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Rank-n logarithmic conformal field theory in the BTZ black hole

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We construct the rank-n finite temperature logarithmic conformal field theory (LCFT) starting from the n-coupled scalar field theory in the Bañados-Teitelboim-Zanelli black hole background. Its zero temperature limit reduces to a rank-n LCFT in the AdS_3 background whose gravity dual is a polycritical gravity. We compute all two-point functions of a rank-n finite temperature LCFT. Using the retarded Green's functions on the boundary, we obtain quasinormal modes of scalar A_n which satisfies the $2n$th order linearized equation. Furthermore, the absorption cross section of A_n indicates a feature of $\ln^{n-1}[\omega\ell]$ correction to the Klein-Gordon mode.

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Critical gravity based on higher-curvature terms in the AdS_{d+1} spacetimes [1–4] has been regarded as a toy model for quantum gravity. At the critical point of avoiding the ghosts, a degeneracy takes place and massive gravitons coincide with massless gravitons. All massive gravitons are replaced by an equal amount of logarithmic modes at the critical point, leading to the critical gravity (log-gravity). According to the dictionary of AdS-LCFT correspondence, one finds that a rank-2 logarithmic conformal field theory (LCFT) is dual to a critical gravity [5–7].

However, one has to face the nonunitarity issue of the log-gravity theory, because it contains higher-derivative terms. In order to resolve this issue, a polycritical gravity with a $2n(n>2)$ derivative was introduced to provide multiple critical points [8] whose CFT dual seems to be a rank-n LCFT. A consistent unitary truncation of polycritical gravity was performed at the linearized level for odd n but not for even n [9].

On the other hand, an n-coupled scalar field model in the AdS_{d+1} spacetimes has been proposed as a toy model for a gravitational dual to a rank-n LCFT [10]. By introducing $n-1$ auxiliary scalar fields, this model could be rewritten as a two-derivative theory. The critical point is obtained when all masses of the n-scalar fields degenerate. The $n-1$ higher-order logarithmic modes appear as logarithmic partners of the Klein-Gordon scalar A_1. This model is compared to the $2n$-derivative Lee-Wick model, where the odd n and the even n cases feature a qualitative difference [11].

In this paper, we will investigate an n-coupled scalar field model on the Bañados-Teitelboim-Zanelli (BTZ) black hole background [12]. Our purpose is twofold. One is to recognize the difference in the AdS-LCFT correspondence between an n-coupled scalar field model on the AdS_3—a rank-n (zero temperature) LCFT and an n-coupled scalar field model on the BTZ black hole—a rank-n finite temperature LCFT. The other is to compute the quasinormal frequencies of a field A_n satisfying $(\nabla_B^2-m^2)^nA_n=0$ and the absorption cross section from scattering A_n off the BTZ black hole by using the retarded Green's function of rank-n finite temperature LCFT. Actually, this computation is a formidable task when using a single scalar equation

$$(\nabla_B^2-m^2)^n\varphi=0. \tag{1}$$

Instead, employing the n-coupled scalar field model to give $(\nabla_B^2-m^2)^nA_n=0$ eventually, one could compute the quasinormal frequencies and absorption cross section in the low-temperature and massless limits.

We take a toy scalar model for the polycritical gravity with n-coupled scalar fields with degenerate masses

$$S=\int d^3x\sqrt{-g}[R-2\Lambda]+S_\Phi, \tag{2}$$

where S_Φ is given by

$$S_\Phi=-\frac{1}{2}\int d^3x\sqrt{-g}\sum_{i,j=1}^{n}[\alpha_{ij}\partial_\mu\Phi_i\partial^\mu\Phi_j+\beta_{ij}\Phi_i\Phi_j] \tag{3}$$

with the n-dimensional matrices α_{ij} and β_{ij} [10]. Now we introduce a background metric $\bar{g}_{\mu\nu}$ of the BTZ black hole with the cosmological constant $\Lambda=-1/\ell^2=-1$ [12]:

$$\begin{aligned}ds_B^2&=\bar{g}_{\mu\nu}dx^\mu dx^\nu\\&=-\frac{(r^2-r_+^2)(r^2-r_-^2)}{r^2}dt^2+\frac{r^2}{(r^2-r_+^2)(r^2-r_-^2)}dr^2\\&\quad+r^2\left(d\phi+\frac{r_+r_-}{r^2}dt\right)^2.\end{aligned} \tag{4}$$

Here, the Arnowitt-Deser-Misner mass $M=r_+^2-r_-^2$, angular momentum $J=2r_+r_-$, and right and left temperature $T_{R/L}=(r_+\pm r_-)/2\pi$.

We consider the perturbation around the background spacetimes (4) of $\Phi_i=\bar{\Phi}_i+A_i$ with $\bar{\Phi}_i=0$. Then the linearized equations are given by

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$$(\nabla_B^2 - m^2)A_1 = 0, \qquad (\nabla_B^2 - m^2)A_p = A_{p-1} \tag{5}$$

with $p = 2, \ldots, n$, which lead to the $2n$th order differential equation for A_n as

$$(\nabla_B^2 - m^2)^n A_n = 0. \tag{6}$$

It is noted that unlike Eq. (1), Eq. (6) was obtained from the recursion relation, which means that the equation for A_p is recursively related to A_1.

We are in a position to compute the bulk-to-boundary propagators so that a rank-n finite temperature LCFT is formed on the boundary. The bulk scalar A_i is represented by bulk-to-boundary propagators K_{ij}, which relate the bulk solution to the boundary source fields $A_{i(b)}$. The propagator K_{ij} is given as

$$K_{ij} = \begin{pmatrix} 0 & 0 & \cdots & 0 & K_1 \\ 0 & 0 & \cdots & K_2 & K_2 \\ \vdots & \vdots & \iddots & \vdots & \vdots \\ K_1 & K_2 & \cdots & K_{n-1} & K_n \end{pmatrix}. \tag{7}$$

Here K_1 and K_p with $p = 2, \ldots, n$ are

$$K_1 = K_{1n} = K_{2n-1} = \cdots = K_{n1}, \tag{8}$$

$$K_p = K_{pn} = K_{p+1n-1} = \cdots = K_{np}, \tag{9}$$

which satisfy the following relations:

$$(\nabla_{\rm B}^2 - m^2)K_1 = 0, \qquad (\nabla_{\rm B}^2 - m^2)K_p = K_{p-1}. \tag{10}$$

In this case, K_1 and K_p correspond to the bulk-to-boundary propagators of the Klein-Gordon mode A_1 and mode A_p, respectively. Importantly, the K_n propagator satisfies

$$(\nabla_{\rm B}^2 - m^2)^n K_n = 0. \tag{11}$$

The solutions $A_i(r, u_+, u_-)$ to (5) are written as

$$A_i = \int du'_+ du'_- \Bigg[\sum_{j=1}^n K_{ij}(r, u_+, u_-; u'_+, u'_-)A_{j(b)}\Bigg] \tag{12}$$

with $u_\pm = \phi \pm t$. It is well known that, in the BTZ black hole background, K_1 can be found to be the solution to the Klein-Gordon equation [13]

$$K_1(r, u_+, u_-; u'_+, u'_-) = \left[\frac{N\pi^2 T_R T_L}{Me^{\pi T_L \delta u_+ + \pi T_R \delta u_-}/4r + r\sinh(\pi T_L \delta u_+)\sinh(\pi T_R \delta u_-)}\right]^{\triangle} \equiv N[f(r, u_+, u_-; u'_+, u'_-)]^{\triangle}, \tag{13}$$

where $\delta u_\pm = u_\pm - u'_\pm$, $\triangle(\triangle - 2) = m^2$, and N is a normalization constant. Here the Hawking temperature T_H is defined by $T_H = 2/(1/T_R + 1/T_L)$. It is worth noting that a general formula for the bulk-to-boundary propagator K_p leads to

$$\begin{aligned} K_p &= \frac{1}{(p-1)!}\frac{d^{p-1}}{(dm^2)^{p-1}}K_1 \\ &= \frac{K_1}{2^{p-1}(p-1)!(\triangle - 1)^{p-1}} \\ &\quad \times \Bigg[\ln^{p-1}[f] + \frac{1}{N}\Bigg\{\sum_{l=1}^{p-1} {}_{p-1}C_l\Big(\frac{\partial^l N}{\partial \triangle^l}\Big)\ln^{p-l-1}[f]\Bigg\} \\ &\quad + \frac{1}{(\triangle - 1)^k N}\Bigg\{\sum_{k=1}^{p-2} b_{kp}\sum_{l=0}^{p'} {}_{p'}C_l\Big(\frac{\partial^l N}{\partial \triangle^l}\Big)\ln^{p'-l}[f]\Bigg\}\Bigg], \end{aligned} \tag{14}$$

where $p = 2, \ldots, n$, ${}_nC_i \equiv \frac{n!}{i!(n-i)!}$, b_{kp} are constant, and $p' = p - k - 1$. Here we observe the highest order logarithmic term of "$\ln^{p-1}[f]K_1$" showing that the bulk-to-boundary operator is determined as the solution to the $2p$th order differential equation $(\nabla_B^2 - m^2)^p K_p = 0$. Now we consider an on-shell bilinear action $S_{\rm eff}$ on the boundary:

$$2S_{\rm eff} = -\lim_{r_s\to\infty}\int_s du_+ du_- \sqrt{-\gamma}\Bigg[\sum_{i,j=1}^n \alpha_{ij}A_i(\hat{n}\cdot\nabla)A_j\Bigg], \tag{15}$$

which leads to a complicated expression (see Ref. [14]) obtained from inserting Eq. (12) with (13) and (14) into (15). Following the AdS-LCFT correspondence, we wish to couple the boundary values $A_{i(b)}$ of the fields to their dual operators $\mathcal{O}_i$ as $\int du_+ du_- \sqrt{-\gamma}\sum_{i=1}^n A_{i(b)}\mathcal{O}_i$ for the symmetric-type coupling. This shows clearly the one-to-one correspondence between $A_{i(b)}$ and $\mathcal{O}_i$. One can derive the two-point functions for the dual conformal operators $\mathcal{O}_i$ as follows:

$$\begin{aligned} \langle\mathcal{O}_i(u_+, u_-)\mathcal{O}_{j-i}(0)\rangle &= \frac{\delta^2 S_{\rm eff}}{\delta A_{i(b)}(u_+, u_-)\delta A_{j-i(b)}(0)} = 0, \\ \xi_{kk'}\langle\mathcal{O}_k(u_+, u_-)\mathcal{O}_{k'}(0)\rangle &= \frac{\delta^2 S_{\rm eff}}{\delta A_{k(b)}(u_+, u_-)\delta A_{k'(b)}(0)} \\ &= 2^{-\delta_{kk'}}\triangle N\Big(\frac{\pi T_L}{\sinh[\pi T_L u_+]}\Big)^{\triangle} \\ &\quad \times\Big(\frac{\pi T_R}{\sinh[\pi T_R u_-]}\Big)^{\triangle}, \end{aligned} \tag{16}$$

where $k' = n - k + 1$ with $k = 1, \ldots, n$ and $i = 1, \ldots, n-1$, $j = i + 1, i + 2, \ldots, n$. For $s = p + q - n - 1 > 0$ with $p, q = 2, \ldots, n$, the two-point functions are given by

$$\zeta_{pq}\langle\mathcal{O}_p(u_+,u_-)\mathcal{O}_q(0)\rangle = \frac{\delta^2 S_{\rm eff}}{\delta A_{p(b)}(u_+,u_-)\delta A_{q(b)}(0)}$$
$$= \triangle N[\tilde{f}(1)]^{\triangle}2^{-\delta_{pq}}\times\Bigg\{\frac{1}{2^s(s)!(\triangle-1)^s}\Bigg(\ln^s[\tilde{f}(\epsilon)]+\frac{1}{N}\sum_{l=1}^{s}{}_s a_l \ln^{s-l}[\tilde{f}(\epsilon)]+\frac{s}{\triangle}\ln^{s-1}[\tilde{f}(\epsilon)]$$
$$+\frac{1}{\triangle N}\sum_{l=1}^{s}(s-l)_s a_l\ln^{s-l-1}[\tilde{f}(\epsilon)]+\frac{1}{(\triangle-1)^k N}\Bigg\{\sum_{k=1}^{s-1}b_{ks+1}\sum_{l=0}^{s-k}{}_{s-k}a_l\ln^{s-l-k}[\tilde{f}(\epsilon)]+\frac{1}{\triangle}$$
$$\times\sum_{k=1}^{s-1}b_{ks+1}\sum_{l=0}^{s-k}(s-l-k)_{s-k}a_l\ln^{s-l-k-1}[\tilde{f}(\epsilon)]\Bigg\}\Bigg)\Bigg\}, \tag{17}$$

where $\tilde{f}(\epsilon)$ and ${}_j a_l$ are, respectively, given by

$$\tilde{f}(\epsilon)=\epsilon\frac{\pi^2 T_R T_L}{\sinh[\pi T_L\delta u_+]\sinh[\pi T_R\delta u_-]},$$
$${}_j a_l = {}_j C_l\Big(\frac{\partial^l N}{\partial\triangle^l}\Big).$$

All correlation functions are zero for $s<0$. We note that this is one of our main results. In (16) and (17), when reducing to the AdS_3 background [10], the parameters $\xi_{kk'}$ and ζ_{pq} are determined to be

$$\xi_{kk'}=2^{-\delta_{kk'}}\frac{\triangle N}{[2(\Delta-1)]^n},\qquad \zeta_{pq}=\frac{2^{-\delta_{pq}}\triangle N}{[2(\Delta-1)]^{n+s}}.$$

At this stage, we wish to point out that if one truncates the theory with odd rank to be unitary, the only nonzero correlation function is given by a reduced matrix as

$$\langle\mathcal{O}^i\mathcal{O}^j\rangle\sim\begin{pmatrix}0 & 0\\ 0 & \text{CFT}\end{pmatrix}, \tag{18}$$

which implies that the remaining sector involves a nontrivial two-point correlator

$$\langle\mathcal{O}^{\log^{\frac{n-1}{2}}}(x)\mathcal{O}^{\log^{\frac{n-1}{2}}}(0)\rangle=\frac{[2(\triangle-1)]^n}{|x|^{2\triangle}}. \tag{19}$$

This defines a unitary CFT, and, thus, the nonunitary issue could be resolved by truncating a rank-n LCFT with $n=3,5,\ldots$. On the contrary, for the even rank of $n=4,6,\ldots$, it reduces to a null matrix

$$\langle\mathcal{O}^i\mathcal{O}^j\rangle\sim\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}, \tag{20}$$

which contains null states only.

On the other hand, the retarded Green's functions are defined by

$$\mathcal{D}^{\rm ret}_{jk}(t,\phi;0,0)=i\Theta(t-0)\bar{\mathcal{D}}_{jk}(u_+,u_-),$$
$$j,k=1,2,\ldots,n,$$

where the commutators evaluated in the equilibrium canonical ensemble are given by

$$\bar{\mathcal{D}}_{jk}=\langle\mathcal{O}_j(u_+-i\epsilon,-u_--i\epsilon)\mathcal{O}_k(0)\rangle-\langle\epsilon\to-\epsilon\rangle.$$

Making the Fourier transform of $\bar{\mathcal{D}}_{jk}(u_+,u_-)$, $\bar{\mathcal{D}}_{jk}(p_+,p_-)$ takes the form

$$\bar{\mathcal{D}}_{jk}=\int du_+du_-e^{i(p_+u_+-p_-u_-)}\bar{\mathcal{D}}_{jk}(u_+,u_-) \tag{21}$$

in the momentum space. Here $p_\pm=(\omega\mp k)/2$. Using (21) together with (16), we obtain the null Green's function as

$$\bar{\mathcal{D}}_{ij-i}(p_+,p_-)=0(j=i+1,\ldots,n).$$

Also, substituting (16) into (21) leads to the CFT-retarded Green's function in the momentum space

$$\bar{\mathcal{D}}_{1n}=\bar{\mathcal{D}}_{2n-1}=\cdots=\bar{\mathcal{D}}_{n1}$$
$$=(2(\triangle-1))^n\frac{(2\pi T_L)^{\triangle-1}(2\pi T_R)^{\triangle-1}}{\Gamma(\triangle)^2}$$
$$\times\sinh\Big[\frac{p_+}{2T_L}+\frac{p_-}{2T_R}\Big]\Big|\Gamma\Big(\frac{\triangle}{2}+i\frac{p_+}{2\pi T_L}\Big)\Big|^2$$
$$\times\Big|\Gamma\Big(\frac{\triangle}{2}+i\frac{p_-}{2\pi T_R}\Big)\Big|^2, \tag{22}$$

where Γ is the gamma function. The log-retarded Green's functions are given by

$$\bar{\mathcal{D}}_{2n}=\bar{\mathcal{D}}_{3n-1}=\cdots=\bar{\mathcal{D}}_{n2}$$
$$=\bar{\mathcal{D}}_{1n}(p_+,p_-)\times\Big\{\frac{n}{\triangle-1}+\ln[2\pi T_L]+\ln[2\pi T_R]$$
$$-2\psi(\triangle)+\frac{1}{2}\psi\Big(\frac{\triangle}{2}+i\frac{p_+}{2\pi T_L}\Big)$$
$$+\frac{1}{2}\psi\Big(\frac{\triangle}{2}-i\frac{p_+}{2\pi T_L}\Big)+\frac{1}{2}\psi\Big(\frac{\triangle}{2}+i\frac{p_-}{2\pi T_R}\Big)$$
$$+\frac{1}{2}\psi\Big(\frac{\triangle}{2}-i\frac{p_-}{2\pi T_R}\Big)\Big\}, \tag{23}$$

where $\psi(A)=\partial\ln[\Gamma(A)]/\partial A$ is the digamma function. In deriving $\bar{D}_{2n}$, we have used the following relation [15]:

$$\langle\mathcal{O}_i(u_+,u_-)\mathcal{O}_j(0)\rangle=\frac{1}{s!}\Big(\frac{\partial}{\partial\triangle}\Big)^s\langle\mathcal{O}_1(u_+,u_-)\mathcal{O}_n(0)\rangle.$$

We notice that the $\log^{n-1}$-Green's functions of $\bar{\mathcal{D}}_{kn}[=\bar{\mathcal{D}}_{k+1n-1}=\cdots=\bar{\mathcal{D}}_{nk}]$ with $k=3,\ldots,n$ can be found along the same line by using the above relation.

In order to derive quasinormal frequencies, we investigate the pole structure of the commutators for the nonrotating BTZ black hole ($T_R=T_L=T_H=1/2\pi\ell$, $r_+=\ell$). It is found that for $i,j=1,2,\ldots,n$ and $s=i+j-n-1\geq 0$, the pole structure of the retarded Green's functions is given by

$$\bar{\mathcal{D}}_{ij}(p_+)\propto\sum_{m=0s}^{s}C_m\Gamma^{(m)}\Big(h_L+i\frac{p_+}{2\pi T_L}\Big)\Gamma^{(s-m)}\Big(h_L-i\frac{p_+}{2\pi T_L}\Big)$$

with $\Gamma^{(m)}=\frac{\partial^m\Gamma}{\partial\triangle^m}$. This is one of our main results. Following Ref. [16], for the rank-n case one can read off quasinormal frequencies of scalar A_n, which satisfies $(\nabla_B^2-m^2)^n A_n=0$:

$$\omega_n^N=k-i4\pi T_L(N+h_L),\quad N=0,1,2,\ldots,\tag{24}$$

from an n-fold pole of the retarded Green's function $\bar{\mathcal{D}}_{nn}(p_+)$. Applying the previous truncation process for the LCFT to $\bar{\mathcal{D}}_{ij}$ leads to

$$\bar{\mathcal{D}}_{ij}\sim\begin{pmatrix}0 & 0\\ 0 & \bar{\mathcal{D}}_{(n+1)/2(n+1)/2}\end{pmatrix}\quad\text{for odd } n,\tag{25}$$

$$\bar{\mathcal{D}}_{ij}\sim\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}\quad\text{for even } n,\tag{26}$$

which implies that, for odd rank, the matrix provides a simple pole of $\omega_s^N=k-i4\pi T_L(N+h_L)$ obtained from the finite temperature CFT, while, for even rank, it contains nothing.

It is well known that the absorption cross section [17] can be written in terms of frequency (ω) and temperatures ($T_{R/L}$, T_H) as

$$\sigma_{\rm abs}^{ij}=\frac{\mathcal{C}_n}{\omega}\bar{\mathcal{D}}_{ij}(\omega),\tag{27}$$

where $\mathcal{C}_n$ is a normalization constant. Here $\bar{\mathcal{D}}_{ij}(\omega)$ is obtained by substituting $p_+=p_-=\omega/2$ for the s wave ($k=0$) into (22) and (23), and $\bar{\mathcal{D}}_{pn}[=\bar{\mathcal{D}}_{p+1n-1}=\cdots=\bar{\mathcal{D}}_{np}]$ with $p=3,4,\ldots,n$. From the expression (27), one can find the absorption cross section

$$\begin{aligned}\sigma_{\rm abs}^{1n}&=\sigma_{\rm abs}^{2n-1}=\cdots=\sigma_{\rm abs}^{n1}\simeq\pi^2\omega\ell^2,\\ \sigma_{\rm abs}^{2n}&=\sigma_{\rm abs}^{3n-1}=\cdots=\sigma_{\rm abs}^{n2}\\ &\simeq[n-2+2\gamma-2\ln 2+2\ln[\omega\ell]]\sigma_{\rm abs}^{1n}\end{aligned}\tag{28}$$

for the low-temperature limit of $\omega\gg T_{R/L}$ and $\triangle=2$.

Here Euler's constant $\gamma=0.5772$, ℓ is the AdS_3 curvature radius, and the normalization constant is fixed to be $\mathcal{C}_n=2^{1-n}$. On the other hand, one finds that the other absorption cross sections of $\sigma_{\rm abs}^{pn}[=\sigma_{\rm abs}^{p+1n-1}=\cdots=\sigma_{\rm abs}^{np}]$ with $p=3,4,\ldots,n$ are given in terms of $\bar{\mathcal{D}}_{pn}[=\bar{\mathcal{D}}_{p+1n-1}=\cdots=\bar{\mathcal{D}}_{np}]$.

Finally, it turns out that, in the low-temperature limit and $\Delta=2$, the general form of the absorption cross sections is given by the power series expansion of $\ln[\omega\ell]$:

$$\sigma_{\rm abs}^{ij}|_{\omega\gg T_{R/L}}=\Big[\sum_{m=0}^{s}a_{sm}^{(n)}\ln^m[\omega\ell]\Big]\sigma_{\rm abs}^{1n},\tag{29}$$

where $s=i+j-n-1\geq 0$ and $a_{sm}^{(n)}$ are some constants to be fixed (see Ref. [14]). For $i,j=n$ and $s=n-1$, (29) leads to the highest-order logarithmic correction $\ln^{n-1}[\omega\ell]$ to the Klein-Gordon mode which corresponds to the mode A_n satisfying $\nabla_B^{2n}A_n=0$.

Applying the previous truncation process to $\sigma_{\rm abs}^{ij}|_{\omega\gg T_{R/L}}$ leads to

$$\sigma_{\rm abs}^{ij}\sim\begin{pmatrix}0 & 0\\ 0 & \sigma_{\rm abs}^{(n+1)/2(n+1)/2}\end{pmatrix}\quad\text{for odd } n,\tag{30}$$

$$\sigma_{\rm abs}^{ij}\sim\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}\quad\text{for even } n,\tag{31}$$

which imply that, in the low-temperature and massless limits, the odd rank case provides the absorption cross section for the Klein-Gordon mode only, while, for even rank, it contains null states only.

In summary, we have constructed the rank-n finite temperature LCFT starting from the n-coupled scalar field theory S_Φ (3) in the BTZ black hole background. Our approach has provided two important quantities of quasinormal frequencies and absorption cross section of scalar A_n, which satisfies the $2n$th order linearized equation $(\nabla_B^2-m^2)^nA_n=0$ around the BTZ black hole. This work shows a usefulness of the AdS-LCFT correspondence for obtaining two observables of quasinormal modes and absorption cross section of A_n without solving the $2n$th order linearized equation directly.

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