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Nonzero θ_{13} signals nonmaximal atmospheric neutrino mixing

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From recent groundbreaking experiments, it is now known that the Pontecorvo-Maki-Nakagawa-Sakata mixing differs significantly from the tribimaximal model in which $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. Flavor symmetry can require that the departures from these two equations are linearly related. T' and A_4, which successfully accommodated the pre-T2K Pontecorvo-Maki-Nakagawa-Sakata matrix, predict that $38.07° \leq \theta_{23} \leq 39.52°$ at 95% C.L. The best fit values, combining the model predictions with T2K, MINOS, Double Chooz, Daya Bay, and RENO data, are $\theta_{23} = 38.7°$ and $\theta_{13} = 8.9°$.

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Of the parameters in the standard model of particle theory, we will focus on the mixing matrices for down-type quarks and for neutrinos, named respectively for Cabibbo-Kobayashi-Maskawa (CKM) [1,2] and for Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [3,4]. Without losing generality, we choose a basis in which the flavor and mass eigenstates coincide for the three up-type quarks and all three charged leptons.

This investigation will consider one of three mixing angles of CKM quark mixing (Θ_{12}) and two of the three mixing angles of PMNS neutrino mixing (θ_{13} and θ_{23}), ignoring for the moment the *CP*-violating phases in both cases.

We recall the values of the angles θ_{13} and θ_{23} listed in the 2010 Review of Particle Physics[1] [5] since these two are, we suggest, both changed by the T2K measurement [6–11]. The values then were

$$36.8° \lesssim \theta_{23} \lesssim 45.0°, \quad 0.0° \leq \theta_{13} \lesssim 11.4°, \tag{1}$$

consistent with vanishing θ_{13} and maximal θ_{23}.

The other angles are not considered to be variables in this analysis, although the superior experimental accuracy of the CKM Gell-Mann-Lévy quark mixing angle [12],

$$\Theta_{12} = (13.03 \pm 0.06)°, \tag{2}$$

played an important role in our investigation of flavor symmetry.

To accommodate the new data, we invoke broken binary tetrahedral (T') flavor symmetry as a promising approach to explaining the mixing angles [13–22].

This flavor symmetry was first used in Ref. [13] solely as a symmetry for quarks, because neutrinos were still believed to be massless. After neutrino masses and mixings were discovered [23], the mixing matrix for neutrinos was measured and found to be very different from the CKM mixing matrix for quarks. A number of theories arose [24–28] to explain this. Eventually, a useful approximation to the empirical PMNS mixing was determined to be the tribimaximal (TBM) matrix [29],

$$U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -1/\sqrt{2} \\ -\sqrt{1/6} & \sqrt{1/3} & 1/\sqrt{2} \end{pmatrix}. \tag{3}$$

Flavor symmetry based on the tetrahedral group, $A_4 = T$, was introduced by Ref. [30] to underpin TBM neutrino mixing. Further investigation revealed that this model could not be extended to quarks because a viable CKM matrix could not be obtained [31]. A_4 is not a subgroup of its double cover [20], T', nevertheless from the viewpoint of kronecker products used in model building [14], A_4 behaves *as if* it were a subgroup. This explains why the larger group can act as a successful flavor symmetry for both quarks and leptons.

We shall consider only the projection on the two-dimensional $\theta_{23} - \theta_{13}$ plane of the three-dimensional $\theta_{12} - \theta_{23} - \theta_{13}$ space. At leading order, requiring $\sin\alpha \sim \alpha^2$ for θ_{13} and $(\frac{\pi}{4} - \theta_{23})$, the calculation of the perturbation of this projection from the TBM matrix in Eq. (3) is independent of the solar neutrino mixing angle θ_{12}. The relevant perturbation away from Eq. (3) was explicitly calculated in Refs. [18,19].

Before T2K, the neutrino mixing angles were all empirically consistent with the TBM values. However, as the experimental accuracy has now improved in recent data from T2K [6–11], MINOS [32–38], Double Chooz [39–43], Daya Bay [44,45], and RENO [46,47], this situation has changed dramatically, as discussed in the global fits of Refs. [48–50]; of these we shall use Fogli *et al.* [49]. These five remarkable experiments have provided us with a

[1] The reader is directed to the references summarized in RPP.

[2] This is a $<1\%$ approximation for θ_{13} and $(\frac{\pi}{4} - \theta_{23})$ since both angles are less than $\alpha = 12° = 0.2094$ radians with $\sin\alpha = 0.2079$.

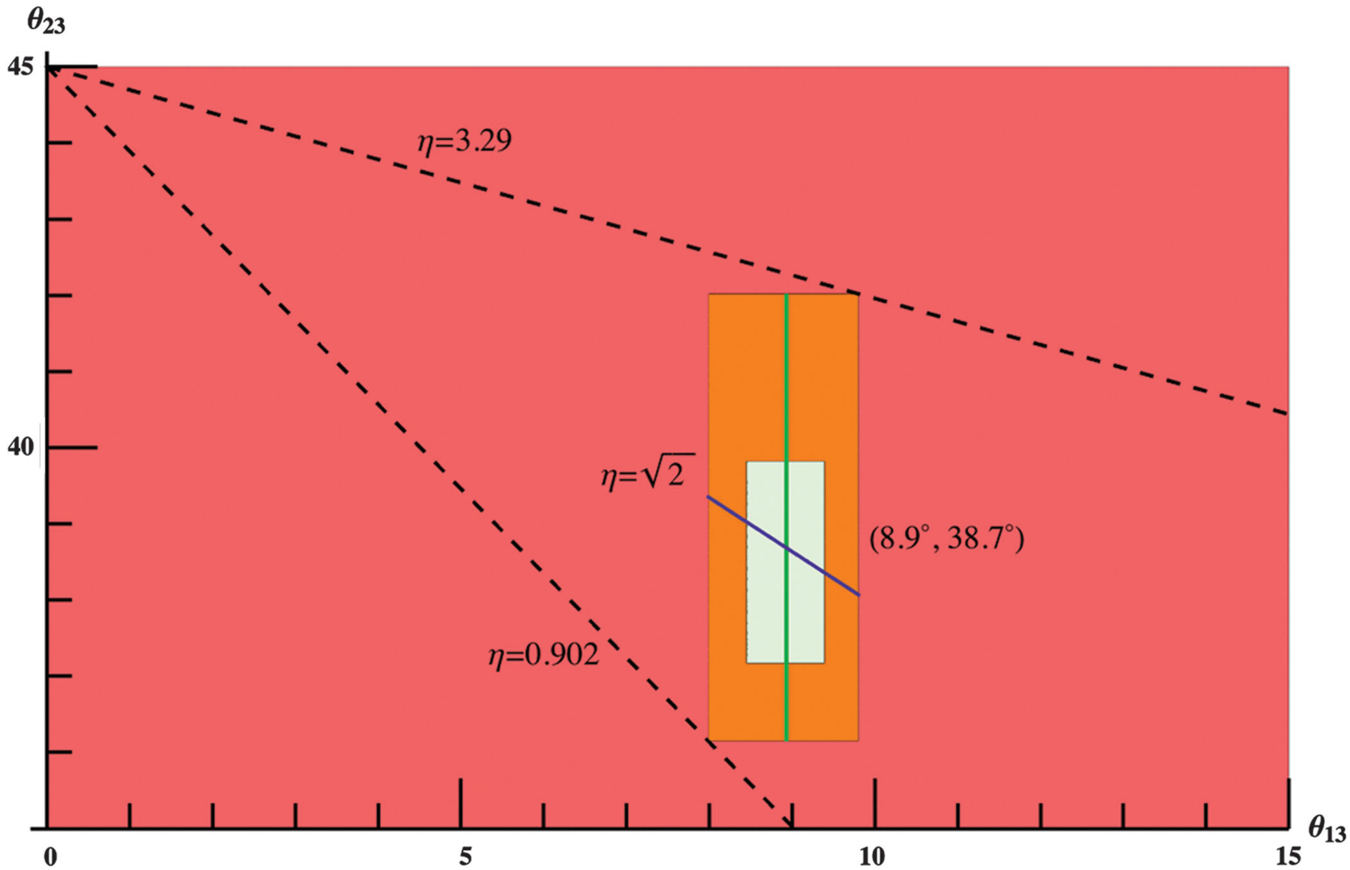

FIG. 1 (color). The global analysis of Ref. [49], incorporating SBL, LBL, solar, and atmospheric neutrino observations, excludes the red-shaded region at 2σ. The same assessment excludes the orange-shaded region at 1σ. The best fit value for θ_{13} is indicated by the vertical green line at $\theta_{13} = 8.9°$. Extreme values of the linear correlation coefficient, η, are indicated by dashed lines at $\eta = 0.902$ and $\eta = 3.29$, while our predicted correlation of $\eta = \sqrt{2}$ is indicated by the solid dark blue line. The intersection of our correlation prediction and the θ_{13} best fit occurs at $\theta_{13} = 8.9°$ and $\theta_{23} = 38.7°$, a close match to the current experimental best fit of $\theta_{23} = 38.4°$.

rich new perspective on mixing angles. From flavor symmetry, it is then possible to predict quantitatively how departures from the TBM values,

$$\theta_{12} = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right), \qquad \theta_{23} = (\pi/4), \qquad \theta_{13} = 0, \quad (4)$$

are related. The model allows one to address this question by relating the perturbations around TBM,

$$\theta_{ij} = (\theta_{ij})_{\mathrm{TBM}} + \epsilon_k, \quad (5)$$

(where ϵ_3 corresponds to θ_{12}, and so on) to the analogous perturbations around the minimal model's prediction for the CKM Gell-Mann-Lévy quark mixing angle,

$$\tan 2(\Theta_{12}) = \left(\frac{\sqrt{2}}{3}\right). \quad (6)$$

The data from KamLAND, LBL accelerators (like T2K and MINOS), solar experiments, SBL accelerators (such as Double Chooz, Daya Bay, and RENO), and Super-Kamiokande, as combined in Ref. [49] indicate (accounting for CP violation)

$$\sin^2\theta_{13} = 0.0241^{+0.0049}_{-0.0048} \quad \text{with} \quad 95\%\text{C.L.} \quad (7)$$

for a normal neutrino mass hierarchy, as favored by T'.

Because Eq. (6) yields a value of $\Theta_{12} = 12.62°$, which while close is significantly below the experimental value, Eq. (2), it is possible to perturb to the empirical Θ_{12} and to track the deviations in the PMNS mixing matrix to the linear relationship,[3]

$$\theta_{13} = \eta\left(\frac{\pi}{4} - \theta_{23}\right), \quad (8)$$

with the sharp prediction[4] that $\eta = \sqrt{2}$. Thus,

[3] A_4 is also capable of producing Eq. (8) with $\eta = \sqrt{2}$, though we give preference in this paper to T' for its capacity to explain CKM mixing.

[4] It is notable that Eq. (8) with $\eta \simeq \sqrt{2}$ appears *en passant* in Ref. [51]; see also Ref. [52] which implies that $\eta \sim 2$. Another, model-independent correlation was developed in Ref. [53], including the three PMNS mixing angles and the CP-violating phase.

$$\theta_{13} = \sqrt{2}\left(\frac{\pi}{4} - \theta_{23}\right). \qquad (9)$$

This prediction is derived in further detail in Ref. [19].

Considering the result, Eq. (8), it requires that, if η is finite as expected, any departure from $\theta_{13} = 0$ signals that $\theta_{23} < \pi/4$. As shown in Fig. 1, the recent experimental data, combined with theory, suggest that $(\theta_{13}, \theta_{23})$ are respectively closer to $(8.9°, 38.7°)$ than to $(0.0°, 45.0°)$. Before T2K, η was unconstrained, $0 \leq \eta < \infty$. With the current global fit data, we find $0.902 \leq \eta \leq 3.29$.

This is in sharp contrast to the previously widespread acceptance of a maximal $\theta_{23} = \pi/4$, which fitted so well with vanishing $\theta_{13} = 0$ in the TBM context.

As the measurement of θ_{13} sharpens experimentally, so will the prediction for θ_{23} from Eq. (9), and measurement of the atmospheric neutrino mixing's departure from maximality will provide an interesting test of the binary tetrahedral flavor symmetry.

Several years ago Super-Kamiokande showed $\theta_{23} > 36.8°$ [54], and current analysis places it at $\theta_{23} \simeq 40.7°$ [55]. Once combined in a global fit of 3ν oscillation, Ref. [49] states the best fit of $\theta_{23} = 38.4°$, tantalizingly close to our central value of $\theta_{23} = 38.7°$.

This suggests to us that the T' flavor symmetry, introduced in Ref. [13], should now be taken much more seriously. As errors in θ_{13} and θ_{23} diminish even further, it will be interesting to see how the prediction of Eq. (9) by T' perseveres, as it would inspire further investigation into other mixing angles for quarks and leptons. This, in turn, may show that T', first mentioned in physics as an example of an $SU(2)$ subgroup [56], is actually a useful approximate symmetry in the physical application of quark and lepton flavors.

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[1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).
[2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[3] B. Pontecorvo, Sov. Phys. JETP **7**, 172 (1958).
[4] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).
[5] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).
[6] K. Abe *et al.* (T2K Collaboration), Phys. Rev. Lett. **107**, 041801 (2011).
[7] F. Dufour (T2K Collaboration), J. Phys. Conf. Ser. **335**, 012053 (2011).
[8] M. Hartz (T2K Collaboration), arXiv:1201.1846.
[9] E. Frank (T2K Collaboration), Proc. Sci. IDM2010 (2011) 103.
[10] A. Izmaylov (T2K Collaboration), arXiv:1112.0273.
[11] T. Nakaya, in Proceedings of Neutrino 2012 Conference, Kyoto, 2012 [Nucl. Phys. B, Proc. Suppl. (to be published)].
[12] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960).
[13] P. H. Frampton and T. W. Kephart, Int. J. Mod. Phys. A **10**, 4689 (1995).
[14] P. H. Frampton and T. W. Kephart, Phys. Rev. D **64**, 086007 (2001).
[15] M.-C. Chen and K. T. Mahanthappa, Phys. Lett. B **652**, 34 (2007).
[16] F. Feruglio, C. Hagedorn, Y. Lin, and L. Merlo, Nucl. Phys. **B775**, 120 (2007); **B836**, 127(E) (2010).
[17] P. H. Frampton and T. W. Kephart, J. High Energy Phys. 09 (2007) 110.
[18] P. H. Frampton, T. W. Kephart, and S. Matsuzaki, Phys. Rev. D **78**, 073004 (2008).
[19] D. A. Eby, P. H. Frampton, and S. Matsuzaki, Phys. Lett. B **671**, 386 (2009).
[20] P. H. Frampton, T. W. Kephart, and R. M. Rohm, Phys. Lett. B **679**, 478 (2009).
[21] D. A. Eby, P. H. Frampton, and S. Matsuzaki, Phys. Rev. D **80**, 053007 (2009).
[22] P. H. Frampton, C. M. Ho, T. W. Kephart, and S. Matsuzaki, Phys. Rev. D **82**, 113007 (2010).
[23] Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1158 (1998); **81**, 4279(E) (1998).
[24] A. Aranda, C. D. Carone, and R. F. Lebed, Phys. Rev. D **62**, 016009 (2000).
[25] X.-G. He and A. Zee, Phys. Lett. B **560**, 87 (2003).
[26] K. S. Babu and X.-G. He, arXiv:hep-ph/0507217.
[27] T. D. Lee, Chinese Phys. C **15**, 1125 (2006).
[28] P. H. Frampton, P. I. Krastev, and J. T. Liu, Mod. Phys. Lett. A **09**, 761 (1994).
[29] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. B **530**, 167 (2002).
[30] E. Ma and G. Rajasekaran, Phys. Rev. D **64**, 113012 (2001).
[31] G. Altarelli and F. Feruglio, Nucl. Phys. **B741**, 215 (2006).
[32] R. Nichol, in Proceedings of Neutrino 2012 Conference, Kyoto, 2012 [Nucl. Phys. B, Proc. Suppl. (to be published)].
[33] P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. Lett. **108**, 191801 (2012).
[34] P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. Lett. **107**, 181802 (2011).
[35] A. Holin (MINOS Collaboration), Proc. Sci. EPSHEP (2011) 088 [arXiv:1201.3645].
[36] A. Habig (MINOS Collaboration), Nucl. Phys. B, Proc. Suppl. **218**, 320 (2011).
[37] M. Orchanian (MINOS Collaboration), arXiv:1109.6795.
[38] J. Evans (MINOS Collaboration), Proc. Sci. ICHEP (2010) 298.

[39] M. Ishitsuka, in Proceedings of Neutrino 2012 Conference, Kyoto, 2012 [Nucl. Phys. B, Proc. Suppl. (to be published)].

[40] H. De Kerret, in *Proceedings of LowNu* (Seoul National University, Seoul, 2011), http://workshop.kias.re.kr/lownu11/.

[41] C. Palomares (DOUBLE-CHOOZ Collaboration), J. Phys. Conf. Ser. **335**, 012055 (2011).

[42] Y. Abe *et al.* (DOUBLE-CHOOZ Collaboration), Phys. Rev. Lett. **108**, 131801 (2012).

[43] C. Palomares (Double-Chooz Collaboration), Proc. Sci. EPS HEP (2009) 275 [arXiv:0911.3227].

[44] F. P. An *et al.* (DAYA-BAY Collaboration), Phys. Rev. Lett. **108**, 171803 (2012).

[45] D. Dwyer, in Proceedings of Neutrino 2012 Conference, Kyoto, 2012 [Nucl. Phys. B, Proc. Suppl. (to be published)].

[46] J. K. Ahn *et al.* (RENO Collaboration), Phys. Rev. Lett. **108**, 191802 (2012).

[47] S.-B. Kim, in Proceedings of Neutrino 2012 Conference, Kyoto, 2012 [Nucl. Phys. B, Proc. Suppl. (to be published)].

[48] D. V. Forero, M. Tortola, and J. W. F. Valle, Phys. Rev. D **86**, 073012 (2012).

[49] G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, and A. M. Rotunno, Phys. Rev. D **86**, 013012 (2012).

[50] M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado, and T. Schwetz, arXiv:1209.3023.

[51] P. F. Harrison and W. G. Scott, Phys. Lett. B **628**, 93 (2005).

[52] K. Fuki and M. Yasue, Nucl. Phys. **B783**, 31 (2007).

[53] S.-F. Ge, D. A. Dicus, and W. W. Repko, Phys. Rev. Lett. **108**, 041801 (2012).

[54] Y. Ashie *et al.* (Super-Kamiokande Collaboration), Phys. Rev. D **71**, 112005 (2005).

[55] Y. Itow, in Proceedings of Neutrino 2012 Conference, Kyoto, 2012 [Nucl. Phys. B, Proc. Suppl. (to be published)].

[56] K. M. Case, R. Karplus, and C. N. Yang, Phys. Rev. **101**, 874 (1956).