PHYSICAL REVIEW D **86,** 125012 (2012)

Chern-Simons spinor electrodynamics in the light-cone gauge

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(Received 4 October 2012; published 3 December 2012)

The one-loop quantum corrections of Chern-Simons spinor electrodynamics in the light-cone gauge has been investigated. We have calculated the vacuum polarization tensor, fermionic self-energy, and on-shell vertex correction with a hybrid regularization consisting of a higher covariant derivative regularization and dimensional continuation. The Mandelstam-Leibbrandt prescription is used to handle the spurious light-cone singularity in the gauge field propagator. We then perform the finite renormalization to define the quantum theory. The generation of the parity-even Maxwell term and the arising of anomalous magnetic moment from quantum corrections are reproduced as in the case of a covariant gauge choice. The Ward identities in the light-cone gauge are verified to satisfy explicitly. Further, we have found the light-cone vector dependent sector of local quantum effective action for the fermion is explicitly gauge invariant, and hence the Lorentz covariance of S-matrix elements of the theory can be achieved.

DOI: 10.1103/PhysRevD.86.125012 PACS numbers: 11.15.Yc, 11.10.Gh, 11.15.Bt, 12.20.Ds

I. INTRODUCTION

The first step in calculating quantum correction of a gauge theory by perturbation theory is choosing a gauge condition to eliminate the nonphysical degrees of freedom caused by gauge symmetry. This process is called gauge fixing. Despite that physically measurable results should be independent of gauge choice, but with different gauge-fixing, the quantum theory presents distinct features in both calculation techniques and the resultant quantum corrections. The usually preferred choice is a Lorentz covariant gauge condition like $\partial_\mu A^\mu = 0$, since the Lorentz covariance can be preserved in the entire calculation process, and further, the propagator of gauge field has a nice analytical structure.

Nevertheless, in certain circumstances, a noncovariant gauge choice turns out to be more convenient than a covariant one, since this kind of gauge choice can somehow approach to physical degrees of freedom straightforwardly at the classical level. Especially, a noncovariant gauge fixing in a non-Abelian gauge theory can make ghost fields decouple from the physical sector in the classical stage, and avoid the notorious Gribov's ambiguity haunted the gauge-fixing procedure [1,2].

However, a noncovariant gauge fixing brings about a spurious singularity in the gauge field propagator [1,2]. This hinders the loop integration in perturbation theory from being performed straightforwardly as in the covariant case. Therefore, a prescription of handling the spurious singularity must be defined so that the denominator of the integrand in a loop integration is quadratic in the loop momentum [1,2]. A number of prescriptions had been proposed [1,2]. Up to now it seems that the most convenient and universal prescription is the n_μ^* prescription suggested by Mandelstam [3] and Leibbrandt [4], which is now termed as the Mandelstam-Leibbrandt (ML) prescription [1]. It has been tested that the ML prescription can give consistent results for any noncovariant gauge choices at one-loop level for gauge theories in both four and three dimensions [2]; although, its applicability in evaluating two-loop and higher order quantum corrections needs to be verified explicitly.

The study on the pure non-Abelian Chern-Simons (CS) gauge theory in the light-cone gauge at one-loop with the ML-prescription was pioneered by Martin and Leibbrandt [5]. A consistent result with the covariant gauge fixing had been achieved: the celebrated finite quantum correction k-shift [6–8] of the gauge coupling is reproduced, and the nonlocal gauge dependent terms are unobservable. Consequently, the topological feature of the theory is preserved. Hence the applicability of the ML-prescription to three-dimensional gauge theory with parity violation had been verified at one-loop order [5].

In this article we shall investigate three-dimensional Chern-Simons spinor electrodynamics [9,10] in the light-cone gauge, i.e., $U(1)$ CS gauge theory coupled with fermions. This model has some distinct features from the pure non-Abelian CS gauge theory, and it is worthy to observe its quantum corrections in the light-cone gauge with the ML prescription. First, it is not a topological field theory since the coupling of gauge field with fermion requires an explicit involvement of the space-time metric, and the theory has local dynamical degrees of freedom. Second, from the one-loop result of four-dimensional gauge theory in the light-cone gauge calculated with the ML-prescription, the light-cone vector dependent part in the local quantum effective action for fermions should take a specific gauge-invariant form [1], determined by the Ward-Takahashi identities in the light-cone gauge, so that the covariance of S-matrix elements of theory can be recovered. It is interesting to check explicitly whether such a result arises in a three-dimensional gauge theory in the light-cone gauge. Third, in contrast to the pure non-Abelian CS gauge theory, which has only one

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dimensionless parameter—the gauge coupling, CS spinor electrodynamics has a parameter with mass dimension—the mass of the fermion. This will make both the tensor structure and form factors of quantum corrections of the theory much more involved.

Furthermore, it has been shown that in the covariant gauge-fixing, CS spinor electrodynamics presents some remarkable quantum effects including the generation of Maxwell (or parity-even) term and the arising of anomalous magnetic moment of the fermion [9]. It is significant to observe these radiative corrections in the light-cone gauge with the ML prescription, since this can not only reveal quantum features of Chern-Simons-matter theory, but also confirm and consolidate the validity of the ML prescription in evaluating quantum corrections of three-dimensional gauge theories in the light-cone gauge.

In Sec. II, we introduce the classical CS spinor electrodynamics with the light-cone gauge-fixing. For later perturbative calculation, we choose a hybrid regularization scheme to derive the Feynmann rules. The hybrid regularization is a combination of higher covariant derivative regularization and dimensional continuation with the Maxwell term as the higher derivative term. Section III contains a calculation on two-point functions at one-loop including the vacuum polarization tensor $\Pi_{\mu\nu}(p)$ and the fermionic self-energy $\Sigma(p)$. We use the ML prescription to handle the spurious singularity of the gauge field propagator. In Sec. IV we display a detailed evaluation of one-loop quantum vertex on the mass-shell of the fermion. Because it requires two light-cone vectors n_μ and n^*_μ to implement the ML prescription, the calculation on the form factors of on-shell vertex correction is much more tedious than the case of covariant gauge-fixing. In Sec. V we perform renormalization on the quantum corrections found in previous two sections, and reveal quantum effects and the structure of local quantum effective action of the theory. The calculation techniques and integration formulas are given in detail in Appendices A and B. In Appendix C we derive the Ward identities of CS spinor electrodynamics in the light-cone gauge. In particular, we show the explicit restriction of the Ward identity on the general form of two-point function of gauge field, and the relation between the gauge field-fermion-fermion vertex correction and the fermionic self-energy.

II. CHERN-SIMONS SPINOR ELECTRODYNAMICS IN THE LIGHT-CONE GAUGE

The Lagrangian density of CS spinor electrodynamics in the light-cone gauge is

$$\mathcal{L} = \frac{1}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho + \bar{\psi}(i\not{\partial} + e\not{A} - m)\psi - \frac{1}{2\xi}(n^\mu A_\mu)^2, \tag{1}$$

where $(n_\mu) = (n_0, n_1, n_2)$ is the light-cone vector, which by definition satisfies $n^2 = 0$, and further $\xi \to 0$. The γ matrices in the Lagrangian density (1) are chosen as follows:

$$\gamma^0 = \sigma_2, \qquad \gamma^1 = i\sigma_3, \qquad \gamma^2 = i\sigma_1. \tag{2}$$

Consequently, the algebra formed by the γ matrices is

$$\begin{aligned} \gamma_\mu\gamma_\nu &= g_{\mu\nu} - i\epsilon_{\mu\nu\rho}\gamma^\rho, \qquad \{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}, \\ [\gamma_\mu, \gamma_\nu] &= -2i\epsilon_{\mu\nu\rho}\gamma^\rho, \qquad (g_{\mu\nu}) = \mathrm{diag}(1, -1, -1). \end{aligned} \tag{3}$$

The gauge-fixing term $-1/(2\xi)(n^\mu A_\mu)^2$ in the Lagrangian density (1) comes from the light-cone gauge condition $n^\mu A_\mu = 0$, $n^2 = 0$ with $\xi \to 0$ in the gauge field propagator.

To investigate the perturbative quantum corrections of CS spinor dynamics, we must choose a regularization scheme to deal with the ultraviolet divergence in loop integration. Usually, the most convenient method is dimensional regularization. However, due to the particular feature of CS term (its kinetic operator $\epsilon^{\mu\nu\rho}\partial_\rho$ being a first-order nonpositive definite differential operator), we must first implement a higher covariant derivative regularization scheme. The simplest gauge invariant higher covariant derivative term is the Maxwell term,

$$\mathcal{L}_\Lambda = -\frac{1}{4\Lambda}F_{\mu\nu}F^{\mu\nu}, \tag{4}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and Λ is the regulator.

To apply dimensional regularization, we should use the 't Hooft-Veltman prescription to define the dimensional continuation of $\epsilon_{\mu\nu\rho}$ tensor and the γ matrices [11,12]. The regularized d-dimensional space is divided into a direct sum of the original three-dimensional space and a $(d-3)$-dimensional space, d being a complex number [5,7]. However, for the Abelian CS theory, the ϵ tensor appears only in the gauge field propagator, and especially, in this work we consider only the perturbative theory at one-loop level. Hence the 't Hooft-Veltman recipe makes no difference with the usual naïve dimensional continuation, and the inconsistency found in Ref. [13] will not arise. The explicit calculations carried out later will confirms this argument.

As a hybrid combination of the dimensional continuation and the higher covariant derivative regularization, the order of removing the regulators after the renormalization is first taking the limit $d \to 3$ and then $\Lambda \to \infty$.

The regularized Lagrangian density $\mathcal{L} + \mathcal{L}_\Lambda$ leads to the following tree-level Feynman rules:

(i) Photon propagator [5]:

$$iG^{(0)}_{\mu\nu}(p,n,\Lambda)=\frac{i\Lambda}{p^2(p^2-\Lambda^2)}\Big[i\Lambda\epsilon_{\mu\nu\rho}p^\rho - \frac{i\Lambda}{n\cdot p}(p_\mu\epsilon_{\nu\alpha\beta}-p_\nu\epsilon_{\mu\alpha\beta})p^\alpha n^\beta - p^2g_{\mu\nu}+\frac{p^2}{n\cdot p}(p_\mu n_\nu+p_\nu n_\mu)\Big] = \frac{i\Lambda}{(p^2-\Lambda^2)n\cdot p}[i\Lambda\epsilon_{\mu\nu\rho}n^\rho - n\cdot p g_{\mu\nu} + (p_\mu n_\nu+p_\nu n_\mu)], \quad (5)$$

where the following one of Martin's identities has been used [5,14],

$$\frac{1}{n\cdot p}\epsilon_{\mu\nu\rho}n^\rho=\frac{1}{p^2}\epsilon_{\mu\nu\rho}p^\rho - \frac{1}{p^2(n\cdot p)}(p_\mu\epsilon_{\nu\alpha\beta}-p_\nu\epsilon_{\mu\alpha\beta})p^\alpha n^\beta. \quad (6)$$

As $\Lambda\to\infty$ at tree-level, the propagator (5) reduces to

$$iG^{(0)}_{\mu\nu}(p,n)=\frac{1}{n\cdot p}\epsilon_{\mu\nu\rho}n^\rho. \quad (7)$$

(ii) Fermionic propagator:

$$iS^{(0)}(p)=i\frac{\not p+m}{p^2-m^2}. \quad (8)$$

(iii) Gauge field-fermion-fermion vertex:

$$-ie\Gamma^{(0)}_\mu(p,q,r)=-ie\gamma_\mu(2\pi)^d\delta^{(d)}(p+q+r). \quad (9)$$

In the following sections we shall calculate one-loop quantum corrections of the theory and show quantum features of CS spinor electrodynamics in the light-cone gauge.

III. ONE-LOOP VACUUM POLARIZATION TENSOR AND FERMIONIC SELF-ENERGY

A. Vacuum polarization tensor

Since the characteristic of the light-cone gauge fixing involves only the $U(1)$ CS gauge field propagator, the vacuum polarization tensor is identical to that in the usual covariant gauge,

$$\begin{aligned} i\Pi^{(1)}_{\mu\nu}(p^2) &= -e^2\int\frac{d^dk}{(2\pi)^d}\frac{\mathrm{Tr}[\gamma_\nu(\not k+\not p+m)\gamma_\mu(\not k+m)]}{(k^2-m^2)[(k+p)^2-m^2]} \\ &= -2e^2\int\frac{d^dk}{(2\pi)^d}\frac{-im\epsilon_{\mu\nu\rho}p^\rho+2k_\mu k_\nu+k_\mu p_\nu+k_\nu p_\mu-g_{\mu\nu}[k\cdot(k+p)-m^2]}{(k^2-m^2)[(k+p)^2-m^2]}, \end{aligned} \quad (10)$$

where we have used γ-matrix algebra listed in (3). The parity-odd part is finite, and one can take the limit $d\to3$ before performing the loop integration. The parity-even part contains the superficially linear and logarithmic divergent terms, which can be evaluated by the dimensional regularization. Using the formula listed in Appendix B, we obtain (after taking the limit $d\to3$)

$$\begin{aligned} \Pi^{(1)}_{\mu\nu}(p) &= i\epsilon_{\mu\nu\rho}p^\rho\Pi_{\mathrm{o}}(p^2)+(p^2g_{\mu\nu}-p_\mu p_\nu)\Pi_{\mathrm{e}}(p^2) \\ &= \frac{e^2}{4\pi}\Big\{i\epsilon_{\mu\nu\rho}p^\rho\frac{m}{p}\ln\Big[\frac{1+p/(2m)}{1-p/(2m)}\Big]-(p^2g_{\mu\nu}-p_\mu p_\nu)\frac{1}{m}\Big[-\frac{m^2}{p^2}+\frac{m}{p}\Big(\frac14+\frac{m^2}{p^2}\Big)\ln\Big(\frac{1+p/(2m)}{1-p/(2m)}\Big)\Big]\Big\}. \end{aligned} \quad (11)$$

In Eq. (11) $p\equiv|p|$, and $\Pi_{\mathrm{o}}(p^2)$ and $\Pi_{\mathrm{e}}(p^2)$ represent the parity odd- and even form factors of the vacuum polarization tensor,

$$\Pi_{\mathrm{o}}(p)=\frac{e^2}{4\pi}\frac{m}{p}\ln\Big[\frac{1+p/(2m)}{1-p/(2m)}\Big], \qquad \Pi_{\mathrm{e}}(p)=\frac{e^2}{4\pi}\frac{1}{m}\Big[\frac{m^2}{p^2}-\frac{m}{p}\Big(\frac14+\frac{m^2}{p^2}\Big)\ln\Big(\frac{1+p/(2m)}{1-p/(2m)}\Big)\Big]. \quad (12)$$

B. Self-energy of fermion

Compared with the case of the covariant Landau gauge [9], the fermionic self-energy has some distinct features due to the presence of the spurious light-cone gauge singularity $1/(n\cdot k)$ in the propagator of $U(1)$ CS gauge field:

$$-i\Sigma^{(1)}(p,m,n,\Lambda,d) = e^2\int\frac{d^dk}{(2\pi)^d}\left\{\frac{\gamma^\nu(\not k+\not p+m)\gamma^\mu\Lambda}{[(k+p)^2-m^2](k^2-\Lambda^2)n\cdot k}[i\Lambda\epsilon_{\mu\nu\rho}n^\rho - n\cdot kg_{\mu\nu}+(k_\mu n_\nu+k_\nu n_\mu)]\right\}$$
$$= e^2\int\frac{d^dk}{(2\pi)^d}\left\{-\frac{\Lambda\gamma^\mu(\not k+\not p+m)\gamma_\mu}{[(k+p)^2-m^2](k^2-\Lambda^2)}+\frac{i\Lambda^2\epsilon^{\mu\nu\rho}n_\rho\gamma_\nu(\not k+\not p+m)\gamma_\mu}{[(k+p)^2-m^2](k^2-\Lambda^2)n\cdot k}\right.$$
$$\left.+\frac{\Lambda[\not n(\not k+\not p+m)\not k+\not k(\not k+\not p+m)\not n]}{[(k+p)^2-m^2](k^2-\Lambda^2)n\cdot k}\right\}. \tag{13}$$

The spurious light-cone gauge singularity $1/(n\cdot k)$ in the integrand brings difficulty in evaluating the loop integration. We use the ML prescription in Minkowskian space to handle the singularity [3,4]:

$$\frac{1}{n\cdot k} = \lim_{\epsilon\to 0}\frac{n^*\cdot k}{(n\cdot k)(n^*\cdot k)+i\epsilon} = \lim_{\epsilon\to 0}\frac{n^*\cdot k}{n_0^2k_0^2+(\mathbf{n}\cdot\mathbf{k})^2+i\epsilon}, \qquad \epsilon>0, \qquad n=(n_\mu)=(n_0,\mathbf{n}),$$
$$n^*=(n^*_\mu)=(n_0,-\mathbf{n}), \qquad n_0>0. \tag{14}$$

Obviously, n^* is also a light-cone vector since $n^{*2}=0$.

We first expand the numerator of each term in the integrand using the γ-matrix algebra (3), and then separate the integrands into the parts with and without the light-cone pole,

$$-i\Sigma^{(1)}(p,m,n,\Lambda,d) = -i(\Sigma_{\rm NP}+\Sigma_{\rm P}), \qquad -i\Sigma_{\rm NP} = e^2\int\frac{d^dk}{(2\pi)^d}\frac{2\Lambda^2+\Lambda[(d-2)\not k-(4-d)\not p-(d-2)m]}{[(k+p)^2-m^2](k^2-\Lambda^2)},$$
$$-i\Sigma_{\rm P} = e^2\int\frac{d^dk}{(2\pi)^d}\frac{2\Lambda^2(n\cdot p-m\not n)+2\Lambda[k\cdot(k+p)\not n+n\cdot p\not k]}{[(k+p)^2-m^2](k^2-\Lambda^2)n\cdot k}. \tag{15}$$

The loop integration will become much easier to carry out if the large-Λ limit can be taken before the integration. However, this operation is only feasible if the integration is finite before and after taking the large-Λ limit. Therefore, we first successively use the identity [7]

$$\frac{1}{(k+p)^2-m^2} = \frac{1}{k^2-m^2}-\frac{2k\cdot p+p^2}{(k^2-m^2)[(k+p)^2-m^2]} \tag{16}$$

to reduce the superficial UV divergent degree of the integrand until the large-Λ limit can be safely taken. For example, a term in $\Sigma_{\rm NP}$ can be calculated as follows:

$$\lim_{\Lambda\to\infty}\int\frac{d^dk}{(2\pi)^d}\frac{\Lambda k_\mu}{(k^2-\Lambda^2)[(k+p)^2-m^2]}$$
$$= \lim_{\Lambda\to\infty}\int\frac{d^dk}{(2\pi)^d}\frac{\Lambda k_\mu}{(k^2-\Lambda^2)}\left[\frac{1}{k^2-m^2}-\frac{2k\cdot p+p^2}{(k^2-m^2)[(k+p)^2-m^2]}\right]$$
$$= -\lim_{\Lambda\to\infty}\int\frac{d^dk}{(2\pi)^d}\frac{\Lambda k_\mu}{(k^2-\Lambda^2)}\frac{2k\cdot p+p^2}{(k^2-m^2)}\left[\frac{1}{k^2-m^2}-\frac{2k\cdot p+p^2}{(k^2-m^2)[(k+p)^2-m^2]}\right]$$
$$= -\lim_{\Lambda\to\infty}\int\frac{d^dk}{(2\pi)^d}\left\{\frac{2\Lambda k\cdot pk_\mu}{(k^2-\Lambda^2)(k^2-m^2)^2}-\frac{\Lambda k_\mu(2k\cdot p+p^2)^2}{(k^2-\Lambda^2)(k^2-m^2)^2}\left[\frac{1}{k^2-m^2}-\frac{2k\cdot p+p^2}{(k^2-m^2)[(k+p)^2-m^2]}\right]\right\}$$
$$= -\lim_{\Lambda\to\infty}\int\frac{d^3k}{(2\pi)^3}\frac{2\Lambda k\cdot pk_\mu}{(k^2-\Lambda^2)(k^2-m^2)^2}$$
$$= -\lim_{\Lambda\to\infty}\frac{2}{3}\Lambda p_\mu\int\frac{d^3k}{(2\pi)^3}\frac{k^2}{(k^2-\Lambda^2)(k^2-m^2)^2}$$
$$= ip_\mu\lim_{\Lambda\to\infty}\left[-\frac{1}{12}\frac{\Lambda(2\Lambda^3-3\Lambda^2m+m^3)}{(\Lambda^2-m^2)^2}\right] = -\frac{i}{6\pi}p_\mu. \tag{17}$$

In above calculation we have used the even and odd property of the integrands. Other terms in $\Sigma_{\rm NP}$ can be evaluated in a similar way. For the terms in $\Sigma_{\rm P}$ we first use the ML prescription shown in (14) to deal with the spurious light-cone pole. Then we choose a convenient Lorentz frame for the light-cone vector n_μ to perform the loop integration in a noncovariant way, and finally express the results in terms of a Lorentz invariant functions with the light-cone vector n_μ and its conjugate

n^*_μ. The explicit calculation techniques are shown in Appendix A. Consequently, we obtain the fermionic self-energy at one-loop order,

$$-i\Sigma^{(1)}(p,m,n) = \lim_{\Lambda\to\infty}\{\lim_{d\to 3}[-i\Sigma^{(1)}(p,m,n,\Lambda,d)]\}$$
$$= \frac{ie^2}{2\pi}\left[\Lambda + \frac{2}{3}m - \frac{5}{6}(\not{p}-m) + \frac{(n\cdot p)\not{n}^* - (n^*\cdot p)\not{n}}{n^*\cdot n} - \frac{m^2\not{n}}{n\cdot p} - \frac{m(n\cdot p - m\not{n})}{n\cdot p}\left(1 - \frac{2(n^*\cdot p)(n\cdot p)}{m^2(n^*\cdot n)}\right)^{1/2}\right]. \tag{18}$$

IV. VERTEX CORRECTION ON MASS SHELL AT ONE-LOOP

In the following we consider the one-loop quantum correction for the vertex $\psi - \bar{\psi} - A$ on the mass-shell of the fermion. That is,

$$-i\bar{u}(p')\Gamma^{(1)}_\mu(p',p,m,n)u(p) = \lim_{\Lambda\to\infty}\bar{u}(p')\bigg\{e^2\int\frac{d^3k}{(2\pi)^3}\frac{\gamma_\rho(\not{k}+\not{p}'+m)\gamma_\mu(\not{k}+\not{p}+m)\gamma^\nu}{[(k+p')^2-m^2][(k+p)^2-m^2]}\frac{1}{k^2-\Lambda^2}$$
$$\times\left[\frac{i\Lambda^2}{n\cdot k}\epsilon^{\nu\rho\lambda}n_\lambda - \Lambda g^{\nu\rho} + \frac{\Lambda}{n\cdot k}(k_\nu n_\rho + k_\rho n_\nu)\right]\bigg\}u(p)$$
$$\equiv -i(\Gamma_{[1]\mu} + \Gamma_{[2]\mu} + \Gamma_{[3]\mu}), \tag{19}$$

where the Dirac spinor $u(p)$ is a solution of the Dirac equation and $\bar{u}(p)$ is its conjugate,

$$(\not{p}-m)u(p) = 0, \qquad \bar{u}(p)(\not{p}-m) = 0. \tag{20}$$

The three parts in (19) are listed as follows:

$$-i\Gamma_{[1]\mu} = \lim_{\Lambda\to\infty}\bar{u}(p')\bigg\{-\Lambda e^2\int\frac{d^3k}{(2\pi)^3}\frac{[-\not{k}\gamma_\nu + 2(k+p')_\nu]\gamma_\mu[-\gamma_\nu\not{k} + 2(k+p)^\nu]}{(k^2-\Lambda^2)[(k+p')^2-m^2][(k+p)^2-m^2]}\bigg\}u(p); \tag{21}$$

$$-i\Gamma_{[2]\mu} = \lim_{\Lambda\to\infty}\bar{u}(p')\bigg\{\Lambda e^2\int\frac{d^3k}{(2\pi)^3}\frac{1}{(n\cdot k)(k^2-\Lambda^2)[(k+p')^2-m^2][(k+p)^2-m^2]}$$
$$\times[\not{n}(\not{k}+\not{p}'+m)\gamma_\mu(\not{k}+\not{p}+m)\not{k} + \not{k}(\not{k}+\not{p}'+m)\gamma_\mu(\not{k}+\not{p}+m)\not{n}]\bigg\}u(p)$$
$$= \lim_{\Lambda\to\infty}\bar{u}(p')\bigg\{\Lambda e^2\int\frac{d^3k}{(2\pi)^3}\frac{1}{(n\cdot k)(k^2-\Lambda^2)[(k+p')^2-m^2][(k+p)^2-m^2]}$$
$$\times[(2n\cdot(k+p') - \not{k}\not{n})\gamma_\mu(k^2+2k\cdot p) + (k^2+2k\cdot p')\gamma_\mu(2n\cdot(k+p) - \not{n}\not{k})]\bigg\}u(p); \tag{22}$$

$$-i\Gamma_{[3]\mu} = \lim_{\Lambda\to\infty}\bar{u}(p')\bigg\{i\Lambda^2 e^2\int\frac{d^3k}{(2\pi)^3}\frac{1}{(n\cdot k)(k^2-\Lambda^2)[(k+p')^2-m^2][(k+p)^2-m^2]}\epsilon_{\nu\rho\lambda}n^\lambda$$
$$\times[-\not{k}\gamma^\rho + 2(k+p')^\rho]\gamma_\mu[-\gamma^\nu\not{k} + 2(k+p)^\nu]\bigg\}u(p). \tag{23}$$

In writing down $\Gamma_{[1]\mu}$, $\Gamma_{[2]\mu}$, and $\Gamma_{[3]\mu}$, we have used the mass shell condition shown in Eq. (20),

$$\bar{u}(p')\gamma_\rho(\not{k}+\not{p}'+m) = \bar{u}(p')[-\not{k}\gamma_\rho + 2(k+p')_\rho], \qquad (\not{p}+\not{k}+m)\gamma_\nu u(p) = [-\gamma_\nu\not{k} + 2(k+p)_\nu]u(p). \tag{24}$$

In the following we calculate $\Gamma_{[1]\mu}$, $\Gamma_{[2]\mu}$, and $\Gamma_{[3]\mu}$:

(i) $\Gamma_{[1]\mu}$

$\Gamma_{[1]\mu}$ can be reduced to the following form with the γ-matrix algebra (3) and the mass shell condition given in Eq. (20),

$$-i\Gamma_{[1]\mu} = \lim_{\Lambda\to\infty}\bar{u}(p')\bigg\{-\Lambda e^2\int\frac{d^3k}{(2\pi)^3}\frac{1}{(k^2-\Lambda^2)[(k+p')^2-m^2][(k+p)^2-m^2]}[k^2\gamma_\mu - 2\not{k}k_\mu + 4k\cdot(p'+p)\gamma_\mu$$
$$+ 4mk_\mu - 4\not{k}(p'_\mu + p_\mu) + 4p'\cdot p\gamma_\mu]\bigg\}u(p). \tag{25}$$

We can take the large-Λ limit before evaluating the integration for the term with the numerator $4p'\cdot p\gamma_\mu$, which vanishes after taking the large-Λ limit. As for other terms, we must first make the decomposition (16) successively until it is feasible to take the large-Λ limit. It can be easily seen that the terms whose numerator linear in k_μ vanishes:

$$\begin{aligned}
&\lim_{\Lambda\to\infty}\int\frac{d^3k}{(2\pi)^3}\frac{\Lambda k_\mu}{(k^2-\Lambda^2)[(k+p')^2-m^2][(k+p)^2-m^2]}\\
&\quad=\lim_{\Lambda\to\infty}\int\frac{d^3k}{(2\pi)^3}\frac{\Lambda k_\mu}{(k^2-\Lambda^2)(k^2-m^2)^2}\left[1-\frac{2k\cdot p'+p'^2}{(k+p')^2-m^2}\right]\left[1-\frac{2k\cdot p+p^2}{(k+p)^2-m^2}\right]\\
&\quad=\lim_{\Lambda\to\infty}\int\frac{d^3k}{(2\pi)^3}\frac{\Lambda k_\mu}{(k^2-\Lambda^2)(k^2-m^2)^2}\left[-\frac{2k\cdot p'+p'^2}{(k+p')^2-m^2}-\frac{2k\cdot p+p^2}{(k+p)^2-m^2}+\frac{(2k\cdot p'+p'^2)(2k\cdot p+p^2)}{[(k+p')^2-m^2][(k+p')^2-m^2]}\right]\\
&\quad=0.
\end{aligned}\tag{26}$$

As for the first two terms whose numerators are quadratic in k_μ, we have from the decomposition (16),

$$\begin{aligned}
\lim_{\Lambda\to\infty}\int\frac{d^3k}{(2\pi)^3}\frac{\Lambda k_\mu k_\nu}{(k^2-\Lambda^2)[(k+p')^2-m^2][(k+p)^2-m^2]}&=\lim_{\Lambda\to\infty}\int\frac{d^3k}{(2\pi)^3}\frac{\Lambda k_\mu k_\nu}{(k^2-\Lambda^2)(k^2-m^2)^2}\\
&\quad\times\left[1-\frac{2k\cdot p'}{(k+p')^2-m^2}-\frac{2k\cdot p}{(k+p)^2-m^2}\right].
\end{aligned}\tag{27}$$

Hence only the first term survives after the large-Λ limit. Thus, we obtain

$$\begin{aligned}
-i\Gamma_{[1]\mu}&=\lim_{\Lambda\to\infty}\left[-\Lambda e^2\int\frac{d^3k}{(2\pi)^3}\frac{\gamma_\mu k^2-2\not{k}k_\mu}{(k^2-\Lambda^2)(k^2-m^2)^2}\right]=\lim_{\Lambda\to\infty}\left[-\frac{1}{3}\Lambda e^2\gamma_\mu\int\frac{d^3k}{(2\pi)^3}\frac{k^2}{(k^2-\Lambda^2)(k^2-m^2)^2}\right]\\
&=\lim_{\Lambda\to\infty}\left[-\frac{1}{3}e^2\gamma_\mu\frac{i}{8\pi}\frac{\Lambda(2\Lambda^3-3\Lambda^2m+m^3)}{(\Lambda^2-m^2)^2}\right]=-\frac{ie^2}{12\pi}\gamma_\mu.
\end{aligned}\tag{28}$$

(ii) $\Gamma_{[2]\mu}$

To evaluate $\Gamma_{[2]\mu}$, we first separate it into the sectors with and without the spurious light-cone singularity $(n\cdot k)^{-1}$, and impose the mass shell conditions $p'^2=p^2=m^2$. Then $\Gamma_{[2]\mu}$ takes the following form:

$$\begin{aligned}
-i\Gamma_{[2]\mu}=\lim_{\Lambda\to\infty}\bar{u}(p')\bigg\{\Lambda e^2\int\frac{d^3k}{(2\pi)^3}\bigg[&\frac{2\gamma_\mu}{(k^2-\Lambda^2)(k^2+2k\cdot p')}+\frac{2\gamma_\mu}{(k^2-\Lambda^2)(k^2+2k\cdot p)}+\frac{2n\cdot p'\gamma_\mu}{n\cdot k(k^2-\Lambda^2)(k^2+2k\cdot p')}\\
&+\frac{2n\cdot p\gamma_\mu}{n\cdot k(k^2-\Lambda^2)(k^2+2k\cdot p)}-\frac{\not{k}\not{n}\gamma_\mu}{n\cdot k(k^2-\Lambda^2)(k^2+2k\cdot p')}-\frac{\gamma_\mu\not{n}\not{k}}{n\cdot k(k^2-\Lambda^2)(k^2+2k\cdot p)}\bigg]\bigg\}u(p).
\end{aligned}\tag{29}$$

Note that the mass shell condition $p'^2=p^2=m^2$ should not be imposed on some terms until the integrations have been performed in order to avoid the artificial infrared divergence caused by implementing the mass shell condition. Using the formula listed in Appendix B, we obtain

$$-i\Gamma_{[2]\mu}=\frac{1}{\pi}ie^2\gamma_\mu-\frac{1}{2\pi}ie^2\frac{\not{n}^*n_\mu-\not{n}n^*_\mu}{n^*\cdot n}.\tag{30}$$

(iii) $\Gamma_{[3]\mu}$

We first use the γ-matrix algebra, $\epsilon_{\mu\nu\rho}\gamma^\rho=i/2[\gamma_\mu,\gamma_\nu]$, to rewrite $\Gamma_{[3]\mu}$ and take the large-Λ limit on those feasible terms to simplify $\Gamma_{[3]\mu}$. Then there appears

$$
\begin{aligned}
-i\Gamma_{[3]\mu} = & \lim_{\Lambda\to\infty} \bar{u}(p')\Big[\Lambda^2 e^2 \int \frac{d^3k}{(2\pi)^3} \frac{4k_\mu}{(k^2-\Lambda^2)[(k+p')^2-m^2][(k+p)^2-m^2]}\Big]u(p) \\
& + \lim_{\Lambda\to\infty} \bar{u}(p')\Big[-\Lambda^2 e^2 \int \frac{d^3k}{(2\pi)^3} \frac{2k^2 n_\mu}{n\cdot k(k^2-\Lambda^2)[(k+p')^2-m^2][(k+p)^2-m^2]}\Big]u(p) \\
& - \bar{u}(p')e^2 \int \frac{d^3k}{(2\pi)^3} \frac{4(p_\mu \not{k}\not{n} + p'_\mu \not{n}\not{k}) - 2m(\not{k}\not{n}\gamma_\mu + \gamma_\mu \not{n}\not{k}) - 2(n\cdot p\not{k}\gamma_\mu + n\cdot p'\gamma_\mu\not{k})}{n\cdot k(k^2+2k\cdot p')(k^2+2k\cdot p)} u(p) \\
& - \bar{u}(p')\Big[4ie^2 \int \frac{d^3k}{(2\pi)^3} \frac{\epsilon^{\nu\rho\lambda} n_\lambda (p_\nu k_\rho + k_\nu p'_\rho + p_\nu p'_\rho)\gamma_\mu}{n\cdot k(k^2+2k\cdot p')(k^2+2k\cdot p)}\Big]u(p) \\
\equiv & -i[V_{(1)\mu} + V_{(2)\mu} + V_{(3)\mu} + V_{(4)\mu}].
\end{aligned} \tag{31}
$$

Using the decomposition (16), taking the large-Λ limit and then putting them on the mass shell, we can calculate $V_{(1)\mu}$ and $V_{(2)\mu}$ as follows:

$$
\begin{aligned}
-iV_{(1)\mu} &= \lim_{\Lambda\to\infty} \bar{u}(p')\Big[\Lambda^2 e^2 \int \frac{d^3k}{(2\pi)^3} \frac{4k_\mu}{(k^2-\Lambda^2)[(k+p')^2-m^2][(k+p)^2-m^2]}\Big]u(p) \\
&= \bar{u}(p')\Big\{-4e^2 \int \frac{d^3k}{(2\pi)^3} \frac{k_\mu}{(k^2-m^2)^2}\Big[-\frac{2k\cdot p' + p'^2}{(k+p')^2-m^2} - \frac{2k\cdot p + p^2}{(k+p)^2-m^2} + \frac{(2k\cdot p' + p'^2)(2k\cdot p + p^2)}{[(k+p')^2-m^2][(k+p)^2-m^2]}\Big]\Big\}u(p) \\
&= \bar{u}(p')\Big[-4e^2 \int \frac{d^3k}{(2\pi)^3} \frac{k_\mu}{(k^2+2k\cdot p')(k^2+2k\cdot p)}\Big]u(p) \\
&= \bar{u}(p')\Big[\frac{ie^2}{4\pi}\frac{p'_\mu + p_\mu}{q}\ln\frac{1+q/(2m)}{1-q/(2m)}\Big]u(p),
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
-iV_{(2)\mu} &= -\lim_{\Lambda\to\infty} \int \frac{d^3k}{(2\pi)^3} \frac{2\Lambda^2 e^2 n_\mu k^2}{(k^2-\Lambda^2)(n\cdot k)[(k+p')^2-m^2][(k+p)^2-m^2]}\Bigg|_{p^2=p'^2=m^2} \\
&= 2e^2 n_\mu \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{(n\cdot k)(k^2-m^2)^2}\Big[-\frac{2k\cdot p' + p'^2}{(k+p')^2-m^2} - \frac{2k\cdot p + p^2}{(k+p)^2-m^2} + \frac{(2k\cdot p' + p'^2)(2k\cdot p + p^2)}{[(k+p')^2-m^2][(k+p)^2-m^2]}\Big]\Bigg|_{p^2=p'^2=m^2} \\
&= 2e^2 n_\mu \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{(n\cdot k)(k^2+2k\cdot p')(k^2+2k\cdot p)} = -2e^2 n_\mu g^{\nu\rho} I_{\nu\rho} \\
&= -\frac{ie^2}{8\pi} n_\mu \Big[\frac{1}{n\cdot(p'+p)}\frac{4m^2-q^2}{q}\ln\frac{1+q/(2m)}{1-q/(2m)} + m\Big(\frac{1}{n\cdot p'} + \frac{1}{n\cdot p}\Big) + \frac{1}{n\cdot n^*}\Big(\frac{n\cdot(p'+p)}{m} - 2\Big)\frac{D(p')^{1/2} - D(p)^{1/2}}{n\cdot(p'-p)}\Big].
\end{aligned} \tag{33}
$$

In above equations, $q_\mu \equiv p'_\mu - p_\mu$ and $D(p) \equiv m^2 n\cdot n^* - 2(n^*\cdot p)(n\cdot p)$. In addition, we have used the integral formula (A16) of $I_{\mu\nu}$ worked out in Appendix A.

To show the explicit symmetry of $V_{(3)\mu}$ and $V_{(4)\mu}$ in p'_μ and p_μ, we express

$$
p'_\mu = \frac{1}{2}(\mathcal{P}_\mu + q_\mu), \qquad p_\mu = \frac{1}{2}(\mathcal{P}_\mu - q_\mu) \tag{34}
$$

in evaluating $V_{(3)\mu}$ and $V_{(4)\mu}$, where $\mathcal{P}_\mu \equiv 1/2(p'_\mu + p_\mu)$. Then

$$
\begin{aligned}
-iV_{(3)\mu} = & -e^2\bar{u}(p')\Big[4(p'_\mu + p_\mu)\int \frac{d^3k}{(2\pi)^3}\frac{1}{(k^2+2k\cdot p')(k^2+2k\cdot p)} + 2q_\mu \int \frac{d^3k}{(2\pi)^3} \frac{\not{n}\not{k} - \not{k}\not{n}}{n\cdot k(k^2+2k\cdot p')(k^2+2k\cdot p)} \\
& - 2m \int \frac{d^3k}{(2\pi)^3} \frac{\not{k}\not{n}\gamma_\mu + \gamma_\mu\not{n}\not{k}}{n\cdot k(k^2+2k\cdot p')(k^2+2k\cdot p)}\Big]u(p),
\end{aligned} \tag{35}
$$

$$
-iV_{(4)\mu} = \bar{u}(p')\Big[-4ie^2\gamma_\mu \epsilon^{\nu\rho\lambda} n_\lambda q_\rho \int \frac{d^3k}{(2\pi)^3} \frac{k_\nu + 1/2(p'_\nu + p_\nu)}{n\cdot k(k^2+2k\cdot p')(k^2+2k\cdot p)}\Big]u(p). \tag{36}
$$

Using the integral formulas (A5) and (A26) for I_2 and $I_{2\mu}$, we have

$$-iV_{(3)\mu} = -ie^2\Big\{\frac{1}{2\pi}(p'_\mu + p_\mu)\frac{1}{q}\ln\frac{1+q/(2m)}{1-q/(2m)} + 4E_1[n\cdot q q_\mu - 2m^2 n_\mu - n\cdot(p'+p)m\gamma_\mu + m\not{n}(p'_\mu + p_\mu)] + 2E_3[q_\mu(\not{n}\not{n}^* - \not{n}^*\not{n}) + 2m(\not{n}n^*_\mu - \not{n}^* n_\mu) - 2mn\cdot n^*\gamma_\mu]\Big\}, \tag{37}$$

$$-iV_{(4)\mu} = -4e^2\gamma_\mu\epsilon^{\nu\rho\lambda}n_\nu q_\rho\Big[\Big(E_1 + \frac{1}{2}I_2\Big)(p'_\lambda + p_\lambda) + E_3 n^*_\lambda\Big], \tag{38}$$

where I_2, E_1, and E_3 are the Lorentz scalar functions constructed from p_μ, p'_μ, n_μ, and n^*_μ and are symmetric in p_μ and p'_μ, and their explicit forms are given in (A5), (A32), and (A34).

The one-loop quantum vertex function $\Gamma^{(1)}_\mu$ on mass shell of the fermion in the light-cone gauge can be obtained by summing up $\Gamma_{[1]\mu}$, $\Gamma_{[2]\mu}$, and $\Gamma_{[3]\mu}$.

V. RENORMALIZATION AND STRUCTURE OF LOCAL QUANTUM EFFECTIVE ACTION

A. Finite renormalization of gauge field propagator and generation of Maxwell term

Equation (11) shows that the vacuum polarization tensor $\Pi_{\mu\nu}(p)$ is finite. The finite renormalization on the propagator of the $U(1)$ CS gauge field can still be performed according to the standard procedure. The inverse of CS gauge field propagator up to one-loop level is

$$[iG^{(1)}_{\mu\nu}(p)]^{-1} = [iG^{(0)}_{\mu\nu}(p)]^{-1} - i\Pi_{\mu\nu}(p) = i\Big[\epsilon_{\mu\nu\lambda}ip^\lambda(1-\Pi_o(p)) + (p^2 g_{\mu\nu} - p_\mu p_\nu)\Pi_e(p) + \frac{1}{\xi}n_\mu n_\nu\Big]. \tag{39}$$

Hence

$$\begin{aligned} iG^{(1)}_{\mu\nu} &= -i\frac{1-\Pi_o(p)}{[1-\Pi_o(p)]^2 - p^2[\Pi_e(p)]^2}\Big[\frac{i}{p^2}\epsilon_{\mu\nu\rho}p^\rho - \frac{i}{(n\cdot p)p^2}(p_\mu\epsilon_{\nu\alpha\beta} - p_\nu\epsilon_{\mu\alpha\beta})p^\alpha n^\beta \\ &\quad - \frac{\Pi_e(p)}{1-\Pi_o(p)}g_{\mu\nu} + \frac{\Pi_e(p)}{1-\Pi_o(p)}\frac{1}{(n\cdot p)}(p_\mu n_\nu + p_\nu n_\mu)\Big] \\ &= \frac{1}{n\cdot p}\epsilon_{\mu\nu\rho}n^\rho\frac{1-\Pi_o(p)}{[1-\Pi_o(p)]^2 - p^2[\Pi_e(p)]^2} + i\Big[g_{\mu\nu} - \frac{1}{n\cdot p}(p_\mu n_\nu + p_\nu n_\mu)\Big]\frac{\Pi_e(p)}{[1-\Pi_o(p)]^2 - p^2[\Pi_e(p)]^2} \\ &= \frac{1}{n\cdot p}\epsilon_{\mu\nu\rho}n^\rho\frac{1}{1+\Pi_1(p)} + i\Big[g_{\mu\nu} - \frac{1}{n\cdot p}(p_\mu n_\nu + p_\nu n_\mu)\Big]\Pi_2(p), \end{aligned} \tag{40}$$

where the Martin identity (6) is employed and

$$\begin{aligned} \Pi_1(p) &= -\Pi_o(p) - \frac{p^2\Pi_e^2(p)}{1-\Pi_o(p)}, \\ \Pi_2(p) &= \frac{\Pi_e(p)}{[1-\Pi_o(p)]^2 - p^2\Pi_e^2(p)}. \end{aligned} \tag{41}$$

We choose the renormalization condition that at $p = 0$,

$$\Pi_{1R}(0) = 0, \tag{42}$$

and define the wave function renormalization constant of the CS gauge field in the usual way,

$$Z_3^{-1} = 1 + \Pi_1(0) = 1 - \Pi_o(0). \tag{43}$$

This gives

$$Z_3 = 1 + \frac{e^2}{4\pi}. \tag{44}$$

Consequently, the one-loop renormalized propagator of the $U(1)$ CS gauge field (i.e., up to the order e^2) is

$$\begin{aligned} iG^{(1)}_{\mu\nu R}(p) &= Z_3^{-1}[iG^{(1)}_{\mu\nu}(p)] \\ &= \frac{1}{n\cdot p}\epsilon_{\mu\nu\rho}n^\rho\frac{1}{1+\Pi_{1R}(p)} \\ &\quad + i\Big[g_{\mu\nu} - \frac{1}{n\cdot p}(p_\mu n_\nu + p_\nu n_\mu)\Big]\Pi_{2R}(p), \end{aligned} \tag{45}$$

where at one-loop level,

$$\begin{aligned} \Pi_{1R}(p) &= \Pi_1(p) - \Pi_1(0) = \Pi_o(0) - \Pi_o(p) \\ &= \frac{e^2}{4\pi}\Big[1 - \frac{m}{p}\ln\frac{1+p/(2m)}{1-p/(2m)}\Big]; \\ \Pi_{2R}(p) &= \Pi_e(p) \\ &= \frac{e^2}{4\pi}\frac{1}{m}\Big[\frac{m^2}{p^2} - \frac{m}{p}\Big(\frac{1}{4} + \frac{m^2}{p^2}\Big)\ln\frac{1+p/(2m)}{1-p/(2m)}\Big]. \end{aligned} \tag{46}$$

Equation (46) shows

$$\Pi_{2R}(0) = -\frac{e^2}{4\pi}\frac{1}{3m} \neq 0. \tag{47}$$

This fact means that the parity-even Maxwell term in the CS spinor electrodynamics is generated by quantum correction, which is a general feature of the CS gauge theory coupled with fermions [9].

B. Renormalization of fermionic propagator

Equation (18) shows that the self-energy is composed of the light-cone vector dependent part $\Sigma^{(1)}_{(\mathrm{I})}$ and the independent one $\Sigma^{(1)}_{(\mathrm{D})}$:

$$\begin{aligned}\Sigma^{(1)}(p, m, n, \Lambda) &= \Sigma^{(1)}_{(\mathrm{I})}(p, m, \Lambda) + \Sigma^{(1)}_{(\mathrm{D})}(p, m, n),\\ \Sigma^{(1)}_{(\mathrm{I})}(p, m, \Lambda) &= \frac{e^2}{2\pi}\Big[-\Lambda - \frac{2}{3}m + \frac{5}{6}(\not{p} - m)\Big],\end{aligned} \tag{48}$$

$$\begin{aligned}\Sigma^{(1)}_{(\mathrm{D})}(p, m, n) = -\frac{e^2}{2\pi}\Big\{&-\frac{(n\cdot p)\not{n}^* - (n^*\cdot p)\not{n}}{n^*\cdot n} + \frac{m^2\not{n}}{n\cdot p}\\ &+\frac{1}{2}\frac{m}{n\cdot p}\Big(1 - \frac{2(n^*\cdot p)(n\cdot p)}{m^2(n^*\cdot n)}\Big)^{1/2}\\ &\times[\not{n}(\not{p} - m) + (\not{p} - m)\not{n}]\Big\}.\end{aligned} \tag{49}$$

We impose the following mass-shell renormalization condition on the light-cone vector independent part $\Sigma_{(\mathrm{I})\mathrm{R}}(p)$:

$$\Sigma_{(\mathrm{I})\mathrm{R}}(p)|_{\not{p}=m_\mathrm{R}} = 0, \qquad \frac{\partial}{\partial\not{p}}\Sigma_{(\mathrm{I})\mathrm{R}}(p)|_{\not{p}=m_\mathrm{R}} = 0. \tag{50}$$

Then $\Sigma_{(\mathrm{I})}(p, m, \Lambda)$ has the following expansion around $\not{p} = m_\mathrm{R}$,

$$\Sigma_{(\mathrm{I})}(p, m, \Lambda) = \delta m - (Z_2^{-1} - 1)(\not{p} - m_\mathrm{R}) + Z_2^{-1}\Sigma_{(\mathrm{I})\mathrm{R}}(p), \tag{51}$$

where Z_2 is the wave function constant of the fermion.

Equations (48) and (51) yield that the renormalized fermionic mass, the wave function renormalization constant of the fermion and the light-cone vector independent part of one-loop fermionic self-energy are as follows:

$$\begin{aligned}m_R &= m - \delta m = \frac{e^2}{2\pi}\Big(\Lambda + \frac{2}{3}m\Big),\\ Z_2 &= 1 + \frac{e^2}{4\pi}\frac{5}{3}, \qquad \Sigma_{(\mathrm{I})\mathrm{R}} = 0.\end{aligned} \tag{52}$$

The light-vector dependent sector $\Sigma_{(\mathrm{D})}(p, m, n)$ is finite. We shall show that combined with the light-cone vector dependent sector in the vertex correction, it contributes to a gauge invariant quantum effective action specific to the light-cone gauge.

C. Finitely renormalized on-shell vertex correction and arising of anomalous magnetic moment of fermion

Collecting the results shown in Eqs. (19), (28), (30)–(33), (37), and (38), we see that the the on-shell vertex correction at one-loop is finite, and consists of the light-cone vector independent sector $\Gamma^{(1)}_{(\mathrm{I})\mu}$ and the dependent sector $\Gamma^{(1)}_{(\mathrm{D})\mu}$:

$$\begin{aligned}\Gamma^{(1)}_{(\mathrm{I})\mu} = \frac{e^2}{4\pi}\Big\{&\Big[-\frac{11}{3} + \frac{2m}{q}\ln\frac{1 + q/(2m)}{1 - q/2m}\Big]\gamma_\mu\\ &-\frac{1}{q}\ln\frac{1 + q/(2m)}{1 - q/2m}i\epsilon_{\mu\nu\rho}q^\nu\gamma^\rho\Big\},\end{aligned} \tag{53}$$

$$\begin{aligned}\Gamma^{(1)}_{(\mathrm{D})\mu} = &\frac{e^2}{2\pi}\frac{\not{n}^* n_\mu - \not{n} n^*_\mu}{n^*\cdot n}\\ &+ \text{nonpolynomial terms in } p_\mu \text{ and } p'_\mu.\end{aligned} \tag{54}$$

In writing down Eq. (53), we have used the three-dimensional analogue of the Gordon identity,

$$\bar{u}(p')(p'_\mu + p_\mu)u(p) = \bar{u}(p')(2m\gamma_\mu - i\epsilon_{\mu\nu\rho}q^\nu\gamma^\rho)u(p). \tag{55}$$

To perform the finite renormalization on the vertex correction, we choose the renormalized light-cone vector independent sector $\Gamma^{(1)R}_{(\mathrm{I})\mu}$ to satisfy

$$\Gamma^{(1)R}_{(\mathrm{I})\mu}(p', p)|_{p'^2=p^2=m^2, q_\mu=0} = 0, \tag{56}$$

and define the vertex renormalization constant Z_1 as follows:

$$\Gamma^{(1)}_{(\mathrm{I})\mu}(p', p) = (Z_1^{-1} - 1)\gamma_\mu + Z_1^{-1}\Gamma^{(1)R}_{(\mathrm{I})\mu}(p', p). \tag{57}$$

Then from Eqs. (53), (56), and (57) we obtain the vertex renormalization constant at one-loop level:

$$Z_1^{-1} - 1 = -\frac{e^2}{4\pi}\frac{5}{3}, \qquad Z_1 = 1 + \frac{e^2}{4\pi}\frac{5}{3}. \tag{58}$$

It is equal to Z_2, the wave function renormalization constant of the fermion, which is a direct consequence of the Ward identity (C14) or (C16).

According to Eq. (57), the light-cone vector independent radiative corrections of the vertex at one-loop is

$$\begin{aligned}\Gamma^{(1)R}_{(\mathrm{I})\mu}(p', p) &= -\gamma_\mu + Z_1[\Gamma^{(1)}_{(\mathrm{I})\mu}(p', p) + \gamma_\mu]\\ &= \gamma_\mu F_1(q^2) + i\epsilon_{\mu\nu\rho}q^\nu\gamma_\rho F_2(q^2),\end{aligned} \tag{59}$$

where

$$\begin{aligned}F_1(q^2) &= \frac{e^2}{4\pi}\Big[-2 + \frac{2m}{q}\ln\frac{1 + q/(2m)}{1 - q/(2m)}\Big],\\ F_2(q^2) &= -\frac{1}{q}\ln\frac{1 + q/(2m)}{1 - q/(2m)}.\end{aligned} \tag{60}$$

Equation (60) shows that at the renormalization point $q^2 = 0$, the form factor $F_2(q^2)$ does not vanish,

$$F_2(0) = -\frac{1}{m}. \tag{61}$$

This actually gives rise to the analogue of Schwinger's result for the anomalous magnetic moment of the fermion in the CS spinor electrodynamics. The term with tensor structure $\epsilon_{\mu\nu\rho}q^\nu\gamma_\rho$ and the form factor $F_2(q^2)$ leads to an interaction Hamiltonian at a higher order when the fermions are in a slowly varying $U(1)$ CS gauge field (since $q_\mu \to 0$),

$$\begin{aligned}\Delta\mathcal{H} &= -\frac{e^2}{4\pi}\frac{1}{m}\epsilon_{\mu\nu\rho}\bar{\psi}(x)\gamma_\rho\bar{\psi}(x)\partial^\nu A^\mu(x)\\ &= \frac{e^2}{8\pi}\frac{1}{m}\bar{\psi}(x)\sigma_{\mu\nu}\bar{\psi}(x)F^{\mu\nu}(x).\end{aligned} \tag{62}$$

This result coincides with that obtained in the covariant gauge [9].

D. Contribution to local quantum effective action from light-cone vector dependent terms

We now turn to the light-cone vector dependent terms appearing in the fermionic self-energy and in the on-shell vertex correction. Equations (49) and (54) lead to the following light-cone vector dependent local fermionic quantum effective action at one-loop order:

$$\begin{aligned}\Gamma^{(1)}_{(\mathrm{D})} &= \frac{e^2}{2\pi}\frac{1}{n\cdot n^*}[i\bar{\psi}(\not{n}^* n_\mu\partial^\mu - \not{n}n^*_\mu\partial^\mu)\psi\\ &\quad - \bar{\psi}(\not{n}^* n_\mu A^\mu - \not{n}n^*_\mu A^\mu)\psi]\\ &= \frac{e^2}{2\pi}\frac{1}{n\cdot n^*}i\bar{\psi}(\not{n}^* n_\mu D^\mu - \not{n}n^*_\mu D^\mu)\psi,\end{aligned} \tag{63}$$

where $D_\mu = \partial_\mu - ieA_\mu$ is the covariant derivative. $\Gamma^{(1)}_{(\mathrm{D})}$ is invariant under the $U(1)$ gauge transformation listed in Eq. (C2). It should be emphasized that this is precisely analogous to the result of a four-dimensional non-Abelian gauge theory coupled with fermions in the light-cone gauge [1].

The nonpolynomial terms in the external momenta given in Eqs. (49) and (54) will contribute to the nonlocal sector of the light-cone vector dependent quantum effective action for the fermion. Unfortunately, unlike the pure non-Abelian CS gauge theory in the light-cone gauge, which has no dimensional parameter [2,5], we are unable to extract out the explicit form of the nonlocal light-cone vector dependent quantum effective action due to the complications of those nonpolynomial terms.

VI. SUMMARY AND CONCLUSION

A complete investigation in the perturbation theory of Chern-Simons spinor electrodynamics in the light-cone gauge ($n\cdot A = 0$, $n^2 = 0$) at one-loop order has been made. We have calculated the vacuum polarization tensor, fermionic self-energy and on-shell vertex correction, and further performed the one-loop renormalization to define the quantum theory. The peculiar features of quantum corrections of Chern-Simons spinor electrodynamics in the light-cone gauge have been revealed. Two typical quantum effects in CS spinor electrodynamics, the generation of the parity-even Maxwell term and the arising of anomalous magnetic moment of the fermion from quantum corrections, have been reproduced as in the case of the covariant gauge fixing. We have also shown that as a consequence of the Ward identities in the light-cone gauge, the wave function renormalization constant of the fermion is equal to the vertex renormalization constant. Further, we have displayed the structure of local quantum effective action for the fermion, and found that its light-cone vector dependent sector is explicit gauge invariant. Especially, it takes exactly the same form as that in a four-dimensional gauge theory coupled with fermions in the light-cone gauge. This result is a natural consequence of the Ward identities for the CS spinor electrodynamics in the light-cone gauge. Therefore, the Lorentz covariance of S-matrix elements will be achieved.

The result summarized above has not only verified the applicability of the ML prescription to three-dimensional gauge theory in the presence of fermions, but it has also shown the gauge independence of the Chern-Simons type of gauge theory in evaluating gauge invariant physical observables.

ACKNOWLEDGMENTS

A partial support from the start-up research grant of Nipissing University is acknowledged.

APPENDIX A: FEYNMAN INTEGRAL WITH SPURIOUS LIGHT-CONE GAUGE SINGULARITY IN LEIBBRANDT-MANDELSTAM PRESCRIPTION

In this appendix we show how the Feynman integrals containing the spurious light-cone pole in three dimensions are evaluated with the ML prescription. Actually, only the following five types of integrals containing the pole are needed for evaluating the fermionic self-energy and on-shell vertex correction:

$$\begin{aligned}iI_1 &= \int\frac{d^3k}{(2\pi)^3}\frac{1}{n\cdot k[(k+p)^2-m^2]};\\ iI_2 &= \int\frac{d^3k}{(2\pi)^3}\frac{1}{n\cdot k(k^2+2k\cdot p')(k^2+2k\cdot p)};\\ I_{1\mu} &= \lim_{d\to 3}\int\frac{d^dk}{(2\pi)^d}\frac{k_\mu}{(n\cdot k)(k^2+2k\cdot p)};\\ I_{\mu\nu} &= \lim_{d\to 3}\int\frac{d^dk}{(2\pi)^d}\frac{k_\mu k_\nu}{n\cdot k(k^2+2k\cdot p')(k^2+2k\cdot p)};\\ I_{2\mu} &= \lim_{d\to 3}\int\frac{d^dk}{(2\pi)^d}\frac{k_\mu}{n\cdot k(k^2+2k\cdot p')(k^2+2k\cdot p)}.\end{aligned} \tag{A1}$$

We adopt the procedure illustrated in Ref. [1] rather than the exponential parametrization used in three-dimensional noncovariant gauge theory [2,5,14]. For the convenience of calculation, we choose the Lorentz frame such that

$$n = (n_0, 0, n_2), \qquad n^* = (n_0, 0, -n_2), \qquad n_0 > 0. \tag{A2}$$

The superficially covariant three-vector notation will be restored at the end of calculation. Since the light-gauge vectors n_μ and n^*_μ satisfy $n^2 = n^{*2} = 0$, there exist

$$\begin{aligned} &n_2 = \pm n_0, \qquad \kappa \equiv \frac{n_2}{n_0} = \pm 1, \qquad n_0^2 = n_2^2 = \frac{1}{2} n \cdot n^*. \\ &p_0^2 - p_2^2 = (p_0 + \kappa p_2)(p_0 - \kappa p_2) = \frac{1}{n_0^2}(n_0 p_0 + n_2 p_2)(n_0 p_0 - n_2 p_2) = \frac{2(n^* \cdot p)(n \cdot p)}{n^* \cdot n}. \end{aligned} \tag{A3}$$

(i) *Evaluation of* I_1

$$\begin{aligned} iI_1 &= \int \frac{d^3k}{(2\pi)^3} \frac{n^* \cdot k}{[(n \cdot k)(n^* \cdot k) + i\epsilon][(k+p)^2 - m^2]} \\ &\equiv \frac{1}{n_0} \int \frac{d^3k}{(2\pi)^3} \frac{k_0 + \kappa k_2}{(k_0^2 - k_2^2)[(k+p)^2 - m^2]} \\ &= \frac{1}{n_0} \int_0^1 dx \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} dk_1 \int_{-\infty}^{\infty} dk_2 \int_{-\infty}^{\infty} dk_0 \bigg[(k_0 + \kappa k_2) \\ &\quad \times \frac{1}{[(k_0 + p_0 x)^2 - (k_2 + p_2 x)^2 - (k_1 + p_1)^2 x + (p_0^2 - p_2^2) x(1-x) - m^2 x]^2} \bigg] \\ &= -\frac{i}{8\pi} \frac{p_0 + \kappa p_2}{n_0} \frac{1}{m} \int_0^1 dx \frac{1}{[1 - (p_0^2 - p_2^2)(1-x)/m^2]^{1/2}} \\ &= -\frac{i}{4\pi} \frac{m}{n_0(p_0 - \kappa p_2)} \bigg[1 - \bigg(1 - \frac{p_0^2 - p_2^2}{m^2}\bigg)^{1/2} \bigg] \\ &= -\frac{i}{4\pi} \frac{m}{n \cdot p} \bigg[1 - \bigg(1 - \frac{2(n^* \cdot p)(n \cdot p)}{m^2 (n^* \cdot n)}\bigg)^{1/2} \bigg]. \end{aligned} \tag{A4}$$

(ii) *Calculation of* I_2

$$\begin{aligned} iI_2 &= \int \frac{d^3k}{(2\pi)^3} \frac{1}{n \cdot k (k^2 + 2k \cdot p')(k^2 + 2k \cdot p)} \\ &\equiv \int \frac{d^3k}{(2\pi)^3} \frac{n^* \cdot k}{[(n^* \cdot k)(n \cdot k) + i\epsilon](k^2 + 2k \cdot p')(k^2 + 2k \cdot p)} \\ &= \frac{1}{2n_0} \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} dk_1 \int_{-\infty}^{\infty} dk_2 \int_{-\infty}^{\infty} dk_0 \int_0^1 dx \int_0^1 dy 2y (k_0 + \kappa k_2) \\ &\quad \times \bigg[\frac{1}{\{[(k^2 + 2k \cdot p')x + (k^2 + 2k \cdot p)(1-x)]y + (k_0^2 - k_2^2)(1-y)\}^3} \\ &\quad + \frac{1}{\{[(k^2 + 2k \cdot p)x + (k^2 + 2k \cdot p')(1-x)]y + (k_0^2 - k_2^2)(1-y)\}^3} \bigg] \\ &= \frac{i}{16\pi} \int_0^1 dx \frac{n \cdot (p' + p)}{n \cdot (p + qx) n \cdot (p' - qx)} \frac{1}{[m^2 - q^2 x(1-x)]^{1/2}} \\ &\quad - \frac{i}{16\pi} \int_0^1 dx \bigg[\frac{1}{n \cdot (p + qx)} \frac{1}{[m^2 - q^2 x(1-x) - 2n^* \cdot (p + qx) n \cdot (p + qx)/(n^* \cdot n)]^{1/2}} \\ &\quad + \frac{1}{n \cdot (p' - qx)} \frac{1}{[m^2 - q^2 x(1-x) - 2n^* \cdot (p' - qx) n \cdot (p' - qx)/(n^* \cdot n)]^{1/2}} \bigg], \end{aligned} \tag{A5}$$

which shows that I_2 is symmetric in p_μ and p'_μ.

(iii) *Calculating $I_{1\mu}$*

According to the Lorentz covariance, $I_{1\mu}$ has the following tensor structure,

$$I_{1\mu}=\lim_{d\to 3}\int\frac{d^dk}{(2\pi)^d}\frac{k_\mu}{(n\cdot k)(k^2+2k\cdot p)}=iK_1p_\mu+iK_2n_\mu K_2+iK_3n^*_\mu, \tag{A6}$$

where the undetermined coefficients K_1, K_2, and K_3 are the functions of Lorentz scalars constructed from p_μ, n_μ, and n^*_μ. Then making the projections of $I_{1\mu}$ on p_μ, n_μ, and n^*_μ, respectively, we have

$$X\equiv I_{1\mu}p^\mu=iK_1m^2+iK_2n\cdot p+iK_3n^*\cdot p=\lim_{d\to 3}\int\frac{d^dk}{(2\pi)^d}\frac{k\cdot p}{(n\cdot k)(k^2+2k\cdot p)}; \tag{A7}$$

$$Y\equiv I_{1\mu}n^\mu=iK_1n\cdot p+iK_3n\cdot n^*=\lim_{d\to 3}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2+2k\cdot p}; \tag{A8}$$

$$Z\equiv I_{1\mu}n^{*\mu}=iK_1n^*\cdot p+iK_2n\cdot n^*=\lim_{d\to 3}\int\frac{d^dk}{(2\pi)^d}\frac{n^*\cdot k}{(n\cdot k)(k^2+2k\cdot p)}. \tag{A9}$$

It is straightforward to evaluate X, Y, and Z using the ML prescription and taking into account the mass shell condition $p^2=m^2$. Note that in the regularized d dimensions, $k_\mu=(k_0,k_\perp,k_2)$ and $k_\perp$ has $d-2$ components. Then,

$$Y=\lim_{d\to 3}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2+2k\cdot p}=\lim_{d\to 3}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m^2}=\frac{i}{4\pi}m, \tag{A10}$$

$$\begin{aligned}X&=\lim_{d\to 3}\int\frac{d^dk}{(2\pi)^d}\frac{k\cdot p}{(n\cdot k)(k^2+2k\cdot p)}=\lim_{d\to 3}\int\frac{d^dk}{(2\pi)^d}\frac{(n^*\cdot k)k\cdot p}{[(n^*\cdot k)(n\cdot k)+i\epsilon](k^2+2k\cdot p)}\\&=\frac{1}{n_0}\lim_{d\to 3}\int_0^1dx\frac{1}{(2\pi)^d}\int_{-\infty}^{\infty}dk_0\int_{-\infty}^{\infty}dk_2\int d^{d-2}k_\perp\\&\quad\times\left[(k_0+\kappa k_2)\frac{k_0p_0-k_2p_2-k_\perp p_\perp}{[(k_0+p_0x)^2-(k_2+p_2x)^2-(k_\perp+p_\perp)^2x+(p_0^2-p_2^2)x(1-x)-m^2x]^2}\right]\\&=\frac{i}{4\pi}\frac{m(p_0+\kappa p_2)}{n_0}\int_0^1dx\left[1-\frac{p_0^2-p_2^2}{m^2}(1-x)\right]^{1/2}=\frac{i}{6\pi}\frac{1}{n\cdot p}\left[m^3-\frac{D(p)^{3/2}}{(n^*\cdot n)^{3/2}}\right],\end{aligned} \tag{A11}$$

$$\begin{aligned}Z&=\lim_{d\to 3}\int\frac{d^dk}{(2\pi)^d}\frac{n^*\cdot k}{(n\cdot k)(k^2+2k\cdot p)}=\lim_{d\to 3}\int\frac{d^dk}{(2\pi)^d}\frac{(n^*\cdot k)^2}{[(n^*\cdot k)(n\cdot k)+i\epsilon](k^2+2k\cdot p)}\\&=\frac{i}{8\pi}\frac{(p_0+\kappa p_2)^2}{m}\int_0^1dx\frac{x}{[1-(1-x)(p_0^2-p_2^2)/m^2]^{1/2}}=\frac{i}{4\pi}m\left[\frac{n^*\cdot p}{n\cdot p}-\frac{1}{3}\frac{m^2n^*\cdot n}{(n\cdot p)^2}+\frac{1}{3}\frac{D(p)^{3/2}}{m(n\cdot p)^2(n^*\cdot n)^{1/2}}\right],\end{aligned} \tag{A12}$$

where $D(p)=m^2n^*\cdot n-2(n^*\cdot p)(n\cdot p)$. Solving the system of algebraic equations for K_1, K_2, and K_3 listed in (A7)–(A9), we have

$$K_1=\frac{1}{4\pi}\frac{1}{D(p)}(n^*\cdot nX-n^*\cdot pY-n\cdot pZ)=\frac{1}{4\pi}\frac{m}{n\cdot p}\left[1-\frac{D(p)^{1/2}}{m(n\cdot n^*)^{1/2}}\right], \tag{A13}$$

$$K_2=\frac{Z}{n^*\cdot n}-\frac{n^*\cdot p}{n\cdot n^*}K_1=\frac{1}{12\pi}\left(\frac{m}{n\cdot p}\right)^2\left[-m+\frac{D(p)^{1/2}}{(n^*\cdot n)^{1/2}}\left(1+\frac{(n^*\cdot p)(n\cdot p)}{m^2n\cdot n^*}\right)\right], \tag{A14}$$

$$K_3=\frac{Y}{n\cdot n^*}-\frac{n^*\cdot p}{n\cdot n^*}K_1=\frac{1}{4\pi}\frac{D(p)^{1/2}}{(n\cdot n^*)^{3/2}}. \tag{A15}$$

(iv) *Evaluating $I_{\mu\nu}$*

$I_{\mu\nu}$ is invariant under the exchanges $\mu\leftrightarrow\nu$ and $p_\mu\leftrightarrow p'_\mu$, respectively. Therefore, the general tensor structure of $I_{\mu\nu}$ should be the following form:

$$
\begin{aligned}
I_{\mu\nu} &= \lim_{d\to 3}\int \frac{d^d k}{(2\pi)^d}\frac{k_\mu k_\nu}{n\cdot k(k^2+2k\cdot p')(k^2+2k\cdot p)} \\
&= iC_1(p'_\mu p'_\nu + p_\mu p_\nu) + iC_2(p'_\mu p_\nu + p_\mu p'_\nu) + iC_3[n^*_\mu(p'_\nu + p_\nu) + n^*_\nu(p'_\mu + p_\mu)] \\
&\quad + iC_4[n_\mu(p'_\nu + p_\nu) + n_\nu(p'_\mu + p_\mu)] + iC_5 n^*_\mu n^*_\nu + iC_6 n_\mu n_\nu + iC_7(n^*_\mu n_\nu + n^*_\nu n_\mu) + iC_8 g_{\mu\nu}, \qquad \text{(A16)}
\end{aligned}
$$

where C_i ($i = 1, 2, \cdots, 8$) are functions of the Lorentz scalars constructed from p_μ, p'_μ, n_μ, and n^*_μ, and are symmetric in p'_μ and p_μ. Then contracting $I_{\mu\nu}$ with the vector n^ν, and using Eq. (B8), we obtain

$$
\begin{aligned}
&C_1 = C_2, \qquad C_8 = -C_4 n\cdot(p'+p) - C_7 n\cdot n', \qquad C_5 = -\frac{n\cdot(p'+p)}{n\cdot n^*}C_3, \\
&C_1 = -\frac{1}{n\cdot(p'+p)}\left[\frac{1}{16\pi}\frac{1}{q}\ln\frac{1+q/(2m)}{1-q/(2m)} + C_3(n\cdot n^*)\right]. \qquad \text{(A17)}
\end{aligned}
$$

Further, $I_{\mu\nu}n^\mu n^\nu$ and Eq. (B8) determine that $C_3 = 0$. Hence

$$
C_1 = C_2 = -\frac{1}{n\cdot(p'+p)}\left[\frac{1}{16\pi}\frac{1}{q}\ln\frac{1+q/(2m)}{1-q/(2m)}\right]; \qquad C_5 = 0. \qquad \text{(A18)}
$$

Consequently, $I_{\mu\nu}$ becomes

$$
\begin{aligned}
I_{\mu\nu} &= \lim_{d\to 3}\int \frac{d^d k}{(2\pi)^d}\frac{k_\mu k_\nu}{n\cdot k(k^2+2k\cdot p')(k^2+2k\cdot p)} \\
&= iC_1(p'_\mu + p_\mu)(p'_\nu + p_\nu) + iC_4[n_\mu(p'_\nu + p_\nu) + n_\nu(p'_\mu + p_\mu)] + iC_6 n_\mu n_\nu + C_7(n^*_\mu n_\nu + n^*_\nu n_\mu) \\
&\quad - [iC_4 n\cdot(p'+p) + iC_7 n\cdot n^*]g_{\mu\nu}. \qquad \text{(A19)}
\end{aligned}
$$

To evaluate the scalar coefficients C_1, C_4, C_6, and C_7, we consider $I_{\mu\nu}(p'^\nu - p^\nu)$,

$$
\begin{aligned}
I_{\mu\nu}(p'^\nu - p^\nu) &= p'_\mu(-2iC_4 n\cdot p - iC_7 n\cdot n^*) + p_\mu(2iC_4 n\cdot p' + iC_7 n\cdot n^*) \\
&\quad + n_\mu[iC_6 n\cdot(p'-p) + iC_7 n^*\cdot(p'-p)] + n^*_\mu iC_7 n\cdot(p'-p) \\
&= \lim_{d\to 3}\int \frac{d^d k}{(2\pi)^d}\frac{k_\mu k\cdot(p'-p)}{n\cdot k(k^2+2k\cdot p')(k^2+2k\cdot p)} \\
&= \frac{1}{2}\left[\lim_{d\to 3}\int \frac{d^d k}{(2\pi)^d}\frac{k_\mu}{n\cdot k(k^2+2k\cdot p)} - \lim_{d\to 3}\int \frac{d^d k}{(2\pi)^d}\frac{k_\mu}{n\cdot k(k^2+2k\cdot p')}\right]. \qquad \text{(A20)}
\end{aligned}
$$

Using the results (A6) and (A15) of $I_{1\mu}$, we obtain the following algebraic equations:

$$
2C_4 n\cdot p' + C_7 n\cdot n^* = \frac{1}{8\pi}\frac{m}{n\cdot p}\left[1 - \frac{D(p)^{1/2}}{m(n\cdot n^*)^{1/2}}\right], \qquad \text{(A21)}
$$

$$
2C_4 n\cdot p + C_7 n\cdot n^* = \frac{1}{8\pi}\frac{m}{n\cdot p'}\left[1 - \frac{D(p')^{1/2}}{m(n\cdot n^*)^{1/2}}\right], \qquad \text{(A22)}
$$

$$
C_7 n\cdot(p'-p) = -\frac{1}{8\pi}\frac{D(p')^{1/2} - D(p)^{1/2}}{(n\cdot n^*)^{1/2}}, \qquad \text{(A23)}
$$

$$
C_6 n\cdot(p'-p) + C_7 n^*\cdot(p'-p) = K_2(p) - K_2(p'), \qquad \text{(A24)}
$$

which yield

$$C_4 = \frac{1}{16\pi}\frac{m}{(n\cdot p)(n\cdot p')} + \frac{i}{16\pi}\frac{1}{(n\cdot n^*)^{1/2}}\frac{D(p')^{1/2}-D(p)^{1/2}}{n\cdot(p'-p)};$$

$$C_6 = -\frac{1}{24\pi}\frac{m^3}{(n\cdot p')(n\cdot p)}\left(\frac{1}{n\cdot p'}+\frac{1}{n\cdot p}\right) - \frac{1}{24\pi}\frac{1}{(n^*\cdot n)^{1/2}}\frac{m^2}{n\cdot(p'-p)}\left[\frac{D(p')^{1/2}}{(n\cdot p')^2}-\frac{D(p)^{1/2}}{(n\cdot p)^2}\right]$$
$$- \frac{1}{24\pi}\frac{1}{(n^*\cdot n)^{3/2}}\frac{1}{n\cdot(p'-p)}\left[\frac{n^*\cdot p'}{n\cdot p'}D(p')^{1/2} - \frac{n^*\cdot p}{n\cdot p}D(p)^{1/2}\right]$$
$$+ \frac{1}{8\pi}\frac{1}{(n^*\cdot n)^{3/2}}\frac{n^*\cdot(p'-p)}{[n\cdot(p'-p)]^2}[D(p')^{1/2}-D(p)^{1/2}];$$

$$C_7 = -\frac{1}{8\pi}\frac{1}{(n\cdot n^*)^{3/2}}\frac{D(p')^{1/2}-D(p)^{1/2}}{n\cdot(p'-p)}. \tag{A25}$$

Then $I_{\mu\nu}$ is given by Eqs. (A18), (A19), and (A25).

(v) *Calculation of $I_{2\mu}$*

We calculate $I_{2\mu}$ in a similar way as evaluating $I_{1\mu}$, whose tensor structure takes the following form:

$$I_{2\mu} = \int\frac{d^3k}{(2\pi)^3}\frac{k_\mu}{(n\cdot k)(k^2+2k\cdot p)(k^2+2k\cdot p')} = iE_1(p'_\mu+p_\mu) + iE_2 n_\mu + iE_3 n^*_\mu, \tag{A26}$$

where E_i, $i = 1, 2, 3$ are functions of the Lorentz scalars constructed from p_μ, p'_μ, n_μ, and n^*_μ, and are symmetric in p_μ and p'_μ. Projecting $I_{2\mu}$ on n^μ, $(p'^\mu - p^\mu)$, and $(p'^\mu + p^\mu)$, respectively, and using the mass-shell condition, $p^2 = p'^2 = m^2$, we have

$$U \equiv n^\mu I_{2\mu} = E_1 n\cdot(p'+p) + n\cdot n^* E_3 = \int\frac{d^3k}{(2\pi)^3}\frac{1}{(k^2+2k\cdot p)(k^2+2k\cdot p')}, \tag{A27}$$

$$V \equiv (p'^\mu - p^\mu)I_{2\mu} = E_2 n\cdot(p'-p) + E_3 n^*\cdot(p'-p) = \int\frac{d^3k}{(2\pi)^3}\frac{k\cdot(p'-p)}{(n\cdot k)(k^2+2k\cdot p)(k^2+2k\cdot p')}$$
$$= \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\frac{1}{(n\cdot k)(k^2+2k\cdot p)} - \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\frac{1}{(n\cdot k)(k^2+2k\cdot p')}, \tag{A28}$$

$$W = (p'^\mu + p^\mu)I_{2\mu} = E_1(4m^2-q^2) + E_2 n\cdot(p'+p) + E_3 n^*\cdot(p'+p) = \int\frac{d^3k}{(2\pi)^3}\frac{k\cdot(p'+p)}{(n\cdot k)(k^2+2k\cdot p)(k^2+2k\cdot p')}. \tag{A29}$$

The scalar function U can be calculated straightforwardly and is given in Eq. (B7). Further, the scalar function V is obtained from I_1 as follows:

$$V = \frac{1}{2}[I_1(p) - I_1(p')]. \tag{A30}$$

Finally, the scalar function W can be evaluated from I_1 and $g^{\mu\nu}I_{\mu\nu}$ by the following algebraic operations:

$$\int\frac{d^3k}{(2\pi)^3}\frac{k\cdot(p'+p)}{(n\cdot k)(k^2+2k\cdot p)(k^2+2k\cdot p')}$$
$$= \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\frac{1}{(n\cdot k)(k^2+2k\cdot p)} + \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\frac{1}{(n\cdot k)(k^2+2k\cdot p')}$$
$$- \int\frac{d^3k}{(2\pi)^3}\frac{k^2}{(n\cdot k)(k^2+2k\cdot p)(k^2+2k\cdot p')} = \frac{1}{2}I_1(p) + \frac{1}{2}I_1(p') - g^{\mu\nu}I_{\mu\nu}. \tag{A31}$$

Thus E_1, E_2, and E_3 can be determined by solving the system of algebraic equations (A27)–(A29),

$$E_1 = \frac{1}{N}\{2(n\cdot p' n^*\cdot p - n\cdot p n^*\cdot p')U - n^*\cdot n[n\cdot(p'+p)V - n\cdot(p'-p)W]\}, \tag{A32}$$

$$E_2 = \frac{1}{N}\{[n\cdot(p'+p)n^*\cdot(p'+p) - (4m^2-q^2)n^*\cdot n]V + n^*\cdot(p'-p)[(4m^2-q^2)U - n\cdot(p'+p)W]\}, \tag{A33}$$

$$E_3 = \frac{1}{N}\{-n\cdot(p'-p)(4m^2-q^2)U - [n\cdot(p'+p)]^2V + n\cdot(p'+p)n\cdot(p'-p)W\}, \tag{A34}$$

where the denominator N reads

$$N = 2n\cdot(p'+p)(n\cdot p' n^*\cdot p - n\cdot p n^*\cdot p') - n\cdot n^* n\cdot(p'-p)(4m^2-q^2). \tag{A35}$$

$I_{2\mu}$ is thus fixed from Eqs. (A26)–(A35).

APPENDIX B: INTEGRATION FORMULA

We list in this appendix the integration formulas needed for evaluating the vacuum polarization tensor, fermionic self-energy, and on-shell vertex correction. In the following, $q_\mu = p'_\mu - p_\mu$, $q = |\mathbf{q}|$, $n = (n_\mu) = (n_0, \mathbf{n})$, and $n^* = (n^*_\mu) = (n_0, -\mathbf{n})$:

$$\int \frac{d^3k}{(2\pi)^3}\frac{1}{(k^2-m^2)[(k+p)^2-m^2]} = \frac{i}{8\pi}\frac{1}{p}\ln\left[\frac{1+p/(2m)}{1-p/(2m)}\right], \tag{B1}$$

$$\lim_{d\to 3}\int \frac{d^dk}{(2\pi)^d}\frac{k_\mu}{(k^2-m^2)[(k+p)^2-m^2]} = -\frac{i}{16\pi}\frac{p_\mu}{p}\ln\frac{1+p/(2m)}{1-p/(2m)}, \tag{B2}$$

$$\begin{aligned}\lim_{d\to 3}\int \frac{d^dk}{(2\pi)^d}\frac{k_\mu k_\nu}{(k^2-m^2)[(k+p)^2-m^2]} &= \frac{i}{16\pi}m\left\{\left[1+\frac{m}{p}\left(1-\frac{p^2}{m^2}\right)\ln\frac{1+p/(2m)}{1-p/(2m)}\right]g_{\mu\nu}\right.\\ &\quad\left.+\left[1+\frac{m}{p}\left(\frac{3}{4}\frac{p^2}{m^2}-1\right)\ln\frac{1+p/(2m)}{1-p/(2m)}\right]\frac{p_\mu p_\nu}{p^2}\right\},\end{aligned} \tag{B3}$$

$$\begin{aligned}\lim_{\Lambda\to\infty}\int \frac{d^3k}{(2\pi)^3}\frac{\Lambda k_\mu}{(k^2-\Lambda^2)[(k+p)^2-m^2]} &= \lim_{\Lambda\to\infty}\left[-\int\frac{d^3k}{(2\pi)^3}\frac{2\Lambda k\cdot p k_\mu}{(k^2-\Lambda^2)(k^2-m^2)^2}\right]\\ &= \lim_{\Lambda\to\infty}\left[-\frac{i}{12}p_\mu\frac{\Lambda(2\Lambda^3-3\Lambda^2m+m^3)}{(\Lambda^2-m^2)^2}\right] = -\frac{i}{6\pi}p_\mu,\end{aligned} \tag{B4}$$

$$\begin{aligned}\lim_{\Lambda\to\infty}\int \frac{d^3k}{(2\pi)^3}\frac{\Lambda}{(k^2-\Lambda^2)[(k+p)^2-m^2]} &= \lim_{\Lambda\to\infty}\int \frac{d^3k}{(2\pi)^3}\frac{\Lambda}{(k^2-\Lambda^2)(k^2-m^2)} = \lim_{\Lambda\to\infty}\int \frac{d^3k}{(2\pi)^3}\frac{\Lambda}{(k^2-\Lambda^2)(k^2-m^2)}\\ &= \frac{i}{4\pi},\end{aligned} \tag{B5}$$

$$\begin{aligned}\lim_{\Lambda\to\infty}\int \frac{d^3k}{(2\pi)^3}\frac{\Lambda^2}{(k^2-\Lambda^2)[(k+p)^2-m^2]} &= \lim_{\Lambda\to\infty}\int \frac{d^3k}{(2\pi)^3}\frac{\Lambda^2}{(k^2-\Lambda^2)(k^2-m^2)} + p^2\int\frac{d^3k}{(2\pi)^3}\frac{1}{(k^2-m^2)^2}\\ &\quad-\int\frac{d^3k}{(2\pi)^3}\frac{(2k\cdot p+p^2)^2}{(k^2-m^2)^2[(k+p)^2-m^2]} = \frac{i}{4\pi}\Lambda,\end{aligned} \tag{B6}$$

$$\int \frac{d^3k}{(2\pi)^3}\frac{1}{(k^2+2k\cdot p')(k^2+2k\cdot p)}\bigg|_{p'^2=p^2=m^2} = \frac{i}{8\pi}\frac{1}{q}\ln\frac{1+q/(2m)}{1-q/(2m)}, \tag{B7}$$

$$\int \frac{d^3k}{(2\pi)^3}\frac{k_\mu}{(k^2+2k\cdot p')(k^2+2k\cdot p)}\bigg|_{p'^2=p^2=m^2} = -\frac{i}{16\pi}\frac{p'_\mu+p_\mu}{q}\ln\frac{1+q/(2m)}{1-q/(2m)}, \tag{B8}$$

$$\lim_{\Lambda\to\infty}\int \frac{d^3k}{(2\pi)^3}\frac{\Lambda k_\mu}{n\cdot k(k^2-\Lambda^2)[(k+p)^2-m^2]} = \frac{i}{4\pi}\frac{n^*_\mu}{n\cdot n^*}, \tag{B9}$$

$$\lim_{\Lambda\to\infty}\int\frac{d^3k}{(2\pi)^3}\frac{\Lambda k^2}{n\cdot k(k^2-\Lambda^2)[(k+p)^2-m^2]}=\lim_{\Lambda\to\infty}\left[-\int\frac{d^3k}{(2\pi)^3}\frac{2\Lambda k^2 k\cdot p}{n\cdot k(k^2-\Lambda^2)(k^2-m^2)^2}\right]=-\frac{i}{2\pi}\frac{n^*\cdot p}{n\cdot n^*},\quad\text{(B10)}$$

$$\int\frac{d^3k}{(2\pi)^3}\frac{1}{(n\cdot k)(k^2+2k\cdot p)}=-\frac{i}{4\pi}\frac{m}{n\cdot p}\left[1-\left(1-\frac{2(n\cdot p)(n^*\cdot p)}{m^2(n\cdot n^*)}\right)^{1/2}\right].\quad\text{(B11)}$$

APPENDIX C: WARD IDENTITIES IN THE LIGHT-CONE GAUGE

The generating functional of the CS spinor electrodynamics in the light-cone gauge is

$$Z[J,\eta,\bar\eta]=\frac{1}{\mathcal{N}}\int\mathcal{D}\bar\psi\mathcal{D}\psi\mathcal{D}A\exp\left[i\int d^3x(\mathcal{L}+\bar\eta\psi+\bar\psi\eta+J_\mu A^\mu)\right],\quad\text{(C1)}$$

where the Lagrangian density $\mathcal{L}$ is given in Eq. (1), and $\bar\eta$, η, and J_μ are the auxiliary external sources for ψ, $\bar\psi$, and A_μ, respectively. Note that $\bar\eta$ and η are the Grassmann variables. $Z[J,\eta,\bar\eta]$ is invariant under the following gauge transformation:

$$\psi'(x)=e^{ie\theta(x)}\psi(x),\qquad\bar\psi'(x)=\bar\psi e^{-ie\theta(x)},\qquad A'_\mu(x)=A_\mu(x)+\partial_\mu\theta(x).\quad\text{(C2)}$$

That is,

$$\delta Z=\frac{1}{\mathcal{N}}\int\mathcal{D}\bar\psi\mathcal{D}\psi\mathcal{D}A\left\{\exp\left[i\int d^3x(\mathcal{L}+\bar\eta\psi+\bar\psi\eta+J_\mu A^\mu)\right]\times i\int d^3y\left(-\frac{1}{\xi}n^\nu A_\nu n^\mu\partial_\mu\theta-ie\theta\bar\eta\psi+ie\theta\bar\psi\eta+J^\mu\partial^\mu\theta\right)\right\}=0.\quad\text{(C3)}$$

Replacing $A_\mu(x)$, $\psi(x)$, and $\bar\psi(x)$ by the functional derivatives $\delta Z/\delta J^\mu(x)$, $\delta Z/\delta\bar\eta(x)$, and $\delta Z/\delta\eta(x)$, respectively, we obtain the identity,

$$\left[\frac{1}{\xi}n^\lambda n^\mu\partial_\lambda\frac{\delta}{i\delta J^\mu(x)}-ie\bar\eta(x)\frac{\delta}{i\delta\bar\eta(x)}+ie\eta(x)\frac{\delta}{i\delta\bar\eta(x)}-\partial_\nu J^\nu(x)\right]Z=0.\quad\text{(C4)}$$

The corresponding Ward identity for the generating functional $W\equiv -i\ln Z$ of the connected Green functions can be straightforwardly derived due to the linearity of the the functional derivative operator in (C4),

$$\left[\frac{1}{\xi}n^\lambda n^\mu\partial_\lambda\frac{\delta}{i\delta J^\mu(x)}-ie\bar\eta(x)\frac{\delta}{i\delta\bar\eta(x)}+ie\eta(x)\frac{\delta}{i\delta\bar\eta(x)}-\partial_\mu J^\mu(x)\right]W=0.\quad\text{(C5)}$$

Acting $\delta/[i\delta J_\nu(y)]$ on the identity (C5) and then setting the external sources J_μ, η, and $\bar\eta$ equal to zero, we obtain the Ward identity for the two-point function of gauge field,

$$\left[\frac{1}{\xi}n^\lambda n^\mu\partial^x_\lambda\frac{\delta^2}{i\delta J^\mu(x)i\delta J^\nu(y)}+i\partial^x_\mu\delta^{(3)}(x-y)\right]W|_{J^\mu=\eta=\bar\eta=0}=0,\qquad n^\lambda n^\mu\partial^x_\lambda[iG_{\mu\nu}(x-y)]=-i\xi\partial^x_\nu\delta^{(3)}(x-y).\quad\text{(C6)}$$

In momentum space it reads as

$$n^\mu G_{\mu\nu}(p)=-i\xi\frac{p_\nu}{n\cdot p}.\quad\text{(C7)}$$

Equation (C7) implies that the tensor structure of two-point function of the $U(1)$ CS gauge field is

$$iG_{\mu\nu}(p)=A(p^2,n\cdot p)\epsilon_{\mu\nu\rho}n^\rho+B(p,n\cdot p)\left[g_{\mu\nu}-\frac{1}{n\cdot p}(p_\mu n_\nu+p_\nu n_\mu)\right]+\xi\frac{p_\mu p_\nu}{(n\cdot p)^2}.\quad\text{(C8)}$$

Further, acting $\delta/i\delta\bar\eta(y)$ and $\delta/i\delta\eta(z)$ on the identity successively, and then letting all the external sources equal to zero, we can obtain the Ward identity relating the three-point function $\langle A_\mu(x)\psi(y)\bar\psi(z)\rangle_C$ and two-point function $\langle\psi(x)\bar\psi(y)\rangle$:

$$\begin{aligned}&\left[\frac{1}{\xi}n^\mu n^\nu\partial^x_\nu\frac{\delta^3}{i\delta J^\mu(x)i\delta\bar\eta(y)i\delta\eta(z)}-e\delta^{(3)}(x-y)\frac{\delta^2}{i\delta\bar\eta(x)i\delta\eta(z)}+e\delta^{(3)}(x-z)\frac{\delta^2}{i\delta\bar\eta(y)i\delta\eta(x)}\right]W|_{J^\mu=\eta=\bar\eta=0}=0,\\&\frac{1}{\xi}n^\mu n^\nu\partial^x_\nu\langle A_\mu(x)\psi(y)\bar\psi(z)\rangle_C-e\delta^{(3)}(x-y)\langle\psi(x)\bar\psi(z)\rangle+e\delta^{(3)}(x-z)\langle\psi(y)\bar\psi(x)\rangle=0,\end{aligned}\quad\text{(C9)}$$

where the subscript C denotes the connected part of the three-point function $\langle A_\mu(x)\psi(y)\bar\psi(z)\rangle$. We further make one-particle-irreducible decomposition on the connected three-point function $\langle A_\mu(x)\psi(y)\bar\psi(z)\rangle_C$, and then Eq. (C9) becomes

$$\frac{1}{\xi}n^{\mu}n^{\nu}\partial_{\nu}^{x}\int d^3x'd^3y'd^3z'[iG_{\mu\lambda}(x-x')][iS(y-y')][iS(z-z')]\Gamma_{\lambda}(x',y',z') - e\delta^{(3)}(x-y)[iS(x-z)] + e\delta^{(3)}(x-z)[iS(y-x)] = 0. \tag{C10}$$

Inserting (C6) and cutting-off the external legs, we obtain the identity between the gauge field-fermion-fermion vertex function and two-point function of the fermion,

$$i\partial_{\mu}^{x}\Gamma^{\mu}(x,y,z) = [iS(z-x)]^{-1}\delta^{(3)}(x-y) - [iS(x-y)]^{-1}\delta^{(3)}(x-z), \tag{C11}$$

which is identical to the case in covariant gauge. In momentum space it reads

$$q^{\mu}\Gamma_{\mu}[p', p, -(p'+p)] = S^{-1}(p') - S^{-1}(p), \tag{C12}$$

where $q_{\mu} \equiv p'_{\mu} - p_{\mu}$. Further, using the fact that the perturbative quantum correction is the quantum fluctuation around a classical background,

$$\Gamma_{\mu}(p', p) = \gamma_{\mu} + \Lambda_{\mu}(p', p), \qquad S^{-1}(p) = \not{p} - m - \Sigma(p), \tag{C13}$$

we finally obtain the identity relating the vertex correction and fermionic self-energy:

$$q^{\mu}\Lambda_{\mu}(p', p) = (p'^{\mu} - p^{\mu})\Lambda_{\mu}(p', p) = -[\Sigma(p') - \Sigma(p)]. \tag{C14}$$

It is equivalent to

$$\Lambda_{\mu}(p) = \lim_{p'_{\mu}\to p_{\mu}}\Lambda_{\mu}(p', p) = -\frac{\partial}{\partial p^{\mu}}\Sigma(p), \tag{C15}$$

which implies

$$\Gamma_{\mu}(p) = -\frac{\partial}{\partial p^{\mu}}S^{-1}(p). \tag{C16}$$

The identity (C15) or (C16) leads to $Z_1 = Z_2$ as in the case of a covariant gauge.

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