PHYSICAL REVIEW D **86,** 123012 (2012)

Cosmic inflation and big bang interpreted as explosions

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(Received 9 July 2012; published 19 December 2012)

It has become common understanding that the recession of galaxies and the corresponding redshift of light received from them can only be explained by an expansion of the space between them and us. In this paper, for the presently favored case of a universe without spatial curvature, it is shown that this interpretation is restricted to comoving coordinates. It is proven by construction that within the framework of general relativity other coordinates exist in relation to which these phenomena can be explained by a motion of the cosmic substrate across space, caused by an explosionlike big bang or by inflation preceding an almost big bang. At the place of an observer, this motion occurs without any spatial expansion. It is shown that in these "explosion coordinates" the usual redshift comes about by a Doppler shift and a subsequent gravitational shift. Making use of this interpretation, it can easily be understood why in comoving coordinates light rays of short spatial extension expand and thus constitute an exemption from the rule that small objects up to the size of the solar system or even galaxies do not participate in the expansion of the universe. It is also discussed how the two interpretations can be reconciled with each other.

DOI: 10.1103/PhysRevD.86.123012 PACS numbers: 04.20.Cv, 98.62.Py, 98.80.Bp, 98.80.Jk

I. INTRODUCTION

After its discovery, the redshift of light from far away galaxies was first explained in the context of special relativity and attributed to a Doppler shift caused by an outward flight of the galaxies in a preexisting invariable space. It was first observed by V. M. Slipher in the years since 1912 (see e.g., Ref. [1]), and in 1918 C. Wirtz interpreted it as being due to a general recessive motion of galaxies [2]. Use of the Doppler formula led to good agreement with observational results and revealed the validity of the Hubble law that, in fact, was first derived in 1927 from general relativity by G. Lemaître [3,4]. Only in 1929 it was formulated by Hubble in the context of astronomical observations [5]. Hubble's interpretation of the cosmological redshift as a Doppler shift became the generally accepted view, and in accordance with it the big bang was considered as a giant explosion.

The successful general relativistic formulation of the basic cosmological equations in comoving coordinates, first by A. Friedman [6] in 1922, later independently by Lemaître [3] in 1927, and 1936 supplemented by a rigorous derivation of the corresponding metric by H. P. Robertson [7] and A. G. Walker [8], brought a second interpretation into play. According to this the recession of galaxies from a distant observer is not caused by a motion relative to their spatial environment but rather by an expansion of the space between them and the observer. Correspondingly inflation and the big bang are sometimes denoted as space explosions.

For a long time the two interpretations coexisted because in those days the most distant observable galaxies gave rise to very small redshifts z only, and at small z the velocity- redshift relation is the same for both interpretations. However, at least since 1998 in the context of the Supernova Cosmology Project [9] more distant cosmic objects (supernovae) were observed and discrepancies between the two interpretations became evident, the emphasis has completely shifted to spatial expansion. By now even for small z the recession of galaxies is exclusively attributed to spatial expansion whereas the explosion perspective is completely ruled out. This has become the standard doctrine in research articles (see e.g., Refs. [10,11]), in textbooks (see e.g., Refs. [12–15]), and in Wikipedia [16], the interpretation as an explosion sometimes being denoted as a *popular* or *common misconception* (see e.g., Refs. [10,14,17,18] or Ref. [15], page 28). In the world wide web a multitude of contributions arguing against the interpretation as an explosion can be found (see e.g., Refs. [19–25]).

In 1934, W. H. McCrea and E. A. Milne [26] showed that the Friedmann-Lemaître equations describing the dynamics of the cosmic scale factor $a(t)$ in general relativity follow in exactly the same form from Newton's laws of motion (more precisely from the corresponding fluid equations by Euler) and of gravity. In this so-called Newtonian cosmology the galaxies are located at the positions $\vec{r}(t) = [a(t)/a(t_0)]\vec{r}(t_0)$ where $\vec{r}$ is the radius vector in a Euclidean space and $\vec{r} = 0$ is our position in the universe, and they move with the velocities $\vec{v} = H(t)\vec{r}$ (where $H(t) = \dot{a}(t)/a(t)$) relative to a preexisting and invariable space. Near the origin the velocities of galaxies and the gravitational field produced by them are so small that the Newtonian equations of motion should asymptotically coincide with equations obtainable from general relativity. This suggests that also in general relativity there should exist coordinates in which the galaxies are moving across space.

In this paper, for the presently favored case of a universe without spatial curvature, it will be shown that the seemingly contradictory interpretations, motion relative to an invariable space and recession due to expansion of space, are both possible and not in contradiction with each other. It will become apparent that they are relative views restricted to special sets of coordinate systems which are related to each other by simple transformations.

It is clear that in the transition from one coordinate system to another also the interpretation of physical phenomena can change. Since in general relativity a huge variety of different coordinate systems is available, it may at first glance appear almost trivial to find a coordinate system which allows for a prescribed interpretation-motion across space instead of expansion of space in our case. There are, however, several restrictions making things less trivial. In order to remain within the framework of general relativity, the signature of the metric of all admissible coordinate systems must be the same.[1] In addition, conditions for excluding radial expansion must be imposed. This will lead us to a set of differential equations to be solved under the observance of specific boundary conditions. It is not obvious that solutions of this problems exist, and even when they exist, it is still not granted that the corresponding metric has the appropriate signature. A simple example may demonstrate the problem.

In Newtonian mechanics one can go from the Cartesian coordinates of an inertial system to any other coordinates and still remain within the framework of the theory, provided that pseudoforces are admitted. Specifically one can go to a rigidly rotating coordinate system. The fact that beyond a certain radius this system rotates with superluminal velocity poses no problem in Newtonian mechanics. In general relativity, however, rigidly rotating coordinates are not admissible because in the superluminal region the metric assumes the wrong signature. A second example is presented in footnote 2.

The main body of this paper consists of proving the existence of explosion coordinates. This is achieved in a constructive way, deriving them by means of a transformation from Robertson Walker coordinates. In Sec. II the determining differential equations and boundary conditions are specified, formal solutions are derived, and properties of them as well as conclusions following from them are discussed. In Sec. III explicit solutions for the most important special cases of cosmic evolution are deduced, thus bringing to an end the envisaged proof of existence. As already mentioned the interpretation of physical phenomena like the redshift of light from far away galaxies must be adjusted. It will be shown that the latter can exactly be described by a Doppler shift and a subsequent gravitational shift. In this context it will be discussed why and how a spontaneous Doppler shift at the place of emission can be reconciled with the fact that in comoving coordinates "... the increase of wavelength from emission to absorption of light does not depend on the rate of change of [the cosmic scale factor] $a(t)$ at the times of emission or absorption, but on the increase of $a(t)$ in the whole period from emission to absorption" [13]. In Sec. III it is also discussed to what extent the cosmic explosion provided by inflation or a big bang can be compared with usual explosions. Section IV deals with some applications and extensions.

It is clear that the purpose of this paper cannot be the replacement of the usual approach with Robertson-Walker coordinates and the corresponding interpretation (also, see Sec. V), especially since it starts off with solutions obtained in them. Rather the paper is meant to provide a supplementation which might even turn out to be useful in specific cases.

II. SYSTEMS WITHOUT RADIAL EXPANSION-GENERAL THEORY

In the case of a spatially uncurved universe the square of the line element in (comoving) Robertson-Walker coordinates is

$$ds^2 = c^2 dt^2 - a^2(t)(dr^2 + r^2 d\Omega) \quad \text{with} \qquad (1)$$
$$d\Omega = d\vartheta^2 + \sin^2\vartheta d\varphi^2.$$

[r is dimensionless and $a(t)$ has the dimension of a length.] The radial expansion of the universe is expressed by the time-dependence of length elements, e.g., $dl_r = a(t)dr$ in radial direction. The underlying coordinates t, r, ϑ, and φ are called *expansion coordinates* in this paper. We are looking for a transformation $t, r, \vartheta, \varphi \to \tau, \rho, \vartheta, \varphi$ to new coordinates τ, ρ, ϑ and φ, the *explosion coordinates*, for which the square of the line element is given by

$$ds^2 = c^2 g_{00}(\rho, \tau) d\tau^2 - d\rho^2 + g_\Omega(\rho, \tau) d\Omega. \qquad (2)$$

[In contrast to r, ρ has the dimension of length.] The radial length element $dl_\rho = d\rho$ is time independent whence in explosion coordinates there is no radial expansion. As we shall see instead of this there is a radial motion. Since ϑ and φ remain unchanged, we have

$$t = t(\rho, \tau), \qquad r = r(\rho, \tau). \qquad (3)$$

Other than in Newtonian cosmology the space is not completely invariable but may involve angular expansion because g_Ω can be time dependent. A corresponding expansion of volumes will be discussed at the end of this section.

A. Derivation of explosion coordinates and corresponding metric

From Eqs. (3) we get

$$dt = t_\rho d\rho + t_\tau d\tau, \qquad dr = r_\rho d\rho + r_\tau d\tau,$$

where t_ρ denotes the partial derivative of the function $t(\rho, \tau)$ with respect to ρ etc. With this, from Eq. (1) we obtain

[1] In this paper SI units are used whence the speed of light is c. Furthermore, the signature of the metric is $(+, -, -, -)$.

$$ds^2 = (c^2 t_\rho^2 - a^2 r_\rho^2) d\rho^2 + (c^2 t_\tau^2 - a^2 r_\tau^2) d\tau^2 + 2(c^2 t_\rho t_\tau - a^2 r_\rho r_\tau) d\tau^2 + \cdots .$$

In order for this to assume the form of Eq. (2), the equations

$$c^2 t_\rho t_\tau = a^2 r_\rho r_\tau, \qquad c^2 t_\rho^2 - a^2 r_\rho^2 = -1, \qquad g_{00} = t_\tau^2 - \frac{a^2 r_\tau^2}{c^2}, \qquad g_\Omega = -a^2 r^2 \tag{4}$$

must be fulfilled, or, equivalently

$$r_\rho = \pm \frac{\sqrt{1 + c^2 t_\rho^2}}{a(t)}, \quad r_\tau = \pm \frac{c^2 t_\rho t_\tau}{a(t)\sqrt{1 + c^2 t_\rho^2}}. \tag{5}$$

In order to make the results in expansion and explosion coordinates comparable we impose the following conditions: 1. the origins of the two coordinate systems permanently coincide, and 2. the coordinate times at the origins are the same,

$$1.\ \rho = 0 \quad \text{for } r = 0, \qquad 2.\ t = \tau \quad \text{at} \quad \rho = 0. \tag{6}$$

Equations (5) are two equations only for the determination of the four derivatives r_ρ, r_τ, t_ρ, and t_τ, since as soon as the transformation functions (3) have been determined from Eqs. (5), the last two of the Eqs. (4) only serve for the evaluation of g_{00} and g_Ω. For Eqs. (5) to have solutions, the integrability condition $r_{\rho\tau} = r_{\tau\rho}$ must be satisfied. The evaluation of it results in a nonlinear second order differential equation for the function $t(\rho, \tau)$,

$$a(t) t_{\rho\rho} + \frac{\dot{a}(t)}{c^2} (c^2 t_\rho^2 + 1) = 0. \tag{7}$$

Once a solution $t(\rho, \tau)$ is found, $r(\rho, \tau)$ can be determined from Eqs. (5). After multiplication with $2at_\rho$ Eq. (7) becomes

$$2a^2(t) t_\rho t_{\rho\rho} + 2a(t)\dot{a}(t) t_\rho^3 + \frac{2a(t)\dot{a}(t)}{c^2} t_\rho = \frac{\partial}{\partial \rho}[a^2(t)(t_\rho^2 + 1/c^2)] = 0.$$

This equation can be integrated once to yield

$$t_\rho = \pm \frac{1}{c} \sqrt{F(\tau)/a^2(t) - 1} \tag{8}$$

and once more to yield

$$\rho = G(\tau) \pm c \int_0^t \frac{dt'}{\sqrt{F(\tau)/a^2(t') - 1}} \Bigg|_{\tau=\text{const}}, \tag{9}$$

where $F(\tau)$ and $G(\tau)$ are integration functions. [Note that the algebraic sign in Eqs. (8) and (9) can be chosen independently from that in the Eqs. (5).] The latter can be chosen in such a way that additional conditions are satisfied. Using the second of the conditions (6), from Eq. (9) we get

$$G(\tau) = \mp c \int_0^\tau \frac{dt'}{\sqrt{F(\tau)/a^2(t') - 1}} \Bigg|_{\tau=\text{const}}. \tag{10}$$

Inserting Eq. (8) and the second of the Eqs. (5) in the third of the Eqs. (4) yields

$$g_{00} = \frac{t_\tau^2}{1 + c^2 t_\rho^2} = \frac{a^2(t) t_\tau^2}{F(\tau)}. \tag{11}$$

Since under observance of both conditions (6) the origins of the two coordinate systems permanently coincide and since $t = \tau$ there, also the metric times must be the same there. (Although this statement needs no separate proof, it will later be validated as a test in special cases.) As a result of this from Eqs. (1) and (2) we get

$$g_{00}(\rho = 0, \tau) \equiv 1. \tag{12}$$

With this and $t_\tau = 1$ for $\rho = 0$ since $t = \tau$ for $\rho = 0$, from Eq. (11) we finally obtain

$$F(\tau) = a^2(\tau). \tag{13}$$

Here, $a(\tau)$ stands for the function $a(t)$ with t replaced by τ. With this and Eq. (10), Eq. (9) becomes

$$\rho = \pm c \int_\tau^t \frac{dt'}{\sqrt{a^2(\tau)/a^2(t') - 1}} \Bigg|_{\tau=\text{const}}. \tag{14}$$

This equation implicitly determines the function $t = t(\rho, \tau)$. Inserting Eqs. (13) and (8) in the first of the Eqs. (5) yields

$$r_\rho = \pm \frac{a(\tau)}{a^2(t)} \quad \text{and} \quad r = R(\tau) \pm a(\tau) \int_0^\rho \frac{d\rho'}{a^2(t(\rho', \tau))} \Bigg|_{\tau=\text{const}}, \tag{15}$$

where $R(\tau)$ is an arbitrary integration function. From the first of the conditions (6) it follows that $R(\tau) = 0$, so finally we get

$$r(\rho, \tau) = a(\tau) \int_0^\rho \frac{d\rho'}{a^2(t(\rho', \tau))} \Bigg|_{\tau=\text{const}}. \tag{16}$$

(The minus sign could be excluded since we must have $r > 0$.)

The Eqs. (14) and (16) determine the solutions of the Eqs. (4) for the transformation functions $r(\rho, \tau)$ and $t(\rho, \tau)$ only implicitly and require the knowledge of a solution $a(t)$ of the Friedman-Lemaître equations. Since according to Eq. (14) their range of validity is restricted to $a(\tau) \geq a(t)$, it is at this point not yet clear whether for each $a(t)$ also explicit solutions exist and in addition lead to the right signature of the metric. Therefore, the proof of existence of solutions is not yet complete. Their determination will in general require numerical methods. However, in special cases also analytical solutions can be obtained. In Sec. III the full solution for two relatively simple but nevertheless representative cases will be derived.

B. Flight velocity of galaxies

In explosion coordinates, the radial position of a galaxy [or an element of the cosmic substrate] is given by $r(\rho, \tau) = \text{const}$, whence $\rho = \rho(\tau)$ and

$$\dot{\rho}(\tau) = -\frac{r_\tau}{r_\rho} = \pm\frac{ca^2(t)t_\tau\sqrt{a^2(\tau)/a^2(t) - 1}}{a^2(\tau)}. \quad (17)$$

[For the last step Eqs. (5), (8), and (13) were used.] This means that the galaxies are moving across space. $\dot{\rho}(\tau)$ is the coordinate velocity, and from it the physical velocity v is obtained according to

$$v = \frac{\sqrt{g_{\rho\rho}}d\rho}{\sqrt{g_{00}}d\tau} = \frac{d\rho}{\sqrt{g_{00}}d\tau} = \frac{\dot{\rho}(\tau)a(\tau)}{a(t)t_\tau}, \quad (18)$$

where at last Eqs. (11) and (13) were used. Insertion of Eq. (17) finally yields

$$v = c\sqrt{1 - a^2(t)/a^2(\tau)}. \quad (19)$$

Thereby a freely disposable sign was chosen such that v is positive.

C. Redshift of light from far away galaxies

(1) In expansion coordinates, the redshift z of light emitted from a galaxy at the radius r_{em} at the time t_{em} and received at the origin $r = 0$ at the time t_0 is given by

$$z + 1 = \frac{\lambda_0}{\lambda_{\text{em}}} = \frac{a(t_0)}{a(t_{\text{em}})}, \quad (20)$$

where λ_{em} or λ_0 is the wavelength of the light at emission or reception, respectively. The radial propagation of light is described by $cdt = a(t)dr$ whence

$$r_{\text{em}} = c\int_{t_{\text{em}}}^{t_0}\frac{dt'}{a(t')}. \quad (21)$$

With this equation the time t_0 of reception can be expressed in terms of the time t_{em} and the location r_{em} of emission.

(2) In explosion coordinates for an observer fixed at the distance $\rho = \rho_{\text{em}}$ from the origin, the light emitted from a galaxy flying past him at the radial velocity v undergoes the longitudinal Doppler-shift

$$\frac{\lambda_\rho}{\lambda_{\text{em}}} = \left(\frac{1 + v/c}{1 - v/c}\right)^{1/2} = \frac{1 + v/c}{\sqrt{1 - v^2/c^2}}. \quad (22)$$

The physical velocity of Eq. (19) is the one that must be inserted in the Doppler formula, and because unlike the recessional velocity $\dot{a}(t)r$ in expansion coordinates it cannot exceed the speed of light, it is always in the range of validity of this formula. With it Eq. (22) becomes

$$\frac{\lambda_\rho}{\lambda_{\text{em}}} = \frac{a(\tau)}{a(t)}\left(1 + \sqrt{1 - a^2(t)/a^2(\tau)}\right), \quad (23)$$

where $\tau = \tau_{\text{em}}$ or $t = t(\rho, \tau) = t_{\text{em}}$ are the times of emission in explosion or expansion coordinates, respectively. On its way from $\rho = \rho_{\text{em}}$ to the observer at $\rho = 0$ the Doppler-shifted light experiences an additional shift in the gravitational field acting in the system of explosion coordinates. For simplicity we assume that this field is time independent as is true for the special cases to be studied below. Then, according to general relativity, the ratio of the wavelength λ_0 observed at $\rho = 0$ and $\tau = \tau_0$ to the wavelength λ_ρ of the Doppler-shifted light at $\rho_{\text{em}} \to \rho$ and $\tau_{\text{em}} \to \tau$ is

$$\frac{\lambda_0}{\lambda_\rho} = \sqrt{\frac{g_{00}(\rho = 0, \tau = \tau_0)}{g_{00}(\rho, \tau)}}. \quad (24)$$

Making use of the Eqs. (11)–(13), from this we get

$$\frac{\lambda_0}{\lambda_\rho} = \frac{1}{\sqrt{g_{00}(\rho, \tau)}} = \frac{a(\tau)}{a(t)t_\tau}. \quad (25)$$

Combining Eqs. (23) and (25) we finally obtain

$$\frac{\lambda_0}{\lambda_{\text{em}}} = \frac{a^2(\tau)}{a^2(t)t_\tau}\left(1 + \sqrt{1 - a^2(t)/a^2(\tau)}\right). \quad (26)$$

Since observers at the origin use the same metric time in both coordinate systems, they also measure the same frequency or redshift of light that is emitted by far away galaxies. Therefore the result (26) must be the same as the result provided by Eqs. (20) and (21), and although not evident this coincidence does not need proof. Nevertheless it is illuminating to see it verified in the special cases considered in Sec. III.

D. Volume expansion rate

One could be tempted to assume that the elimination of radial expansion in explosion coordinates leads to increased angular expansion. Indeed the structure of $g_\Omega(\rho, \tau)$ in explosion coordinates [see Eq. (32) for example] suggests that there could still be a volume expansion due to a time dependence of azimuthal distances. In order to clarify this issue we calculate an expansion rate that involves both radial and angular properties, the volume expansion rate $E = (d\Delta V/dT)/\Delta V$. In this ΔV is the volume of a spherical shell with infinitesimally small thickness $\Delta\rho$, and $T = \sqrt{g_{00}}\tau$ is the time measured on clocks in the system of explosion coordinates whence

$$E = \frac{1}{\Delta V}\frac{d\Delta V}{\sqrt{g_{00}}d\tau}. \quad (27)$$

According to Eq. (2) and the last of the Eqs. (4)

$$\Delta V = 4\pi g_\Omega \Delta\rho = 4\pi a^2(t) r^2 \Delta\rho,$$

with $t = t(\rho, \tau)$ and $r = r(\rho, \tau)$. From this, for fixed ρ we obtain

$$\frac{1}{\Delta V}\frac{d\Delta V}{d\tau} = \frac{2[\dot{a}(t) r t_\tau + a(t) r_\tau]}{a(t) r},$$

and using Eqs. (11) and (13) we finally get

$$E = \frac{2a(\tau)}{a(t)}\left(\frac{\dot{a}(t)}{a(t)} + \frac{r_\tau}{r t_\tau}\right). \tag{28}$$

For obtaining the expansion rate E at $\rho = 0$ we consider the volume

$$V = 4\pi \int_0^\rho a^2(t) r^2 d\rho'$$

of a sphere of radius ρ. Expanding around $\rho = 0$ we have

$$r(\rho, \tau) = r(0, \tau) + r_\rho(0, \tau)\rho + \cdots = r_\rho(0, \tau)\rho + \cdots$$

because $r(0, \tau) = 0$ according to the first of the conditions (6). Furthermore, according to the first of the Eqs. (15) we have

$$r_\rho(0, \tau) = \frac{a(\tau)}{a^2(t)}\bigg|_{\rho=0} = \frac{1}{a(\tau)}$$

since $t = \tau$ for $\rho = 0$ according to the second of the conditions (6). In consequence

$$r(\rho, \tau) = \frac{\rho}{a(\tau)} + \mathcal{O}(\rho^2).$$

Finally, we get

$$a(t(\rho, \tau)) = a(t(0, \tau)) + \mathcal{O}(\rho) = a(\tau) + \mathcal{O}(\rho).$$

Altogether we have

$$V = 4\pi \int_0^\rho [\rho'^2 + \mathcal{O}(\rho'^3)] d\rho' = \frac{4\pi\rho^3}{3} + \mathcal{O}(\rho^4).$$

From this we get $dV/d\tau|_{\rho=0} = 0$ and $E|_{\rho=0} = 0$, *in explosion coordinates at the origin there is no volume expansion*. Since in an expanding universe each point can be chosen as the origin, in explosion coordinates at each point for a local observer there is no expansion. The expansion around points at some distance from the origin, described by Eq. (28), can therefore be regarded as virtual.

III. SYSTEMS WITHOUT RADIAL EXPANSION-SPECIAL CASES

For simplicity our analysis of special cases will be restricted to situations in which the gravitational field is static in explosion coordinates. Since Newtonian cosmology is a very good approximation in the neighborhood of the origin $r = 0$, the corresponding conditions can be determined with it. According to the Hubble law $v = H(t) r$ the accelerating (or decelerating) field, equaling the gravitational field (an inflation field being included), is

$$\dot{v}(t) = \dot{H}(t) r + H\dot{r}(t) = (\dot{H}(t) + H^2) r.$$

It becomes time independent for

$$\dot{H}(t) + H^2 = \begin{cases} 0 \\ A^2 = \text{const} \\ -A^2 = \text{const} \end{cases}.$$

In the first case, we obtain

$$H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{1}{t} \rightarrow a(t) = \alpha t,$$

where $\alpha = \text{const}$, i.e., the universe is expanding with constant velocity. In the second case two solutions $H(t)$ exist, firstly $H = A$, i.e., a universe with constant expansion rate, and secondly

$$H(t) = A\coth(At) \rightarrow a(t) = a_0 \sinh(At).$$

In the third case we obtain

$$H(t) = A\tanh(At) \rightarrow a(t) = a_0 \cosh(At).$$

In the following we shall only consider the cases of constant expansion velocity and constant expansion rate.

A. Universe with constant expansion velocity

For a long time the recent evolution of the universe up to the present state was considered to be best represented by a model that assumes the domination of matter and exhibits a decelerated expansion with $a(t) \sim t^{3/2}$. Only shortly before the end of the last century it was detected that the expansion is rather slightly accelerated [9]. The case of a universe without acceleration or deceleration, $a(t) = \alpha t$ with constant α, lies in between, represents a fairly good approximation-which is best at the time of the transition from decelerated to accelerated expansion-and enables a full analytic solution of our problem.

1. Derivation of explosion coordinates

With $a(t) = \alpha t$ the integral in Eq. (14) can readily be evaluated and is given by

$$\int_\tau^t \frac{dt'}{\sqrt{a^2(\tau)/a^2(t') - 1}} = \frac{1}{2}\int_\tau^t \frac{dt'^2}{\sqrt{\tau^2 - t'^2}} = -\sqrt{\tau^2 - t^2}.$$

With this, resolving Eq. (14) with respect to t and choosing a sign such that $t \geq 0$, we obtain

$$t = \frac{1}{c}\sqrt{c^2\tau^2 - \rho^2}. \tag{29}$$

With $a(t) = \alpha t$ and this result Eq. (16) becomes

$$r = \alpha\tau \int_0^\rho \frac{d\rho'}{\alpha^2 t'^2(\rho',\tau)} = \frac{\tau c^2}{\alpha} \int_0^\rho \frac{d\rho'}{c^2\tau^2 - \rho'^2} = \frac{c}{2\alpha} \ln\frac{c\tau+\rho}{c\tau-\rho}. \qquad (30)$$

The transformation $t, r, \vartheta, \varphi \to \tau, \rho, \vartheta, \varphi$, providing a line element of the form (2), is given by Eqs. (29) and (30), and the inverse transformation is

$$\tau = t\cosh\frac{\alpha r}{c}, \qquad \rho = ct\sinh\frac{\alpha r}{c}. \qquad (31)$$

Inserting Eqs. (29) and (30) in the third and fourth of the Eqs. (4) yields

$$g_{00} = 1, \qquad g_\Omega = -\frac{c^2\tau^2 - \rho^2}{4}\left(\ln\frac{c\tau+\rho}{c\tau-\rho}\right)^2. \qquad (32)$$

[In the derivation of the first equation, for the sake of brevity Eq. (12) was used. One can easily make sure that the same result is obtained without it.] Inserting the results (32) in Eq. (2) yields

$$ds^2 = c^2 d\tau^2 - d\rho^2 - \frac{c^2\tau^2-\rho^2}{4}\left(\ln\frac{c\tau+\rho}{c\tau-\rho}\right)^2 d\Omega. \qquad (33)$$

2. Flight velocity of galaxies and associated redshift

The flight velocity of a galaxy at $r = \text{const}$ can be obtained from Eqs. (19) and (29) with $a(t) = \alpha t$. Alternatively, from Eqs. (31) we immediately get

$$\rho = c\tau\tanh\frac{\alpha r}{c} \quad \text{and} \quad v = \frac{\rho}{\tau} = c\tanh\frac{\alpha r}{c}. \qquad (34)$$

(1) For expansion coordinates, with $a(t) = \alpha t$ the Eqs. (20) and (21) yield

$$\frac{\lambda_0}{\lambda_{em}} = \frac{t_0}{t_{em}} \quad \text{and} \quad r_{em} = \frac{c}{\alpha}\int_{t_{em}}^{t_0}\frac{dt}{t} = \frac{c}{\alpha}\ln\frac{t_0}{t_{em}}$$

whence

$$\frac{\lambda_0}{\lambda_{em}} = e^{\alpha r/c}. \qquad (35)$$

(2) In explosion coordinates from Eqs. (22) and (34) we get

$$\frac{\lambda_\rho}{\lambda_{em}} = \left[\frac{1+\tanh(\alpha r/c)}{1-\tanh(\alpha r/c)}\right]^{1/2} = e^{\alpha r/c}. \qquad (36)$$

According to the first of the Eqs. (32) there is no additional gravitational redshift of the light on its way from $\rho = \rho_{em}$ to $\rho = 0$, i.e. $\lambda_0/\lambda_\rho = 1$, and therefore we obtain

$$\frac{\lambda_0}{\lambda_{em}} = e^{\alpha r/c}, \qquad (37)$$

exactly the same result as in expansion coordinates, Eq. (35). It should be noted, however, that, following from Eqs. (31), (1), and (33), in the two coordinate systems the distance from the point of emission as well as the time of emission differ from each other.

(3) The case of constant expansion velocity is particularly suited for demonstrating the compatibility of the two interpretations at issue, because in explosion coordinates the redshift is completely due to the Doppler effect. According to Eqs. (34) the spatial grid of expansion coordinates r is moving relative to that of explosion coordinates ρ at a speed that increases with the distance from the origin. We now consider a light ray emitted at $r = r_{em}$ and directed toward the origin. From the viewpoint of the system S_{expl} of explosion coordinates, observers at rest in the system S_{expa} of expansion coordinates are moving relative to the emitting galaxy at $r = r_{em}$ at a velocity that increases with decreasing r. Therefore in S_{expl} they appear to observe a Doppler shift that continuously increases as r approaches zero. This way an observer in S_{expl} can readily understand why for observers in S_{expa} the redshift appears to be accumulated on the way of light from emission to reception.

3. View of an external observer

According to the theory of chaotic inflation [27,28], in the case $k = 0$ (no spatial curvature) considered in this paper a Friedman-Lemaître universe could be finite and embedded in an infinite super-universe. This so-called multiverse is filled with a "foam" of fluctuating quantum fields from which numerous universes of all kinds can emerge by inflation or have already done so. It is interesting to find out how the situation, so far considered from an internal observer, would be seen by an external observer located at some distance from our universe.

Considerably simplifying the above model we consider a toy model of the universe in which a finite spherical section of a Friedman-Lemaître universe is surrounded by a sufficiently large bubble of true vacuum, uninfiltrated by any externally generated gravitational fields. In contrast to the situation outside a collapsing star, due to Birkhoff's theorem (time-independence of all metric coefficients in the vacuum surrounding a spherically symmetric mass or energy distribution, see Ref. [29] or e.g., Ref. [30]) the metric of the external space is not of the Schwarzschild type but must be pseudo-Euclidean. The reason is that it has been of this kind before the universe emerged by a creation out of nothing, a process tolerated by general relativity when the big bang (or, rather, an almost big bang) is preceded and triggered by an inflation field (dark energy, inflaton). Therefore, in polar spatial coordinates ρ, ϑ, and φ the corresponding line element is

$$ds^2 = c^2 d\tau^2 - d\rho^2 - \rho^2 d\Omega. \qquad (38)$$

The case $a(t) = \alpha t$ represents a fairly good approximation to the state of our universe from the end of the period of matter dominance until now, i.e., for the time interval $t_0/2 \leq t \leq t_0$ where t_0 is the present age of the universe. In this time interval the pressure can be neglected and matter can be treated as pressureless dust. The only forces acting on the matter elements are gravitational forces [including the action of dark energy], and we can therefore assume that in comoving coordinates the boundary of the universe [which must lie well beyond our horizon] is at rest, $r = R =$ const, and is constituted by matter elements moving at the velocity (34) with $r = R$.

Since according to Eqs. (33) and (38) the external and internal coordinates are both not only Gaussian normal but also employ the same radial metric,[2] it is rather obvious that the velocity of the boundary seen by an external observer is the same as that seen by an internal observer. A more physical proof of this result is the following. We consider the propagation of light emitted with frequency ν_0 at the origin and directed toward an observer outside the universe, at rest at $\rho = \rho_{\rm obs} =$ const. According to the Eqs. (33) and (38), on its whole way inside and outside the universe the propagation of light is described by the equation $\dot\rho(\tau) = c$ or $\rho = c(\tau - \tau_0)$, respectively. In consequence, the time between the arrival of successive wave crests at the place of the observer in "outer space" equals the time between their emission at the origin whence $\nu_{\rm obs} = \nu_0$. An observer comoving with the boundary of the universe sees the origin receding from him at the velocity v given by the second of Eqs. (34). He therefore observes a redshift of the light from the origin, and according to Eq. (22) he measures the frequency

$$\nu = \nu_0\left(\frac{1 - v/c}{1 + v/c}\right)^{1/2}.$$

The light leaves him with the same frequency as it had on its arrival. Since the boundary moves toward the external observer, the latter will observe a blueshift of the light from the boundary and measure the frequency

$$\nu_{\rm obs} = \nu\left(\frac{1 + u/c}{1 - u/c}\right)^{1/2} = \nu_0\left(\frac{1 - v/c}{1 + v/c}\right)^{1/2}\left(\frac{1 + u/c}{1 - u/c}\right)^{1/2},$$

where u is the velocity of the boundary in his (external) coordinate frame. As we said above we must have $\nu_{\rm obs} = \nu_0$, and with this the last equation yields

$$u = v = c \tanh\frac{\alpha R}{c}, \qquad (39)$$

where at last Eq. (34) and $r = R$ was used.

[2]For a smooth connection with the external coordinates, as internal coordinates also standard coordinates ($ds^2 = g_{00}(\rho, \tau)c^2 d\tau^2 + g_{\rho\rho}(\rho, \tau)d\rho^2 + \rho^2 d\Omega$) could be envisaged. However, for $a(t) \sim t$, at some critical ρ a coordinate singularity occurs, and for all $\rho > \rho_{\rm crit}$ standard coordinates do not exist. This provides another example that compatibility with the requirements of general relativity may prevent the existence of coordinates with specified properties.

4. Numerical values of characteristic velocities

(1) In the comoving coordinates of the Robertson-Walker metric, according to Hubble's law the present recessional velocity of the boundary of the observable universe is $v = H_0 d_0$ where $d_0 = a(t_0) r_{bo} \approx 3.5ct_0$ is its metric distance from us and t_0 the age of the universe. With $H_0 = 0.7 \cdot 3.24 \cdot 10^{-18}\ {\rm s}^{-1}$ it becomes $v_{\rm rec} = 3.45c$. A reasonable assumption about the outer boundary R of the universe is that at present it is two times as far away from us as the present boundary of the observable universe, i.e., $d \approx 7ct_0$. (This way inhomogeneities propagating from outer space into the universe cannot have spoiled the homogeneity and isotropy inside the observable universe, see Ref. [15], page 231). The corresponding recessional velocity is

$$v_{\rm rec} = 6.9c. \qquad (40)$$

(2) In explosion coordinates the velocity at which the present boundary of the observable universe moves away from us is given by the second of the Eqs. (34). Following from $a(t) = \alpha t$ we have $\alpha r_{\rm bo}/c = a(t_0) r_{\rm bo}/(ct_0) = d_0/(ct_0) = 3.5$, and with this we obtain $v_{\rm bo} = 0.998c$. At the outer boundary of the universe we have $\alpha R/c = d/(ct_0) = 7$, and according to Eq. (39) the velocity at which it moves is

$$u = v = 0.999998c. \qquad (41)$$

B. Universe with constant expansion rate

We consider a second case in which a full analytic solution for explosion coordinates can be obtained, namely, $a(t) = \alpha e^{Ht}$ with constant expansion rate H. In this case which describes an *inflationary expansion*, in addition to the Doppler shift there is a gravitational blueshift of the light from far away galaxies.

1. Derivation of explosion coordinates

In the present case, instead of using Eqs. (14) and (16) it is easier to go back to Eq. (7), i.e.,

$$t_{\rho\rho} + Ht_\rho = -H/c^2.$$

After solving the homogeneous equation, the solution of the inhomogeneous equation can be obtained by variation of constants and is given by

$$t(\rho, \tau) = \tau + \frac{1}{H}\ln\cos w \quad \text{with} \quad w = \frac{H\rho}{c}. \qquad (42)$$

An integration function was chosen such as to satisfy the second of the conditions (6). With this result and $a(t) = \alpha e^{Ht}$ from Eq. (16) we get

$$r(\rho, \tau) = \frac{c\mathrm{e}^{-H\tau}}{\alpha H}\tan w. \tag{43}$$

Inversion of the functions $r(\rho, \tau)$ and $t(\rho, \tau)$ given in Eqs. (42) and (43) yields

$$\rho = \frac{c}{H}\arcsin\left(\frac{\alpha H r}{c}\mathrm{e}^{Ht}\right), \qquad \tau = t - \frac{1}{2H}\ln\left(1 - \frac{\alpha H^2 r^2}{c^2}\mathrm{e}^{2Ht}\right). \tag{44}$$

Inserting $a(t) = \alpha \mathrm{e}^{Ht}$ and Eqs. (42) and (43) in Eq. (11) and the fourth of the Eqs. (4) we obtain

$$g_{00} = \cos^2 w, \qquad g_\Omega = -\frac{c^2}{H^2}\sin^2 w. \tag{45}$$

From the first equation it follows that Eq. (12) is satisfied as it should according to the general theory.

2. Flight velocity of the cosmic substrate and associated redshift

From the Eqs. (19) and (42), and $a(t) = \alpha \mathrm{e}^{Ht}$ we obtain the flight velocity

$$v = c\sin w. \tag{46}$$

(1) In the present case Eqs. (20) and (21) become

$$\frac{\lambda_0}{\lambda_{\mathrm{em}}} = \mathrm{e}^{H(t_0 - t_{\mathrm{em}})} \quad \text{with}$$
$$r_{\mathrm{em}} = \frac{c}{\alpha}\int_{t_{\mathrm{em}}}^{t_0}\mathrm{e}^{-Ht}dt = \frac{c}{\alpha H}(\mathrm{e}^{-Ht_{\mathrm{em}}} - \mathrm{e}^{-Ht_0})$$

and in combination yield

$$\frac{\lambda_0}{\lambda_{\mathrm{em}}} = \left(1 - \frac{\alpha H r_{\mathrm{em}}}{c}\mathrm{e}^{Ht_{\mathrm{em}}}\right)^{-1}.$$

From this, with $r_{\mathrm{em}} \to r$, $t_{\mathrm{em}} \to t$ and $\alpha Hr/c = \mathrm{e}^{-H\tau}\tan w$ according to Eq. (43) and using Eq. (42), for expansion coordinates we obtain the result

$$\frac{\lambda_0}{\lambda_{\mathrm{em}}} = \frac{1}{1 - \sin w}. \tag{47}$$

(2) In the system of explosion coordinates, from Eqs. (22) and (46) we obtain

$$\frac{\lambda_\rho}{\lambda_{\mathrm{em}}} = \frac{1 + \sin w}{\cos w}, \tag{48}$$

and from Eq. (24) and the first of the Eqs. (45) we get

$$\frac{\lambda_0}{\lambda_\rho} = \sqrt{\frac{\cos^2 0}{\cos^2 w}} = \frac{1}{\cos w}. \tag{49}$$

Combing the results (48) and (49) we obtain the same result (47) as in the system of expansion coordinates.

3. Volume expansion rate

According to Eq. (45) the volume of a spherical shell of small thickness $\Delta\rho$ is given by

$$\Delta V(\rho, \tau) = 4\pi\frac{c^2}{H^2}\Delta\rho\sin^2 w.$$

Because it is time independent, the expansion rate $E = d\Delta V/(\Delta V\sqrt{g_{00}}d\tau)$ vanishes everywhere. This means that in the present case, exponential inflationary evolution, the cosmic substrate is moving relative to a completely invariable space.

4. Acceleration of the cosmic substrate and comparison with ordinary explosions

From the first of the Eqs. (44) and from Eq. (46) with

$$\left.\frac{\partial\rho}{\partial t}\right|_r = c\tan\frac{H\rho}{c} = c\tan w$$

for simplicity in terms of the time t we obtain

$$\dot v(t) = c\cos w\left.\frac{\partial w}{\partial t}\right|_r = H\cos w\left.\frac{\partial\rho}{\partial t}\right|_r = cH\sin w = Hv.$$

From this there follows an exponentially growing acceleration of the cosmic substrate along its radial trajectories. Accordingly the inflation preceding an almost big bang describes the actual explosion, whereas the subsequent dynamical processes are basically consequences of it. (In the case of a big bang without inflation the dynamics of explosion is compressed into a singular instant.) Like in ordinary explosions the accelerated substance–here dark energy, there, e.g., a chemical explosive like dynamite or gunpowder-simultaneously is the blasting agent driving the explosion, and for an external observer it moves outward filling a previously empty space. What differs from ordinary explosions is that there are no shattered fragments of a casing, and there is no precursive shock wave. Instead there will be a precursive front of the inflation field, a weak discontinuity, which propagates at the speed of light.

C. Hubble Law

In expansion coordinates the distance from the origin of a galaxy or an element of the cosmic substrate is $d = a(t)r$. From this follows the Hubble law

$$\dot d(t) = Hd(t) \quad \text{with} \quad H = \frac{\dot a(t)}{a(t)}. \tag{50}$$

In the explosion coordinates belonging to the case of constant expansion velocity, $a(t) = \alpha t$, according to the second of the Eqs. (34) we can write the flight velocity of galaxies etc., in the forms

$$v = H_{\mathrm{expl}}\rho \quad \text{with} \quad H_{\mathrm{expl}} = \frac{1}{\tau}. \tag{51}$$

Formally, this is identical with the Hubble law

$$\dot{d}(t) = Hd(t) \quad \text{with} \quad H = \frac{1}{t}$$

obtained in expansion coordinates. Physically the two laws are different in that the underlying lengths as well as proper times are measured differently.

Let us now consider the case of constant expansion rate H. In expansion coordinates Eq. (50) with $H = \text{const}$ holds. In explosion coordinates according to Eq. (46) we have

$$v = H_{\text{expl}}\rho \quad \text{with}$$
$$H_{\text{expl}} = \frac{c\sin(H\rho/c)}{\rho} = H\left(1 - \frac{H^2\rho^2}{6c^2} + \cdots\right), \quad (52)$$

where at last H_{expl} was expanded with respect to $H\rho/c$. In contrast to the result for expansion coordinates, H_{expl} is weakly space dependent. This is not in contradiction to the result obtained for expansion coordinates or from observations although a space-dependent Hubble parameter appears unusual. The reason is that measurements concerning far away objects involve the application of a theory related to the specific coordinate system in use, and this theory must appropriately be adjusted in the transition to explosion coordinates. (For example, the luminosity distance must be redefined.)

IV. APPLICATIONS AND EXTENSIONS

(1) In more general cases than the ones considered in the last section g_{00} will depend on ρ and τ. Therefore, in addition to the Doppler shift a time-dependent gravitational redshift of light will occur. While the Doppler shift can still be calculated from the Doppler formula (22), no generally valid formula for the shift effect of time-dependent gravitation is known (at least to the author of this paper). However, for all cases covered by Eq. (2) the latter can be derived by making use of the equivalence of the two different conceptions of galaxy recession. Combining Eqs. (20), (21), and (23), with $t_{\text{em}} \to t$ as in Eq. (23) we obtain

$$\frac{\lambda_0}{\lambda_\rho} = \frac{\lambda_0}{\lambda_{\text{em}}}\frac{\lambda_{\text{em}}}{\lambda_\rho} = \frac{a(t_0)}{a(\tau)(1 + \sqrt{1 - a^2(t)/a^2(\tau)})}$$
$$\text{with} \quad r = r_{\text{em}} = c\int_{t_{\text{em}}}^{t_0} \frac{dt'}{a(t')}. \quad (53)$$

(2) Objects that are small in relation to cosmic distances like atoms, the solar system or even galaxies do not participate in the spatial expansion of the universe associated with usual theory. In expansion coordinates this phenomenon is not easily comprehensible. A first important proof of it is implicitly contained in one of A. Einstein's last papers [31]. It is shown there that a star in static equilibrium can smoothly be embedded in an expanding universe which means that the radius of the star does not expand. From this it is often derived that cosmic expansion is restricted to distances of cosmic scale. (According to Ref. [32] "only distances between clusters of galaxies and greater distances are subject to the expansion.") What cannot be understood on this basis is, why, on the other hand, light rays of much smaller spatial extensions are still subject to expansion. (This applies in particular to the incoherent light emitted by far away galaxies etc. However, in the usual derivation of the cosmic redshift only monochromatic wave trains of infinite extension are considered.) This different behavior is especially difficult to understand in expansion coordinates [33], but it is very easily understood in explosion coordinates: Completely independent of the radial extension of a radially directed light ray the frequency of it is redshifted due to the Doppler effect and the gravitational field.

(3) The transformation from expansion to explosion coordinates can only in special cases be analytically expressed and will in general involve numerical calculations. A formulation of the general relativistic equations for the dynamic evolution of the universe in explosion coordinates may offer an easier approach to solutions, at least in special cases.

(4) The introduction of explosion coordinates in a universe with negative spatial curvature ($k = -1$) is certainly feasible in a similar way as in the case $k = 0$ considered in this paper. In the case of positive spatial curvature the situation is different. Nevertheless it may be possible to successfully impose the condition $g_{\rho\rho} = 0$ also in this case, although this would appear somewhat artificial.

V. CONCLUSION

It was shown in this paper that explaining the recession of galaxies (or other manifestations of matter at earlier stages) by an expansion of space or as an explosion and after-explosionlike motion relative to space is equivalent. Both interpretations are relative in that their validity is restricted to specific coordinate systems. The transition between them can be performed by simple transformations.

Certainly, the usual approach using expansion coordinates has the major merits. Most important is its simplicity provided by the fact that many physical properties are related to one simple time-dependent parameter, the scale factor $a(t)$. Furthermore, the coordinate time is equal to the proper time and is thus valid for the whole universe. For people unfamiliar with the field the concept of an expanding space may occasionally provide difficulties, but this is by far surpassed by the aforementioned advantages.

Our approach by explosion coordinates aimed at an alternative interpretation and consequently is based on a reformulation of known results. Nevertheless it has its merits as well. For one, it better fits the view of an external observer. Also, the interpretation of the big bang or inflation as giant explosions, and the restriction of recessional velocities to values below the velocity of light are more intuitive. Furthermore, there exist problems which are easier to handle in explosion coordinates. Finally, the close affinity to Newtonian cosmology may provide advantages in some cases.

The two interpretations should by no means be confounded. As well as the Doppler effect must not be used for explaining the redshift in expansion coordinates, their corresponding distances and recessional velocities must not be employed when Doppler effect and gravitational blueshift are evaluated in explosion coordinates.

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