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Renormalization flow of axion electrodynamics

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We study the renormalization flow of axion electrodynamics, concentrating on the nonperturbative running of the axion-photon coupling and the mass of the axion(-like) particle. Due to a nonrenormalization property of the axion-photon vertex, the renormalization flow is controlled by photon and axion anomalous dimensions. As a consequence, momentum-independent axion self-interactions are not induced by photon fluctuations. The nonperturbative flow towards the ultraviolet exhibits a Landau-pole-type behavior, implying that the system has a scale of maximum UV extension and that the renormalized axion-photon coupling in the deep infrared is bounded from above. Even though gauge invariance guarantees that photon fluctuations do not decouple in the infrared, the renormalized couplings remain finite even in the deep infrared and even for massless axions. Within our truncation, we also observe the existence of an exceptional renormalization group trajectory, which is extendable to arbitrarily high scales, without being governed by a UV fixed point.

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I. INTRODUCTION

The existence of a fundamental pseudoscalar particle is strongly motivated by the Peccei-Quinn solution to the strong *CP* problem [1]. The axion, being the pseudo-Nambu-Goldstone boson of a spontaneously as well as anomalously broken axial U(1) symmetry [2], receives a small coupling to electromagnetism in generic models [3,4], very similar to the neutral pion in QCD. As a consequence, the effective theory below the QCD scale contains photons and axions as light fundamental degrees of freedom. Their interaction is governed by a dimension-five operator with a coupling being inversely proportional to the (presumably high) scale of Peccei-Quinn symmetry breaking. For reviews, see, e.g., Ref. [5].

The resulting effective theory—axion electrodynamics—actually has a wide range of applications. For instance, it also occurs in the context of macroscopic media exhibiting the magnetoelectric effect [6], as well as in any hypothetical theory beyond the standard model with axion-like degrees of freedom [7,8], and, of course, for the description of pion decay into two photons.

In the present work, we take a more general viewpoint on axion electrodynamics and study its renormalization properties in the effective framework with pure photon and axion degrees of freedom. There are several motivations for this study. First, experiments actively searching for the axion cover a wide range of scales: solar observations such as CAST [9] or the Tokio helioscope [10] search for axions emitted by the sun through the Primakoff process with a typical keV momentum scale [11]. Even higher momentum scales are probed by the stellar cooling rate in the helium-burning phase of horizontal branch stars and other astrophysical considerations as reviewed in Refs. [12–14], with hints of possible observable effects even at the TeV scale [15]. By contrast, purely laboratory-based experiments, such as the classical light-shining-through-walls [16–22] (see also Ref. [23] for an overview) or polarimetry setups [17,24,25] using optical lasers work with momentum transfers on the μeV scale. Searches which are sensitive to axion(-like) dark matter operate at similar scales [26]. A sizable running of the couplings over these nine orders of magnitude would have severe implications for the comparison of experimental results [27]. Even though the coupling is expected to be weak, photon and axion fluctuations could potentially lead to sizable renormalization effects because of their small mass. In particular, photon fluctuations strictly speaking never decouple as their masslessness is granted by gauge invariance.

Another motivation is of a more conceptual type: even though QCD-type models of axion electrodynamics do have a physical cutoff, approximately given by the scale of chiral symmetry breaking $\sim\mathcal{O}(1)$ GeV, it remains an interesting question as to whether axion electrodynamics could be a self-consistent fundamental quantum field theory. Within a nonperturbative context, the negative mass dimension of the axion-photon coupling does not prohibit the existence of the theory as a fundamental theory; it only precludes the possibility of the coupling becoming asymptotically free. If the theory can only exist as an effective theory—as naively expected—its breakdown in the ultraviolet can put restrictions on the physically admissible values of the coupling and the axion mass, much in the same way as the Higgs boson mass in the standard model is bounded from above by UV renormalization arguments (see, e.g., Ref. [28]).

Since the axion-photon interaction as a dimension-five operator is perturbatively nonrenormalizable, a nonperturbative treatment is required. This work is thus based on the functional renormalization group (RG), allowing one to explore a controlled approach to the potentially strongly coupled UV.

The motivating questions are answered by the present study with little surprise, though with interesting lessons to be learned from field theory: in the physically relevant weak-coupling parameter regime, the renormalization effects remain tiny despite the presence of massless fluctuations. Furthermore, no indications for generic UV completeness are found beyond perturbation theory. Nevertheless, the present study yields an interesting example of renormalization theory revealing several unusual properties: we identify a nonrenormalization property of the axion self-interactions and show that the RG flow to lowest order is determined solely by the anomalous dimensions of the photon and the axion field. In addition to explicit solutions to the RG equations, we also identify an exceptional RG trajectory which exemplifies a new class of potentially UV-controlled flows.

Despite the nondecoupling of massless modes in the deep infrared, our RG flow predicts that the physical couplings reach finite IR values. Explicit solutions demonstrate how the massless modes effectively decouple by means of a power law. Finally, the generic UV incompleteness of axion electrodynamics puts an upper bound on the axion-photon interaction for small axion masses in much the same way as the Higgs boson mass is bounded from above in the standard model.

Our article is organized as follows. In Sec. II we introduce axion electrodynamics in Minkowski and Euclidean space. Section III is devoted to the quantization of axion electrodynamics using the Wetterich equation, i.e., the RG flow equation for the one-particle irreducible generating functional. The RG flow is investigated in the theory space spanned by the axion mass and axion-photon coupling in Sec. IV, where the general structure of the flow is discussed, and explicit solutions are worked out as well. Phenomenological implications in the context of the QCD axion as well as for more general axion-like particles (ALPs) are summarized in Sec. V. We conclude in Sec. VI and defer some technical details (Appendix A) as well as further conceptual considerations (Appendix B) to the appendices.

II. AXION ELECTRODYNAMICS

We consider an axion-photon field theory with classical Lagrangian in Minkowski space (with metric $\eta = (+,-,-,-)$),

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu a\partial^\mu a - \frac{1}{2}\bar{m}^2a^2 - \frac{1}{4}\bar{g}aF_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{1}$$

where $\bar{m}$ denotes the axion mass and $\bar{g}$ the axion-photon coupling. The latter has an inverse mass dimension. This Lagrangian—considered as the definition of an effective classical field theory—serves as a starting point for a variety of axion-photon phenomena, such as the Primakoff effect [11] or axion-photon oscillations giving rise to light-shining-through-wall signatures [16], polarimetry effects [29,30] or higher-frequency generation [31]. For the QCD axion, the mass and coupling parameters are related: $\bar{g} \sim \bar{m}$. Particular implications for the example of a QCD axion will be discussed in Sec. V. In the following, we take a different viewpoint and consider this Lagrangian as a starting point for an (effective) quantum field theory.

Let us carefully perform the rotation to Euclidean space where the fluctuation calculation will ultimately be performed. We use the standard conventions

$$x^0|_{\rm M} = -ix_4|_{\rm E}, \qquad A^0|_{\rm M} = -iA_4|_{\rm E}, \qquad \partial^0|_{\rm M} = i\partial_4|_{\rm E}, \qquad \mathcal{L}|_{\rm M} = -\mathcal{L}|_{\rm E}, \tag{2}$$

where $|_{\rm M/E}$ marks the quantities in Minkowski/Euclidean space, respectively. As a consequence, we get, for instance, $\partial_\mu a\partial^\mu a|_{\rm M} = -\partial_\mu a\partial_\mu a|_{\rm E}$, and $F_{\mu\nu}F^{\mu\nu}|_{\rm M} = F_{\mu\nu}F_{\mu\nu}|_{\rm E}$, as usual. The axion-photon coupling contains the structure $aF_{\mu\nu}\tilde{F}^{\mu\nu} = \frac{1}{2}a\epsilon_{\mu\nu\kappa\lambda}F^{\mu\nu}F^{\kappa\lambda}$, which includes $\partial^0A^1\partial^2A^3|_{\rm M}$ and index permutations thereof as building blocks. As only one timelike component appears, the Minkowskian and Euclidean versions of the axion-photon couplings differ by a factor of i. A possible further but irrelevant factor of (-1) may or may not arise depending on the conventions for the Levi-Civita symbol.

The resulting Euclidean Lagrangian finally reads

$$\mathcal{L}|_{\rm E} = \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2}\partial_\mu a\partial_\mu a + \frac{1}{2}\bar{m}^2a^2 + \frac{1}{4}i\bar{g}aF_{\mu\nu}\tilde{F}_{\mu\nu}. \tag{3}$$

For a computation of photon fluctuations in the continuum, gauge fixing is necessary. As there are no minimally coupled charges in pure axion electrodynamics, we can perform a complete Coulomb-Weyl gauge-fixing procedure, imposing the gauge conditions

$$\nabla\cdot\mathbf{A} = 0, \qquad A_0 = 0, \tag{4}$$

such that only the two transversal degrees of freedom of the photon field remain. The corresponding gauge-fixing action in Minkowski space reads

$$\mathcal{L}_{\rm GF} = \frac{1}{2\alpha}(\eta^{\mu\nu} - n^\mu n^\nu)(\partial_\mu A_\nu)(\partial^\kappa A^\lambda)(\eta_{\kappa\lambda} - n_\kappa n_\lambda) + \frac{1}{2\beta}(n_\mu A^\mu)^2, \tag{5}$$

where $n_\mu = (1,0,0,0)$. The translation to Euclidean space using Eq. (2) is straightforward. The two gauge parameters α, β can be chosen independently. In the present work, we will exclusively consider a Landau-gauge limit, $\alpha, \beta \to 0$, corresponding to an exact implementation of the gauge-fixing conditions.

III. QUANTIZATION OF AXION ELECTRODYNAMICS

As the axion-photon coupling has an inverse mass dimension, i.e., $[\bar{g}] = -1$, axion electrodynamics does not belong to the class of perturbatively renormalizable theories. Without any further prerequisites, quantum fluctuations in this theory can still be dealt with consistently in the framework of effective field theories. This may potentially require the fixing of further physical parameters corresponding to higher-order operators. If the theory featured a non-Gaussian UV fixed point with suitable properties, axion electrodynamics could even be asymptotically safe [32,33] and thus nonperturbatively renormalizable.

A suitable framework to deal with quantum fluctuations in either scenario is the functional renormalization group. It allows us to study the renormalization flow of rather general classes of theories specified in terms of their degrees of freedom and their symmetries. For this, a convenient tool is the Wetterich equation [34],

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr}[\partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1}], \qquad \partial_t = k\frac{d}{dk}, \tag{6}$$

for the effective average action Γ_k, which interpolates between a microscopic or bare UV action $S_\Lambda = \Gamma_{k\to\Lambda}$ and the full quantum effective action $\Gamma = \Gamma_{k\to 0}$. The effective average action Γ_k governs the dynamics of the field expectation values after having integrated out quantum fluctuations from a UV scale Λ down to the infrared scale k. The infrared regulator function R_k specifies the details of the regularization of quantum fluctuations to be integrated out near an infrared momentum shell with momentum k. $\Gamma_k^{(2)}$ denotes the second functional derivative of the effective average action with respect to the fields, and the trace contains a summation/integration over all discrete/continuous indices, reducing to a momentum integral in the simplest case. Thus the Wetterich equation is a one-loop equation from a technical point of view, but nevertheless includes effects at higher loop order in perturbation theory since it is the full nonperturbative propagator that enters the loop diagram. Since its derivation does not rely on the existence of a small parameter, it is applicable also in the nonperturbative regime. For reviews of the functional RG see, e.g., Ref. [35].

In the present work, we study the renormalization flow of axion electrodynamics as parametrized by a class of (Euclidean) action functionals of the form

$$\Gamma_k = \int d^4x \bigg[\frac{Z_F}{4}(F_{\mu\nu}(x))^2 + \frac{Z_a}{2}(\partial_\mu a(x))^2 + \frac{\bar{m}_k^2}{2} a(x)^2 + \frac{i\bar{g}_k}{4} a(x) F_{\mu\nu}(x) \tilde{F}_{\mu\nu}(x) \bigg] + Z_F \int d^4x \mathcal{L}_{\mathrm{GF}}, \tag{7}$$

where we allow the mass and the coupling constant to be scale-dependent. Also, the wave function renormalizations $Z_{F/a}$ are implicitly assumed to be scale-dependent. In principle, the gauge-fixing parameters α, β could also run with k. However, we use the known fact that the Landau-gauge limit is a fixed point of the RG [36]. Further terms beyond this simple truncation of full axion electrodynamics are induced by the operators present in our truncation, even if set to zero at a UV scale Λ, and are generically expected to couple back into the flow of the couplings considered here.

The flow equation involves the second functional derivative of the action $\Gamma_k^{(2)}$, corresponding to the inverse propagator, which is matrix-valued in field space:

$$\Gamma_k^{(2)} = \begin{pmatrix} \Gamma^{aa} & \Gamma^{aA} \\ \Gamma^{Aa} & \Gamma^{AA} \end{pmatrix}. \tag{8}$$

In momentum space, the corresponding components read

$$\Gamma^{aa} = \frac{\delta^2 \Gamma_k}{\delta a(p) \delta a(-q)} = (2\pi)^4 \delta^{(4)}(q-p)[\bar{m}_k^2 + Z_a q^2], \tag{9a}$$

$$\Gamma_\lambda^{aA} = \frac{\delta^2 \Gamma_k}{\delta a(p) \delta A_\lambda(-q)} = i\bar{g}_k \varepsilon^{\mu\nu\gamma\delta} \delta_{\nu\lambda} A_\delta(q-p) q_\mu (q_\gamma - p_\gamma), \tag{9b}$$

$$\Gamma_\kappa^{Aa} = \frac{\delta^2 \Gamma_k}{\delta A_\kappa(p) \delta a(-q)} = i\bar{g}_k \varepsilon^{\mu\nu\gamma\delta} \delta_{\nu\kappa} A_\delta(q-p) p_\mu (p_\gamma - q_\gamma), \tag{9c}$$

$$\Gamma_{\kappa\lambda}^{AA} = \frac{\delta^2 \Gamma_k}{\delta A_\kappa(p) \delta A_\lambda(-q)} = i\bar{g}_k \varepsilon^{\mu\nu\gamma\delta} \delta_{\nu\lambda} \delta_{\delta\kappa} q_\mu p_\gamma a(q-p) + (2\pi)^4 \delta^{(4)}(q-p) \times Z_F \bigg[(q^2 \delta_{\kappa\lambda} - q_\kappa q_\lambda) + (\delta_{\kappa\mu} - n_\kappa n_\mu) \frac{q_\mu q_\nu}{\alpha} (\delta_{\nu\lambda} - n_\nu n_\lambda) + \frac{n_\kappa n_\lambda}{\beta} \bigg]. \tag{9d}$$

Note that Γ^{aa} and the second term of Γ^{AA} denote the field-independent inverse propagators, whereas Γ^{aA}, Γ^{Aa} and the first term of Γ^{AA} correspond to vertices, since they contain powers of the external fields. The regulator R_k is chosen diagonal in field space with the components

$$R_k^{aa}(q) = Z_a q^2 r(q^2/k^2),$$

$$R_{k\mu\nu}^{AA}(q) = Z_F q^2\left[\left(\delta_{\kappa\lambda} - \frac{q_\kappa q_\lambda}{q^2}\right) + (\delta_{\kappa\mu} - n_\kappa n_\mu)\frac{q_\mu q_\nu}{\alpha q^2} \times (\delta_{\nu\lambda} - n_\nu n_\lambda) + \frac{n_\kappa n_\lambda}{\beta q^2}\right] r(q^2/k^2). \quad (10)$$

The photon propagator in the Landau gauge then takes the form

$$(\Gamma^{(2)} + R_k)^{-1})^{AA}_{\kappa\lambda}(q)|_{a=0} = \frac{1}{Z_F q^2(1 + r(q^2/k^2))}\left(\delta_{\kappa\lambda} - \frac{1}{q^2 - (n\cdot q)^2} \times (q_\kappa q_\lambda + q^2 n_\kappa n_\lambda) + \frac{n\cdot q}{q^2 - (n\cdot q)^2}(n_\kappa q_\lambda + n_\lambda q_\kappa)\right), \quad (11)$$

where $r(y)$ denotes a regulator shape function, specifying the details of the regularization scheme; see below. Evaluating the Wetterich equation and determining the flow of Γ_k from a UV cutoff Λ down to $k = 0$ corresponds to a quantization of axion electrodynamics in the path-integral framework. If a cutoff $\Lambda \to \infty$ limit existed, axion electrodynamics would even be UV-complete. In the following, we determine the flow of Γ_k truncated down to the form of Eq. (7) as a nonperturbative approximation to the full effective action.

IV. RG FLOW OF AXION ELECTRODYNAMICS

We are interested in the RG flow of all scale-dependent parameters of Γ_k in Eq. (7). Let us start with the flow of the wave function renormalizations Z_a and Z_F. Their flow can be extracted by projecting the right-hand side of the Wetterich equation onto the corresponding kinetic operators:

$$\partial_t Z_a = \frac{1}{\Omega}\left[\frac{\partial^2}{\partial q^2}\int d^4p \frac{\delta^2 \partial_t \Gamma_k}{\delta a(p)\delta a(-q)}\right]_{a,A,q\to 0}, \quad (12)$$

$$\partial_t Z_F = \frac{1}{\Omega}\left[\frac{4}{3}\frac{\partial}{\partial q^2}\int d^4p n_\kappa n_\lambda \frac{\delta^2 \partial_t \Gamma_k}{\delta A_\kappa(p)\delta A_\lambda(-q)}\right]_{a,A,q\to 0}, \quad (13)$$

where Ω denotes the spacetime volume. Similar projection prescriptions also exist for the flow of the (unrenormalized or bare) axion mass and axion-photon coupling,

$$\partial_t \bar{m}_k^2 = \frac{1}{\Omega}\left[\int d^4p \frac{\delta^2 \partial_t \Gamma_k}{\delta a(p)\delta a(-q)}\right]_{a,A,q\to 0}, \quad (14)$$

$$\partial_t \bar{g}_k = \frac{1}{\Omega}\frac{i}{24}\epsilon^{\kappa\,\lambda}_{\alpha\,\beta}\frac{\partial}{\partial r_\alpha}\frac{\partial}{\partial q_\beta} \times \int d^4p\left[\frac{\delta^3 \partial_t \Gamma_k}{\delta a(p)\delta A_\kappa(r)\delta A_\lambda(-q)}\right]\Bigg|_{a,A,q\to 0}. \quad (15)$$

It is straightforward to verify that—within our truncation—the RG flow of the latter quantities vanishes exactly,

$$\partial_t \bar{m}_k^2 = 0, \qquad \partial_t \bar{g}_k = 0. \quad (16)$$

The reason for this nonrenormalization property is obvious from the structure of the axion-photon vertex. From the first term of Eq. (9d), we observe that the vertex with an external axion field has a nontrivial momentum structure $\sim qpa(q - p)$. Together with the antisymmetric Lorentz structure this vertex can only generate operators containing derivatives of the axion field $\sim\partial a$. This also ensures that no nonderivative axion-fermion couplings can be generated by integrating out photon fluctuations, as is in accordance with the axion being the pseudo-Goldstone boson of a spontaneously broken symmetry. In other words, an axion mass term or a higher-order self-interaction potential cannot be generated directly from the axion-photon vertex (nor from possible higher-order axion derivative terms, which are of the form X^n with $X = \partial_\mu a\partial^\mu a$, which is the only purely axionic tensor structure that can be generated within our truncation; see, e.g., Ref. [37]). A simple proof of Eq. (16) can be based on the observation that both the axion mass term as well as the axion-photon coupling are operators which are nonzero for constant axion fields, $a = a_c$. Hence, a projection of the right-hand side of the flow equation onto $a = a_c$ suffices in order to extract the flow of these operators. However, for a constant axion, the axion-photon coupling can be written as

$$\int d^4x a_c F_{\mu\nu}\tilde{F}_{\mu\nu} = 2a_c\int d^4x \partial_\mu(A_\mu \tilde{F}_{\mu\nu}), \quad (17)$$

i.e., as a total derivative of an Abelian Chern-Simons current. Therefore, it can be eliminated from the action and thus cannot contribute to the renormalization flow of the nonderivative axion terms of the theory.[1]

The nonrenormalization property extends to a full axion potential,

$$\partial_t V_k(a^2) = 0, \quad (18)$$

which holds as long as the potential is purely mass-like at some initial scale Λ, $V_\Lambda(a^2) = \frac{1}{2}\bar{m}_\Lambda^2 a^2$. By the same arguments, it also extends to a whole class of general axion-photon interaction operators $\mathcal{O}_k$ which contain a factorizable set of nonderivative axion terms,

$$\partial_t \mathcal{O}_k = 0, \quad \text{for } \mathcal{O}_k = W_k(a)\mathcal{T}_k(a, A_\mu), \quad (19)$$

where $W_k(a)$ is a local function of $a(x)$, and $\mathcal{T}_k$ can also depend on derivatives of a and A_μ. Again, this nonrenormalization holds as long as the interactions are purely of axion-electrodynamics-type at some initial scale, $\mathcal{O}_\Lambda \sim aF_{\mu\nu}\tilde{F}_{\mu\nu}$. Of course, as soon as axion self-interactions are present at some scale, all parameters of the potential V_k get

[1] A similar observation was made in Ref. [38] in order to inversely argue that the renormalization flow of the topological charge in a non-Abelian gauge theory can be properly formulated if the topological charge θ is temporarily considered as a spacetime-dependent field.

renormalized through these self-interactions. Similarly, if higher-order axion-photon interactions are present at some scale, many possible operators $\mathcal{O}_k$ can be generated. In other words, the pure axion mass term and the axion-photon coupling are partial fixed points of the RG flow of axion electrodynamics in terms of bare quantities.

The only renormalization effects in our truncation therefore arise from the running wave function renormalizations. The physical observables can be expressed in terms of the (dimensionful) renormalized couplings, given by

$$m_{\rm R}^2 = \frac{\bar{m}_k^2}{Z_a}, \qquad g_{\rm R}^2 = \frac{\bar{g}_k^2}{Z_F^2 Z_a}. \tag{20}$$

In order to investigate the structure of the RG flow and in particular to search for fixed points at which the theory becomes scale-free, it is convenient to introduce the renormalized dimensionless axion mass and axion-photon coupling,

$$m^2 = \frac{\bar{m}_k^2}{k^2 Z_a} = \frac{m_{\rm R}^2}{k^2}, \qquad g^2 = \frac{\bar{g}_k^2 k^2}{Z_F^2 Z_a} = g_{\rm R}^2 k^2. \tag{21}$$

Defining the anomalous dimensions of the axion and the photon field

$$\eta_a = -\partial_t \ln Z_a, \qquad \eta_F = -\partial_t \ln Z_F, \tag{22}$$

the β functions of mass and coupling can be written as

$$\partial_t g^2 = \beta_{g^2} = (2 + 2\eta_F + \eta_a) g^2, \tag{23}$$

$$\partial_t m^2 = \beta_{m^2} = (\eta_a - 2) m^2. \tag{24}$$

From the projections (12) and (13) of the flow onto the kinetic operators, the anomalous dimensions can be extracted:

$$\eta_a = \frac{g^2}{6(4\pi)^2}\left(2 - \frac{\eta_F}{4}\right), \tag{25}$$

$$\eta_F = \frac{g^2}{6(4\pi)^2}\left(\frac{(2-\frac{\eta_a}{4})}{(1+m^2)^2} + \frac{(2-\frac{\eta_F}{4})}{1+m^2}\right). \tag{26}$$

For these specific forms, we have used the linear regulator shape function $r(y) = (\frac{1}{y} - 1)\theta(1-y)$ [39]. The corresponding results for arbitrary shape functions are given in Appendix A. These are the central results of the present paper, diagrammatically represented in Fig. 1.

As axion electrodynamics belongs to the perturbatively nonrenormalizable theories according to naive power counting, the β or η functions given above do not exhibit the same degree of universality as β functions of marginal couplings in renormalizable theories. This implies that typically even the leading-order (i.e., "one-loop") β function coefficients are scheme-dependent. In the present case, we can change the numerical value of the prefactors in Eqs. (25) and (26) by varying the regulator shape function $r(y)$. Nevertheless, the sign of the prefactors cannot be

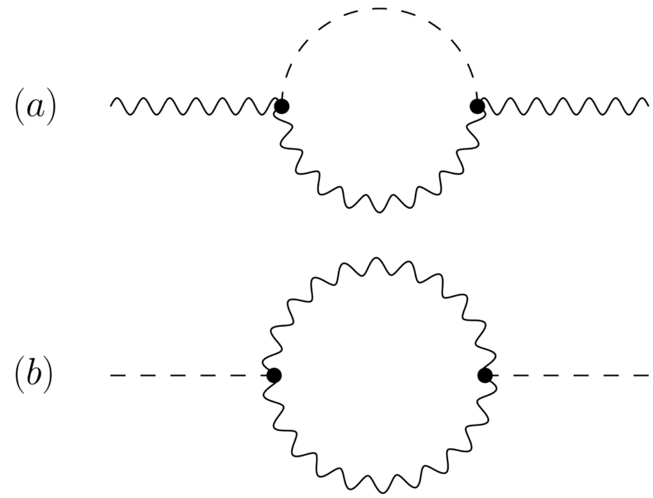

FIG. 1. Schematic representation of the diagrammatic contributions to η_F (a) and η_a (b). Dashed lines denote axions, curly lines photons. As implied by the exact flow equation, all internal propagators and vertices are considered as fully dressed at the scale k. Appropriate insertions of $\partial_t R_k$ at one of the internal lines in each diagram and a corresponding sum over insertions is understood implicitly. The contribution to η_F is thus to be understood as two distinct diagrams, associated with the contributions $\sim\eta_a$ and $\sim\eta_F$ in Eq. (26).

changed for admissible regulators as is visible from the explicit representations given in Appendix A. Furthermore, even for theories exhibiting this type of nonuniversality in their β functions, the existence of fixed points and the critical exponents determining the universality class of the fixed point are universal.

Let us first discuss the resulting β functions in various simple limits. At weak coupling, the anomalous dimensions are small, $\eta \sim g^2$. This implies that the anomalous dimensions on the right-hand sides of Eqs. (25) and (26), signaling a typical RG "improvement," can be neglected. Further assuming a heavy-axion limit, $m^2 \gg 1$, corresponding to a renormalized axion mass being larger than a given scale under consideration, $m_{\rm R}^2 \gg k^2$, the axions decouple, yielding

$$\eta_F \to 0, \qquad \eta_a \to \frac{g^2}{3(4\pi)^2}, \qquad \text{for } m \gg 1, g \ll 1. \tag{27}$$

The axion anomalous dimension remains finite due to the fact that the massless photons never strictly decouple from the flow. In the present weak-coupling heavy-axion limit, the remaining flow of the coupling reduces to

$$\partial_t g^2 = \left(2 + \frac{g^2}{3(4\pi)^2}\right) g^2, \qquad \text{for } m \gg 1, g \ll 1, \tag{28}$$

which can be straightforwardly integrated from a high UV scale Λ to some low scale k. In terms of the dimensionful renormalized coupling, the solution reads

$$g_{\mathrm{R}}^2(k) = \frac{g_{\mathrm{R}}^2(\Lambda)}{1 + \frac{1}{6(4\pi)^2}(\Lambda^2 - k^2) g_{\mathrm{R}}^2(\Lambda)}, \quad \text{for } m \gg 1,\, g \ll 1, \tag{29}$$

where $g_{\mathrm{R}}^2(\Lambda)$ denotes the initial coupling value at the UV cutoff. This value has to satisfy the weak coupling condition $g_{\mathrm{R}}^2(\Lambda)\Lambda^2 \ll 1$. In the present weak-coupling heavy-axion limit, we conclude that the photon-axion coupling undergoes a finite renormalization even in the deep infrared (IR) limit, despite the fact that photonic fluctuations never strictly decouple in the deep IR. The photons still *effectively* decouple, as their low-momentum contributions to the flow vanish according to the power law $\sim k^2 g_{\mathrm{R}}^2(\Lambda)$ for $k \to 0$. The same conclusion holds for the axion mass. Inserting the coupling solution (29) into the flow for the mass (24), the solution for the renormalized mass reads

$$m_{\mathrm{R}}^2(k) = \frac{m_{\mathrm{R}}^2(\Lambda)}{1 + \frac{1}{6(4\pi)^2}(\Lambda^2 - k^2) g_{\mathrm{R}}^2(\Lambda)}, \quad \text{for } m \gg 1,\, g \ll 1. \tag{30}$$

Axion electrodynamics therefore exhibits a remarkable IR stability. Quantitatively, both the coupling as well as the axion mass run to slightly smaller values towards the infrared in the present limit.

Let us now turn to the massless axion limit. As is obvious from Eq. (24), the massless theory is an RG fixed point, $m_* = 0$. This follows from the nonrenormalization of the axion potential, which also implies that the axion will presumably not exhibit the fine-tuning problem typically associated with massive scalars. As long as $\eta_a < 2$, this fixed point is IR-attractive. For weak coupling, the anomalous dimensions reduce to

$$\eta_F \to \frac{2g^2}{3(4\pi)^2}, \quad \eta_a \to \frac{g^2}{3(4\pi)^2}, \quad \text{for } m = 0,\, g \ll 1, \tag{31}$$

yielding the coupling flow

$$\partial_t g^2 = \left(2 + 5\frac{g^2}{3(4\pi)^2}\right) g^2, \quad \text{for } m = 0,\, g \ll 1. \tag{32}$$

Integrating the flow analogously to Eq. (29) leads us to

$$g_{\mathrm{R}}^2(k) = \frac{g_{\mathrm{R}}^2(\Lambda)}{1 + \frac{5}{6(4\pi)^2}(\Lambda^2 - k^2) g_{\mathrm{R}}^2(\Lambda)}, \quad \text{for } m = 0,\, g \ll 1. \tag{33}$$

The conclusion is similar to the heavy-axion case: even though there is no decoupling of any massive modes, axion electrodynamics shows a remarkable IR stability. The fluctuations of the massless degrees of freedom induce only a finite renormalization of the axion-photon coupling yielding smaller couplings towards the IR.

In the intermediate region for finite but not too heavy masses, $m_* = 0$ remains an IR-attractive fixed point for weak coupling, but this massless point is so weakly attractive that decoupling of the axions typically sets in first and the flow ends up in the heavy-axion limit.

Let us now turn to arbitrary values of the coupling. For this purpose, we solve Eqs. (25) and (26) for η_a, η_F and insert the result into Eqs. (23) and (24), yielding the obviously nonperturbative flow equations

$$\partial_t g^2 = \beta_{g^2} = 2g^2 \frac{13g^4 - 384\pi^2 g^2(21 + 17m^2 + 4m^4) - 147456\pi^4(1 + m^2)^2}{g^4 - 384\pi^2 g^2(1 + m^2) - 147456\pi^4(1 + m^2)^2}, \tag{34}$$

$$\partial_t m^2 = \beta_{m^2} = 6m^2 \frac{-g^4 + 128\pi^2 g^2(4m^4 + 7m^2 + 3) - 49152\pi^4(1 + m^2)^2}{-g^4 + 384\pi^2 g^2(1 + m^2) + 147456\pi^4(1 + m^2)^2}. \tag{35}$$

In the heavy-axion limit the decoupling of the axion fluctuations still simplifies the system considerably, since the suppression of $\eta_F(m \to \infty) \to 0$ still persists, leading again to

$$\partial_t g^2 = \left(2 + \frac{g^2}{3(4\pi)^2}\right) g^2, \quad \text{for } m \gg 1, \tag{36}$$

as in Eq. (28). Also the corresponding mass flow is equivalent to that of the heavy-axion weak-coupling limit. The resulting flow exhibits no signature of a fixed point apart from the Gaussian one at $g = 0$. On the contrary, the β function for the running coupling is somewhat similar to that of one-loop QED. The integrated flow will therefore give rise to a Landau pole of the running coupling at high scales. Fixing the renormalized dimensionful coupling to some value at the scale $k = 0$, i.e., $g_{\mathrm{R}}^2(k = 0) = g_{\mathrm{R}0}^2$, the scale of the Landau pole μ_{L}, where $g_{\mathrm{R}}^2(\mu_{\mathrm{L}}) \to \infty$, can be directly read off from Eq. (29):

$$\mu_{\mathrm{L}} \simeq \frac{\sqrt{6}(4\pi)}{g_{\mathrm{R}0}} \simeq \frac{31}{g_{\mathrm{R}0}}, \tag{37}$$

which is an order of magnitude larger than the inverse coupling in the deep IR. For this estimate to hold, the dimensionless mass has to be large during the whole flow. With regard to Eq. (30), this is true as long as $1 \ll m^2 = m_{\mathrm{R}}^2/k^2$ is satisfied for all k. This is always true if $m_{\mathrm{R}}^2(k = 0)/\mu_{\mathrm{L}}^2 \gg 1$ (this criterion could even be relaxed a bit). Going towards larger scales, $m_{\mathrm{R}}(k)$ increases with k and diverges even at $k = \mu_{\mathrm{L}}$.

A different structure becomes visible in the massless limit for arbitrary coupling. Whereas $m_* = 0$ is still a fixed point of the RG, the nonperturbative flow of the coupling becomes

$$\partial_t g^2 = 2g^2 \frac{13g^4 - 8064\pi^2 g^2 - 147456\pi^4}{g^4 - 384\pi^2 g^2 - 147456\pi^4}, \tag{38}$$

for $m = 0$.

Starting from small coupling values, the β_{g^2} function runs into a singularity ($\beta_{g^2} \to \infty$) for

$$g^2_{\text{sing}} = 64\pi^2(3 + 3\sqrt{5}), \quad \text{for } m = 0, \tag{39}$$

where the denominator changes sign. For even larger couplings, the β_{g^2} function returns from $-\infty$ and has a zero at

$$g^2_* = 64\pi^2\left(\frac{3(21 + \sqrt{493})}{13}\right), \quad \text{for } m = 0. \tag{40}$$

For $g^2 \to \infty$, the β_{g^2} function approaches the simple form

$$\partial_t g^2 \simeq 26g^2, \quad \text{for } g^2 \to \infty, m = 0. \tag{41}$$

The massless β_{g^2} function is plotted in Fig. 2.

Fixing the renormalized dimensionful coupling in the deep IR as above, $g^2_{\text{R}}(0) = g_{\text{R0}}$, we can integrate the flow until the running coupling hits the singularity of the β_{g^2} function. The corresponding scale μ'_{L} can be viewed as an "RG-improved" Landau pole, giving a nonperturbative estimate of the scale of maximum UV extension of quantum axion electrodynamics. From a numerical integration, we obtain

$$\mu'_{\text{L}} \simeq \frac{14.844}{g_{\text{R0}}}, \quad \text{for } m = 0. \tag{42}$$

This is of the same order of magnitude as the scale μ_{L} found above for the heavy-mass limit. As a rough estimate of the regularization scheme dependence of this result, let us note that in the case of an exponential shape function $r(y) = (\exp(y) - 1)^{-1}$ we obtain $\mu'_{\text{Lexp}} \simeq \frac{10}{g_{\text{R0}}}$.

For finite values of the axion mass, the flow of the coupling generically exhibits the same features as for the massless case. The position of the singularity in β_{g^2} is shifted towards larger values of the coupling for increasing mass. It is straightforward to verify that the β functions (34) and (35) do not support a nontrivial fixed point for physically admissible positive values of g^2 and m^2. At the same time, the mass flow exhibits the same singularity in β_{m^2} as the denominators in Eqs. (34) and (35) are identical.

We conclude that generic flows in the physically admissible parameter space cannot be extended beyond a scale of maximum UV extension. Phenomenological implications of the existence of such a maximum UV scale will be discussed in the next section. Whether or not substantial extensions of our truncation are able to modify this conclusion is hard to predict. For instance, an inclusion of axion self-interactions would add another sector to the theory which generically suffers from a triviality problem and is thus not expected to change our conclusions. Qualitative modifications of our β functions could potentially arise from momentum-dependent axion self-interactions. These are induced by photon fluctuations even within our truncation, and can couple into the flow of η_a, thus also contributing to the β functions for the axion mass and axion-photon coupling, as well as further momentum-dependent axion self-interactions.

For the remainder of this section, let us concentrate on an oddity of the present flow. There is in fact one exceptional RG trajectory which can be extended to all scales. This trajectory requires the singularity induced by the denominator of the β function to be canceled by a zero of the numerator. Remarkably, there exists a mass and coupling value in the physically admissible region, where the singularities in both the mass and the coupling flow are canceled. This exceptional point in theory space is given by

$$m^2_{\text{exc}} = \frac{1}{2}(\sqrt{5} - 1), \qquad \frac{g^2_{\text{exc}}}{6(4\pi)^2} = 2(3 + \sqrt{5}). \tag{43}$$

The RG trajectory which passes through this point, say at a scale Λ_{exc}, has a standard IR behavior exhibiting the typical decrease towards smaller renormalized mass and couplings. Towards the UV, the β functions do not exhibit a fixed point but approach the simple form

$$\partial_t g^2 \simeq 26g^2, \qquad \partial_t m^2 \simeq 6m^2, \tag{44}$$

which resembles a pure dimensional scaling with large anomalous dimensions. In fact, this behavior corresponds to $\eta_a \to 8$, and $\eta_F \to 8$. It implies that both dimensionless and dimensionful renormalized couplings increase strongly towards the UV without hitting a Landau pole singularity. Instead the growth of the couplings remains controlled on all scales and approaches infinity at infinite UV cutoff scale.

As expected, the scaling dimensions are nonuniversal, and in fact show a considerable regulator dependence: employing an exponential shape function yields $\partial_t g^2 \simeq \frac{31}{2} g^2$ and $\partial_t m^2 \simeq \frac{5}{2} m^2$, corresponding to $\eta_a = 4.5 = \eta_F$.

An important consequence of this exceptional flow is that the physical IR values are completely fixed in terms of

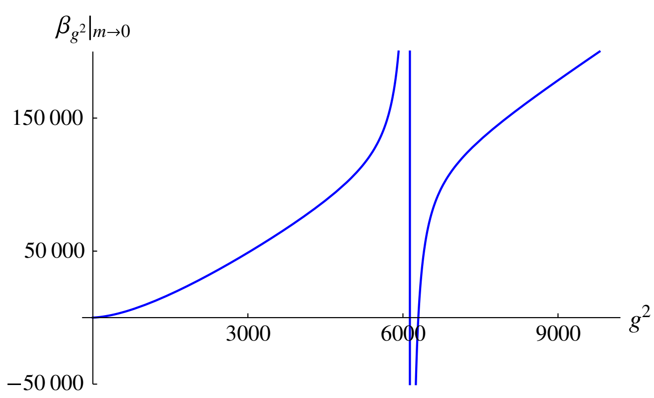

FIG. 2 (color online). The β function β_{g^2} in the massless limit clearly exhibits an IR-attractive Gaussian fixed point, a singularity at $g = g_{\text{sing}}$, and an asymptotic behavior $\sim 26g^2$.

the scale $\Lambda_{\rm exc}$. Numerically integrating the flow towards $k \to 0$ yields for the linear regulator

$$g_{\rm R}(k=0) \equiv g_{\rm R0} \simeq \frac{20.36}{\Lambda_{\rm exc}}, \quad (45)$$

$$m_{\rm R}(k=0) \equiv m_{\rm R0} \simeq 0.4657\Lambda_{\rm exc}, \quad (46)$$

implying the dimensionless combination $g_{\rm R0}m_{\rm R0} \simeq 9.484$. Of course, our observation of this exceptional trajectory requires a critical discussion: the fact that the anomalous dimensions become comparatively large may be interpreted as a signature that explicit momentum dependencies of the propagators and vertices become important at larger couplings. If so, the exceptional trajectory might just be an artifact of our truncation which assumes tree-level-type propagators and vertices.

On the other hand, we cannot exclude the possibility that this exceptional trajectory is a simple projection of a legitimate UV-extendable trajectory in the full theory space. The singularities encountered on the nonexceptional trajectories could then either be an artifact of the truncation or signal the necessity to introduce other microscopic degrees of freedom for reaching specific coupling values in the IR. If so, the observed exceptionality could reflect the restrictions on the physical parameters induced by the true UV behavior (potentially controlled by a UV fixed point). Our truncated RG flow could then be quantitatively reliable below $\Lambda_{\rm exc}$. The strong regulator dependence observed above should be read as a hint that if such an exceptional trajectory exists within the full theory space, it might still change considerably when operators beyond our present truncation are taken into account. In particular, the large anomalous dimensions seem to call for the inclusion of higher-derivative operators.

In the rather speculative case that the trajectory exists with the same qualitative features in the full theory, we would have discovered a (to our knowledge) first example of a UV-complete theory with a high-energy behavior that is controlled by neither a fixed point nor a limit cycle; see, e.g., Ref. [40]. Though this theory would not fall into the class of asymptotically safe systems due to the lack of a UV fixed point, it would be asymptotically controllable. The number of physical parameters of such systems would then correspond to the dimensionality of the exceptional manifold, i.e., the analog of Eq. (43) including all possible further couplings. We emphasize that in this speculative case as well further properties required for a legitimate field theory such as unitarity would have to be critically examined. In particular, a large positive anomalous dimension, as observed here, implies a strongly UV-divergent propagator $\sim(p^2)^{1-\eta/2}$, which might result in cross sections increasing as a large power of the momentum. Whether such behavior can be reconciled with requirements such as perturbative unitarity within standard quantum field theory remains to be investigated.

V. PHENOMENOLOGICAL IMPLICATIONS

A. QCD axion

Axion electrodynamics occurs naturally as a low-energy effective theory in the context of the Peccei-Quinn solution of the strong *CP* problem. The QCD axion develops a generic two-photon coupling, induced by its mixing with the mesonic π^0, η and η' degrees of freedom. Effective axion electrodynamics therefore arises during the chiral phase transition of QCD at around a typical QCD scale which we choose to be $\Lambda_{\rm QCD} \simeq 1$ GeV. At that scale, the relation between axion coupling and mass is essentially fixed by the corresponding pion scales,

$$g_{\rm R}(\Lambda_{\rm QCD}) = C\frac{m_{\rm R}(\Lambda_{\rm QCD})}{m_\pi f_\pi}, \quad (47)$$

where m_π and f_π denote the pion mass and decay constant, respectively, and C is a numerical dimensionless constant that depends on the microscopic axion model (typically defined in the context of a grand unified scenario). For instance, for the KSVZ axion [3], the coupling mass relation yields $g_{\rm R}(\Lambda_{\rm QCD}) \simeq \frac{0.4}{\rm GeV^2} m_{\rm R}(\Lambda_{\rm QCD})$. Other axion models typically lie within an order of magnitude of this relation [4,41].

Axion searches in experiments or astrophysical/cosmological observations actually do not test the parameters occurring in Eq. (47) directly—as they do not operate near the QCD scale—but typically at much lower scales $k_{\rm obs}$. They range from ~keV momentum scales of the stellar evolution theory of horizontal-branch stars, via ~eV scales for solar energy loss or direct helioscope observations, to ~μeV scales in light-shining-through-walls experiments. In other words, a proper comparison of such observational results with Eq. (47) requires taking the finite renormalization of axion electrodynamics between $\Lambda_{\rm QCD}$ and $k_{\rm obs}$ into account. For simplicity, we estimate the maximum renormalization effect by choosing $k_{\rm obs} = 0$. In order to illustrate our findings, we plot in Fig. 3 the renormalized ratio

$$c_{\rm R} = \frac{g_{\rm R}(k_{\rm obs})/m_{\rm R}(k_{\rm obs})}{g_{\rm R}(\Lambda_{\rm QCD})/m_{\rm R}(\Lambda_{\rm QCD})} \quad (48)$$

inspired by Eq. (47) as well as the ratio of the product

$$p_{\rm R} = \frac{g_{\rm R}(k_{\rm obs})m_{\rm R}(k_{\rm obs})}{g_{\rm R}(\Lambda_{\rm QCD})m_{\rm R}(\Lambda_{\rm QCD})} \quad (49)$$

as dark-blue (dashed) and purple (solid) lines, respectively. The ratios are plotted as a function of the logarithm of the initial coupling $g_{\rm R}(\Lambda_{\rm QCD})$ in units of GeV. For small couplings, the renormalization of both coupling and mass remain unobservably small, implying that both ratios $c_{\rm R}$, $p_{\rm R} \to 1$. It is only for larger couplings that the renormalization of both towards smaller values becomes visible, implying a significant decrease of $p_{\rm R}$. Most importantly, the proportionality between mass and coupling remains essentially unaffected. Numerically, the ratio c_R changes only on the 10^{-5} level.

We conclude that exclusion bounds derived from the nonobservation of axion effects are not modified by renormalization effects of axion electrodynamics along the lines of constant g_R/m_R. As the physically relevant parameter space for the QCD axion lies below $g_R \lesssim 10^{-9}$ (GeV)$^{-1}$, we conclude from Fig. 3 that the renormalization effects within the effective theory of axion electrodynamics are completely irrelevant.

The existence of a scale of maximum UV extension μ_L also seems to be of no relevance for the QCD axion: for instance, for an axion coupling in the range of $g \sim \mathcal{O}(10^{-16})$ GeV where the axion could also provide a substantial part of the dark matter of the universe, the scale μ_L would lie well beyond a typical GUT scale. In turn, this demonstrates that axion electrodynamics is a consistent effective (quantum) field theory at low energies within the QCD axion scenario.

For completeness, let us mention that the QCD axion would be on the exceptional trajectory found above if $g_R(\Lambda_{QCD}) \simeq 1.9\ \mathrm{GeV}^{-1}$ and $m_R(\Lambda_{QCD}) \simeq 5.0$ GeV. This is, of course, beyond the parameter regime conventionally considered for the QCD axion.

B. General ALPs

Apart from the QCD axion, massive scalar bosons may arise as pseudo-Nambu-Golstone bosons in a variety of contexts [7,8]. In the case of a pseudoscalar boson, the emergence of a coupling to photons of axion-electrodynamics-type is a natural consequence. Since the coupling-mass relation in this general case is not necessarily fixed as in Eq. (47), a more general parameter space for such axion-like particles can be investigated.

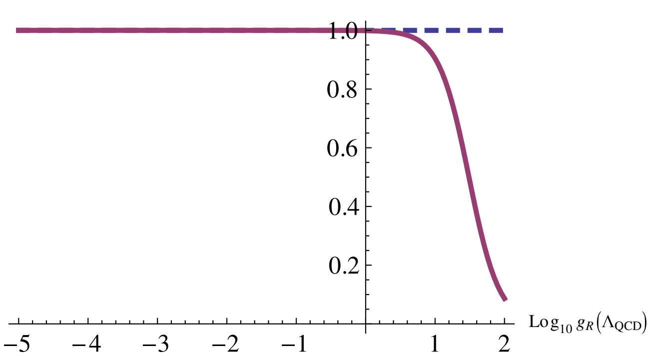

FIG. 3 (color online). Plot of the renormalized coupling ratios c_R of (48) (blue/dashed line) and p_R of (49) (purple/solid line) as a function of the logarithm of the initial axion-photon coupling $g_R(\Lambda_{QCD})$ in units of GeV. For small couplings, the renormalization of both coupling and mass remain unobservably small. It is only for larger couplings that the renormalization of both towards smaller values becomes visible (purple/solid line), while the proportionality between mass and coupling remains essentially unaffected (blue/dashed line).

A priori, the mass-coupling space seems rather unrestricted. Depending on the mass generation mechanism, the ALP mass can vary strongly from very light masses arising from anomalous breakings, as in the QCD-axion case, to explicit masses of the order of the high-scale Λ_{UV} (such as the Fermi, GUT, or Planck scale). Depending on the value of the coupling, the low-energy mass may be very different from the high-energy mass due to renormalizations of the type (30), but, in general, the renormalized low-energy mass still remains rather unconstrained.

This can be different for the possible values of the effective low-energy ALP-photon coupling. If the effective low-energy theory still exhibits a scale of maximum UV extension μ_L as in Eqs. (37) or (42), then that scale must necessarily be higher than the microscopic high-energy scale Λ_{UV}. Otherwise, the effective low-energy theory could not possibly arise from that unknown microscopic theory. As a consequence, the low-energy coupling is bounded from above due to the renormalization flow in the effective theory,

$$\max g_R(k \to 0) \equiv g_{R0,\max} = \frac{N_R}{\mu_L} \leq \frac{N_R}{\Lambda_{UV}}, \qquad (50)$$

where N_R is a number that depends on the ALP mass as well as on further degrees of freedom in the effective low-energy theory. In the case that this effective theory is well-approximated by axion electrodynamics, we found that $N_R = \mathcal{O}(10)$ ($N_R \simeq 31$ for large masses and $N_R \simeq 14.8$ for the massless case).

Our conclusion that g_{R0} should be suppressed by the high-scale Λ_{UV} looks rather trivial, as it seems to follow standard power-counting arguments for a higher-dimensional operator. However, we stress that the statement is actually stronger: even for unusually enhanced couplings at the high scale (invalidating naive power counting), axion electrodynamics as an effective theory ensures that the low-energy coupling is suppressed by renormalization effects and obeys the bound (50).

We expect the precise number N_R to be modified by further degrees of freedom, such as the standard model fermions contributing to the flow at higher scales. As long as they leave the anomalous dimension η_a in Eq. (25) positive, we expect a mere quantitative influence on N_R. Of course, these conclusions no longer hold if the additional degrees of freedom render the effective theory asymptotically free or safe, or if the system sits on the exceptional trajectory as discussed above.

VI. CONCLUSIONS

We have analyzed the renormalization flow of axion electrodynamics considered as an effective quantum field theory. From a field theory viewpoint, we have revealed several interesting properties: nonrenormalization properties protect the flow of the axion mass and axion-photon coupling if the interactions initially are of pure axion-electrodynamics-type. In this case, the flow of the corresponding renormalized quantities is solely determined by the axion and photon anomalous dimensions. These nonrenormalization properties are in line with but go beyond the fact that the axion can be understood as a pseudo-Nambu-Goldstone boson of a broken Peccei-Quinn symmetry.

Towards the infrared, the flow remains well-controlled even in the presence of massless degrees of freedom. Even though massless photons strictly speaking never decouple, their contribution to the RG flow effectively decouples, leading to finite and predictable IR observables. By contrast, the UV of axion electrodynamics exhibits a triviality problem somewhat similar to ϕ^4 theory: insisting on sending the cutoff $\Lambda \to \infty$ is only possible for the free theory $g \to 0$. From a more physical viewpoint, fixing the coupling to a finite value at a finite scale implies that axion electrodynamics can be treated as a quantum field theory only up to a scale of maximum UV extension μ_L. The value of this scale is, of course, not universal. For our regularization scheme, this scale is about an order of magnitude larger than the inverse IR coupling and depends weakly on the axion mass.

A behavior different from this generic case is only found on an exceptional RG trajectory which remains free of singularities on all finite scales. From within our truncated RG flow, it is difficult to decide whether this trajectory is a mere artifact of the truncated theory space or a remnant of a valid trajectory of an interacting UV-controlled theory. In the latter case, it could be the first example of a predictive and consistent theory without being associated with an obvious UV fixed point. At the present level of approximation, however, we consider this exceptional trajectory as an oddity, the status, physical relevance and consistency of which still has to be carefully examined.

For the QCD axion, our findings demonstrate that the typical proportionality between axion-photon coupling and axion mass is not relevantly renormalized by low-energy axion-photon fluctuations. In fact, possible renormalizations of mass and coupling largely cancel out of the proportionality relation even at stronger coupling. As a consequence, phenomenological bounds on these parameters remain essentially unaffected by the RG flow. By contrast, the absolute values of mass or coupling can undergo a sizeable renormalization; however, the required coupling strength is not part of the natural QCD axion regime.

For more general axion-like particles the existence of a generic maximum scale of UV extension induces an upper bound on possible values of the axion-photon coupling. Standard lines of perturbative reasoning [42] suggest that the renormalized coupling should be of order $g_R \sim 1/\Lambda_{UV}$, where Λ_{UV} denotes the microscopic scale where the axion sector is coupled to the standard model particles. Our RG study now demonstrates that couplings of that size are not only natural, but are in fact bounded by $g_R \lesssim \mathcal{O}(10)/\Lambda_{UV}$ due to renormalization effects in the axion-photon sector. In view of the rather unconstrained ALP parameter space at large masses and comparatively large couplings (see, e.g., the compilation in Ref. [14]), our bound could become of relevance for ALP searches at hadron colliders above $m_R \gtrsim 1$ GeV and couplings above $g_R \gtrsim 10^{-3}/(\mathrm{GeV})^{-1}$.

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APPENDIX A: RG FLOWS FOR GENERAL REGULATOR SHAPE FUNCTIONS

For generality, we present the flows of the wave function renormalizations for general regulator shape functions $r(y)$. For the flow of the axion wave function, we obtain

$$\partial_t Z_a = -\frac{g^2}{2(2\pi)^4}\int d^4p \frac{-\eta_F r_k(\frac{p^2}{k^2}) + \partial_t r_k(\frac{p^2}{k^2})}{Z_F^2 p^2(1+r_k[\frac{p^2}{k^2}])^3}. \quad \text{(A1)}$$

The flow of the photon wave function is given by

$$\partial_t Z_F = -\frac{g^2}{2}\int\frac{d^4p}{(2\pi)^4} \times \Bigg(\frac{-Z_a\eta_a p^2 r_k(\frac{p^2}{k^2}) + Z_a p^2\partial_t r_k(\frac{p^2}{k^2})}{(m^2+Z_a p^2[1+r_k(\frac{p^2}{k^2})])^2 Z_F[1+r_k(\frac{p^2}{k^2})]} + \frac{-\eta_F r_k(\frac{p^2}{k^2}) + \partial_t r_k(\frac{p^2}{k^2})}{Z_F[1+r_k(\frac{p^2}{k^2})]^2(m^2+Z_a p^2[1+r_k(\frac{p^2}{k^2})])}\Bigg). \quad \text{(A2)}$$

Inserting the linear regulator shape function $r(y) = (\frac{1}{y}-1)\theta(1-y)$, the momentum integrals can be performed analytically. The results can be expressed in terms of the anomalous dimensions and are given by Eqs. (25) and (26).

APPENDIX B: EUCLIDEAN AXION ELECTRODYNAMICS WITH IMAGINARY COUPLING

In the main text, we have emphasized that the physically admissible parameter space is constrained to positive masses and couplings (squared), g^2, $m^2 > 0$. For $m^2 < 0$ and in absence of any further axion potential, the Euclidean action is unbounded from below along the direction of large axion-field amplitude. Of course, this could be cured by adding a stabilizing potential, but this route will not be followed in this work.

A more interesting case is provided by the case of imaginary axion-photon couplings, $g^2 < 0$. In this case, the Euclidean action would violate Osterwalder-Schrader reflection positivity, such that a corresponding Minkowskian theory can be expected to violate unitarity. Still, the Euclidean theory could be regarded as a valid field theory description of some suitable statistical system. In this case, the action would be stable along the axion-amplitude direction, but unstable towards the formation of large electromagnetic fields with large $\mathbf{E}\cdot\mathbf{B}$. This instability could be cured by higher photon self-interaction such as

$$\mathcal{L}_F = f_1(F_{\mu\nu}F_{\mu\nu})^2 + f_2(F_{\mu\nu}\tilde{F}_{\mu\nu})^2, \tag{B1}$$

with positive constants $f_{1,2}$ carrying a mass dimension of -4. Actions of this type are familiar from fluctuation-induced nonlinear QED contributions to electrodynamics [43,44]. Also within axion electrodynamics, we expect these contributions to be generated by mixed axion-photon fluctuations (some properties of such amplitudes have, for instance, been studied in Ref. [45]). In the following, we simply assume that these terms are suitably generated either within axion electrodynamics or provided by an exterior sector coupling to photons. Then, the parameter region where $g^2 < 0$ can become physically admissible as well.

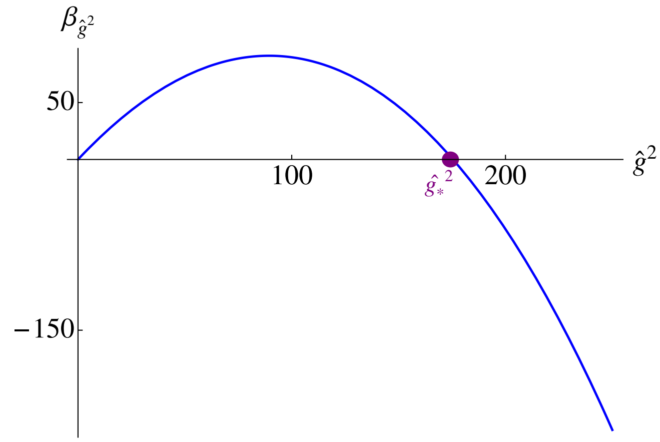

FIG. 4 (color online). The β function for the coupling $\hat{g}$ in the limit $m \to 0$ clearly shows a UV-attractive non-Gaussian fixed point at $\hat{g} = \hat{g}_* = 13.25$, as well as an infrared-attractive Gaussian fixed point.

In that case, the β function for $\hat{g} = -ig$ is given by

$$\beta_{\hat{g}^2} = 2\hat{g}^2 \frac{13\hat{g}^4 + 384\pi^2\hat{g}^2(21 + 17m^2 + 4m^4) - 147456\pi^4(1 + m^2)^2}{\hat{g}^4 + 384\pi^2\hat{g}^2(1 + m^2) - 147456\pi^4(1 + m^2)^2}. \tag{B2}$$

The mass β function now takes the form

$$\beta_{m^2} = 6m^2 \frac{-\hat{g}^4 - 128\pi^2\hat{g}^2(4m^4 + 7m^2 + 3) - 49152\pi^4(1 + m^2)^2}{-\hat{g}^4 - 384\pi^2\hat{g}^2(1 + m^2) + 147456\pi^4(1 + m^2)^2}. \tag{B3}$$

In this system, we find a UV-attractive fixed point at $(\hat{g}, m) \approx (13.25, 0)$. The critical exponents, defined as eigenvalues of the stability matrix $\frac{\partial \beta_{g_i}}{\partial g_j}$ with $g_i = (\hat{g}, m)$, multiplied by an additional negative sign, are $(\theta_1, \theta_2) = (2.16, 1.20)$. Thus this fixed point is UV-attractive in both directions.

Clearly the system also admits a Gaussian fixed point with critical exponents given by the canonical dimensions. Accordingly this fixed point is IR-attractive in the coupling.

We conclude that, setting the axion mass to zero, the system admits the construction of a complete RG trajectory, extending from $\hat{g} = 0$ at $k \to 0$ to $\hat{g} = \hat{g}_*$ for $k \to \infty$; see Fig. 4. Thus the axion-photon system with an imaginary coupling has the potential to provide a simple example of an asymptotically safe quantum field theory, albeit without an immediate physical application.

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