Effects of a *CPT*-even and Lorentz-violating nonminimal coupling on electron-positron scattering

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We propose a new *CPT*-even and Lorentz-violating nonminimal coupling between fermions and Abelian gauge fields involving the *CPT*-even tensor $(K_F)_{\mu\nu\alpha\beta}$ of the standard model extension. We thus investigate its effects on the cross section of the electron-positron scattering by analyzing the process $e^+ + e^- \longrightarrow \mu^+ + \mu^-$. Such a study was performed for the parity-odd and parity-even nonbirefringent components of the Lorentz-violating $(K_F)_{\mu\nu\alpha\beta}$ tensor. Finally, by using experimental data available in the literature, we have imposed upper bounds as tight as 10^{-12} $(\mathrm{eV})^{-1}$ on the magnitude of the *CPT*-even and Lorentz-violating parameters while nonminimally coupled.

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I. INTRODUCTION

The standard model extension (SME) is a large theoretical framework that includes terms of Lorentz and *CPT* violation in the structure of the usual standard model [1]. This model was proposed after the verification about the possibility of having spontaneous violation of Lorentz symmetry in the context of string theories [2]. The Lorentz-violating (LV) terms are generated as vacuum expectation values of tensor quantities, keeping the coordinate invariance of the extended theory [3]. This model has been scrutinized in many respects in the latest years, with studies embracing the fermion and gauge sectors, and gravitation extension [4]. The fermion sector [5] was much examined, mainly in connection with *CPT*-violating tests to impose some upper bounds on the magnitude of the LV terms [6], dealing with other interesting aspects as well [7]. The Abelian gauge sector of the SME is composed of a *CPT*-odd [8] and a *CPT*-even sector, both intensively investigated in the latest years [9–15].

Besides the investigations undertaken into the structure of the SME, some other works were proposed to examine Lorentz-violating developments out of this broad framework. Some of them involve nonminimal coupling terms that modify the vertex interaction between fermions and photons. *CPT*-odd nonminimal couplings as $gv_\mu \tilde{F}^{\mu\nu}$ and $g\gamma_5 b_\mu \tilde{F}^{\mu\nu}$ were considered some time ago in the context of the Dirac equation, with interesting consequences in the nonrelativistic limit, involving topological phases [16,17], corrections on the hydrogen spectrum [18]. Such nonminimal coupling has been reassessed in connection with its implications on the Aharonov-Bohm-Casher problem [19], the Bhabha cross section [20], and other respects [21]. Recently, other types of nonminimal coupling, defined in the context of the Dirac equation, have been proposed for investigating the generation of topological and geometrical phases [22].

Theoretical studies about cross section evaluation in the presence of Lorentz-violating terms were accomplished by some authors [23], searching to elucidate the route for evaluating the cross section for a general scattering. Very recently, some authors performed a study on the Bhabha scattering [20], determining the effects induced by the nonminimal *CPT*-odd coupling on the Bhabha cross section. The results were compared with some available data concerning this scattering [24] and used to impose the upper bound $|gv_\mu| \leq 10^{-12}$ $(\mathrm{eV})^{-1}$.

In this work, we reassess a well-known quantum electrodynamics process, the $e^+ + e^- \longrightarrow \mu^+ + \mu^-$ scattering, in the presence of a new Lorentz violating *CPT*-even nonminimal coupling involving the fermion and gauge sectors. First, we calculate the scattering amplitude, considering new Feynman diagrams due to the emergence of a new vertex in the theory. In order to evaluate the total cross section, we first calculate the unpolarized squared amplitude, using the Casimir trick. We specialize our evaluations for the parity-odd and parity-even subsectors of the *CPT*-even gauge sector. At the end, following the approach of Refs. [20,24], we compare the cross section results with the experimental data, finding an upper limit for the magnitude for the new nonminimal coupling as tight as $|\lambda(K_F)| \leq 10^{-12}$ $(\mathrm{eV})^{-1}$.

II. THE THEORETICAL MODEL

We are interested in analyzing some aspects of a modified quantum electrodynamics, whose fermion sector is governed by the generalized Dirac equation,

$$(i\gamma^\mu D_\mu - m)\Psi = 0, \tag{1}$$

in which the usual covariant derivative is supplemented by a nonminimal *CPT*-even coupling term, that is,

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$$D_\mu = \partial_\mu + ieA_\mu + \frac{\lambda}{2}(K_F)_{\mu\nu\alpha\beta}\gamma^\nu F^{\alpha\beta}, \tag{2}$$

where $(K_F)_{\mu\nu\alpha\beta}$ is the tensor that embraces the 19 LV terms belonging to the *CPT*-even gauge sector of the SME. This tensor possesses the same symmetries of the Riemann tensor: $(K_F)_{\alpha\nu\rho\varphi} = -(K_F)_{\nu\alpha\rho\varphi}$, $(K_F)_{\alpha\nu\rho\varphi} = -(K_F)_{\alpha\nu\varphi\rho}$, $(K_F)_{\alpha\nu\rho\varphi} = (K_F)_{\rho\varphi\alpha\nu}$, and a double null trace, $(K_F)^{\alpha\beta}{}_{\alpha\beta} = 0$, implying 19 components. Using the symmetries of the tensor $(K_F)_{\mu\nu\alpha\beta}$ in the Dirac (1) equation, one obtains

$$\left[i\gamma^\mu\partial_\mu - e\gamma^\mu A_\mu + \frac{\lambda}{2}(K_F)_{\mu\nu\alpha\beta}\sigma^{\mu\nu}F^{\alpha\beta} - m\right]\Psi = 0, \tag{3}$$

with

$$\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu) = \frac{i}{2}[\gamma^\mu, \gamma^\nu], \tag{4}$$

whose components, σ^{0i} and σ^{ij}, are

$$\sigma^{0i} = i\begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}, \qquad \sigma^{ij} = -\begin{pmatrix} \epsilon_{ijk}\sigma^k & 0 \\ 0 & \epsilon_{ijk}\sigma^k \end{pmatrix}. \tag{5}$$

This new coupling, represented by $(\lambda K_F)_{\mu\nu\alpha\beta}$, has mass dimension $[\lambda K_F] = -1$, which leads to a nonrenormalizable theory at power counting. This respect does not pose a problem for this investigation, once we are interested in analyzing the tree-level scattering process.

We now present the Lagrangian of the modified QED,

$$\mathcal{L}_{\rm modQED} = \mathcal{L}_{\rm QED} + \mathcal{L}_I^{\rm new}, \tag{6}$$

where $\mathcal{L}_{\rm QED}$ is the usual Lagrangian density of QED in the Lorenz gauge,

$$\mathcal{L}_{\rm QED} = \bar\psi(i\not\partial - e\not A - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2, \tag{7}$$

and $\mathcal{L}_I^{\rm new}$ represents the new interaction produced by the nonminimal coupling, to be regarded

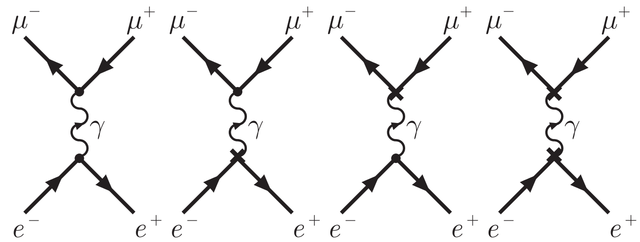
$$\mathcal{L}_I^{\rm new} = \frac{\lambda}{2}(K_F)_{\mu\nu\alpha\beta}\bar\psi\sigma^{\mu\nu}\psi F^{\alpha\beta}. \tag{8}$$

In the next steps we will consider the Feynman gauge, $\xi = 1$. The theory represented by Lagrangian (6) has, besides the usual vertex, $\bullet \to -ie\gamma^\mu$, an additional LV vertex, represented as

$$\times \to \lambda V_\beta = \lambda(K_F)_{\mu\nu\alpha\beta}\sigma^{\mu\nu}q^\alpha, \tag{9}$$

in the momentum space.

We are interested in analyzing how the electron-positron scattering, $e^+ + e^- \to \mu^+ + \mu^-$, is altered by this new vertex. This process may be depicted by the following tree-level Feynman diagrams:



The tensor K_F is composed of birefringent and nonbirefringent components. Without loss of generality, we restrain our investigation to the nonbirefringent sector [25], represented by nine coefficients and parametrized by a symmetric and traceless rank-2 tensor defined by the contraction

$$\kappa^{\mu\nu} = (K_F)_\alpha{}^{\mu\alpha\nu}, \tag{10}$$

which fulfills

$$(K_F)^{\lambda\nu\delta\rho} = \frac{1}{2}[g^{\lambda\delta}\kappa^{\nu\rho} - g^{\nu\delta}\kappa^{\lambda\rho} + g^{\nu\rho}\kappa^{\lambda\delta} - g^{\lambda\rho}\kappa^{\nu\delta}]. \tag{11}$$

Hence, the interaction (9) is rewritten as

$$\mathcal{L}_I^{\rm new} = \lambda\kappa_{\nu\beta}\bar\psi\sigma^{\mu\nu}\psi F_\mu{}^\beta, \tag{12}$$

which implies the following vertex:

$$\lambda V^\mu = \lambda q^\beta(\kappa^{\nu\mu}\sigma_{\beta\nu} - \kappa_{\nu\beta}\sigma^{\mu\nu}). \tag{13}$$

The components of the tensor can be classified by their parity properties: κ_{00}, κ_{ij} are parity even, while κ_{0i} is parity odd.

III. THE CROSS SECTION EVALUATION

In this section, we evaluate the differential and total cross section for the process,

$$e^+ + e^- \to \mu^+ + \mu^-, \tag{14}$$

where the particles are labeled with momentum and spin variables as $e^+(p_1; s_1)$, $e^-(p_2; s_2)$, $\mu^+(p_1'; s_1')$, and $\mu^-(p_2'; s_2')$. We work in the center of mass frame, in which it holds $p_1 = (E, \mathbf{p})$, $p_2 = (E, -\mathbf{p})$, $p_1' = (E, \mathbf{p}')$, and $p_2' = (E, -\mathbf{p}')$, with p_1, p_2, and p_1', p_2' being the momenta of the incoming and outgoing particles, respectively. Transfer momentum $(q = p_1 + p_2)$ is $q^\beta = (\sqrt{s}, 0)$, where $\sqrt{s}$ is the energy in the center of mass. In this frame, it holds $|\mathbf{p}'|^2 = |\mathbf{p}|^2 - m_\mu^2 + m_e^2$, and

$$\not p_2 = \gamma^0\not p_1\gamma^0 = \gamma^0\not p\gamma^0, \quad \not p_2' = \gamma^0\not p_1'\gamma^0 = \gamma^0\not p'^0, \tag{15}$$

with m_μ, m_e being the masses of the muon and the electron, respectively. The vertex components are $V^0 = 0$, and

$$V^i = \sqrt{s}(\kappa_{00}\sigma^{0i} - \kappa_{ij}\sigma^{0j} - \kappa_{0j}\sigma^{ij}). \tag{16}$$

Note that it holds $\kappa^{00} = \kappa^{ii} = \frac{3}{2}\kappa_{\rm tr}$, $\kappa^{ij} = -(\kappa_{e-})^{ij} + \frac{1}{2}\kappa_{\rm tr}\delta^{ij}$, $\kappa^{0i} = -\kappa^i$, where $\kappa_{\rm tr}$ and $(\kappa_{e-})_{ij}$ correspond to the isotropic and anisotropic parity-even components of the *CPT*-even sector, respectively, while κ^i represents the parity-odd components in accordance with Ref. [12]. These vertex components can be read as

$$V^i = V^i_{+I} + V^i_{+A} + V^i_{-}, \tag{17}$$

where $V^i_{+I} = \sqrt{s}\kappa_{00}\sigma^{0i}$ is the part associated with the parity-even isotropic coefficient, $V^i_{+A} = -\sqrt{s}\kappa_{ij}\sigma^{0j}$ is related to the anisotropic parity-even component, and $V^i_{-} = -\sqrt{s}\kappa_{0j}\sigma^{ij} = \sqrt{s}\kappa_j\sigma^{ij}$ is the contribution stemming from the parity-odd components.

In this scenario, the differential cross section (in natural units) is given by

$$\frac{d\sigma}{d\Omega} = \frac{|\mathbf{p}'|}{(8\pi)^2 s|\mathbf{p}|}|\mathcal{M}|^2. \tag{18}$$

The scattering amplitude is read off from the Feynman diagrams,

$$\mathcal{M} = \sum_{a,b}[\bar{v}^{s_2}(p_2)\Gamma^{\mu}_{(a)}u^{s_1}(p_1)]\frac{1}{q^2}[\bar{u}^{s'_1}(p'_1)\Gamma_{(b)\mu}v^{s'_2}(p'_2)], \tag{19}$$

where $a, b = 0, 1$ and $\Gamma^{\mu}_{(a)}$ defined by

$$\Gamma^{\mu}_{(0)} = -ie\gamma^{\mu}, \qquad \Gamma^{\mu}_{(1)} = \lambda V^{\mu}, \tag{20}$$

stands for the usual and new vertices. Here, $u^{s_1}(p_1)$, $\bar{v}^{s_2}(p_2)$ are the spinors for the electron and the positron, while $\bar{u}^{s'_1}(p'_1)$, $v^{s'_2}(p'_2)$ represent the muon and antimuon spinors. For evaluating the unpolarized cross section, the relevant quantity is $\langle|\mathcal{M}|^2\rangle$, defined as $|\mathcal{M}|^2 = \sum \mathcal{M}\mathcal{M}^*$, where the sum is over the spin indices, s_1, s_2, s'_1, s'_2. This squared amplitude is carried out by means of the Casimir's trick, based on the use of spinor completeness relations and the trace properties of γ matrices. Knowing that

$$\mathcal{M}^* = \sum[\bar{u}^{s_1}(p_1)\bar{\Gamma}^{\mu}_{(a)}v^{s_2}(p_2)]\frac{1}{q^2}[\bar{v}^{s'_2}(p'_2)\bar{\Gamma}_{(b)\mu}u^{s'_1}(p'_1)], \tag{21}$$

the squared amplitude is written as

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{4q^4}\sum \bar{v}^{s_2}(p_2)\Gamma^{\mu}_{(a)}u^{s_1}(p_1)\bar{u}^{s_1}(p_1)\bar{\Gamma}^{\nu}_{(b)}v^{s_2}(p_2) \times \bar{u}^{s'_1}(p'_1)\Gamma_{(c)\mu}v^{s'_2}(p'_2)\bar{v}^{s'_2}(p'_2)\bar{\Gamma}_{(d)\nu}u^{s'_1}(p'_1), \tag{22}$$

where $\bar{\Gamma}^{\mu}_{(i)} = \gamma^0\Gamma^{\mu\dagger}_{(i)}\gamma^0$, and the sum is over the spin indices and over a, b, c, d. Using the relation,

$$\bar{v}^{s_2}(p_2)\Gamma^{\mu}_{(a)}u^{s_1}(p_1)\bar{u}^{s_1}(p_1)\bar{\Gamma}^{\nu}_{(b)}v^{s_2}(p_2) = \mathrm{tr}(\Gamma^{\mu}_{(a)}u^{s_1}(p_1)\bar{u}^{s_1}(p_1)\bar{\Gamma}^{\nu}_{(b)}v^{s_2}(p_2)\bar{v}^{s_2}(p_2)), \tag{23}$$

the spin sum yields

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{4q^4}L^{\mu\nu}_T(M_T)_{\mu\nu}, \tag{24}$$

$$L^{\mu\nu}_T = L^{\mu\nu}_{(00)} + L^{\mu\nu}_{(01)} + L^{\mu\nu}_{(10)} + L^{\mu\nu}_{(11)}, \tag{25}$$

$$M^{\mu\nu}_T = M^{\mu\nu}_{(00)} + M^{\mu\nu}_{(01)} + M^{\mu\nu}_{(10)} + M^{\mu\nu}_{(11)}, \tag{26}$$

with

$$L^{\mu\nu}_{(ab)} = \mathrm{tr}[\Gamma^{\mu}_{(a)}(\not{p}_1 + m_e)\bar{\Gamma}^{\nu}_{(b)}(\not{p}_2 - m_e)], \tag{27}$$

$$M_{(ab)\mu\nu} = \mathrm{tr}[\Gamma_{(a)\mu}(\not{p}'_2 - m_\mu)\bar{\Gamma}_{(b)\nu}(\not{p}'_1 + m_\mu)]. \tag{28}$$

Remember that the Latin indices inside parentheses (a, b) can assume only two values, 0 or 1, corresponding to the usual and new nonminimal vertex, properly defined in Eqs. (16) and (20).

Next, in order to facilitate our evaluations and better discuss our results, we proceed to separate the contributions coming from the parity-odd and parity-even coefficients.

A. Parity-odd contribution

To calculate parity-odd contributions to the cross section, we restrict the vertex (16) to

$$V^i_{-} = \sqrt{s}\sigma^{ij}\kappa_j, \tag{29}$$

where $\kappa^i = (K_F)^{0jij}$. Using the trace technique, and using identity (15), we show that

$$L^{ij}_{(01)} = L^{ij}_{(10)} = M^{ij}_{(01)} = M^{ij}_{(10)} = 0. \tag{30}$$

The nonnull terms of the tensors (27) and (28) are

$$L^{ij}_{(00)} = e^2\,\mathrm{tr}[\gamma^i\not{p}_1\gamma^j\not{p}_2 - m_e^2\gamma^i\gamma^j], \tag{31}$$

$$L^{ij}_{(11)} = \lambda^2\,\mathrm{tr}[V^i\not{p}_1V^j\not{p}_2 - m_e^2V^iV^j], \tag{32}$$

$$M^{ij}_{(00)} = e^2\,\mathrm{tr}[\gamma^i\not{p}'_1\gamma^j\not{p}'_2 - m_\mu^2\gamma^i\gamma^j], \tag{33}$$

$$M^{ij}_{(11)} = \lambda^2\,\mathrm{tr}[V^i\not{p}'_1V^j\not{p}'_2 - m_\mu^2V^iV^j], \tag{34}$$

while $L^{0\mu}_{(ab)} = L^{\mu 0}_{(ab)} = M^{0\mu}_{(ab)} = M^{\mu 0}_{(ab)} = 0$. These latter terms are explicitly carried out:

$$L^{ij}_{(00)} = e^2(2s\delta^{ij} - 8p^ip^j), \quad M^{ij}_{(00)} = e^2(2s\delta^{ij} - 8p'^ip'^j), \tag{35}$$

$$L^{ij}_{(11)} = 8\lambda^2 s\varepsilon^{i\kappa m}\varepsilon^{jln}p^np^m\kappa^\kappa\kappa^l, \tag{36}$$

$$M^{ij}_{(11)} = 8\lambda^2 s\varepsilon^{i\kappa m}\varepsilon^{jln}p'^np'^m\kappa^\kappa\kappa^l. \tag{37}$$

The squared amplitude is

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{4s^2}[L^{ij}_{(00)}M^{ij}_{(00)} + L^{ij}_{(11)}M^{ij}_{(00)} + L^{ij}_{(00)}M^{ij}_{(11)} + L^{ij}_{(11)}M^{ij}_{(11)}]. \tag{38}$$

The differential cross section is obtained replacing these results in Eqs. (38) and (18). The total cross section is obtained by integration,

$$\sigma = \frac{|\mathbf{p}'|}{(8\pi)^2 s|\mathbf{p}|}\int \langle|\mathcal{M}|^2\rangle d\Omega. \tag{39}$$

Taking the background as fixed, we integrate only on the angular variables of the scattered particles, that is,

$$\int \langle|\mathcal{M}|^2\rangle d\Omega = \frac{1}{4s^2}\bigg[L_{00}^{ij}\int M_{00}^{ij}d\Omega + L_{11}^{ij}\int M_{00}^{ij}d\Omega + L_{00}^{ij}\int M_{11}^{ij}d\Omega + L_{11}^{ij}\int M_{11}^{ij}d\Omega\bigg]. \tag{40}$$

These integrals provide

$$\int M_{(00)}^{ij}d\Omega = \frac{16e^2}{3}(s+2m_\mu^2)\pi\delta^{ij}, \tag{41}$$

$$\int M_{(11)}^{ij}d\Omega = \frac{8\lambda^2}{3}\pi s(s-4m_\mu^2)(\delta^{ij}\boldsymbol{\kappa}^2 - \kappa^i\kappa^j), \tag{42}$$

where the integral $\int p'^i p'^j d\Omega = \frac{1}{3}(s-4m_\mu^2)\pi\delta^{ij}$ was used. In the ultrarelativistic limit, we take $m_e = m_\mu = 0$. The resulting cross section (at second order) is

$$\sigma = \sigma_{\mathrm{QED}}\bigg[1 + \frac{1}{4e^2}\lambda^2(3s\boldsymbol{\kappa}^2 - 4(\mathbf{p}\cdot\boldsymbol{\kappa})^2)\bigg]. \tag{43}$$

The results can be presented in two ways, concerning the beam orientation in relation to the background vector, κ^i. For the case where the beam is perpendicular to the background, $\boldsymbol{\kappa}\cdot\mathbf{p} = 0$, we achieve

$$\sigma = \sigma_{\mathrm{QED}}\bigg(1 + \frac{3s}{4e^2}\lambda^2|\boldsymbol{\kappa}|^2\bigg), \tag{44}$$

while for the case where the beam is parallel to the background, $\boldsymbol{\kappa}\cdot\mathbf{p} = |\boldsymbol{\kappa}|\sqrt{s}/2$, the total cross section is

$$\sigma = \sigma_{\mathrm{QED}}\bigg(1 + \frac{s}{2e^2}\lambda^2|\boldsymbol{\kappa}|^2\bigg). \tag{45}$$

Experimental data from Ref. [24] for the $e^+ + e^- \to \mu^+ + \mu^-$ scattering yields

$$\frac{\sigma - \sigma_{\mathrm{QED}}}{\sigma_{\mathrm{QED}}} = \pm\frac{2s}{\Lambda_\pm^2}, \tag{46}$$

where $\sqrt{s} = 29$ GeV and $\Lambda_+ = 170$ GeV with 95% confidence level. Comparing (44) and (45) with (46), we obtain the following upper bound:

$$|\lambda\boldsymbol{\kappa}| < 3\times 10^{-12}\ (\mathrm{eV})^{-1}. \tag{47}$$

B. Parity-even contribution

We begin considering the parity-even and isotropic contribution, whose associated vertex is $V_{+I}^i = \sqrt{s}\kappa_{00}\sigma^{0i}$. In this case, the elements of the tensors (27) and (28) are

$$L_{(00)}^{\mu\nu} = e^2\,\mathrm{tr}[\gamma^\mu \not{p}_1\gamma^\nu\not{p}_2 - m_e^2\gamma^\mu\gamma^\nu], \tag{48}$$

$$L_{(01)}^{\mu\nu} = ie\lambda m_e\,\mathrm{tr}[\gamma^\mu\not{p}_1 V_{+I}^\nu - \gamma^\mu V_{+I}^\nu\not{p}_2], \tag{49}$$

$$L_{(10)}^{\mu\nu} = -ie\lambda m_e\,\mathrm{tr}[V_{+I}^\mu\not{p}_1\gamma^\nu - V_{+I}^\mu\gamma^\nu\not{p}_2], \tag{50}$$

$$L_{(11)}^{\mu\nu} = \lambda^2\,\mathrm{tr}[V_{+I}^\mu\not{p}_1 V_{+I}^\nu\not{p}_2 - m_e^2 V_{+I}^\mu V_{+I}^\nu]. \tag{51}$$

The components of tensor $M_{(ab)}^{\mu\nu}$ are written in the same way, changing p_1, p_2, m_e by p_1', p_2', m_μ. In this case,

$$L_{(ab)}^{0\mu} = L_{(ab)}^{\mu 0} = M_{(ab)}^{0\mu} = M_{(ab)}^{\mu 0} = 0, \tag{52}$$

remaining as nonnull only the components $L_{(ab)}^{ij}$, $M_{(ab)}^{ij}$, given as

$$L_{(01)}^{ij} = L_{(01)}^{ij} = 4e\lambda\kappa_{00}sm\delta^{ij}, \tag{53}$$

$$M_{(01)}^{ij} = M_{(01)}^{ij} = 4e\lambda\kappa_{00}sM\delta^{ij}, \tag{54}$$

$$L_{(11)}^{ij} = 8s\lambda^2(\kappa_{00})^2(m^2\delta^{ij} + p^i p^j), \tag{55}$$

$$M_{(11)}^{ij} = 8s\lambda^2(\kappa_{00})^2(m^2\delta^{ij} + p'^i p'^j). \tag{56}$$

The squared amplitude now is

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{4s^2}(L_{(00)}^{ij} + 2L_{(01)}^{ij} + L_{(11)}^{ij})(M_{(00)}^{ij} + 2M_{(01)}^{ij} + M_{(11)}^{ij}). \tag{57}$$

Proceeding with the integration evaluations, and taking the ultrarelativistic limit ($m_e = m_\mu = 0$), the total cross section (at second order) is

$$\sigma = \sigma_{\mathrm{QED}}\bigg(1 + \frac{s}{e^2}|\lambda\kappa_{00}|^2\bigg). \tag{58}$$

By using the same conditions as in Eq. (46), we achieve

$$|\lambda\kappa_{00}| < 2.5\times 10^{-12}\ (\mathrm{eV})^{-1}. \tag{59}$$

We continue regarding the anisotropic parity-even contribution, whose vertex is $V_{-A}^i = -\sqrt{s}\kappa^{ij}\sigma^{0j}$. In this case, for turning feasible the evaluations, we consider the ultrarelativistic limit ($m_e = m_\mu = 0$) well in the beginning. The operators (27) and (28) are rewritten as

$$L_{(00)}^{\mu\nu} \approx e^2\,\mathrm{tr}[\gamma^\mu\not{p}_1\gamma^\nu\not{p}_2], \tag{60}$$

$$L_{(11)}^{\mu\nu} \approx \lambda^2\,\mathrm{tr}[V_{-A}^\mu\not{p}_1 V_{-A}^\nu\not{p}_2], \tag{61}$$

with components of the tensor $M_{(ab)}^{\mu\nu}$ read similarly by changing p_1, p_2, m by p_1', p_2', M. Some evaluations lead to

$$L_{(ab)}^{0\mu} = L_{(ab)}^{\mu 0} = M_{(ab)}^{0\mu} = M_{(ab)}^{\mu 0} = 0, \tag{62}$$

$$L_{(11)}^{ij} = 8s\lambda^2\kappa^{ik}\kappa^{jl}p^l p^k, \tag{63}$$

$$M_{(11)}^{ij} = 8s\lambda^2\kappa^{ik}\kappa^{jl}p'^l p'^k, \tag{64}$$

implying

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{4s^2}[L_{00}^{ij}M_{00}^{ij} + 8s(\kappa)^{ik}(\kappa)^{jl}(p^l p^k M_{00}^{ij} + L_{00}^{ij}p'^l p'^k)]. \tag{65}$$

Doing the corresponding integrations in the solid angle, we achieve

$$\int\langle|\mathcal{M}|^2\rangle d\Omega = \frac{16\pi e^4}{3}\left[1 + \frac{\lambda^2}{4e^2}(s(\kappa^2)^{ii} + 4(\kappa^{ij}p^j)^2)\right], \tag{66}$$

where $(\kappa^2)^{ii} = \kappa^{ij}\kappa^{ji}$. Choosing a beam direction so that $\kappa^{ij}p_j = 0$, we attain

$$\int\langle|\mathcal{M}|^2\rangle d\Omega = \frac{16e^4\pi}{3}\left(1 + \frac{\lambda^2 s}{4e^2}(\kappa^2)^{ii}\right). \tag{67}$$

This evaluation leads to

$$\sigma = \sigma_{\mathrm{QED}}\left(1 + \frac{\lambda^2 s}{4e^2}(\kappa^2)^{ii}\right), \tag{68}$$

implying the following upper bound:

$$|\lambda\kappa^{ij}| < 5 \times 10^{-12}\ (\mathrm{eV})^{-1}. \tag{69}$$

We notice that the upper bound on the parity-even parameters have the same order of magnitude as the one on the parity-odd coefficients.

IV. CONCLUSIONS

In this work, we have studied the influence of a Lorentz-violating *CPT*-even nonminimal coupling in the context of the Dirac equation, focusing specifically on the $e^+ + e^- \rightarrow \mu^+ + \mu^-$ scattering process. This new coupling implied the insertion of a new vertex, increasing the number of Feynman diagrams representing the level tree process. We have carried out the contributions of the nonminimally *CPT*-even LV terms on the unpolarized cross section, using the Casimir's trick. This evaluation was performed with details for the parity-odd and parity-even coefficients in the ultrarelativistic limit ($m_e = m_\mu = 0$). Comparing the attained results with scattering data in the literature [24], we have succeeded in imposing upper bounds at the level of 10^{-12} $(\mathrm{eV})^{-1}$ on the parity-odd and parity-even nonbirefringent coefficients of the quantity $\lambda(K_F)_{\mu\nu\alpha\beta}$, representing a good route to constrain the strength of this new nonminimal coupling in a relativistic environment. It is important to mention that these bounds should not be directly compared with the upper bounds imposed on the coefficients of the dimensionless *CPT*-even tensor $(K_F)_{\mu\nu\alpha\beta}$ in Refs. [12,13]. The bounds here achieved restrain the dimensional quantity $\lambda(K_F)_{\mu\nu\alpha\beta}$, representing a constraint on the way the *CPT*-even is coupled to the fermion sector.

Although we have restricted our study to the nonbirefringent sector of the *CPT*-even tensor $(K_F)_{\mu\nu\alpha\beta}$, we could have considered the ten birefringent components of the tensor $(K_F)_{\mu\nu\alpha\beta}$ as well. The point is that these coefficients contribute to the modified cross section also in second order, implying the same upper bound attained on the nonbirefringent components. This reasoning allows one to extend the bounds here achieved to all the components of the tensor (K_F), that is $|\lambda(K_F)| \leq 10^{-12}$ $(\mathrm{eV})^{-1}$, circumventing some cumbersome and unnecessary evaluations.

An interesting investigation concerns the possible connections between this dimension-5 nonminimal coupling and the higher-dimensional operators belonging to the photon sector presented in Ref. [15]. The proposed nonminimal coupling is a dimension-5 operator which is not contained in the framework of Ref. [15], once this term refers to the interaction between fermions and photons. The connection begins to appear when one performs the radiative corrections generated by this nonminimal coupling. Indeed, the evaluation of the one-loop vacuum polarization diagram of the photon leads to operators with dimension-4 and -6. The dimension-4 operator is exactly the *CPT*-even term $(K_F)_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$. The operators of dimension-6 are second order in K_F and could be encompassed in Ref. [15]. The fact that the dimension-4 operator can be generated by radiative corrections allows one to use the existing bounds on the *CPT*-even $(K_F)_{\mu\nu\alpha\beta}$ to attain even better bounds on the magnitude of the quantity $\lambda(K_F)_{\mu\nu\alpha\beta}$. The detailed analysis of this issue is under development now [26].

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