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Anomalous Majorana neutrino masses from torsionful quantum gravity

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The effect of quantum torsion in theories of quantum gravity is usually described by an axionlike field which couples to matter and to gravitation and radiation gauge fields. In perturbation theory, the couplings of this torsion-descent axion field are of derivative type and so preserve a shift symmetry. This shift symmetry may be broken, if the torsion-descent axion field mixes with other axions, which could be related to moduli fields in string-inspired effective theories. In particular, the shift symmetry may break explicitly via nonperturbative effects, when these axions couple to fermions via chirality-changing Yukawa couplings with appropriately suppressed coefficients. We show how in such theories an effective right-handed Majorana neutrino mass can be generated at two loops by gravitational interactions that involve global anomalies related to quantum torsion. We estimate the magnitude of the gravitationally induced Majorana mass and find that it is highly model dependent, ranging from the multi-TeV to keV scale.

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I. INTRODUCTION

The recent discovery [1] of the Higgs boson at the CERN Large Hadron Collider constitutes an important milestone for the ultraviolet (UV) completion of the Standard Model (SM). Although the so-called Higgs mechanism may well explain the generation of most of the particle masses in the SM, the origin of the small neutrino masses still remains an open issue. In particular, the observed smallness of the light neutrino masses may naturally be explained through the seesaw mechanism [2], which necessitates the Majorana nature of the light (active) neutrinos and postulates the presence of heavy right-handed Majorana partners of mass M_R. The right-handed Majorana mass M_R is usually considered to be much larger than the lepton or quark masses. The origin of M_R has been the topic of several extensions of the SM in the literature, within the framework of quantum field theory [2–4] and string theory [5].

Recently, a potentially interesting radiative mechanism for generating gauge-invariant fermion masses at three loops has been studied in Ref. [6]. The mechanism utilizes global anomalies triggered by the possible existence of scalar or pseudoscalar fields in U(1) gauge theories and by heavy fermions F whose masses may not result from spontaneous symmetry breaking. One-loop quantum effects of the heavy fermions F give rise to a global chiral anomaly given by

$$a(x)F_{\mu\nu}(x)^{\star}F^{\mu\nu}(x), \tag{1}$$

where $a(x)$ is a pseudoscalar field, $F_{\mu\nu}$ denotes the U(1) gauge field Maxwell tensor and ${}^{\star}F^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ is its dual. Moreover, it was assumed in Ref. [6] that the pseudoscalar field $a(x)$ couples to chirality-changing fermion bilinears, $\bar{f}i\gamma_5 f$, via the Yukawa-type couplings

$$y_a a(x)\bar{f}i\gamma_5 f. \tag{2}$$

The fermions f are assumed to have zero bare masses. However, as was explicitly demonstrated in Ref. [6], the fermions f can receive a nonzero mass at the three-loop level, through the anomalous interaction (1) and the chirality-changing Yukawa couplings (2).

It was further suggested in Ref. [6] that this mass-generating mechanism can also be applied to create low-scale fermion masses by pure quantum-gravity effects. In this case, the role of the U(1) gauge field strength tensor $F_{\mu\nu}$ will be played by the Riemann curvature tensor $R_{\mu\nu\rho\sigma}$, and hence the role of the gauge fields by the gravitons. Such a gravitationally generating mass mechanism could straightforwardly be applied to fermions without SM quantum charges, such as Majorana right-handed neutrinos, which we restrict our attention to here. In such a framework of quantum gravity, the operator (1) is expected to be replaced by an operator of the form

$$a(x)R_{\mu\nu\rho\sigma}{}^{\star}R^{\mu\nu\rho\sigma}, \tag{3}$$

where $a(x)$ is an appropriate pseudoscalar field and ${}^{\star}R^{\mu\nu\rho\sigma} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}R_{\alpha\beta}{}^{\rho\sigma}$ denotes the dual Riemann curvature tensor.

It is the purpose of this paper to present explicit scenarios of quantum gravity and provide reliable estimates of the gravitationally induced right-handed Majorana mass M_R. Although quantum gravitational interactions are nonrenormalizable, nevertheless there are aspects of the theory that can be exact, in a path integral sense, and these are related to some aspects of torsionful manifolds. Torsion appears as a nonpropagating form-valued pseudoscalar field $b(x)$ in a quantum-gravity path integral and as such it can be integrated out exactly. As we explain in this paper, the effect of torsion would result in anomalous operators analogous to (3), which are instrumental in the generation of Majorana fermion masses.

Nevertheless, in addition to the torsional field $b(x)$, the presence of extra pseudoscalar fields $a(x)$ are required for generating a chirality-violating Majorana mass M_R. The reason is that the couplings of the torsional axion field $b(x)$ are of derivative type and preserve a shift symmetry: $b(x) \to b(x) + c$, where c is an arbitrary constant. As a consequence, chirality is conserved in the massless limit of the right-handed neutrinos, thus forbidding the generation of a Majorana mass M_R. However, the shift symmetry may break explicitly via nonperturbative effects, when these axions couple to fermions via chirality-changing Yukawa couplings y_a of the form (2). The size of the Yukawa coupling y_a is highly model dependent, implying a wide range of values for the gravitationally induced Majorana mass scale M_R.

This paper is organized as follows. After this introductory section, Sec. II reviews some basic properties of manifolds with quantum torsion, within field-theoretic and string-theoretic frameworks. In Sec. III we present an explicit effective field-theoretic scenario of quantum torsion that can give rise to an anomalous Majorana neutrino mass generation at two loops. Finally, Sec. IV summarizes our conclusions and presents possible future directions.

II. PROPERTIES OF QUANTUM TORSION

Quantum field theories in space-times with torsion exhibit some interesting properties, which have been known for some time [7]. In theories with fermions, torsion is introduced necessarily in the first-order Palatini formalism, where vierbein and spin connections are treated as independent variables (Einstein-Cartan theory) [8]. In what follows, we discuss various cases where torsion appears in the spectrum of a quantum-gravity model. We shall examine first string theory models and then proceed to argue that certain properties of quantum torsion, such as those related to chiral (axial) anomalies, are generic to field theories with fermions and can arise in ordinary local field-theoretic models, not only strings. Let us commence our discussion from string theory, not as much for historical purposes, but because this provides a concrete UV complete theoretical framework for quantum gravity.

A. Quantum torsion and KR axions in string theories

In string theories, torsion is introduced as a consequence of the existence of the antisymmetric tensor field $B_{\mu\nu} = -B_{\nu\mu}$ existing in the gravitational multiplet of the string. Indeed, as a result of the stringy "gauge" symmetry $B_{\mu\nu} \to B_{\mu\nu} + \partial_{[\mu} B_{\nu]}$, the low-energy string effective action depends only on the field strength

$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}, \tag{4}$$

where the symbol $[\ldots]$ denotes antisymmetrization of the appropriate indices. In fact, it can be shown [9] that the terms involving the field strength perturbatively to each order in the Regge slope parameter α' can be assembled, in such a way that only torsionful Christoffel symbols, such as $\bar{\Gamma}^{\mu}_{\nu\rho}$, appear. In this formalism, the torsionful Christoffel symbol $\bar{\Gamma}^{\mu}_{\nu\rho}$ is defined as

$$\bar{\Gamma}^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + \frac{\kappa}{\sqrt{3}} H^{\mu}_{\nu\rho} \neq \bar{\Gamma}^{\mu}_{\rho\nu}, \tag{5}$$

where $\Gamma^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\rho\nu}$ is the ordinary, torsion-free, symmetric connection, and κ is the gravitational constant given by

$$\kappa^2 = 8\pi G_N = \frac{8\pi}{M_P^2}, \tag{6}$$

where G_N and M_P are Newton's constant and the Planck mass, respectively. Consequently, terms involving the generalized Riemann curvature tensor $\bar{R}_{\mu\nu\rho\sigma}$ appear in the effective action. In the context of (super)string theories [10], anomaly cancellation requires that Lorentz and gauge Chern-Simons terms are added to the field strength of the $\mathbf{B}$ field so that one may define a new field strength three-form $\mathbf{H} = \mathrm{d}\mathbf{B} + \frac{\alpha'}{8\kappa}(\Omega_L - \Omega_V)$ such that the following Bianchi identity is implied:

$$\mathrm{d}\mathbf{H} = \frac{\alpha'}{8\kappa} \mathrm{Tr}(\mathbf{R} \wedge \mathbf{R} - \mathbf{F} \wedge \mathbf{F}), \tag{7}$$

where $\mathbf{R}$ denotes the gravitational Riemann curvature four-form without H torsion and $\mathbf{F}$ the gauge field strength two-form, which includes the torsion. The $\wedge$ symbol denotes appropriate contractions with the vierbeins $\mathbf{e}^a_\mu$, while the trace is taken over all possible group-theoretic structures. To lowest order in α', where we shall restrict our attention here, the effective action in a four-dimensional space-time (obtained after compactification and up to a total divergence) reads [9]

$$\begin{aligned} S^{(4)} &= \int d^4x \sqrt{-g} \Big(\frac{1}{2\kappa^2} R - \frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} \Big) \\ &= \int d^4x \sqrt{-g} \Big(\frac{1}{2\kappa^2} \bar{R} - \frac{1}{3} \kappa^2 H_{\mu\nu\rho} H^{\mu\nu\rho} \Big), \end{aligned} \tag{8}$$

where in the second line we used the generalized torsionful connection (5). The gravitational constant κ^2 contains all the appropriate compactification volume factors and string coupling terms; in particular we have the following relation from string theory:

$$\frac{1}{g_s^2} M_s^2 V^{(c)} = \frac{1}{2\kappa^2}, \tag{9}$$

with $M_s = 1/\sqrt{\alpha'}$ the string mass scale and g_s the string coupling assumed weak $g_s < 1$. We assume constant dilations ϕ in four dimensions for our purposes here. In general, for nonconstant dilations $g_s = \exp(\phi)$ are a field-dependent quantity.

In four dimensions, we may define the dual of $\mathbf{H}$, $\mathbf{Y} = {}^\star\mathbf{H}$, or equivalently in components,

$$Y_\sigma = -3\sqrt{2}\partial_\sigma b = \sqrt{-g}\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}, \quad (10)$$

after adopting the normalizations of Ref. [11]. The field $b(x)$ is a form-valued pseudoscalar field, with a canonically normalized kinetic term, which we call from now on the Kalb-Ramond (KR) axion [12], in order to distinguish it from other axionlike fields coming, e.g., from the moduli sector of string theory, which we shall also employ in our analysis. Using the definition (10) we may rewrite the Bianchi identity (7) in the form

$$\nabla_\sigma Y^\sigma = \frac{\alpha'}{32\kappa}\sqrt{-g}\epsilon_{\mu\nu\lambda\sigma}(R_{ad}{}^{\mu\nu}R^{\lambda\sigma ad} - F^{\mu\nu}F^{\lambda\sigma}), \quad (11)$$

where ∇_σ is the torsion-free gravitational covariant derivative and $R_{...}$ are the components of the torsion-free curvature tensor. The latin indices a, d are tangent space indices as usual. Using (11) in (8) and performing a partial integration, we arrive at the following form of the string-inspired four-dimensional effective action with H torsion [11]:

$$S^{(4)} = \int d^4x\sqrt{-g}\Big[\frac{1}{2\kappa^2}R - \frac{1}{2}\partial_\mu b(x)\partial^\mu b(x) + \frac{\alpha'\sqrt{2}}{192\kappa}b(x)\epsilon_{\mu\nu\rho\lambda}(R_{ad}{}^{\mu\nu}R^{\rho\lambda ad} - F^{\mu\nu}F^{\rho\lambda})\Big]. \quad (12)$$

The close relation of the H torsion to the appearance of an axionlike field in the effective action is not unique to string theory. In the next subsection we proceed to discuss a field-theoretical case where similar effects take place.

B. Quantum torsion and KR axions in field theory

As observed in Ref. [11] in the context of QED in torsionful manifolds, one obtains similar couplings of the torsion-induced axion to gravity and gauge fields as in (12), by employing *quantum anomalies* of the axial fermion current. Indeed, let us consider Dirac QED fermions in a torsionful space-time. The Dirac action reads

$$S_\psi = \frac{i}{2}\int d^4x\sqrt{-g}(\bar\psi\gamma^\mu\bar{\mathcal{D}}_\mu\psi - (\bar{\mathcal{D}}_\mu\bar\psi)\gamma^\mu\psi), \quad (13)$$

where $\bar{\mathcal{D}}_\mu = \bar\nabla_\mu - ieA_\mu$, with e the electron charge and A_μ the photon field. The overline above the covariant derivative, i.e., $\bar\nabla_\mu$, denotes the presence of torsion, which is introduced through the torsionful spin connection: $\bar\omega_{ab\mu} = \omega_{ab\mu} + K_{ab\mu}$, where $K_{ab\mu}$ is the contorsion tensor. The latter is related to the torsion two-form $\mathbf{T}^a = \mathbf{d}\mathbf{e}^a + \bar\omega^a \wedge \mathbf{e}^b$ via [7,11]: $K_{abc} = \frac{1}{2}(\mathrm{T}_{cab} - \mathrm{T}_{abc} - \mathrm{T}_{bcd})$. The presence of torsion in the covariant derivative in the Dirac-like action (13) leads, apart from the standard terms in manifolds without torsion, to an additional term involving the axial current $J_5^\mu \equiv \bar\psi\gamma^\mu\gamma^5\psi$:

$$S_\psi \ni -\frac{3}{4}\int d^4\sqrt{-g}S_\mu\bar\psi\gamma^\mu\gamma^5\psi = -\frac{3}{4}\int S\wedge{}^\star J^5, \quad (14)$$

where $\mathbf{S} = {}^\star\mathbf{T}$ is the dual of $\mathbf{T}$: $S_d = \frac{1}{3!}\epsilon^{abc}{}_dT_{abc}$.

We next remark that the torsion tensor can be decomposed into its irreducible parts [7], of which S_d is the pseudoscalar axial vector:

$$T_{\mu\nu\rho} = \frac{1}{3}(T_\nu g_{\mu\rho} - T_\rho g_{\mu\nu}) - \frac{1}{3!}\epsilon_{\mu\nu\rho\sigma}S^\sigma + q_{\mu\nu\rho}, \quad (15)$$

with $\epsilon_{\mu\nu\rho\sigma}q^{\nu\rho\sigma} = q^\nu_{\rho\nu} = 0$. This implies that the contorsion tensor undergoes the following decomposition:

$$K_{abc} = \frac{1}{2}\epsilon_{abcd}S^d + \hat K_{abc}, \quad (16)$$

where $\hat K$ includes the trace vector T_μ and the tensor $q_{\mu\nu\rho}$ parts of the torsion tensor.

The gravitational part of the action can then be written as

$$S_G = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}(R + \hat\Delta) + \frac{3}{4\kappa^2}\int \mathbf{S}\wedge{}^\star\mathbf{S}, \quad (17)$$

where $\hat\Delta = \hat K^\lambda{}_{\mu\nu}\hat K^{\nu\mu}{}_\lambda - \hat K^{\mu\nu}{}_\nu\hat K_{\mu\lambda}{}^\lambda$, with the hatted notation defined in (16).

In a quantum-gravity setting, where one integrates over all fields, the torsion terms appear as nonpropagating fields and thus they can be integrated out exactly. The authors of Ref. [11] have observed though that the classical equations of motion identify the axial-pseudovector torsion field S_μ with the axial current, since the torsion equation yields

$$K_{\mu ab} = -\frac{1}{4}e^c_\mu\epsilon_{abcd}\bar\psi\gamma_5\tilde\gamma^d\psi. \quad (18)$$

From this it follows $\mathbf{d}^\star\mathbf{S} = 0$, leading to a conserved "torsion charge" $Q = \int{}^\star\mathbf{S}$. To maintain this conservation in quantum theory, they postulated $\mathbf{d}^\star\mathbf{S} = 0$ at the quantum level, which can be achieved by the addition of judicious counterterms. This constraint, in a path-integral formulation of quantum gravity, is then implemented via a delta function constraint, $\delta(d^\star\mathbf{S})$, and the latter via the well-known trick of introducing a Lagrange multiplier field $\Phi(x) \equiv (3/\kappa^2)^{1/2}b(x)$. Hence, the relevant torsion part of the quantum-gravity path integral would include a factor

$$\int D\mathbf{S}Db\exp\Big[i\int\frac{3}{4\kappa^2}\mathbf{S}\wedge{}^\star\mathbf{S} - \frac{3}{4}\mathbf{S}\wedge{}^\star\mathbf{J}^5 + \Big(\frac{3}{2\kappa^2}\Big)^{1/2}bd^\star\mathbf{S}\Big] = \int Db\exp\Big[-i\int\frac{1}{2}\mathbf{d}b\wedge{}^\star\mathbf{d}b + \frac{1}{f_b}\mathbf{d}b\wedge{}^\star\mathbf{J}^5 + \frac{1}{2f_b^2}\mathbf{J}^5\wedge\mathbf{J}^5\Big], \quad (19)$$

where

$$f_b = (3\kappa^2/8)^{-1/2} = \frac{M_P}{\sqrt{3\pi}} \quad (20)$$

and the nonpropagating $\mathbf{S}$ field has been integrated out. The reader should notice that, as a result of this integration, the corresponding *effective* field theory contains a *nonrenormalizable* repulsive four-fermion axial current-current interaction.[1]

We may partially integrate the second term in the exponent on the right-hand side of (19) and take into account the well known field-theoretic result that in QED the axial current is not conserved at the quantum level, due to anomalies, but its divergence is obtained by the one-loop result [14]:

$$\nabla_\mu J^{5\mu} = \frac{e^2}{8\pi^2} F^{\mu\nu}\tilde{F}_{\mu\nu} - \frac{1}{192\pi^2} R^{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma} \equiv G(\mathbf{A}, \omega). \tag{21}$$

Observe that in (21) the torsion-free spin connection has been used. This can be achieved by the addition of proper counterterms in the action [11], which can convert the anomaly from the initial $G(\mathbf{A}, \bar{\omega})$ to $G(\mathbf{A}, \omega)$. Using (21) in (19) one can then obtain for the effective torsion action in QED

$$\int Db \exp\left[-i\int \frac{1}{2}\mathbf{d}b \wedge {}^\star\mathbf{d}b - \frac{1}{f_b} bG(\mathbf{A}, \omega) + \frac{1}{2f_b^2}\mathbf{J}^5 \wedge \mathbf{J}^5\right]. \tag{22}$$

Thus, even in ordinary field theories, we obtain the coupling of the KR axion to the curvature and gauge field strengths: $bG(\mathbf{A}, \omega)$, exactly as we obtained in the string case (12). In addition, the torsion leads to repulsive four-fermion interactions involving the axial current. Crucial to the above derivation was, however, the postulation of the conservation of the torsion charge at the quantum level, as expressed by the constraint $\mathbf{d}^\star\mathbf{S} = 0$. The resulting axion field has originated from the Lagrange multiplier field implementing this constraint. In the subsequent section we present an alternative derivation of this result.

C. Alternative proof of the connection of axions to torsion in field theory

We shall now provide a different proof concerning the fundamental geometrical properties of the torsion at the quantum level. To this end, we concentrate on the work of Ref. [15], which discusses fermionic torsion in first-order Palatini formalism of fermions in curved space-times. Although the motivation of that work was an attempt to connect some aspects of loop quantum-gravity theories to Ashtekar canonical formulation of quantum gravity, nevertheless the used formalism will allow us to discover the above-mentioned torsion-induced axions from a different viewpoint.

As noted in Refs. [15,16], in the case of Dirac fermions in a manifold with torsion, if one uses the naive version of the Dirac action (13), decomposes the torsion tensor in its irreducible parts, and uses the equations of motion, then the trace vector part T_μ is found proportional to the axial fermion current J^5_μ, which is *inconsistent* with the respective Lorentz transformation properties. Such an inconsistency is remedied by adding to the action a *total derivative* term which can be expressed solely in terms of *topological invariants*, namely the so-called Nieh-Yan invariant density [17], and a *total divergence* of the fermion axial current:

$$\begin{aligned} S_{\text{Holst}} &= -i\frac{\eta}{2}\int d^4x[I_{\text{NY}} + \partial_\mu J^{5\mu}], \\ I_{\text{NY}} &= \epsilon^{\mu\nu\rho\sigma}\left[K^a{}_{\mu\nu}K_{\rho\sigma a} - \frac{1}{2}\Sigma_{\mu\nu}{}^{ab}\bar{R}_{\rho\sigma ab}\right] \\ &= \epsilon^{\mu\nu\rho\sigma}\partial_\mu K_{\nu\rho\sigma}, \end{aligned} \tag{23}$$

where η is a constant real parameter, $\Sigma_{\mu\nu}{}^{ab} = \frac{1}{2}e^a_{[\mu}e^b_{\nu]}$, the overline above the curvature tensor denotes the inclusion of torsion and $K_{\mu\nu\rho}$ is the contorsion tensor defined previously. Notice that in the last equality for the Nieh-Yan topological invariant we took into account the fact that, for the torsion (18), the term quadratic in the contorsion in I_{NY} vanishes.[2] Then, on account of (18), we observe that in the case with fermionic torsion, both terms in the space-time integrand of the Holst action turn out to be proportional to the divergence of the axial current, which can be expressed in terms of the (torsion-free) curvature and gauge field strengths through the anomaly equation (21).[3]

By promoting the constant parameter η into a pseudoscalar field [19],

$$\eta \to \eta(x), \tag{24}$$

[1]This term will induce a cubic term in the equations of motion for the fermions. Under the assumption of formation of a (Lorentz-violating) fermionic condensate of the axial current, it was recently argued [13] that Dirac fermions may lead to C- and *CPT*-violating differences between the fermion-antifermion populations in the finite temperature environment of the early Universe. In contrast, for a Majorana spinor of interest to us, the Majorana condition $\psi^c = \psi$ entails that a Majorana fermion is its own antiparticle and so there is *no* such violation of *CPT*.

[2]We note in passing that the parameter η in (23) is related to the so-called Immirzi-Barbero parameter of loop quantum gravity. Such completions of torsionful-space-time gravity theories, with the above topological invariant terms, have been included for consistency in the canonical quantization of supergravity theories [18], where the various fermion fields, including gravitinos, contribute to torsion.

[3]In the case of string-inspired H torsion and in the presence of fermions, the contortion tensor contains two parts, the fermion-dependent ones (18) and the ones proportional to the H torsion. In that case, through the anomaly equation (21) and Bianchi identity (7), both terms in the expression for the Holst action (23) are proportional to the same form total derivative terms, up to algebraic proportionality factors. The final result for the integrand of the Holst action is again given by the anomaly term (21), $G(\mathbf{A}, \omega)$, up to proportionality constants, which can be absorbed in the normalization of the parameter η.

we notice that the term involving the divergence of the axial current in (23) yields, upon using the anomaly equation (21), a similar term in the effective action as the one involving the axion fields in string theory or in QED through implementing the appropriate constraints by Lagrange multipliers, and thus we can identify the nonconstant field $\eta(x)$ with the axion $b(x)$:

$$\eta(x) \equiv b(x). \tag{25}$$

The kinetic terms for the field η are obtained by using the equations of motion and identifying extra contributions in the contorsion involving derivatives of the field. This identification has also been conjectured in Ref. [19] without having prior knowledge of the works of Ref. [11]. Indeed, it was shown in Ref. [19] that the torsionful spin connection in a theory with fermions and a nonconstant $\eta(x) \equiv b(x)$ is modified compared to the constant η case as follows:

$$\bar{\omega}_\mu^{ab} = \omega_\mu{}^{ab}(e) + \frac{1}{4}\epsilon^{ab}{}_{cd}e^c_\mu(\kappa J^{5d} - 2\eta^{df}\partial_f b(x)), \tag{26}$$

where η^{df} is the Minkowski metric on the tangent space. The quadratic parts of the torsionful spin connection then yield [19] kinetic terms for the field $b(x)$ and an effective action of the form (19), which, upon using the anomaly equation (21), implies the axion-curvature couplings mentioned above in (22).

Consequently, we seem to have established that the presence of the coupling of axion to spatial curvature and the dual of the curvature tensor is a rather generic feature of torsionful theories of space-time. We next proceed to apply the above ideas to the problem of gauge-invariant Majorana mass generation, without spontaneous symmetry breaking, for right-handed Majorana neutrinos, which carry no Standard Model charges, and are thus susceptible to the effects of torsion.

III. ANOMALOUS MAJORANA MASS GENERATION FROM QUANTUM TORSION

An important aspect of the coupling of the torsion KR field $b(x)$ to the fermionic matter discussed above is its shift symmetry, characteristic of an axion field. Indeed, by shifting the field $b(x)$ by a constant, $b(x) \to b(x) + c$, the action (22) only changes by total derivative terms, such as $cR^{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma}$ and $cF^{\mu\nu}\tilde{F}_{\mu\nu}$. These terms are irrelevant for the equations of motion and the induced quantum dynamics, provided the fields fall off sufficiently fast to zero at space-time infinity. Our scenario for the anomalous Majorana mass generation through torsion consists of augmenting the effective action (22) by terms that break such a shift symmetry.

To illustrate this last point, we first couple the KR axion $b(x)$ to another pseudoscalar axion field $a(x)$. In string-inspired models, such pseudoscalar axion $a(x)$ may be provided by the string moduli [20,21]. The proposed coupling occurs through a mixing in the kinetic terms of the two fields. To be specific, we consider the action

$$\mathcal{S} = \int d^4x\sqrt{-g}\Big[\frac{1}{2}(\partial_\mu b)^2 + \frac{b(x)}{192\pi^2 f_b}R^{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma} + \frac{1}{2f_b^2}J^5_\mu J^{5\mu} + \gamma(\partial_\mu b)(\partial^\mu a) + \frac{1}{2}(\partial_\mu a)^2 - y_a i a(\bar{\psi}_R{}^C\psi_R - \bar{\psi}_R\psi_R{}^C)\Big], \tag{27}$$

where $\psi_R{}^C = (\psi_R)^C$ is the charge-conjugate right-handed fermion ψ_R, $J^5_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi$ is the axial current of the four-component Majorana fermion $\psi = \psi_R + (\psi_R)^C$, and γ is a real parameter to be constrained later on. Here, we have ignored gauge fields, which are not of interest to us, and the possibility of a nonperturbative mass M_a for the pseudoscalar field $a(x)$. Moreover, we remind the reader that the *repulsive* self-interaction fermion terms are due to the existence of torsion in the Einstein-Cartan theory. The Yukawa coupling y_a of the axion moduli field a to right-handed sterile neutrino matter ψ_R may be due to nonperturbative effects. These terms *break* the shift symmetry: $a \to a + c$.

Before proceeding with the evaluation of the anomalous Majorana mass, it is convenient to diagonalize the axion kinetic terms by redefining the KR axion field as follows:

$$b(x) \to b'(x) \equiv b(x) + \gamma a(x). \tag{28}$$

This implies that the effective action (27) now becomes

$$\mathcal{S} = \int d^4x\sqrt{-g}\Big[\frac{1}{2}(\partial_\mu b')^2 + \frac{1}{2}(1-\gamma^2)(\partial_\mu a)^2 + \frac{1}{2f_b^2}J^5_\mu J^{5\mu} + \frac{b'(x) - \gamma a(x)}{192\pi^2 f_b}R^{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma} - y_a i a(\bar{\psi}_R{}^C\psi_R - \bar{\psi}_R\psi_R{}^C)\Big]. \tag{29}$$

Thus we observe that the b' field has decoupled and can be integrated out in the path integral, leaving behind an axion field $a(x)$ coupled both to matter fermions and to the operator $R^{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma}$, thereby playing now the role of the torsion field. We observe though that the approach is only valid for

$$|\gamma| < 1; \tag{30}$$

otherwise the axion field would appear as a ghost, i.e., with the wrong sign of its kinetic terms, which would indicate an instability of the model. This is the only restriction of the parameter γ. In this case we may redefine the axion field so as to appear with a canonical normalized kinetic term, implying the effective action:

$$\mathcal{S}_a = \int d^4x\sqrt{-g}\Big[\frac{1}{2}(\partial_\mu a)^2 - \frac{\gamma a(x)}{192\pi^2 f_b\sqrt{1-\gamma^2}}R^{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma} - \frac{iy_a}{\sqrt{1-\gamma^2}}a(\bar{\psi}_R{}^C\psi_R - \bar{\psi}_R\psi_R{}^C) + \frac{1}{2f_b^2}J^5_\mu J^{5\mu}\Big]. \tag{31}$$

Evidently, the action $\mathcal{S}_a$ in (31) corresponds to a canonically normalized axion field $a(x)$, coupled *both* to the curvature of space-time, *à la* torsion, with a modified coupling $\gamma/(192\pi^2 f_b\sqrt{1-\gamma^2})$, and to fermionic matter with chirality-changing Yukawa-like couplings of the form $y_a/\sqrt{1-\gamma^2}$.

The mechanism for the anomalous Majorana mass generation is shown in Fig. 1. We may now estimate the two-loop Majorana neutrino mass in quantum gravity with an effective UV energy cutoff Λ. Adopting the effective field-theory framework of Ref. [22], we first notice that the energy (E) dependence of the curvature R is E^2, since it contains two derivatives ∂_μ, with $i\partial_\mu \to p_\mu \sim E$. Therefore, the operator $a(x)R_{\mu\nu\lambda\rho}\tilde{R}^{\mu\nu\lambda\rho}$ gives rise to an E^4 dependence. Likewise, on naive dimensional grounds, the couplings of the linearized gravitons $h_{\mu\nu}$ and $h_{\rho\sigma}$ to chiral fermions ψ_R and ψ_R^C both grow as E, as their kinetic terms are proportional to a single power of $i\partial_\mu$. This gives rise to another energy factor E^2. Collecting all the energy factors resulting from the gravitational interactions and the loop momenta, we find that the two-loop graph in Fig. 1 exhibits a UV cutoff dependence Λ^6. This leads to a gravitationally induced Majorana mass M_R:

$$M_R \sim \frac{1}{(16\pi^2)^2}\frac{y_a\gamma\kappa^4\Lambda^6}{192\pi^2 f_b(1-\gamma^2)} = \frac{\sqrt{3}y_a\gamma\kappa^5\Lambda^6}{49152\sqrt{8}\pi^4(1-\gamma^2)}, \tag{32}$$

where in the second step we took into account (20). In a UV complete theory such as strings, Λ and M_P are related, since Λ is proportional to M_s and the latter is related to M_P (or κ) through (9).

It is interesting to provide a numerical estimate of the anomalously generated Majorana mass M_R. Assuming that $\gamma \ll 1$, the size of M_R may be estimated from (32) to be

$$M_R \sim (3.1\times 10^{11}\ \text{GeV})\left(\frac{y_a}{10^{-3}}\right)\left(\frac{\gamma}{10^{-1}}\right)\left(\frac{\Lambda}{2.4\times 10^{18}\ \text{GeV}}\right)^6. \tag{33}$$

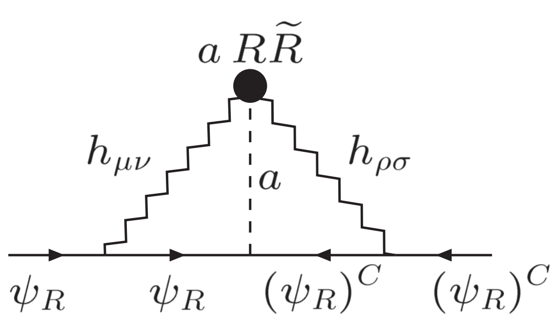

FIG. 1. Typical Feynman graph giving rise to anomalous fermion mass generation. The black circle denotes the operator $a(x)R_{\mu\nu\lambda\rho}\tilde{R}^{\mu\nu\lambda\rho}$ induced by torsion.

Obviously, the generation of M_R is highly model dependent. Taking, for example, the quantum-gravity scale to be $\Lambda = 10^{17}$ GeV, we find that M_R is at the TeV scale, for $y_a = 10^{-3}$ and $\gamma = 0.1$. However, if we take the quantum-gravity scale to be close to the grand unified theory (GUT) scale, i.e., $\Lambda = 10^{16}$ GeV, we obtain a right-handed neutrino mass $M_R \sim 16$ keV, for the choice $y_a = \gamma = 10^{-3}$. This is in the preferred ballpark region for the sterile neutrino ψ_R to qualify as a warm dark matter [23].

In a string-theoretic framework, many axions might exist that could mix with each other. Such a mixing can give rise to reduced UV sensitivity of the two-loop graph shown in Fig. 1. To make this point explicit, let us therefore consider a scenario with a number n axion fields, $a_{1,2,\ldots,n}$. Of this collection of n pseudoscalars, only a_1 has a kinetic mixing term γ with the KR axion b and only a_n has a Yukawa coupling y_a to right-handed neutrinos ψ_R. The other axions $a_{2,3,\ldots,n}$ have a next-to-neighbor mixing pattern. In such a model, the kinetic terms of the effective action are given by

$$\mathcal{S}_a^{\text{kin}} = \int d^4x\sqrt{-g}\Big[\frac{1}{2}\sum_{i=1}^{n}((\partial_\mu a_i)^2 - M_i^2) + \gamma(\partial_\mu b)(\partial^\mu a_1) - \frac{1}{2}\sum_{i=1}^{n-1}\delta M_{i,i+1}^2 a_i a_{i+1}\Big], \tag{34}$$

where the mixing mass terms $\delta M_{i,i+1}^2$ are constrained to be $\delta M_{i,i+1}^2 < M_i M_{i+1}$, so as to obtain a stable positive mass spectrum for all axions. As a consequence of the next-to-neighbor mixing, the UV behavior of the off-shell transition $a_1 \to a_n$, described by the propagator matrix element $\Delta_{a_1a_n}(p)$, changes drastically, i.e., $\Delta_{a_1a_n}(p) \propto 1/(p^2)^n \sim 1/E^{2n}$. Assuming, for simplicity, that all axion masses and mixings are equal, i.e., $M_i^2 = M_a^2$ and $\delta M_{i,i+1}^2 = \delta M_a^2$, the anomalously generated Majorana mass may be estimated to be

$$M_R \sim \frac{\sqrt{3}y_a\gamma\kappa^5\Lambda^{6-2n}(\delta M_a^2)^n}{49152\sqrt{8}\pi^4(1-\gamma^2)}, \tag{35}$$

for $n \le 3$, and

$$M_R \sim \frac{\sqrt{3}y_a\gamma\kappa^5(\delta M_a^2)^3}{49152\sqrt{8}\pi^4(1-\gamma^2)}\frac{(\delta M_a^2)^{n-3}}{(M_a^2)^{n-3}}, \tag{36}$$

for $n > 3$. It is then not difficult to see that three axions $a_{1,2,3}$ with next-to-neighbor mixing as discussed above would be sufficient to obtain a UV finite result for M_R at the two-loop level. Of course, beyond the two loops, M_R will depend on higher powers of the energy cutoff Λ, i.e., $\Lambda^{n>6}$, but if $\kappa\Lambda \ll 1$, these higher-order effects are expected to be subdominant.

In the above n-axion-mixing scenarios, we note that the anomalously generated Majorana mass term will only depend on the mass-mixing parameters δM_a^2 of the axion fields and not on their masses themselves, as long as $n \le 3$. Instead, for axion-mixing scenarios with $n > 3$, the induced Majorana neutrino masses are proportional to the factor $(\delta M_a^2/M_a^2)^n$, which gives rise to an additional suppression for heavy axions with masses $M_a \gg \delta M_a$.

The possible existence of heavy pseudoscalar fields, with masses M_a as large as TeV, has been considered, for instance in Ref. [24], within complicated GUTs, with extra strong interactions, confining at very short distance scales. Phenomenological implications and astrophysical constraints of such heavy axions have been analyzed in Ref. [25], for a wide range of axion couplings. Specifically, for the Peccei-Quinn axion scale[4] $f_a \sim 10^{12}$ GeV, supernovae data exclude heavy pseudoscalar axionlike particles with masses in the region 100 eV $\le M_a \le$ 1 GeV. Heavier axions are allowed provided their coupling is greater than $f_a \ge 10^{15}$ GeV.

In the multiaxion models outlined above, the anomalously generated Majorana neutrino mass M_R will still depend on the Yukawa coupling y_a and the torsion-axion kinetic mixing coefficient γ, besides the assumed UV completion scale Λ of quantum gravity. In order to get an estimate of the size of M_R, we treat the axion masses M_a and mass mixings δM_a as free parameters to be constrained by phenomenology. Let us assume an n-axion-mixing model with $n \ge 3$, in which the axion mass mixings δM_a and their masses M_a are of the same order, i.e., $\delta M_a/M_a \sim 1$. In this case, employing (35), we may estimate the Majorana neutrino mass M_R to be

$$\frac{M_R}{M_a} \sim \frac{10^{-3} y_a \gamma}{1-\gamma^2}\frac{(\delta M_a)^6}{M_a M_P^5} \sim \frac{10^{-3} y_a \gamma}{1-\gamma^2}\left(\frac{\delta M_a}{M_P}\right)^5. \quad (37)$$

For axion masses $M_a \le 1$ TeV considered in the literature thus far, we find that $M_R/M_{a \lesssim} 10^{-83}$, which implies extraordinary small Majorana masses M_R. An obvious caveat to this result would be to have ultraheavy axion masses M_a close to the GUT scale and/or fine-tune the torsion-axion kinetic mixing parameter γ to 1, in a way such that the factor $\frac{\gamma}{1-\gamma^2}$ compensates for the mass suppression $(\delta M_a/M_P)^5$ in (37). The latter possibility, however, might result in an unnaturally large (nonperturbative) "effective" Yukawa coupling $y_a^{\rm eff} \equiv y_a/\sqrt{1-\gamma^2} \gtrsim \sqrt{4\pi}$ in (31), which will bring us outside the perturbative framework that we have been considering here. A more detailed phenomenological and astrophysical analysis of all possible axion-mixing scenarios may be given elsewhere.

IV. CONCLUSIONS

We have shown how, in theories of quantum gravity with torsion, an effective right-handed Majorana neutrino mass M_R can be generated at two loops by gravitational interactions that involve global anomalies. The global anomalies result, after integrating out a formed-valued pseudoscalar field b, the so-called Kalb-Ramond axion, which describes the effect of quantum torsion. The KR axion b couples to both matter and to gravitation and radiation gauge fields. In perturbation theory, this torsion-descent axion field b has derivative couplings, leading to an axion shift symmetry: $b \to b + c$, where c is an arbitrary constant. If another axion field a or fields are present in the theory, the shift symmetry may be broken, giving rise to axion masses and chirality-changing Yukawa couplings to massless fermions, such as right-handed Majorana neutrinos ψ_R.

We have estimated the magnitude of the two-loop gravitationally induced Majorana mass M_R and found that it is highly model dependent. Its size generically depends on three parameters: the value of the Yukawa coupling y_a to ψ_R, the kinetic mixing term γ between the KR axion b and the other axion field a, and the assumed quantum-gravity scale Λ of UV completion. In the present study, we have assumed that Λ is considerably smaller than the Planck mass, i.e., $\kappa\Lambda \ll 1$. The anomalously generated Majorana neutrino mass M_R can take values, ranging from the multi-TeV to keV scale.

The radiative fermion mass mechanism discussed in this paper may be used to account for the generation of other gauge-invariant masses, such as those pertinent to vectorlike quarks and leptons. It would be interesting to investigate, whether this radiative mechanism can be consistently extended to supergravity theories with quantum torsion and analyze the possible consequences of such theories on Majorana fermion masses.

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[4]The Peccei-Quinn symmetry breaking scale f_a usually defines the strength of the axion interactions with the SM matter (nucleons) and photons. Adopting the notations of Ref. [25], the relevant interaction Lagrangian may be written down as

$$\mathcal{L}_{\rm int} = -i y_{ak} a(x) \bar{\psi}_k \gamma_5 \psi_k + \frac{1}{4} g_{a\gamma\gamma} a(x) F_{\mu\nu}\tilde{F}^{\mu\nu} + \ldots,$$

where $y_{ak} = C_k m_k/f_a$ is the Yukawa coupling of the axion $a(x)$ to the fermion (nucleon) species ψ_k of mass m_k, $g_{a\gamma\gamma} = C_\gamma \alpha_{\rm em}/(2\pi f_a)$ is the axion-photon-photon coupling, and $C_{k,\gamma}$ are model-dependent dimensionless parameters which are usually, but not necessarily, of order one. Notice that there is no bare mass for the Majorana fermion ψ in our case, which the pseudoscalar field $a(x)$ couples to. Therefore, the scale f_a cannot be defined, but one should instead constrain directly the dimensionless Yukawa coupling y_a in (31). A detailed phenomenological study may appear elsewhere. Of course, one can always assume that the axion moduli fields couple, in addition to the Majorana neutrinos, also to the SM matter, such as nucleons and photons as above, in which case the phenomenological analysis and constraints of Ref. [25] apply.

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