{3}$, we find

$$N\xi \simeq 1.83 \times 10^5\sqrt{\alpha} \quad \text{Metric}. \quad (32)$$

With α of order unity we can still allow for relatively large values of N satisfying (32). The phenomenologically large value of ξ is common to the case of Higgs inflation [38], and other earlier approaches [39–44]. A more complete treatment for all these models would require to discover in the future a mechanism for generating such a large coupling.

The knowledge of the nonminimal coupling allows us to estimate the initial and final value of the composite glueball field φ, which reads

$$(\varphi_{\rm end}^{\rm SYM})^{\frac{1}{3}} \sim e^{\frac{\sqrt{\alpha N^2\xi}}{3N}}\Lambda, \quad (\varphi_{\rm ini}^{\rm SYM})^{\frac{1}{3}} \sim e^{\frac{\sqrt{121\alpha N^2\xi}}{3N}}\Lambda \quad \text{Palatini}$$
$$(\varphi_{\rm end}^{\rm SYM})^{\frac{1}{3}} \sim 1.8\Lambda, \quad (\varphi_{\rm ini}^{\rm SYM})^{\frac{1}{3}} \sim 570\Lambda \quad \text{Metric with} \quad (\alpha\xi)^{-1} \ll \frac{1}{3}. \quad (33)$$

In the large N limit, also the Palatini formulation leads to initial and final values for φ within a few times Λ.

It is possible to further relate the strongly coupled scale Λ with the Planck mass recalling that in the large field regime (23) we expect that on/near the ground state, we have $N^2\xi\Lambda^2 \simeq M_{\rm P}^2$. This corresponds to assuming that on the vacuum $\Omega = 1$. Assuming for the reduced Planck mass the value 2.44×10^{18} GeV, we obtain

$$\Lambda \simeq \frac{0.57}{\sqrt{N}\alpha^{1/4}} \times 10^{16}\ \text{GeV}. \quad (34)$$

These results are encouraging and indicate that it is possible to conceive an inflationary scenario driven by a SYM-like composite inflaton. This value is not only consistent with the results found in Refs. [4,5] but shows that it is possible to lower the scale of composite inflation by increasing the number of underlying colors. We recall that α is given by the underlying theory and is expected to be of order unity [45].

D. Inflaton scattering and its unitarity constraint

Next, we turn to the interesting question of the constraints set by tree-level unitarity of the inflaton field. According to the potential given above, the ground state reads

$$\langle\varphi\rangle = \Lambda^3. \quad (35)$$

It is worth noting here that the potential evaluated on the ground state has zero energy. In addition, we are interested in the large field regime which can be well approximated by setting $\mathcal{M} = 0$. At the minimum of the potential, the following relation naturally holds:

$$M_{\rm P}^2 \simeq N^2\xi\Lambda^2 \Rightarrow \Omega = \frac{\varphi^{\frac{1}{3}}}{\Lambda}. \quad (36)$$

For later convenience in this section, we introduce the field ϕ possessing unity canonical dimension related to φ as follows:

$$\varphi = \phi^3. \quad (37)$$

In the Einstein frame, we obtain

$$\mathcal{S}^{\rm E}_{\rm SYM} = \int d^4x\sqrt{-g}\bigg[-\frac{M_{\rm P}^2}{2}g^{\mu\nu}R_{\mu\nu} + \frac{9N^2}{2\alpha}\frac{\Lambda^2}{\phi^2}\Big(2+\frac{2}{3}f\alpha\xi\Big)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 4N^2\alpha\Lambda^4\Big[\ln\Big(\frac{\phi}{\Lambda}\Big)\Big]^2\bigg]. \tag{38}$$

Violation of tree-level unitarity of the scattering amplitude concerns the inflaton field fluctuations $\delta\phi$ around its classical time-dependent background $\phi_c(t)$ during the inflationary period

$$\phi(\vec{x},t) = \phi_c(t) + \delta\phi(\vec{x},t). \tag{39}$$

In first approximation it is possible to neglect the time dependence of the classical field and write

$$\phi(\vec{x}) = \phi_c + \delta\phi(\vec{x}). \tag{40}$$

To estimate the actual cutoff of the tree-level scattering amplitude we analyze independently the kinetic and potential terms for the inflaton in the Einstein frame. Expanding the kinetic term around the classical background, we obtain

$$\frac{3N^2}{2\alpha}\frac{\Lambda^2}{\phi_c^2}(6+2f\alpha\xi)(\partial\delta\phi)^2\sum_{n=0}^{\infty}(n+1)\frac{(-\delta\phi)^n}{\phi_c^n}. \tag{41}$$

It is possible to canonically normalize the first term of the series, i.e., the kinetic term for a free field, by rescaling the fluctuations as follows:

$$\frac{\delta\phi}{\phi_c} = \frac{\sqrt{\alpha}\delta\tilde{\phi}}{\sqrt{3}N\Lambda\sqrt{(6+2f\alpha\xi)}}. \tag{42}$$

Under this field redefinition, we find

$$\frac{1}{2}(\partial\delta\tilde{\phi})^2\sum_{n=0}^{\infty}(n+1)\frac{(-\sqrt{\alpha}\delta\tilde{\phi})^n}{(18+6f\alpha\xi)^{\frac{n}{2}}(N\Lambda)^n}. \tag{43}$$

For the potential term the higher-order operators are also, respectively, of the form

$$\frac{(\sqrt{\alpha}\delta\tilde{\phi})^n}{(18+6f\alpha\xi)^{\frac{n}{2}}(N\Lambda)^n}. \tag{44}$$

This implies that the tree-level cutoff for unitarity is

$$\sqrt{\frac{18+6f\alpha\xi}{\alpha}}N\Lambda. \tag{45}$$

This result shows that the cutoff is background independent. In the metric formulation, the cutoff is $N\Lambda\sqrt{6\xi} \sim \sqrt{6}M_{\rm P}$, i.e., a little higher than the Planck scale. Such a very high cutoff means that unitarity does not constrain the range of validity of the effective theory for composite inflation, which is limited, however, from the compositeness scale of the theory.

III. ORIENTIFOLD INFLATION

We wish now to deform the supersymmetric effective action to describe inflation driven by the gauge dynamics of SU(N) gauge theories with one Dirac fermion in either the two-index antisymmetric or symmetric representation of the gauge group. Following [14] we start by recalling the trace and axial anomalies for the orientifold theories,

$$\vartheta^\mu_\mu = 2N\Big[N\pm\frac{4}{9}\Big](F+\bar{F}) = -3\Big[N\pm\frac{4}{9}\Big]\frac{1}{32\pi^2}G^a_{\mu\nu}G^{a,\mu\nu}, \tag{46}$$

$$\partial^\mu J_\mu = i\frac{4N}{3}[N\mp 2](\bar{F}-F) = [N\mp 2]\frac{1}{16\pi^2}G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}, \tag{47}$$

where the top (bottom) sign is for the antysimmetric (symmetric) theory and

$$\varphi = -\frac{3}{32\pi^2 N}\tilde{\psi}^{\alpha,[i,j]_\mp}\psi_{\alpha,[i,j]_\mp}, \tag{48}$$

and F is given in Eq. (10). The gluino field of supersymmetric gluodynamics is replaced here by two Weyl fields, $\tilde{\psi}^{\alpha,[i,j]_\mp}$ and $\psi_{\alpha,[i,j]_\mp}$, which can be combined into one Dirac spinor. The top (bottom) sign for the bracket in $\tilde{\psi}^{\alpha,[i,j]_\mp}$ indicates antisymmetric (symmetric) color indices. The color-singlet field φ is bilinear in $\tilde{\psi}^{\alpha,[i,j]_\mp}$ and $\psi_{\alpha,[i,j]_\mp}$.

A. $1/N$: Orientifold effective action and then inflation

In this limit we can drop subleading $1/N$ terms in the expressions for the trace and chiral anomaly. Then it is clear that the anomalous currents map into the ones of SYM. Therefore, the effective action built to saturate at the tree-level trace and axial anomaly has the same form as in Eq. (13). Hence, by keeping only the leading-N terms only, one recovers the supersymmetry-based bosonic properties, i.e., degeneracy of the opposite-parity mesons and the vanishing of the vacuum energy. Of course in this limit the symmetric and the antisymmetric orientifold theories are indistinguishable.

To parametrize the $1/N$ corrections at the effective Lagrangian level, we use the results of Sannino and Shifman [14] and write

$$\mathcal{L}_{\rm OI} = \mathcal{F}(N)\bigg\{\frac{1}{\alpha}(\varphi\bar{\varphi})^{-2/3}\partial_\mu\bar{\varphi}\partial^\mu\varphi - \frac{4\alpha}{9}(\varphi\bar{\varphi})^{2/3}(\ln\bar{\Phi}\ln\Phi - \beta)\bigg\}, \tag{49}$$

where β is a numerical real and positive [14] parameter and the field Φ and its complex conjugate are defined in (52),

$$\beta = O(1/N), \tag{50}$$

and

$$\mathcal{F}(N) = N^2(1+\beta') \quad \text{with} \quad \beta' = \mathcal{O}(1/N). \tag{51}$$

However, the sign of β' is unknown. In Ref. [14], one did not have to take into account the leading $1/N$ corrections to $\mathcal{F}(N)$ since this function drops out from any physical quantity. However, when coupled to gravity these corrections cannot be neglected. We have also

$$\Phi = \varphi^{1+\epsilon_1}\bar{\varphi}^{-\epsilon_2}, \qquad \bar{\Phi} = \bar{\varphi}^{1+\epsilon_1}\varphi^{-\epsilon_2}, \tag{52}$$

where $\epsilon_{1,2}$ are parameters of order $O(1/N)$,

$$\epsilon_1 = \mp\frac{7}{9N}, \qquad \epsilon_2 = \mp\frac{11}{9N}. \tag{53}$$

The top (bottom) sign corresponds to the two-index antisymmetric (symmetric) theory. The scale and chiral dimensions of $\bar{\Phi}$ and Φ are engineered to saturate the axial and trace anomalies for the orientifold theories.

For the purpose of investigating the inflationary paradigm we restrict the potential to the real part of the field φ and write

$$\mathcal{L}_{\rm OI} \to \frac{\mathcal{F}(N)}{\alpha}(\varphi)^{-4/3}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V_{\rm OI}(\varphi), \tag{54}$$

with

$$V_{\rm OI}(\varphi) = \frac{4\alpha'}{9}\varphi^{\frac{4}{3}}\left(\ln\left(\frac{\varphi}{\Lambda^3}\right)^2 - \frac{\beta}{(1+\epsilon_A)^2}\right),$$
$$\alpha' \equiv \mathcal{F}(N)\alpha(1+\epsilon_A)^2, \qquad \epsilon_A \equiv \epsilon_1 - \epsilon_2. \tag{55}$$

To leading order in $1/N$, we have

$$\alpha' = N^2\alpha(1 + 2\epsilon_A + \beta' + \mathcal{O}(1/N^2)),$$
$$\frac{\beta}{(1+\epsilon_A^2)} = \beta + \mathcal{O}(1/N^2). \tag{56}$$

Adding gravity, we have

$$\mathcal{S}^{\rm J}_{\rm OI} = \int d^4x\sqrt{-g}\left[-\frac{\mathcal{M}^2 + N^2\xi\varphi^{\frac{2}{3}}}{2}g^{\mu\nu}R_{\mu\nu} + \mathcal{L}_{\rm OI}\right]. \tag{57}$$

For the nonminimal coupling to gravity we have assumed, for simplicity, the same as used for SYM. Neglected terms in $1/N$ in the nonminimal coupling could be re-absorbed in a redefinition of the function $\mathcal{F}$. With this choice the only $1/N$ corrections come from the gauge sector.

The action in the Einstein frame reads

$$\mathcal{S}^{\rm E}_{\rm OI} = \int d^4x\sqrt{-g}\left[-\frac{M_{\rm P}^2}{2}g^{\mu\nu}R_{\mu\nu} + \frac{\mathcal{F}(N)}{\alpha}\Omega^{-2}\left(1 + \frac{\alpha f N^4}{\mathcal{F}(N)}\frac{\xi^2}{3M_{\rm P}^2}\Omega^{-2}\varphi^{\frac{2}{3}}\right) \times g^{\mu\nu}\varphi^{-\frac{4}{3}}\partial_\mu\varphi\partial_\nu\varphi - \Omega^{-4}V_{\rm OI}(\varphi)\right], \tag{58}$$

with the same Ω as in the SYM case.

B. Orientifold slow roll parameters

In the large field regime $\varphi^{2/3} \gg \frac{\mathcal{M}^2}{N^2\xi}$, we obtain

$$\epsilon_{\rm OI} \simeq \epsilon_{\rm SYM}\left[1 + \frac{2}{(\ln\frac{\varphi}{\Lambda^3})^2}\beta - \frac{3}{3+f\alpha\xi}\beta'\right], \quad \text{with}$$
$$\epsilon_{\rm SYM} = \frac{1}{(\ln\frac{\varphi}{\Lambda^3})^2((\alpha\xi)^{-1} + \frac{f}{3})}, \tag{59}$$

and $(\alpha\xi)^{-1} = \xi^{-1}/\alpha$ defined first in (24). The value of the field at the end of inflation can be determined by setting $\epsilon_{\rm OI}(\varphi_{\rm end}) = 1$. We use perturbation theory in the small parameters β and β' and search for a solution to this condition of the type,

$$\varphi_{\rm end} = \varphi_{\rm end}^{\rm SYM} + \beta\varphi_1 + \beta'\varphi_1', \quad \text{with}$$
$$\varphi_{\rm end}^{\rm SYM} = \Lambda^3 e^{\frac{\sqrt{3\alpha\xi}}{\sqrt{3+f\alpha\xi}}}. \tag{60}$$

The solution reads

$$\varphi_{\rm end} = \varphi_{\rm end}^{\rm SYM}\left[1 - \frac{1}{2\alpha\xi((\alpha\xi)^{-1} + \frac{f}{3})^{3/2}}\beta' + \sqrt{(\alpha\xi)^{-1} + \frac{f}{3}\beta}\right]. \tag{61}$$

The number of e-foldings reads

$$\mathcal{N}_{\rm OI} \simeq \left[\frac{(\ln\frac{\varphi}{\Lambda^3})^2}{2\alpha\xi}\left(1 - \frac{2\ln\ln\frac{\varphi}{\Lambda^3}}{(\ln\frac{\varphi}{\Lambda^3})^2}\beta + \beta'\right)\right]_{\varphi_{\rm end}}^{\varphi_{\rm ini}}. \tag{62}$$

We fix the initial value of the inflaton field $\varphi_{\rm ini}$ by requiring a total of 60 e-foldings during inflation and obtain

$$\varphi_{\rm ini} = \varphi_{\rm ini}^{\rm SYM}\left[1 + (1+\ln(11))\sqrt{(\alpha\xi)^{-1} + \frac{f}{3}}\frac{\beta}{11} - \frac{11}{2\alpha\xi((\alpha\xi)^{-1} + \frac{f}{3})^{3/2}}\beta'\right]. \tag{63}$$

For $(\alpha\xi)^{-1} \ll 1/3$ and in the metric case we get the following range for inflation:

$$\frac{\varphi_{\rm ini}^{1/3}}{\Lambda} \simeq 570(1 + 0.06\beta),$$
$$\frac{\varphi_{\rm end}^{1/3}}{\Lambda} \simeq 1.8(1 + 0.19\beta) \quad \text{Metric.} \tag{64}$$

To take the limit above we have assumed $\alpha\xi$ large at any N although strictly speaking at extremely large N this approximation may break down. However, for any large but finite N we expect this result to hold.

Perhaps the most relevant result is that the slow-roll parameters are insensitive to the breaking of holomorphicity induced by ϵ_A, i.e., the corrections coming from the mismatch between trace and axial anomaly coefficients.

The irrelevance of ϵ_A is due to the fact that all the slow-roll parameters are defined as ratios of derivatives of the potential divided by the potential itself. And ϵ_A appears only in a function multiplying the overall potential to leading order in $1/N$. Therefore, the results are valid for both orientifold field theories.

IV. ONE-FLAVOR QCD INFLATION

Another way to depart from a supersymmetric theory is to add soft supersymmetric breaking operators such as the mass for the gluino. Following Masiero and Veneziano [46] one can, therefore, add the gluino mass term

$$\Delta\mathcal{L}_m = -\frac{m}{2g^2}\lambda^\alpha\lambda_\alpha + \text{H.c.} \tag{65}$$

At the effective Lagrangian level, it reads

$$\Delta\mathcal{L}_m = 4\frac{m}{2\lambda}N^2(\varphi + \bar\varphi) \tag{66}$$

with $\lambda \equiv g^2N/8\pi^2$ the 't Hooft coupling. We assume here that the mass parameter is real and positive. If this were not the case one can render it real and positive by redefining the vacuum angle θ. We will also assume that softness condition $m/\lambda \ll \Lambda$. One can, however, start immediately from the orientifold theory where the mass term reads

$$\Delta\mathcal{L}_m = -\frac{m}{g^2}\psi^\alpha\tilde\psi_\alpha + \text{H.c.} \tag{67}$$

The color indices for the gluino and the orientifold field theories are (implicitly) contracted to obtain color singlet operators, while the spin indices are explicit and contracted. In the large N limit this term, at the effective Lagrangian level, maps exactly in the one above [14]. Since for three colors the orientifold field theory with antisymmetric matter is QCD with one flavor we can, therefore, study nonminimally coupled inflation driven by a QCD-like theory even featuring a light fermion mass.

The effective Lagrangian augmented with the quark mass reads [14]

$$\mathcal{L}_{1F-QCD} = \mathcal{F}(N)\Big[\frac{1}{\alpha}(\varphi\bar\varphi)^{-2/3}g^{\mu\nu}\partial_\mu\bar\varphi\partial_\nu\varphi - \frac{4\alpha}{9}(\varphi\bar\varphi)^{2/3}(\ln\bar\Phi\ln\Phi - \beta)\Big] + \frac{4m}{3\lambda}N^2(\varphi+\bar\varphi), \tag{68}$$

where

$$\Phi = \varphi^{1+\epsilon_1}\bar\varphi^{-\epsilon_2}, \qquad \bar\Phi = \bar\varphi^{1+\epsilon_1}\varphi^{-\epsilon_2}, \qquad \epsilon_1 = \frac{7}{9N}, \qquad \epsilon_2 = \frac{11}{2N}, \qquad \beta = \mathcal{O}(1/N), \tag{69}$$

$$V_{1F-QCD} = \frac{4\alpha'}{9}\varphi^{\frac{4}{3}}\Big(\ln\Big(\frac{\varphi}{\Lambda^3}\Big)^2 - \beta\Big) - \frac{8N^2m}{3\lambda}\varphi. \tag{70}$$

Here α' assumes the same form of (56). The associated slow-roll parameter epsilon expanded at the leading order in β, β' [defined in (51)], ϵ_A, and $\frac{m}{\lambda}$ reads

$$\epsilon_{1F-QCD} \simeq \epsilon_{SYM}\Big[1 + \frac{2}{(\ln\frac{\varphi}{\Lambda^3})^2}\beta - \frac{3}{3+f\alpha\xi}\beta' + \frac{2(6+\ln(\frac{\varphi}{\Lambda^3}))}{\varphi^{1/3}\alpha(\ln(\frac{\varphi}{\Lambda^3}))^2}\frac{m}{\lambda}\Big]. \tag{71}$$

The dependence on $\epsilon_A \equiv \epsilon_1 - \epsilon_2$ is subleading to the order we are investigating and, therefore, does not appear here. This is so since it would necessarily come multiplied by $\frac{m}{\lambda}$.

Imposing that the end of inflation occurs for $\epsilon_{1F-QCD} = 1$ we obtain to the first order in β, β', and $\frac{m}{\lambda}$,

$$\varphi_{\rm end} = \varphi_{\rm end}^{\rm SYM}\Big[1 - \frac{1}{2\alpha\xi((\alpha\xi)^{-1}+\frac{f}{3})^{3/2}}\beta' + \sqrt{(\alpha\xi)^{-1}+\frac{f}{3}}\beta + \frac{\alpha^{-1}}{\exp[(3\sqrt{\xi^{-1}+\frac{f}{3}})^{-1}]}\times\Big(1+6\sqrt{\xi^{-1}+\frac{f}{3}}\Big)\frac{m}{\lambda}\Big]. \tag{72}$$

In the metric formulation it collapses to:

$$\varphi_{\rm end} = \Lambda^3 e^{\sqrt{3}}\Big[1 + \frac{\beta}{\sqrt{3}} + \frac{(1+2\sqrt{3})}{\alpha e^{\frac{1}{\sqrt{3}}}}\frac{m}{\lambda}\Big] \quad \text{Metric}, \tag{73}$$

anticipating that ξ assumes a very large value.

However, for the initial value of the inflaton, obtained by requiring 60 e-foldings, we get the same results for the coefficients of β and β' as in (63) while the coefficient for $\frac{m}{\lambda}$ is cumbersome to write down explicitly. We, therefore, opt for providing the result directly in the metric formulation which reads

$$\varphi_{\rm ini} \simeq \varphi_{\rm ini}^{\rm SYM}\Big[1 + \frac{1+\ln(11)}{11\sqrt{3}}\beta + \frac{0.46}{\alpha}\frac{m}{\lambda}\Big]. \tag{74}$$

In the metric approach we have the following range for inflation:

$$\frac{\varphi_{\rm ini}^{1/3}}{\Lambda} \simeq 570\Big(1 + 0.06\beta + \frac{0.15}{\alpha}\frac{m}{\lambda}\Big), \qquad \frac{\varphi_{\rm end}^{1/3}}{\Lambda} \simeq 1.8\Big(1 + 0.19\beta + \frac{0.83}{\alpha}\frac{m}{\lambda}\Big) \quad \text{Metric}. \tag{75}$$

These results show that the inflationary slow-roll parameters are not sensitive to the axial anomaly departure from the supersymmetric limit. Furthermore, we learn that the effects of a small nonzero negative vacuum energy induced by the presence of a positive and real β term, of order $1/N$, leads to higher values of the inflaton field with respect to the SYM values. Finally the effects of a quark mass are similar to the $1/N$ corrections, albeit the sign of the corrections are sensitive to the θ angle choice. Here we have chosen a value of θ leading to the same sign of the β term.

V. CONCLUSIONS

We explored the paradigm of a nonminimally coupled composite inflation further by considering orientifold field theories. We have shown that the advantage of using these theories is that at large number of colors they share certain super Yang-Mills properties. Due to these properties we were able to use for inflation the bosonic part of the Veneziano-Yankielowicz effective theory. We have include the $1/N$ and fermion mass corrections at the effective Lagrangian level. This allowed us to determine the associated corrections on the inflationary slow-roll parameters. Additionally, we showed that the scale of composite orientifold inflation is of the order of 10^{16} GeV and that unitarity for inflaton scattering leads to a cutoff at the Planck scale.

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