Renormalizability of generalized quantum electrodynamics

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In this work we present the study of the renormalizability of the Generalized Quantum Electrodynamics (GQED$_4$). We begin the article by reviewing the on-shell renormalization scheme applied to GQED$_4$. Thereafter, we calculate the explicit expressions for all the counter-terms at one-loop approximation and discuss the infrared behavior of the theory as well. Next, we explore some properties of the effective coupling of the theory which would give an indictment of the validity regime of theory: $m^2 \leq k^2 < m_P^2$. Afterwards, we make use of experimental data from the electron anomalous magnetic moment to set possible values for the theory free parameter through the one-loop contribution of Podolsky mass-dependent term to Pauli's form factor $F_2(q^2)$.

I. INTRODUCTION

Effective theories may or may not be unitary. In fact, the unitarity is lost when a particle, retained in an effective theory, can lower its energy by the emission of other particles which have been eliminated in deriving the effective theory [1]. The imprint of effective theories is the appearance of higher order derivatives terms in the Lagrangian density, reflecting the momentum-dependence of self-energies or form factors. Once the effective theory is rendered ultraviolet (UV) finite, we may consider it as an extension of the class of potentially interesting and consistent models because its UV dynamics is well defined. Motivated by the search of possible fundamental theories, one naturally expects the complete suppression of nonphysical, nonunitary processes.

Higher derivative (HD) theories [2] were proposed in an attempt to tame the ultraviolet behavior of physically relevant models [3]. However, it was soon recognized that they have a Hamiltonian which is not bounded from below [4] and that the addition of such terms leads to the existence of negative norm states (or ghosts states)—induces an indefinite metric in the space of states—jeopardizing thus the unitarity [5]. Despite the fact that many attempts to overcome these ghost states have been proposed, no one has been able to give a general method to deal with them [6–8]. In fact, in conventional gauge theories the gauge-fixing term, the Faddeev-Popov-De Witt ghosts, and the original Lagrangian density are invariant under Becchi-Rouet-Stora-Tyutin (BRST) transformation. Its purpose is to remove all the negative norm states and get a unitary theory, physical states are thus defined as those which have zero ghost number. In the light of this idea it was proposed that the HD theories present a BRST symmetry, which is seen as an intrinsic feature of these HD theories [9]. However, by imposing this symmetry we were led to an unitary but trivial resulting theory through the quartet mechanism [10], since all physical states, excepted the vacuum, appear in zero-norm combinations.

It is known that HD theories have, as a field theory, better renormalization properties than the conventional ones. This idea appeared to be quite successful in the case of the attempt to quantize gravity, where the Einstein action is supplied by terms containing higher powers of curvature leading to a renormalizable [11] and asymptotically free theory [6]. Also, a new impetus in exploring appealing quantum theories such as $f(R)$-gravity [12], which may explain the accelerating universe.

One of the first HD theories was the Generalized Electrodynamics [13] originally proposed as to get rid of some pathologies inherent in the Maxwell theory. As pointed out in several works [13,14], it is clear that Maxwell's theory is not the only one to describe the Electromagnetic Field and therefore considered as an appealing theory. In fact, the Podolsky's theory is the only possible linear, Lorentz and $U(1)$ invariant generalization of Maxwell's theory [15]. Actually, it is impressive that the simplest $U(1)$ generalization possesses a richness of interesting features. Concerning the gauge symmetry, it was proved in Ref. [16] that the generalized Lorenz condition, $\Omega[A] = (1 + m_P^{-2}\Box)\partial^\mu A_\mu$, is the correct gauge condition which completely fix the gauge freedom for the Podolsky's theory. Such generalized condition is attached with the correct true degrees of freedom for the theory. A recent study of the finite-temperature free Podolsky's theory has showed a correction to the Stefan-Boltzmann law and by using cosmic microwave background data it was possible to set a thermodynamical limit to the Podolsky's parameter m_P [17].

The issue of quantization of Generalized Electrodynamics in the generalized Lorenz condition were dealt in Ref. [18] through functional methods. In contrast to the results obtained in the usual Lorenz condition [19] it was found that, in the one-loop approximation, the electron self-energy and the vertex function are both ultraviolet finite. Therefore, a natural continuation of these studies would be a discussion upon the renormalizability of the Generalized Quantum Electrodynamics (GQED$_4$). Efforts have also been made to analyze the GQED$_4$ at thermodynamic equilibrium [20]; so far, the current analysis lies upon formal aspects of the theory.

It is worth noticing some points about the free gauge field Green's function [18]:

$$iD_{\mu\nu}(k) = \frac{1}{k^2}\left[\eta_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right]$$
$$- \frac{1}{k^2 - m_P^2}\left[\eta_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2 - m_P^2}\right]$$
$$+ (1-2\xi)\frac{1}{(k^2 - m_P^2)k^2}k_\mu k_\nu + \frac{1}{(k^2 - m_P^2)^2}k_\mu k_\nu.$$

$$(1)$$

The above expression shows explicitly the contributions of the ghost states—even Maxwell's theory is correctly quantized only in an indefinite-metric space [21], but they can safely be excluded from the asymptotic states by means of gauge symmetry without upsetting the unitary time evolution in the physical, positive norm sector. Actually there are sufficient reasons to dismiss any quantum field theory, such as Einstein gravity, that had the presence of HD quantum corrections and ghosts states; nevertheless, in this paper we will assume that, although there is not a formal proof upon the details of physical space for Podolsky's theory, it could be performed an analysis, for instance through either the generalized Kubo-Martin-Schwinger boundary conditions [22], through BRST symmetry and quartet mechanism [9], or even by the quantization scheme proposed by Hawking and Hertog [7], leading therefore to an acceptable and well-defined theory to the Generalized Electrodynamics.[1]

This work is addressed to the issue of renormalizability of the Generalized Quantum Electrodynamics, the plan of the paper is as follows: In Sec. II, we review and apply to the GQED$_4$ the on-shell renormalization prescription with the appropriated renormalization conditions. In Sec. III, we compute the expressions for the counter-terms, and the infrared behavior of the theory is also taken into account. In Sec. IV, we present a proper discussion on the effective coupling of the theory. In Sec. V, we calculate, at one-loop approximation, the Podolsky's contribution to the electron anomalous magnetic moment and accordingly to

experimental data we set a value to the theory's free parameter m_P. We present our remarks and prospects in Sec. VI. The Minkowski spacetime is concerned in the whole work, with the metric signature $(+, -, -, -)$.

II. RENORMALIZATION SCHEDULE

As noticed before, the GQED$_4$ shows a better UV behavior than the usual theory [18], however it contains negative norm states, which induce the lost of unitarity. This is a major issue and certainly cannot be neglected, and requires a detailed study; nevertheless, once we have identified the primitive divergences on the 1PI functions [18] and the theory rendered UV finite (at the light of effective theories) we are motivated to retain some attention regarding the underlying aspects of the renormalized behavior of the present theory. We expect to obtain from comparison of our results to the regular expressions for the effective coupling and one-loop contribution for the anomalous magnetic moment, an energy regime of validity of the theory and a bound value for the free parameter, respectively.

In the following, we shall give a brief derivation on the so-called on-shell renormalization scheme [23] by employing it into the GQED$_4$. Although the renormalization procedure be a well-known subject in the quantum field theory, we would like to point out some relevant features in which the discussion for GQED$_4$ may differ from the usual one. The first part of the analysis relies by determining formally the constants Z_i under suitable renormalization conditions. Therefore, to this aim, we define the gauge-fixed renormalized Lagrangian, in the gauge choice of the so-called generalized Lorenz condition [16]: $\Omega[A] = (1 + m_P^{-2}\Box)\partial^\mu A_\mu$, and also introduce the counter-terms as the following prescription:

$$\mathcal{L} = \bar\psi(i\hat\partial - m + e\hat A)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2m_P^2}\partial^\mu F_{\mu\beta}\partial_\alpha F^{\alpha\beta}$$
$$- \frac{1}{2\xi}[(1 + m_P^{-2}\Box)\partial^\mu A_\mu]^2 + \delta_{Z_2}\bar\psi i\hat\partial\psi$$
$$- \delta_{Z_0}\bar\psi m\psi + \delta_{Z_1}e\bar\psi\hat A\psi - \delta_{Z_3}\frac{1}{4}F_{\mu\nu}F^{\mu\nu}; \quad (2)$$

with the following definition: $\delta_{Z_i} = Z_i - 1$. The relations between the bare and renormalized quantities are, as usual, as follows:

$$A^{(0)} = Z_3^{1/2}A^{(r)}, \qquad \psi^{(0)} = Z_2^{1/2}\psi^{(r)},$$
$$\bar\psi^{(0)} = Z_2^{1/2}\bar\psi^{(r)}, \quad (3)$$

and[2]

$$Z_2 m^{(0)} = Z_0 m, \qquad Z_3^{1/2}e^{(0)} = Z_1 Z_2^{-1}e,$$
$$m_P^{(0)} = Z_3^{1/2}m_P. \quad (4)$$

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[1]The arguments upon unitarity and consistency of HD theories presented on these works can be suitably extended for the Generalized Electrodynamics.

[2]It could also be introduced: $m_0 = Z_m m$, with $Z_m = \frac{Z_0}{Z_2}$.

Here, the Podolsky's parameter m_P has not a constant associated with its renormalization, in the same sense as the ξ parameter (gauge Ward-Fradkin-Takahashi identity [18]).

As in the usual QED$_4$, we obtain here from the bare Ward-Fradkin-Takahashi identity [18]:

$$ik_\mu \tilde{\Gamma}^\mu(p, p'; q = p' - p) = \mathcal{S}^{-1}(p - p') - \mathcal{S}^{-1}(p), \quad (5)$$

that the ratio Z_1/Z_2 must be finite if the theory is renormalizable. Thus, the finiteness of the ratio Z_1/Z_2 implies that order-by-order in perturbation theory the equality $Z_1 = Z_2$ is identically satisfied. Thereby, the coupling constant e is determined by Z_3 only: $e_0 = Z_3^{-1/2}e$.

According to the Lagrangian (2) we obtain here new Schwinger-Dyson-Fradkin equations for the theory, which we shall denote by the suffix$^{(R)}$. More precisely, the original self-energy functions [18] are changed by adding the counter-terms δ_{Z_i}. Now, by a formal derivation it is not complicated to show that the photon sector has the following renormalized self-energy function:

$$\Pi^{(R)}(k) = \Pi(k) + \delta_{Z_3}; \quad (6)$$

where $\Pi(k)$ is the polarization scalar written in terms of the renormalized quantities [18].

We proceed now to the first renormalization condition, which comes to ensure the Maxwell photon behavior for the propagator Eq. (1), in the gauge $\xi = 1$. Thus, it is written as the following:

$$i\mathcal{D}_{\mu\nu}(k) = \frac{1}{k^2}\eta_{\mu\nu}, \quad \text{when } k^2 \to 0. \quad (7)$$

By means of the above condition we find the expression for the counter-term δ_{Z_3}:

$$\delta_{Z_3} = Z_3 - 1 = -\Pi(k^2)|_{k^2 \to 0}. \quad (8)$$

Considering next the fermionic sector, we have that its renormalized self-energy function is written as:

$$i\Sigma^{(R)}(p, m) = i\Sigma(p, m) - im\delta_{Z_0} + i\delta_{Z_2}\hat{p}; \quad (9)$$

where the function $\Sigma(p)$ is the radiative correction of the fermionic 1PI function: $\Gamma(p) = \hat{p} - m - \Sigma^{(R)}(p, m)$; where: $\Gamma(x, y) = -\frac{\delta^2\Gamma}{\delta\psi(y)\delta\bar{\psi}(x)}$. As a matter of calculation, we can write the electron self-energy function in the following general way: $\Sigma(p, m) = \Sigma_1(p^2)\hat{p} + \Sigma_2(p^2)I$. In order to fix the fermionic counter-terms we must impose two renormalization conditions. Which are outlined by[3]:

$$\frac{\partial\Gamma(p)}{\partial\hat{p}} = 1, \quad \Gamma(p) = \hat{p} - m_F, \quad \text{when } \hat{p} \to m_F. \quad (10)$$

These immediately lead to the following relations for the counter-term δ_{Z_2}:

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[3]m_F is defined as the zero of the electron 1PI function.

$$\delta_{Z_2} = Z_2 - 1 = -\Sigma_1(p^2)|_{p^2 \to m_F^2} - 2m_F^2 \frac{\partial\Sigma_1(p^2)}{\partial p^2}\bigg|_{p^2 \to m_F^2}$$

$$- 2m_F \frac{\partial\Sigma_2(p^2)}{\partial p^2}\bigg|_{p^2 \to m_F^2}, \quad (11)$$

whereas for the counter-term δ_{Z_0} one gets:

$$m\delta_{Z_0} = m(Z_0 - 1)$$

$$= \Sigma_2(p^2)|_{p^2 \to m_F^2} - 2m_F^2\bigg[m_F \frac{\partial\Sigma_1(p^2)}{\partial p^2}\bigg|_{p^2 \to m_F^2}$$

$$+ \frac{\partial\Sigma_2(p^2)}{\partial p^2}\bigg|_{p^2 \to m_F^2}\bigg]. \quad (12)$$

Finally, we look on the renormalization condition which determines the counter-term δ_{Z_1}. Using the so-called Gordon decomposition, we can write the vertex part Λ in terms of the Dirac and Pauli form factors, as:

$$\Lambda^\rho(p, p') = \gamma^\rho F_1(q^2) + \frac{i}{2m}\sigma^{\rho\nu}q_\nu F_2(q^2), \quad (13)$$

where $q = p' - p$, is the transferred momentum and $\sigma^{\rho\nu} = \frac{i}{2}[\gamma^\rho, \gamma^\nu]$. Therefore, as a result of the on-shell condition for the vertex part: $p'^2 = p^2 = m^2$, and $q^2 \to 0$, we have:

$$F_1(q^2)|_{q^2 \to 0} = 0, \quad (14)$$

which results in determining the counter-term δ_{Z_1}.

The above discussion was in fact necessary to elaborate some details on the derivation of the formal expressions for the counter-terms δ_{Z_i}, to thus we become clear on some points of the renormalized structure of the present theory.[4] In the next section we shall present the one-loop expressions for the three self-energy functions at a general-ξ, and then calculate the explicit expressions for the counter-terms δ_{Z_i}. Afterwards, we discuss the infrared behavior of these quantities.

III. RENORMALIZATION CONSTANTS AND INFRARED BEHAVIOR

We begin this section by presenting the explicit expressions for the one-loop radiative corrections functions at a general ξ [18]. The expression for the polarization scalar reads as [23]:

$$\Pi^{(1)}(k) = -\frac{\alpha}{3\pi}\bigg[\frac{2}{\epsilon_{UV}} - \frac{1 + 2\gamma}{2}$$

$$+ 6\int_0^1 dy\, y(1 - y)\ln\bigg[\frac{4\pi\mu^2}{m^2 - y(1 - y)k^2}\bigg]\bigg]. \quad (15)$$

Where $\epsilon_{UV} = 4 - d$, $\epsilon_{UV} \to 0^+$, is the ultraviolet dimensional regularization parameter and μ the t'Hooft

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[4]Technically it is not complicated to see through a perturbative graph analysis that the counter-terms are sufficient to absorb all the primitive divergences order-by-order.

mass, and $\alpha = \frac{e^2}{4\pi}$, is the fine-structure constant. The photon sector will be further investigate in the Sec. IV, on the discussion about the theory's effective coupling.

Unlike Π, the electron self-energy and vertex part are both gauge-dependent. Thus, at one-loop calculation, the electron self-energy expression can be casted into the following form [18]:

$$\Sigma^{(1,\xi)}(p) = \frac{\alpha}{2\pi} \int_0^1 dz(\hat{p}(1-z) - 2m)\ln\left|\frac{z - (1-z)z\frac{p^2}{m^2} + (1-z)\frac{p^2}{m^2}}{z - (1-z)z\frac{p^2}{m^2}}\right| + \frac{\alpha}{4\pi}\left(\frac{p^2}{m^2}\right)\int_0^1 dx \int_0^{1-x} dy((1-y)\hat{p} + m)y^2$$

$$\times \left\{\frac{(\xi-1)}{y - (1-y)y\frac{p^2}{m^2}} + \frac{(1-2\xi)}{y - (1-y)y\frac{p^2}{m^2} + x\frac{m_P^2}{m^2}} + \frac{\xi}{y - (1-y)y\frac{p^2}{m^2} + (1-y)\frac{m_P^2}{m^2}}\right\}$$

$$+ \frac{\alpha}{4\pi} \int_0^1 dx \int_0^{1-x} dy(2m - (1+3y)\hat{p})\left\{(\xi-1)\ln\left|\frac{y - (1-y)y\frac{p^2}{m^2}}{y - (1-y)y\frac{p^2}{m^2} + x\frac{m_P^2}{m^2}}\right| + \xi\ln\left|\frac{y - (1-y)y\frac{p^2}{m^2} + (1-y)\frac{m_P^2}{m^2}}{y - (1-y)y\frac{p^2}{m^2} + x\frac{m_P^2}{m^2}}\right|\right\}.$$

(16)

The form factors explicit expressions of the vertex part Λ are given by [18]:

$$F_1(q^2) = (Z_1 - 1)^{(1)} + \frac{\alpha}{2\pi} \int_0^1 dx \int_0^{1-x} dy \ln\left|\frac{(x+y)^2 + (1-x-y)\frac{m_P^2}{m^2} - xy\frac{q^2}{m^2}}{(x+y)^2 - xy\frac{q^2}{m^2}}\right|$$

$$+ \frac{3\alpha}{2\pi} \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz\left[(\xi-1)\ln\left|\frac{(x+y)^2 + z\frac{m_P^2}{m^2} - \frac{q^2}{m^2}xy}{(x+y)^2 - xy\frac{q^2}{m^2}}\right| + \xi\ln\left|\frac{(x+y)^2 + z\frac{m_P^2}{m^2} - xy\frac{q^2}{m^2}}{(x+y)^2 + (1-x-y)\frac{m_P^2}{m^2} - xy\frac{q^2}{m^2}}\right|\right]$$

$$+ \frac{\alpha}{4\pi} \int_0^1 dx \int_0^{1-x} dy\left(2((x+y)(x+5y) - 2(1-x-y)^2 - 4x) + 2\frac{q^2}{m^2}[1-x-y+xy]\right)\left\{\frac{1}{(x+y)^2 - xy\frac{q^2}{m^2}}\right.$$

$$\left. - \frac{1}{(x+y)^2 + (1-x-y)\frac{m_P^2}{m^2} - xy\frac{q^2}{m^2}}\right\} + \frac{\alpha}{2\pi} \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz\left(2(x+y)(2-x-y) - (1-x-y)^2\right.$$

$$+ \frac{q^2}{2m^2}(1 - 3x - 3y + 6xy)\right)\left\{\frac{(\xi-1)}{(x+y)^2 - xy\frac{q^2}{m^2}} + \frac{(1-2\xi)}{(x+y)^2 + z\frac{m_P^2}{m^2} - xy\frac{q^2}{m^2}} + \frac{\xi}{(x+y)^2 + (1-x-y)\frac{m_P^2}{m^2} - xy\frac{q^2}{m^2}}\right\}$$

$$+ \frac{\alpha}{4\pi} \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz\left((4(x - xy + y)((x+y)(1-y) - x^2) + (x^2+y^2)^2)\right.$$

$$\left. - \frac{q^2}{m^2}(x(1-y) + y(1-x))(1 - (1-x-y)^2) + 2\left(\frac{q^2}{m^2}\right)^2 xy(1-x)(1-y)\right)$$

$$\times \left\{\frac{(\xi-1)}{[(x+y)^2 - xy\frac{q^2}{m^2}]^2} + \frac{(1-2\xi)}{[(x+y)^2 + z\frac{m_P^2}{m^2} - xy\frac{q^2}{m^2}]^2} + \frac{\xi}{[(x+y)^2 + (1-x-y)\frac{m_P^2}{m^2} - xy\frac{q^2}{m^2}]^2}\right\},$$

(17)

and

$$F_2(q^2) = \frac{2\alpha}{\pi} m^2 \int_0^1 dx \int_0^{1-x} dy(x - xy - y^2)\left[\frac{1}{m^2(x+y)^2 - q^2xy} - \frac{1}{m^2(x+y)^2 + m_P^2(1-x-y) - q^2xy}\right].$$

(18)

With these explicit expressions for: Π, Σ and Λ, at one-loop approximation, we now proceed to the calculation for the counter-terms δ_{Z_i}. These quantities are obtained through a rather lengthy but straightforward calculation, only requiring that the renormalization conditions be satisfied.

A. Renormalization constant Z_3

We begin from the simplest counter-term calculation; which is from the gauge sector. From the relations (8) and (15) one can easily find:

$$\delta_{Z_3}^{(1)} = (Z_3 - 1)^{(1)} = \frac{\alpha}{3\pi}\left[\frac{2}{\epsilon_{UV}} - \frac{1 + 2\gamma}{2} + \ln\left[\frac{4\pi\mu^2}{m^2}\right]\right]. \tag{19}$$

Actually, the counter-term δ_{Z_3} is the only one which has an ultraviolet term and is infrared divergence free.

B. Renormalization constant Z_0

In the fermionic sector, we first compute the counter-term related with the massive fermionic sector by calculating the Eq. (12). One can easily cast the Eq. (15) in the form $\Sigma(p, m) = \Sigma_1(p^2)\hat{p} + \Sigma_2(p^2)I$. Therefore, from Eq. (12), the resulting expression is:

$$\delta_{Z_2}^{(1)} = \frac{\alpha}{2\pi}\frac{(3 - \xi)}{\epsilon_{IR}} + \frac{\alpha}{24\pi}(36 + 18b - 6\xi + 12b^2\xi) + \frac{\alpha}{8\pi}(-2b^3\xi + 3b^2(\xi - 1) + 2b\xi - 2\xi + 6)\log(b)$$

$$+ \frac{\alpha}{8\pi}\frac{b}{\sqrt{b(b-4)}}(2b^3\xi + b^2(3 - 7\xi) - 6b - 12)\left[\ln\left(\frac{b - 2 + \sqrt{(b-4)b}}{b - 2 - \sqrt{(b-4)b}}\right) + \ln\left(\frac{b - \sqrt{b(b-4)}}{b + \sqrt{b(b-4)}}\right)\right]. \tag{21}$$

Such expression is also an ultraviolet finite quantity at order-α.

At last, it is only remaining the vertex renormalization constant Z_1 to be determined which follows from the condition (14) and (17); and, after some simple integral manipulations, one finds the same expression than Eq. (21). A result which is in agreement with the equality $Z_1 = Z_2$ at order-α.

Although there is a massive sector on the propagator Eq. (1), the expressions of the following counter-terms: δ_{Z_0} Eq. (20), δ_{Z_2} and δ_{Z_1} Eq. (21), reveal that all of them have an infrared divergence which comes from the usual massless QED$_4$ sector contribution. Nevertheless, it possesses a simple ξ-dependent structure which is easily recognized as being:

$$I_{IR} = \frac{\alpha}{2\pi}\frac{(3 - \xi)}{\epsilon_{IR}}. \tag{22}$$

By a renormalization group analysis in QED$_4$, in $p^2 \ll m^2$ regime, it is known that there is unique gauge choice in which the electron propagator behaves asymptotically free and it is also infrared safe; this gauge choice is known as Fried-Yennie gauge [24], and stands as $\xi = 3$. The same statement holds here to the GQED$_4$ [see Eq. (22)]. Therefore, as it turns out from one-loop calculation, we can conclude that the choice $\xi = 3$, besides the Π expression, yields to ultraviolet and infrared finite expressions for GQED$_4$.

$$\delta_{Z_0}^{(1)} = \frac{\alpha}{2\pi}\frac{(3 - \xi)}{\epsilon_{IR}} + \frac{\alpha}{4\pi}(1 + 4b - (2b^2 + \xi - 3)\log(b))$$

$$+ \frac{\alpha}{2\pi}\frac{b((b - 2)b - 5)}{\sqrt{b(b-4)}}\ln\left[\frac{b + \sqrt{b(b-4)}}{b - \sqrt{b(b-4)}}\right]. \tag{20}$$

Where we have introduced the infrared dimensional parameter: $\epsilon_{IR} = d - 4$, $\epsilon_{IR} \rightarrow 0^-$. It was considered the region: $b = \frac{m_P^2}{m^2} > 4$. Otherwise, the logarithm must be replaced by an arctan function in the above expression. We have here obtained an ultraviolet finite counter-term, which apparently might indicate a finite renormalization constant; however, it is, in fact, infrared divergent at order-α. Anyhow, we will present a proper discussion regarding this issue right below.

C. Renormalization constant Z_2 and Z_1

The counter-term δ_{Z_2} can be obtained from Eq. (11). Hence, follows that, at order-α, it has the following expression for $b > 4$:

IV. GQED$_4$ EFFECTIVE COUPLING

As it has been mentioned earlier, although the renormalization constant Z_3 Eq. (19) does not depend on Podolsky's parameter m_P, it might there to be another quantities that may be sensitive to these effects in order-α. A suitable scenario to investigate such effects would be the Coulomb scattering [25]. In fact, the Coulomb potential of a point-like charge is changed in Generalized Electrodynamics as: $\phi(r) = \frac{e}{r}(1 - e^{-m_P r})$. A proper way of representing an important class of these modifications is to introduce some invariant quantities, such as the so-called running coupling constant. Therefore, it is the purpose of this section to examine the effects of Podolsky Electrodynamics into the usual running coupling constant in the Coulomb scattering.

In fact, we are interested in modifications of the Coulomb scattering amplitude at $k^2/m^2 \gg 1$ regime. In this approximation, one defines the running coupling constant as follows:

$$\alpha_R(k^2) = \alpha\left\{1 + \left(1 - \frac{k^2}{k^2 - m_P^2}\right)\left[\frac{\alpha}{3\pi}\left[\frac{2}{\epsilon} - \frac{1 + 2\gamma}{2}\right.\right.\right.$$

$$\left.\left.\left. + \ln\left[\frac{4\pi\mu^2}{m^2}\right]\right] + \frac{\alpha}{3\pi}\ln\left(\frac{k^2}{m^2}\right)\right]\right\},$$

for which one immediately gets:

$$\alpha_R(k^2) = \alpha_R(m^2)\left[1 + \frac{\alpha_R(m^2)}{3\pi}\frac{1}{1-\frac{k^2}{m_P^2}}\ln\left[\frac{k^2}{m^2}\right]\right], \quad (23)$$

where $\alpha_R(m^2) = Z_3\alpha$, with Z_3 given by Eq. (19).

Furthermore, the modifications to the Coulomb scattering can also be obtained in higher-orders of perturbation theory in the regime $k^2/m^2 \gg 1$. For this purpose, we can sum an important class of diagrams through the leading logarithmic approximation; which corresponds to the most divergent set of logarithms. Hence, by means of this approximation we can cast the running coupling constant expression into the following form:

$$\frac{1}{\alpha_R(k^2)} = \frac{1}{\alpha_R(m^2)} - \frac{1}{3\pi}\frac{1}{1-\frac{k^2}{m_P^2}}\ln\left[\frac{k^2}{m^2}\right]. \quad (24)$$

We see in (24) that the running coupling constant expression clearly has a pole at $k^2 = m_P^2$; and, in comparison to the QED$_4$ expression [25] it provides a validity regime: $m^2 \leq k^2 < m_P^2$, where the GQED$_4$ theory is in fact well-defined. The aim of the next section is to determine a bound value for the parameter m_P^2, which therefore will be important to define the accessible energy regime to the theory.

V. ELECTRON MAGNETIC MOMENT AND PODOLSKY'S PARAMETER

The electron anomalous magnetic moment value is the most accurate test of particle physics up-to-date. It was calculated initially at one-loop by Schwinger [26], and at two-loop in Refs. [10,27]. The subsequent orders calculation was summarized by Kinoshita *et al.* [28].

We have as purpose in this section, based on experimental grounds, to make use of this precise data to set a bound value to the Podolsky's parameter m_p in high-energy physics. Thereafter, we shall proceed into the calculation of the Pauli's form factor $F_2(q^2)$, Eq. (18), which can be written as the sum of two contributions:

$$F_2(q^2) = F_{\text{QED}}(q^2) + F_{\text{POD}}(q^2). \quad (25)$$

The first term, $F_{\text{QED}}(q^2)$, is the well-known one-loop contribution calculated by Schwinger [26], and it reads:

$$F_{\text{QED}}(0) = \frac{\alpha}{2\pi}. \quad (26)$$

Whereas the second term $F_{\text{POD}}(q^2)$ gives a new and interesting contribution. We have to solve here the following integral:

$$F_{\text{POD}}(0) = \frac{\alpha}{\pi}\int_0^1 du\,\frac{u^3 - u^2}{u^2 + \frac{m_P^2}{m^2}(1-u)}, \quad (27)$$

Hence, assuming the condition $b > 4$, with $b = \frac{m_P^2}{m^2}$, one obtains:

$$F_{\text{POD}}(0) = \frac{\alpha}{2\pi}\left[2b - 1 + (2-b)b\ln(b) - \frac{b}{\sqrt{b(b-4)}}\right.$$
$$\times (2 + b(b-4))\left\{\ln\left(\frac{\sqrt{b(b-4)}+2-b}{\sqrt{b(b-4)}-2+b}\right)\right.$$
$$\left.\left.+ \ln\left(\frac{\sqrt{b(b-4)}+b}{\sqrt{b(b-4)}-b}\right)\right\}\right]. \quad (28)$$

We claim here to state that, since we have a perfect agreement of the QED$_4$ results with experiments, the Podoslky contribution Eq. (28) must be at most equal to the experimental error in the electron anomalous magnetic moment value. The experimental value of electron anomalous magnetic moment is $a_{\text{exp}} = 1,15965218073 \times 10^{-3} \pm 2,8 \times 10^{-13}$ [29]. Therefore, from the expression (28) we find that for values: $m_P \geq 3,7595 \times 10^{10}$ eV the theory is compatible with the experimental data.[5]

VI. CONCLUDING REMARKS

This paper presents a proper study regarding the renormalizability, and subsequent consequences, of the Generalized Quantum Electrodynamics. It was evoked initially arguments about the consistency of Generalized Electrodynamics, the construction of the physical subspace through a couple of possible prescriptions, where it is always claimed the presence of a powerful symmetry: BRST symmetry, reflection positivity and etc.

We had successfully applied the renormalization program on GQED$_4$, and subsequent quantities were computed at one-loop approximation. Once we had an UV finite theory [18] and also identified the primitive divergences, it was suitable to examine the theory at renormalizability level, what took us to develop the renormalization prescription for the Lagrangian density. Although we had chosen a condition where the photon propagator was massless, there were some quantities affected by the theory's free parameter m_P.

Next we presented the expressions for all four counter-terms of theory. Although the fermionic and vertex counter-terms: δ_{Z_0}, δ_{Z_2}, δ_{Z_1}, have ultraviolet finite expressions, an analysis allowed us to identify that all expressions had the same infrared term, introduced by the integration over the Feynman parameters. However, such infrared term had a simple dependence in the gauge parameter ξ as: $(3 - \xi)$, which is similar to the Fried-Yennie gauge, $\xi = 3$. This gauge choice is well-known as being infrared safe in QED$_4$. The same conclusion occurs in GQED$_4$.

Although the photon field does not feel the effects from the Podolsky contribution at order-α, we studied

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[5]Actually, we find a consistent value for m_P in the region $b > 4$ only.

the scenario of the Coulomb scattering, which actually is sensitive to these effects. Actually, the invariant quantity studied here was the effective coupling of the theory, i.e., the running coupling constant. Hence, through its explicit and m_P-dependent expression, we were able to find a energy range of validity for the GQED$_4$: $m^2 \leq k^2 < m_P^2$.

The last part of work was addressed to the discussion and evaluation of the Podolsky contribution to the electron anomalous magnetic moment at one-loop approximation. Firstly we obtained the known result, the QED$_4$ contribution: $F_2(0) = \frac{\alpha}{2\pi}$. Next, we proposed the possibility of using the experimental data of the electron anomalous magnetic moment to set a limit to the Podolsky parameter m_P. Therefore, we found a consistent value for this parameter as being: $m_P \geq 3,7595 \times 10^{10}$ eV. It is worth to stress here that, as the theory has a natural energy scale, which is the electron mass m, it provides a sensitive result to the Podolsky parameter value.

There are numerous extensions one may consider. For instance, owing its rich interacting structure, the Scalar Generalized Electrodynamics and its non-abelian version would be an appealing extensions, once we now have good insights about the behavior of the complete theory. A study of HD of gravitational fields should also be interesting, it continues being an open issue, and revisited in several different approaches and in different dimensionality. Still in the GQED$_4$ context, it might be an interesting case the study of scattering process with external fields of the HD theories [30]. However, we believe that the remaining major issue involving GQED$_4$, is to formulate the theory following the causal perturbation theory [31]; which by itself is well-established, leading to a well-defined and ultraviolet finite theory. These issues and others will be further elaborated, investigated and reported elsewhere.

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[1] S. Weinberg, Physica (Amsterdam) **96A**, 327 (1979); A. Pich, arXiv:hep-ph/9806303.

[2] M. Ostrogradski, Mem. Acad. St. Petersburg VI **4**, 385 (1850); R. Weiss, Proc. R. Soc. A **169**, 102 (1938); J. S. Chang, Proc. Cambridge Philos. Soc. **44**, 76 (1948).

[3] W. Thirring, Phys. Rev. **77**, 570 (1950).

[4] A. Pais and G. E. Uhlenbeck, Phys. Rev. **79**, 145 (1950).

[5] W. Heisenberg, Nucl. Phys. **4**, 532 (1957).

[6] E. S. Fradkin and A. A. Tseytlin, Nucl. Phys. **B201**, 469 (1982); I. L. Buchbinder, S. D. Odintsov, and I. L. Shapiro, *Effective Action in Quantum Gravity* (IOP, Bristol, 1992).

[7] S. W. Hawking and T. Hertog, Phys. Rev. D **65**, 103515 (2002).

[8] F. S. Gama, M. Gomes, J. R. Nascimento, A. Yu. Petrov, and A. J. da Silva, Phys. Rev. D **84**, 045001 (2011); P. Mukherjee and B. Paul, Phys. Rev. D **85**, 045028 (2012).

[9] V. O. Riveles, Phys. Lett. B **577**, 137 (2003); M. Faizal and M. Khan, Eur. Phys. J. C **71**, 1603 (2011).

[10] T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. **66**, 1 (1979).

[11] K. Stelle, Phys. Rev. D **16**, 953 (1977).

[12] T. P. Soritiou and V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010).

[13] B. Podolsky, Phys. Rev. **62**, 68 (1942); B. Podolsky and C. Kikuchy, Phys. Rev. **65**, 228 (1944); B. Podolsky and P. Schwed, Rev. Mod. Phys. **20**, 40 (1948).

[14] J. Frenkel, Phys. Rev. E **54**, 5859 (1996); R. R. Cuzinatto, C. A. M. de Melo, L. G. Medeiros, and P. J. Pompeia, Int. J. Mod. Phys. A **26**, 3641 (2011).

[15] R. R. Cuzinatto, C. A. M. de Melo, and P. J. Pompeia, Ann. Phys. (Amsterdam) **322**, 1211 (2007).

[16] C. A. P. Galvão and B. M. Pimentel, Can. J. Phys. **66**, 460 (1988).

[17] C. A. Bonin, R. Bufalo, B. M. Pimentel, and G. E. R. Zambrano, Phys. Rev. D **81**, 025003 (2010).

[18] R. Bufalo, B. M. Pimentel, and G. E. R. Zambrano, Phys. Rev. D **83**, 045007 (2011).

[19] A. E. S. Green, Ph.D. thesis, University of Cincinnati, 1948; Report No. AAT 0151695; R. E. Martin, Ph.D. thesis, University of Cincinnati, 1960.

[20] C. A. Bonin and B. M. Pimentel, Phys. Rev. D **84**, 065023 (2011).

[21] N. Nakanishi, Prog. Theor. Phys. Suppl. **51**, 1 (1972).

[22] J. Polonyi and A. Siwek, Phys. Rev. D **81**, 085040 (2010); **84**, 085014 (2011).

[23] N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (John Wiley & Sons, New York, 1980), 3rd ed.; S. Pokorski, *Gauge Field Theories* (Cambridge University Press, Cambridge, England, 2000), 2nd ed.

[24] H. M. Fried and D. R. Yennie, Phys. Rev. **112**, 1391 (1958); G. S. Adkins, Phys. Rev. D **47**, 3647 (1993).

[25] A. I. Akhiezer and V. B. Berestetskii, *Quantum Electrodynmics* (Interscience Publishers, New York, 1965), 2nd ed.

[26] J. Schwinger, Phys. Rev. **73**, 416 (1948); **76**, 790 (1949).

[27] R. Karplus and N. M. Kroll, Phys. Rev. **77**, 536 (1950); C. M. Sommerfield, Phys. Rev. **107**, 328 (1957); Ann. Phys. (N.Y.) **5**, 26 (1958); A. Petermann,

Nucl. Phys. **3**, 689 (1957); Helv. Phys. Acta **30**, 407 (1957).

[28] *Quantum Electrodynamics*, edited by K. Kinoshita (World Scientific, Singapore, 1990), 1st ed.; T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, Phys. Rev. D **83**, 053002 (2011); **84**, 053003 (2011); **85**, 033007 (2012); **85**, 093013 (2012).

[29] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).

[30] E. S. Fradkin, D. M. Gitman, and Sh. M. Shavartsman, *Quantum Electrodynamics with Unstable Vacuum* (Springer-Verlag, Germany, 1991).

[31] G. Scharf, *Finite Quantum Electrodynamics* (Spring-Verlag, Berlin, 1995), 2nd ed.