$Y(4260)$ as a mixed charmonium-tetraquark state

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Using the QCD sum rule approach we study the $Y(4260)$ state assuming that it can be described by a mixed charmonium-tetraquark current with $J^{PC} = 1^{--}$ quantum numbers. For the mixing angle around $\theta \approx (53.0 \pm 0.5)°$, we obtain a value for the mass which is in good agreement with the experimental mass of the $Y(4260)$. For the decay width into the channel $Y \to J/\psi\pi\pi$ we find the value $\Gamma_{Y\to J/\psi\pi\pi} \approx (4.1 \pm 0.6)$ MeV, which is much smaller than the total experimental width $\Gamma \approx (95 \pm 14)$ MeV. However, considering the experimental upper limits for the decay of the $Y(4260)$ into open charm, we conclude that we cannot rule out the possibility of describing this state as a mixed charmonium-tetraquark state.

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I. INTRODUCTION

Many of the charmonium-like states recently observed in e^+e^- collisions by *BABAR* and Belle collaborations do not fit the quarkonia interpretation, and have stimulated an extensive discussion about exotic hadron configurations. The production mechanism, masses, decay widths, spin-parity assignments, and decay modes of these states, called X, Y, and Z states, have been discussed in some reviews [1–5]). Among these states, the $Y(4260)$ is particularly interesting. It was first observed by *BABAR* collaboration in the e^+e^- annihilation through initial state radiation [6], and it was confirmed by CLEO and Belle collaborations [7]. The $Y(4260)$ was also observed in the $B^- \to Y(4260)K^- \to J/\Psi\pi^+\pi^-K^-$ decay [8], and CLEO reported two additional decay channels: $J/\Psi\pi^0\pi^0$ and $J/\Psi K^+K^-$ [7].

Since the mass of the $Y(4260)$ is higher than the $D^{(*)}\bar{D}^{(*)}$ threshold, if it was a normal $c\bar{c}$ charmonium state, it should decay mainly to $D^{(*)}\bar{D}^{(*)}$. However, the observed Y state do not match the peaks in $e^+e^- \to D^{(*)\pm}D^{(*)\mp}$ cross sections measured by Belle [9] and *BABAR* [10,11]. Besides, the $\Psi(3S)$, $\Psi(2D)$, and $\Psi(4S)$ $c\bar{c}$ states have been assigned to the well-established $\Psi(4040)$, $\Psi(4160)$, and $\Psi(4415)$ mesons, respectively, and the prediction from quark models for the $\Psi(3D)$ state is 4.52 GeV. Therefore, the mass of the $Y(4260)$ is not consistent with any of the 1^{--} $c\bar{c}$ states [2,3,12].

There are many theoretical interpretations for the $Y(4260)$: tetraquark state [13], hadronic molecule of D_1D, D_0D^* [14], $\chi_{c1}\omega$ [15], $\chi_{c1}\rho$ [16], $J/\psi f_0(980)$ [17], a hybrid charmonium [18], a charm baryonium [19], a cusp [20–22], etc. Within the available experimental information, none of these suggestions can be completely ruled out. However, there are some calculations, within the QCD sum rules (QCDSR) approach [3,23–25], that cannot explain the mass of the $Y(4260)$ supposing it to be a tetraquark state [26], or a D_1D, D_0D^* hadronic molecule [26], or a $J/\psi f_0(980)$ molecular state [27].

In this work we use again the QCDSR approach to the $Y(4260)$ state including a new possibility: the mixing between two and four-quark states. This will be implemented following the prescription suggested in Ref. [28] for the light scalar mesons. The mixing is done at the level of the currents and was extended to the charm sector in Ref. [29], in order to study the $X(3872)$ as a mixed charmonium-molecular state. In particular, in Ref. [29], the mass and the decay width of the $X(3872)$, into 2π and 3π, were evaluated with good agreement with the experimental values. Agreement with the experimental results has been also obtained, applying this same approach, in the study of the $X(3872)$ radiative decay [30], and also in the $X(3872)$ production rate in B decay [31].

In the next sections we consider a mixed charmonium-tetraquark current and use the QCDSR method to study both mass and decay width of the $Y(4260)$.

II. CONSTRUCTING THE TWO-QUARK AND FOUR-QUARK OPERATOR

In order to define a mixed charmonium-tetraquark current we have to define the currents associated with charmonium and four-quarks (tetraquark) states. For the charmonium part we use the conventional vector current:

$$j'^{(2)}_\mu(x) = \bar{c}_a(x)\gamma_\mu c_a(x), \tag{1}$$

while the tetraquark part is interpolated by [26]

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$$j_\mu^{(4)}(x) = \frac{\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}}[(q_a^T(x)C\gamma_5 c_b(x))(\bar{q}_d(x)\gamma_\mu\gamma_5 C\bar{c}_e^T(x)) + (q_a^T(x)C\gamma_5\gamma_\mu c_b(x))(\bar{q}_d(x)\gamma_5 C\bar{c}_e^T(x))]. \quad (2)$$

As in Refs. [28,29], we define the normalized two-quark current as

$$j_\mu^{(2)} = \frac{1}{\sqrt{2}}\langle\bar{q}q\rangle j_\mu^{\prime(2)}, \quad (3)$$

and from these two currents we build the following mixed charmonium-tetraquark $J^{PC} = 1^{--}$ current for the $Y(4260)$ state:

$$j_\mu(x) = \sin(\theta) j_\mu^{(4)}(x) + \cos(\theta) j_\mu^{(2)}(x). \quad (4)$$

III. THE TWO-POINT CORRELATION FUNCTION

To obtain the mass of a hadronic state using the QCDSR approach, the starting point is the two-point correlation function

$$\Pi_{\mu\nu}(q) = i\int d^4x e^{iq\cdot x}\langle 0|T[j_\mu(x)j_\nu^\dagger(0)]|0\rangle = -\Pi_1(q^2)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \Pi_0(q^2)\frac{q_\mu q_\nu}{q^2}, \quad (5)$$

where $j_\mu(x)$ is the mixed charmonium-tetraquark interpolating current defined in Eq. (4). The functions $\Pi_1(q^2)$ and $\Pi_0(q^2)$ are two independent invariant functions associated with spin-1 and spin-0 mesons, respectively.

According to the principle of duality, Eq. (5) can be evaluated in two ways: in the operator product expansion (OPE) side, we calculate the correlation function in terms of quarks and gluon fields using the Wilson's operator product expansion. The phenomenological side is evaluated by inserting, in Eq. (5), a complete set of intermediate states with 1^{--} quantum numbers. In this side, we parametrize the coupling of the vector state Y with the current defined in Eq. (4) through the coupling parameter λ_Y

$$\langle 0|j_\mu(x)|Y\rangle = \lambda_Y\epsilon_\mu, \quad (6)$$

where ϵ_μ is the polarization vector. Using Eq. (6), we can write the phenomenological side of Eq. (5) as

$$\Pi_{\mu\nu}^{\rm fen}(q) = \frac{\lambda_Y^2}{M_Y^2 - q^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \cdots, \quad (7)$$

where m_Y is the mass of the Y state and the dots, in the second term in the rhs of Eq. (7), denotes the higher resonance contributions which will be parametrized, as usual, through introduction of the continuum threshold parameter s_0 [32].

The OPE side can be written in terms of a dispersion relation

$$\Pi^{\rm OPE}(q^2) = \int_{4m_c^2}^\infty ds\frac{\rho^{\rm OPE}(s)}{s - q^2}, \quad (8)$$

where $\rho^{\rm OPE}(s)$ is given by the imaginary part of the correlation function: $\pi\rho^{\rm OPE}(s) = {\rm Im}[\Pi^{\rm OPE}(s)]$. In this side, we work at leading order in α_s in the operators and we consider the contributions from the condensates up to dimension 8. Although we will consider only a part of the of the dimension 8 condensates (related to the quark condensate times the mixed condensate), in Ref. [33] it was shown that this is the most important dimension 8 condensate contribution.

Considering the current in Eqs. (4) and (5) in the OPE side can be written as

$$\Pi_{\mu\nu}(q) = \frac{\langle\bar{q}q\rangle^2}{2}\cos^2(\theta)\Pi_{\mu\nu}^{22}(q) + \sin^2(\theta)\Pi_{\mu\nu}^{44}(q) + \frac{\langle\bar{q}q\rangle}{\sqrt{2}}\sin(\theta)\cos(\theta)[\Pi_{\mu\nu}^{24}(q) + \Pi_{\mu\nu}^{42}(q)], \quad (9)$$

with

$$\Pi_{\mu\nu}^{ij}(q) = i\int d^4x e^{iq\cdot x}\langle 0|T[j_\mu^i(x)j_\nu^{j\dagger}(0)]|0\rangle. \quad (10)$$

Clearly $\Pi_{\mu\nu}^{22}(q)$ and $\Pi_{\mu\nu}^{44}(q)$ are, respectively, the correlation functions of the J/ψ and $[cq][\bar{c}\bar{q}]$ tetraquark state.

After making a Borel transform in both sides, and transferring the continuum contributions to the OPE side, the sum rule in the $g_{\mu\nu}$ structure for the vector meson can be written as

$$\lambda_Y^2 e^{-m_Y^2/M_B^2} = \frac{\langle\bar{q}q\rangle^2}{2}\cos^2(\theta)\Pi_1^{22}(M_B^2) + \sin^2(\theta)\Pi_1^{44}(M_B^2) + \frac{\langle\bar{q}q\rangle}{\sqrt{2}}\sin(\theta)\cos(\theta)[\Pi_1^{24}(M_B^2) + \Pi_1^{42}(M_B^2)], \quad (11)$$

where

$$\Pi_1^{22}(M_B^2) = \int_{4m_c^2}^{s_0} ds e^{-s/M_B^2}\rho_{\rm pert}^{22}(s) + \Pi_{\langle G^2\rangle}^{22}(M_B^2), \quad (12)$$

$$\Pi_1^{44}(M_B^2) = \int_{4m_c^2}^{s_0} ds e^{-s/M_B^2}(\rho_{\rm pert}^{44}(s) + \rho_{\langle\bar{q}q\rangle}^{44}(s) + \rho_{\langle G^2\rangle}^{44}(s) + \rho_{\langle\bar{q}Gq\rangle}^{44}(s) + \rho_{\langle\bar{q}q\rangle^2}^{44}(s) + \rho_{\langle 8\rangle}^{44}) + \Pi_{\langle 8\rangle}^{44}(M_B^2), \quad (13)$$

$$\Pi_1^{24}(M_B^2) = \int_{4m_c^2}^{s_0} ds e^{-s/M_B^2}\rho_{\langle\bar{q}q\rangle}^{24}(s) + \Pi_{\langle\bar{q}Gq\rangle}^{24}(M_B^2). \quad (14)$$

The expressions for the spectral density $\rho(s)$ appearing in Eqs. (12)–(14) for the charmonium and tetraquark states, as well as the mixed terms are listed in the Appendix.

By taking the derivative of Eq. (11) with respect to $1/M_B^2$ and dividing the result by Eq. (11), we obtain

$$m_Y^2 = -\frac{\frac{dK(M_B^2,\theta)}{d(1/M_B^2)}}{K(M_B^2,\theta)}, \quad (15)$$

where

$$K(M_B^2,\theta) \equiv \frac{\langle\bar{q}q\rangle^2}{2}\cos^2(\theta)\Pi_1^{22}(M_B^2) + \sin^2(\theta)\Pi_1^{44}(M_B^2) + \frac{\langle\bar{q}q\rangle}{\sqrt{2}}\sin(\theta)\cos(\theta)[\Pi_1^{24}(M_B^2) + \Pi_1^{42}(M_B^2)].$$

Equation (15) will be used to extract the mass of the charmonium-tetraquark state.

A. Numerical analysis

In Table I we list the values of the quark masses and condensates that we have used in our numerical analysis. For a consistent comparison with results obtained for the others works using QCD sum rules, these parameters values used here are the same values used in Refs. [25,27,34,35].

The continuum threshold is a physical parameter that, in the QCDSR approach, should be related to the first excited state with the same quantum numbers. In some known cases, like the ρ and J/ψ, the first excited state has a mass approximately 0.5 GeV above the ground state mass. Since in our study we do not know the experimental spectrum for the hadrons studied, we will fix the continuum threshold range starting with the smaller value which provides a valid Borel window, as explained below. Using this criterion, we obtain s_0 in the range $4.6 \leq \sqrt{s_0} \leq 4.8$ GeV.

Reliable results can be extracted from the sum rule if is possible to determine a valid Borel window. Such Borel window is obtained by imposing a good OPE convergence, the dominance of the pole contribution, and a good Borel stability. To determine the minimum value of the Borel mass we adopt the criterion for which the contribution of the higher dimension condensate should be smaller than 15% of the total contribution. Thus, $M^2_{B\,\mathrm{min}}$ is such that

$$\left|\frac{\text{OPE summed up to dim}n - 1(M^2_{B\,\mathrm{min}})}{\text{total contribution}(M^2_{B\,\mathrm{min}})}\right| = 0.85. \qquad (16)$$

In Fig. 1 we plot the relative contributions of all the terms in the OPE side. We have used $\sqrt{s_0} = 4.70$ GeV and $\theta = 53°$. For others θ values outside the range $52.5° \leq \theta \leq 53.5°$, we do not have a good OPE convergence. From this figure we see that the contribution of the dimension-8 condensates is smaller than 15% of the total contribution for values of $M_B^2 \geq 2.4$ GeV^2, indicating a good OPE

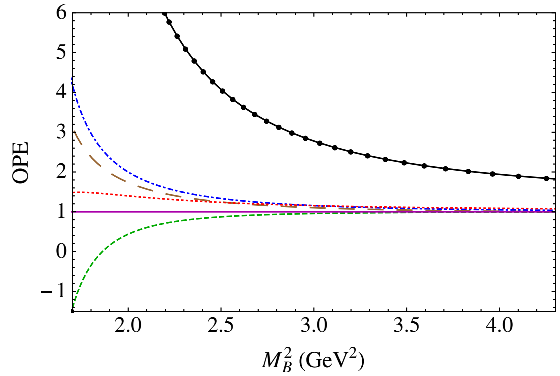

FIG. 1 (color online). The OPE convergence in the region $2.0 \leq M_B^2 \leq 6.0$ GeV^2 for $\sqrt{s_0} = 4.70$ GeV. We plot the relative contributions start with perturbative contribution (line with circles), and each other line represents the relative contribution after the addition of one extra condensate in expansion: $+\langle\bar{q}q\rangle$ (dot-dashed line), $+\langle G^2\rangle$ (long-dashed line), $+\langle\bar{q}g\sigma.Gq\rangle$ (dotted line), $+\langle\bar{q}q\rangle^2$ (dashed line), and $\langle\bar{q}q\rangle\langle\bar{q}g\sigma.Gq\rangle$ (solid line).

convergence. Therefore, we fix the lower value of M_B^2 in the sum rule window as: $M^2_{B\,\mathrm{min}} = 2.4$ GeV^2.

To determine the maximum value of the Borel mass ($M^2_{B\,\mathrm{max}}$) we must analyse the pole-continuum contribution. Unlike the pole contribution, the continuum contribution increases with M_B^2 due to the dominance of the perturbative contribution. Therefore, the maximum value of the Borel mass is determined in the point that the pole contribution is equal to the continuum contribution.

In Fig. 2 we see a comparison between the pole and continuum contributions. It is clear that the pole contribution is equal to the continuum contribution for $M_B^2 = 2.90$ GeV^2. Therefore, for $\sqrt{s_0} = 4.70$ GeV^2 and $\theta = 53°$ the Borel window is: $2.4 \leq M_B^2 \leq 2.90$ GeV^2.

After we have determined the Borel window, we can calculate the ground state mass, which is shown, as a function of M_B^2, in the Fig. 3. From this figure we see that there is a very good stability in the ground state mass in the determined Borel window, which are represented through the crosses in Fig. 3.

Varying the value of the continuum threshold in the range $\sqrt{s_0} = 4.70 \pm 0.10$ GeV, the mixing angle in the range $\theta = (53.0 \pm 0.5)°$, and the other parameters as indicated in Table I, we get:

$$m_Y = (4.26 \pm 0.13)\ \mathrm{GeV}, \qquad (17)$$

which is in a very good agreement with the experimental mass of the $Y(4260)$.

Once we have determined the mass, we can use this value in Eq. (11) to estimate the meson-current coupling parameter, defined in Eq. (6). We have used the same values of the s_0, θ and Borel window used for the mass calculation. Thus, we get:

$$\lambda_Y = (2.00 \pm 0.23) \times 10^{-2}\ \mathrm{GeV}^5. \qquad (18)$$

TABLE I. Quark masses and condensates values.

Parameters	Values
$m_c(m_c)$	(1.23 ± 0.05) GeV
$\langle\bar{q}q\rangle$	$-(0.23 \pm 0.03)^3$ GeV^3
$\langle\bar{q}g\sigma.Gq\rangle$	$m_0^2\langle\bar{q}q\rangle$
m_0^2	(0.8 ± 0.1) GeV^2
$\langle g_s^2G^2\rangle$	(0.88 ± 0.25) GeV^4

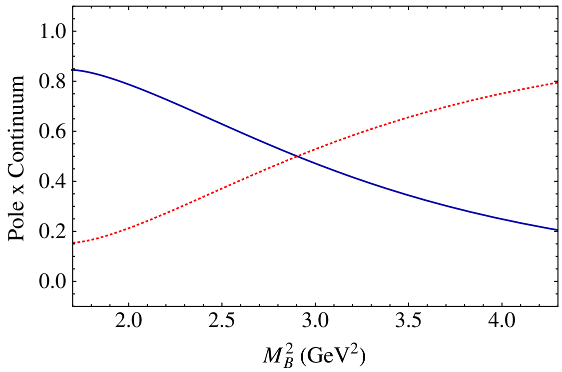

FIG. 2 (color online). The pole contribution (divided by the total, pole plus continuum, contribution) represented by solid line and the continuum contribution (dotted line) for the $\sqrt{s_0} = 4.70$ GeV.

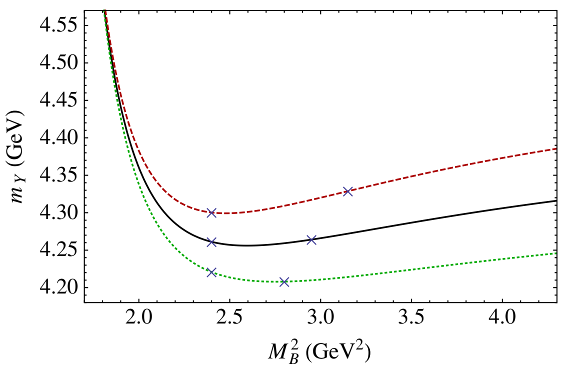

FIG. 3 (color online). The mass as a function of the sum rule parameter M_B^2 for $\sqrt{s_0} = 4.60$ GeV (dotted line), $\sqrt{s_0} = 4.70$ GeV (solid line), $\sqrt{s_0} = 4.80$ GeV (long-dashed line). The crosses indicate the valid Borel window.

The parameter λ_Y gives a measure of the strength of the coupling between the current and the state. The result in Eq. (18) has the same order of magnitude as the coupling obtained for the $X(3872)$ [34], for example.

IV. THE VERTEX FUNCTION AND THE DECAY WIDTH OF THE $Y(4260)$

The QCDSR technique can also be used to extract coupling constants and form factors. In particular, in Ref. [36] the authors determined the form factors and coupling constants in many hadronic vertices containing charmed mesons, in the framework of QCD sum rules. In this section, we will use the QCDSR approach to determine the coupling constant associated with the vertices $YJ/\psi\sigma$ and $YJ/\psi f_0(980)$ to estimate the decay width of the process $Y \to J/\psi\pi\pi$. We are assuming that the two pions in the final state come from the σ and $f_0(980)$ mesons.

We start with the coupling constant associated with the vertex $YJ/\psi\sigma$. To determine the coupling we must evaluate the vertex function (three-point function) defined as

$$\Pi_{\mu\nu}(p, p', q) = \int d^4x d^4y e^{ip'\cdot x} e^{iq\cdot y} \Pi_{\mu\nu}(x, y), \quad (19)$$

with $p = p' + q$ and $\Pi_{\mu\nu}(x, y)$ given by

$$\Pi_{\mu\nu}(x, y) = \langle 0|T\{j_\mu^\psi(x) j^\sigma(y) j_\nu^{Y\dagger}(0)\}|0\rangle. \quad (20)$$

The interpolating fields appearing in Eq. (20) are the currents for J/ψ, σ, and $Y(4260)$, respectively. The currents for J/ψ and Y were defined by Eqs. (1) and (4). For the meson σ, we have

$$j^\sigma(x) = \frac{1}{\sqrt{2}}(\bar{u}_a(x)u_a(x) + \bar{d}_a(x)d_a(x)). \quad (21)$$

Although there are conjectures [37] and lattice calculations [38] proposing that the σ itself could be a tetraquark state, there are also lattice calculations [39] and QCDSR calculations [40] that find it difficult to explain the light scalars as tetraquark states. Therefore, here we use a simple quark-antiquark current to describe the σ.

As in the case of the two-point function studied in the previous section, the three-point correlation function defined by Eq. (19) can also be described in terms of hadronic degrees of freedom (phenomenological side) or in terms of quarks and gluons fields (OPE side). In order to evaluate the phenomenological side of the sum rule we insert, in Eq. (19), intermediate states for Y, J/ψ, and σ. Using the definitions:

$$\begin{aligned} \langle 0|j_\mu^\psi|J/\psi(p')\rangle &= m_\psi f_\psi \epsilon_\mu(p'), \\ \langle 0|j^\sigma|\sigma(q)\rangle &= A_\sigma, \\ \langle Y(p)|j_\nu^Y|0\rangle &= \lambda_Y \epsilon_\nu^*(p), \end{aligned} \quad (22)$$

we obtain the following relation:

$$\Pi_{\mu\nu}^{(\mathrm{phen})}(p, p', q) = \frac{\lambda_Y m_\psi f_\psi A_\sigma g_{Y\psi\sigma}(q^2)}{(p^2 - m_Y^2)(p'^2 - m_\psi^2)(q^2 - m_\sigma^2)} \times ((p'\cdot p)g_{\mu\nu} - p'_\nu q_\mu - p'_\nu p'_\mu) + \cdots, \quad (23)$$

where the dots stand for the contribution of all possible excited states. The form factor, $g_{Y\psi\sigma}(q^2)$, is defined by the generalization of the on-mass-shell matrix element, $\langle J/\psi\sigma|Y\rangle$, for an off-shell σ meson:

$$\langle J/\psi\sigma|Y\rangle = g_{Y\psi\sigma}(q^2)(p'\cdot p\epsilon^*(p')\cdot\epsilon(p) - p'\cdot\epsilon(p)p\cdot\epsilon^*(p')), \quad (24)$$

which can be extracted from the effective Lagrangian that describes the coupling between two vector mesons and one scalar meson:

$$\mathcal{L} = ig_{Y\psi\sigma}V_{\alpha\beta}A^{\alpha\beta}\sigma, \quad (25)$$

where $V_{\alpha\beta} = \partial_\alpha Y_\beta - \partial_\beta Y_\alpha$ and $A^{\alpha\beta} = \partial^\alpha\psi^\beta - \partial^\beta\psi^\alpha$, are the tensor fields of the Y and ψ fields, respectively.

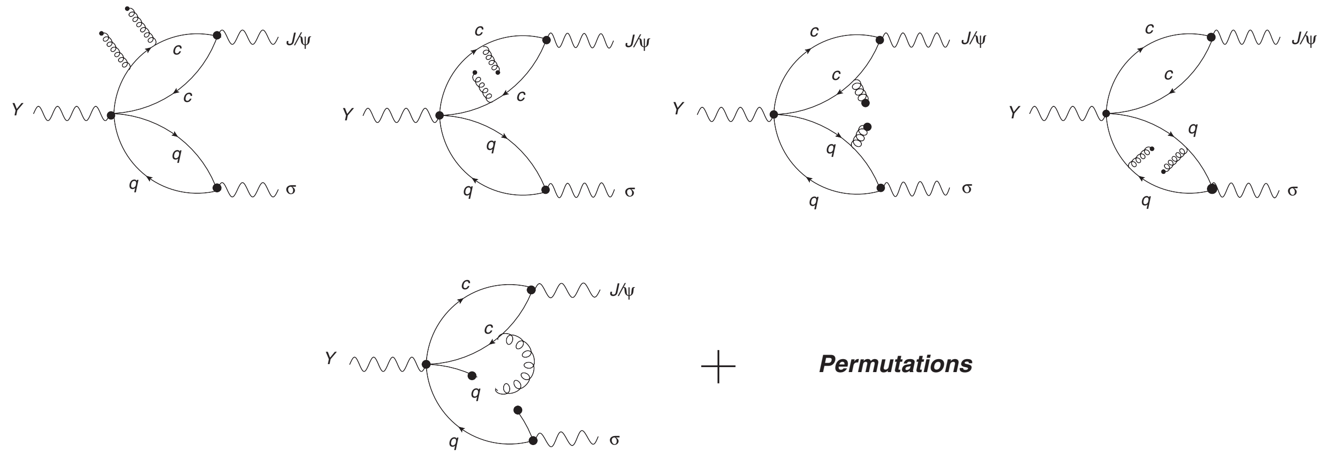

FIG. 4. Diagrams which contribute to the OPE side of the sum rule for the structure $p'_\nu q_\mu$.

In the OPE side, we work at leading order in α_s and we consider the condensates up to dimension five, as shown in Fig. 4. We have chosen to work in the $p'{}_\nu q_\mu$ structure since it has more terms contributing for the OPE. Taking the limit $p^2 = p'^2 = -P^2$ and doing the Borel transform to $P^2 \to M^2$, we get the following expression for the sum rule in the structure $p'{}_\nu q_\mu$:

$$\frac{\lambda_Y A_\sigma m_\psi f_\psi}{(m_Y^2 - m_\psi^2)} g_{Y\psi\sigma}(Q^2)(e^{-m_\psi^2/M^2} - e^{-m_Y^2/M^2}) + B(Q^2)e^{-s_0/M^2} = (Q^2 + m_\sigma^2)\Pi^{(\mathrm{OPE})}(M^2, Q^2), \tag{26}$$

where $Q^2 = -q^2$, and $B(Q^2)$ gives the contribution to the pole-continuum transitions [29,41–43]. $\Pi^{(\mathrm{OPE})}(M^2, Q^2)$ is given by

$$\Pi^{(\mathrm{OPE})}(M^2, Q^2) = \frac{\sin(\theta)}{32^4\sqrt{2}\pi^2}\int_0^1 d\alpha e^{\frac{-m_c^2}{\alpha(1-\alpha)M^2}} \times \left\{\frac{m_c\langle\bar{q}g\sigma.Gq\rangle}{Q^2}\left[\frac{2\alpha(1-\alpha)-1}{\alpha(1-\alpha)}\right] - \frac{\langle g_s^2G^2\rangle}{2^5\pi^4}\right\}. \tag{27}$$

The sine present in Eq. (27) indicates that only the tetraquark part of current in Eq. (4) contributes to the OPE side. In fact, the charmonium part of the current gives only disconnected diagrams that are not considered.

In Eq. (26) m_ψ and f_ψ are the mass and decay constant of the J/ψ and m_σ is the mass of the σ meson. Their values are: $m_\psi = 3.1$ GeV, $f_\psi = 0.405$ GeV [44], and $m_\sigma = 0.478$ GeV [45], which is the mean value of the values quoted for m_σ in [44]. We will use the range in Ref. [44] to evaluate the uncertainties. The parameters λ_Y and A_σ represent, respectively, the coupling of the Y and σ states with the currents defined in Eqs. (6) and (22). The value of λ_Y is given in Eq. (18), while A_σ was determined in Ref. [46] and its value is $A_\sigma = 0.197$ GeV^2.

Similarly to what was done to get m_Y in Eq. (15), one can use Eq. (26) and its derivative with respect to M^2 to eliminate $B(Q^2)$ from these equations and to isolate $g_{Y\psi\sigma}(Q^2)$. A good sum rule must be as much independent of the Borel mass as possible. Therefore, we have to determine a region in the Borel mass where the form factor is independent of M^2. In Fig. 5 we show $g_{Y\psi\sigma}(Q^2)$ as a function of both M^2 and Q^2. Notice that in the region $7.0 \le M^2 \le 10.0$ GeV^2, the form factor is clearly stable, as a function of M^2, for all values of Q^2.

The squares in Fig. 6 show the Q^2 dependence of $g_{Y\psi\sigma}(Q^2)$, obtained for $M^2 = 8.0$ GeV^2. For other values of the Borel mass, in the range $7.0 \le M^2 \le 10.0$ GeV^2, the results are equivalent. Since we are interested in the coupling constant, which is defined as value of the form factor at the meson pole: $Q^2 = -m_\sigma^2$, we need to extrapolate the form factor for a region of Q^2 where the QCDSR is not valid. This extrapolation can be done by parametrizing the QCDSR results for $g_{Y\psi\sigma}(Q^2)$ using a monopole form:

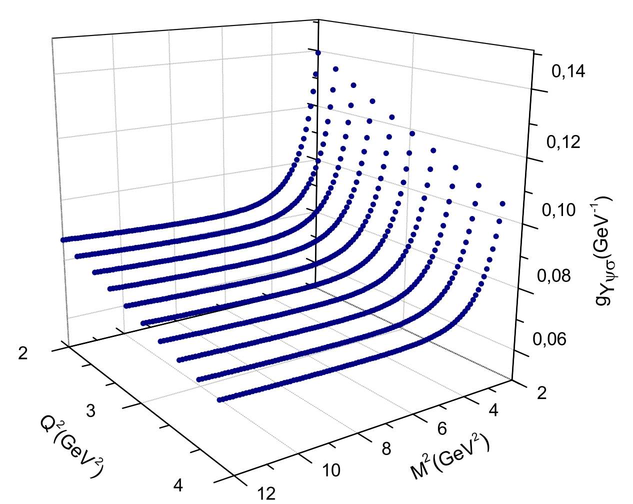

FIG. 5 (color online). $g_{Y\psi\sigma}(Q^2)$ values obtained by varying both Q^2 and M^2.

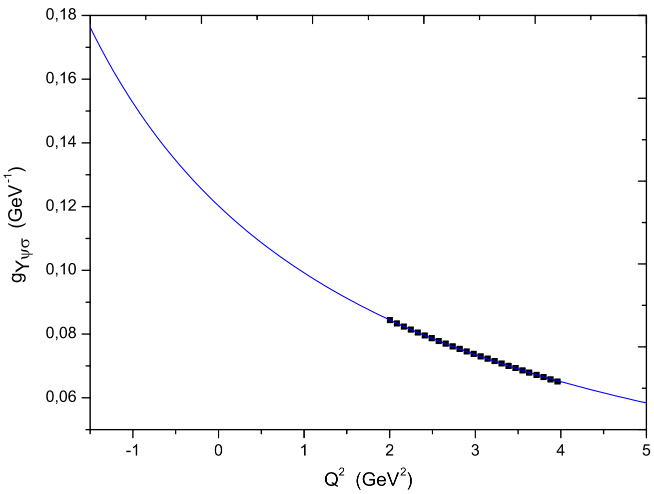

FIG. 6 (color online). QCDSR results for $g_{Y\psi\sigma}(Q^2)$, as a function of Q^2, for $\sqrt{s_0} = 4.76$ GeV (squares). The solid line gives the parametrization of the QCDSR results through Eq. (28).

$$g_{Y\psi\sigma}(Q^2) = \frac{g_1}{g_2 + Q^2}. \tag{28}$$

We do the fit for $\sqrt{s_0} = 4.74$ GeV. We notice that the results do not depend much on this parameter. The results are:

$$g_1 = (0.58 \pm 0.04)\,\text{GeV}; \quad g_2 = (4.71 \pm 0.06)\,\text{GeV}^2. \tag{29}$$

The solid line in Fig. 6 shows that the parametrization given by Eq. (28) reproduces very well the QCDSR results for $g_{Y\psi\sigma}(Q^2)$, in the interval $2.0 \leq Q^2 \leq 4.0$ GeV^2, where the QCDSR is valid.

The coupling constant, $g_{Y\psi\sigma}$ is given by using $Q^2 = -m_\sigma^2$ in Eq. (28). We get:

$$g_{Y\psi\sigma} = g_{Y\psi\sigma}(-m_\sigma^2) = (0.13 \pm 0.01)\ \text{GeV}^{-1}. \tag{30}$$

The error in the coupling constant given above comes from variations in s_0 in the range $4.6 \leq s_0 \leq 4.8$ GeV^2, and in the mixing angle $52.5° \leq \theta \leq 53.5°$.

In Table II, we show the other values of the coupling constant corresponding to the values of $\sqrt{s_0}$ that we have considered in our calculations.

TABLE II. Monopole parametrization of the QCDSR results for the chosen structure, for different values of $\sqrt{s_0}$.

$\sqrt{s_0}$ (GeV)	$g_{Y\psi\sigma}(Q^2)$ (GeV^{-1})	$g_{Y\psi\sigma}(Q^2 = -m_\sigma^2)$ (GeV^{-1})
4.6	$\frac{0.53}{Q^2+4.77}$	0.12
4.7	$\frac{0.57}{Q^2+4.71}$	0.13
4.8	$\frac{0.63}{Q^2+4.66}$	0.14

The decay width for the process $Y(4260) \to J/\psi\sigma \to J/\psi\pi\pi$ in the narrow width approximation is given by

$$\frac{d\Gamma}{ds}(Y \to J/\psi\pi\pi) = \frac{1}{8\pi m_Y^2}|\mathcal{M}|^2 \frac{m_Y^2 - m_\psi^2 + s}{2m_Y^2} \times \frac{\Gamma_\sigma(s) m_\sigma}{\pi} \frac{p(s)}{(s - m_\sigma^2)^2 + (m_\sigma \Gamma_\sigma(s))^2}, \tag{31}$$

with $p(s)$ given by

$$p(s) = \frac{\sqrt{\lambda(m_Y^2, m_\psi^2, s)}}{2m_Y}, \tag{32}$$

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$, and $\Gamma_\sigma(s)$ is the s-dependent width of an off-shell σ meson [45]:

$$\Gamma_\sigma(s) = \Gamma_{0\sigma}\sqrt{\frac{\lambda(s, m_\pi^2, m_\pi^2)}{\lambda(m_\sigma^2, m_\pi^2, m_\pi^2)}}\frac{m_\sigma^2}{s}, \tag{33}$$

where $\Gamma_{0\sigma}$ is the experimental value for the decay of the σ meson into two pions. Its value is $\Gamma_{0\sigma} =$ (0.4–0.7) GeV [44].

The invariant amplitude squared can be obtained from the matrix element in Eq. (24). We get:

$$|\mathcal{M}|^2 = g_{Y\psi\sigma}^2(s) f(m_Y, m_\psi, s), \tag{34}$$

where $g_{Y\psi\sigma}(s)$ is the form factor in the vertex $YJ/\psi\sigma$, given in Eq. (28) using $s = -Q^2$, and

$$f(m_Y, m_\psi, s) = \frac{1}{3}\left(m_Y^2 m_\psi^2 + \frac{1}{2}(m_Y^2 + m_\psi^2 - s)^2\right).$$

Therefore, the decay width for the process $Y(4260) \to J/\psi\pi\pi$ is given by

$$\Gamma = \frac{m_\sigma}{16\pi^2 m_Y^4} I, \tag{35}$$

where we have defined

$$I = \int_{(2m_\pi)^2}^{(m_Y - m_\psi)^2} ds\, g_{Y\psi\sigma}^2(s)\Gamma_\sigma(s)(m_Y^2 - m_\psi^2 + s) \times f(m_Y, m_\psi, s)\frac{p(s)}{(s - m_\sigma^2)^2 + (m_\sigma\Gamma_\sigma(s))^2}. \tag{36}$$

Hence, taking variations on s_0, θ, $\Gamma_{0\sigma}$, and m_σ in the same intervals given above, we obtain from Eqs. (30)–(35) the following value for the decay width

$$\Gamma_\sigma(Y \to J/\psi\pi\pi) = (1.0 \pm 0.4)\ \text{MeV}. \tag{37}$$

The considered decay can also proceed through the $f_0(980)$ intermediate state. In order to estimate the decay width through this intermediate state, we have to determine the coupling constant associated with the vertex $Y \to J/\psi f_0(980)$. Therefore, we have to evaluate the vertex function given in Eq. (19) with the σ meson current replaced by the interpolating current for the $f_0(980)$ meson. Similarly to what was done for the σ intermediate state, we also consider a simple quark-antiquark current to describe the $f_0(980)$. In particular, we consider the $f_0(980)$ as a quark-antiquark state with a mixture of strange and light components. Hence, the interpolating current for $f_0(980)$ is given by

$$j^{f_0} = \cos(\alpha)\bar{s}s + \frac{\sin(\alpha)}{\sqrt{2}}(\bar{u}u + \bar{d}d). \tag{38}$$

This current was used in Refs. [47–49] to study different hadronic D decays into $f_0(980)$. The angle used in these references was $\alpha \approx 37°$, which we are using here.

Since the interpolating field for the $Y(4260)$ in Eq. (4) has no strange quarks, only the light component of the current in Eq. (38) contributes to the vertex function. Comparing the currents in Eqs. (38) and (21), we see that the light part of them differ only by the factor $\sin(\alpha)$. Therefore, the OPE side of the sum rules is just what we have in Eq. (27) multiplying by $\sin(\alpha)$.

In the phenomenological side we have only to replace m_σ by m_{f_0} and A_σ by A_{f_0}, where $A_{f_0} = \langle 0|j^{f_0}|f_0(980)\rangle$ was determined in Ref. [49] and its value is $A_{f0} = (0.19 \pm 0.02)\ \mathrm{GeV}^2$. We are using $m_{f_0} = (990 \pm 20)$ MeV [44].

In Table III, we show the coupling constant values, $g_{Y\psi f_0(980)}$, and its corresponding form factor, calculated for different values of $\sqrt{s_0}$.

We can now estimate the decay width for the process $Y \to J/\psi\pi\pi$, considering that the two pions in the final state come from the $f_0(980)$ meson. Using Eq. (35) with f_0 parameters instead of σ ones, which means $m_{f_0} = (990 \pm 20)$ MeV and $\Gamma_{0f_0} = (40\text{–}100)$ MeV [44], and taking variations in $4.6 \le \sqrt{s_0} \le 4.8$ GeV and $52.3° \le \theta \le 53.5°$, we obtain

$$\Gamma_{f_0}(Y \to J/\psi\pi\pi) = (3.1 \pm 0.2)\ \mathrm{MeV}, \tag{39}$$

leading to the following decay width into this channel:

$$\Gamma(Y \to J/\psi\pi\pi) = (4.1 \pm 0.6)\ \mathrm{MeV}, \tag{40}$$

which is consistent with the lower bound given in Ref. [5]: $\Gamma(Y \to J/\psi\pi\pi) > 508$ keV at 90% CL.

In addition, we can also give an estimate of the decay width in the channel $Y \to J/\psi KK$, that has also been observed. In order to do this, we substitute in Eq. (33) the σ and π parameters by the $f_0(980)$ and K mesons parameters. Using $m_K = (493.677 \pm 0.016)$ MeV [44] and Eq. (35) with the form factors $g_{Y\psi f_0}$ listed in Table III and taking variations on $\sqrt{s_0}$, θ, and Γ_{0f_0} in the ranges given above, we get for the decay width $Y \to J/\psi KK$:

$$\Gamma_{f_0}(Y \to J/\psi KK) = (1.3 \pm 0.4)\ \mathrm{MeV}. \tag{41}$$

TABLE III. Coupling constant $g_{Y\psi f_0(980)}$ values and its corresponding form factors, for different values of $\sqrt{s_0}$.

$\sqrt{s_0}$ (GeV)	$g_{Y\psi f_0}(Q^2)$ (GeV^{-1})	$g_{Y\psi f_0}(Q^2 = -m_{f_0}^2)$ (GeV^{-1})
4.6	$\frac{0.28}{Q^2+2.06}$	0.26
4.7	$\frac{0.29}{Q^2+2.09}$	0.26
4.8	$\frac{0.29}{Q^2+2.12}$	0.26

V. SUMMARY AND CONCLUSIONS

In summary, we have used the QCDSR approach to study the two-point and three-point functions of the $Y(4260)$ state, by considering a mixed charmonium-tetraquark current. In the determination of the mass, we work with the two-point function at leading order in α_s and we consider the contributions from the condensates up to dimension 8. A very good agreement with the experimental value of the mass of the $Y(4260)$ is obtained for the mixing angle around $\theta \approx (53.0 \pm 0.5)°$.

To evaluate the width of the decay $Y(4260) \to J/\psi\pi\pi$, we work with the three-point function also at leading order in α_s and we consider the contributions from the condensates up to dimension five. We assume that the two pions in the final state come from the σ and $f_0(980)$ scalar mesons. The obtained value for the width is $\Gamma_{Y\to J/\psi\pi\pi} \approx (4.1 \pm 0.6)$ MeV, which is much smaller than the total experimental width: $\Gamma_{\mathrm{exp}} \approx (95 \pm 14)$ MeV [44].

To compare the decay width into the $J/\psi\pi\pi$ channel with the total width we have to consider other possible decay channels. With the mixed current, the main decay channel of the $Y(4260)$ should be into D mesons, mostly due to the charmonium part of the current, but also from the tetraquark part through quark rearrangement. Therefore, the total width of the $Y(4260)$ should be given by the sum of the partial widths of all these channels. Unfortunately, the approach used here does not allow us to evaluate the decay channels into D mesons, since one can only use the QCDSR approach to study properties of the low-lying state. Therefore, the charmonium part of the current can only be used to study the decay of J/ψ.

However, if one considers the experimental upper limits, from *BABAR* [11] and CLEO [50] collaborations, for the branching ratios

$$\frac{\mathcal{B}(Y(4260) \to X)}{\mathcal{B}(Y(4260) \to J/\psi\pi\pi)}, \tag{42}$$

where $X = D\bar{D}$, $D\bar{D}^*$, and $D^*\bar{D}^*$, one can see that the width obtained here, for the $J/\psi\pi\pi$ channel, is consistent with the total experimental width of the $Y(4260)$. Therefore, we conclude that it is a possibility to explain the $Y(4260)$ exotic state as a mixed charmonium-tetraquark state.

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APPENDIX: THE SPECTRAL DENSITIES FOR CHARMONIUM AND TETRAQUARK

Next, we list all the spectral densities that appear in Eqs. (12)–(14) for charmonium $\Pi_1^{22}(M_B^2)$, tetraquark $\Pi_1^{44}(M_B^2)$ state as well as the mixed terms $\Pi_1^{24}(M_B^2)$ and $\Pi_1^{42}(M_B^2)$. The contributions for the last two are equal, that is, $\Pi_1^{24}(M_B^2) = \Pi_1^{42}(M_B^2)$.

For the charmonium contribution, the spectral densities are written below [24]

$$\rho_{22}^{\rm pert}(s) = \frac{s\langle\bar{q}q\rangle^2}{2^3\pi^2}(1 + 2m_c^2/s)\sqrt{1 - 4m_c^2/s}, \tag{A1}$$

$$\Pi_{22}^{\langle G^2\rangle}(M_B^2) = -\frac{\langle g_s^2G^2\rangle\langle\bar{q}q\rangle^2}{3\cdot 2^6\pi^2}\int_0^1 d\alpha\left\{2 + \frac{m_c^2(1 - 7\alpha - 2\alpha^2)}{\alpha(1-\alpha)^2M_B^2} + \frac{4m_c^4}{M_B^4(1-\alpha)^3}\right\}e^{-\frac{m_c^2}{M_B^2\alpha(1-\alpha)}}. \tag{A2}$$

For the tetraquark we have

$$\rho_{44}^{\rm pert}(s) = -\frac{1}{3\cdot 2^{10}\pi^6}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{\rm min}}^{1-\alpha}\frac{d\beta}{\beta^3}F^3(1-\alpha-\beta)(2m_c^2(1-\alpha-\beta)^2 - 3F(1+\alpha+\beta)), \tag{A3}$$

$$\rho_{44}^{\langle\bar{q}q\rangle}(s) = 0, \tag{A4}$$

$$\begin{aligned}\rho_{44}^{\langle G^2\rangle}(s) = &-\frac{\langle g_s^2G^2\rangle}{3^2\cdot 2^{11}\pi^6}\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta^3}\times[2m_c^4\alpha(1-\alpha-\beta)^3 - 3m_c^2F(1-\alpha-\beta)\\ &\times(2\alpha^2 + \alpha(8+3\beta) + \beta(1+\beta) - 2) + +6F^2\beta(1-2\alpha-2\beta)],\end{aligned} \tag{A5}$$

$$\begin{aligned}\rho_{44}^{\langle\bar{q}Gq\rangle}(s) = &-\frac{\langle\bar{q}Gq\rangle}{3\cdot 2^7\pi^4}\left\{3m_c\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}\frac{d\alpha}{\alpha^2}\int_{\beta_{\rm min}}^{1-\alpha}\frac{d\beta}{\beta}F[\alpha^2 - \alpha(1+\beta) - 2\beta^2]\right.\\ &\left.+ m_s\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\left[16m_c^2 + 2H\left(\frac{1-\alpha}{\alpha}\right) - \int_{\beta_{\rm min}}^{1-\alpha}\frac{d\beta}{\beta}(m_c^2(9 - 3\alpha - 5\beta) + 7F)\right]\right\},\end{aligned} \tag{A6}$$

$$\rho_{44}^{\langle\bar{q}q\rangle^2}(s) = \frac{s\langle\bar{q}q\rangle^2}{3^2\cdot 2^4\pi^2}(1 - 16m_c^2/s)\sqrt{1 - 4m_c^2/s}, \tag{A7}$$

$$\rho_{44}^{\langle 8\rangle}(s) = -\frac{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}{3\cdot 2^5\pi^2}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\,\alpha(5 - 6\alpha), \tag{A8}$$

$$\begin{aligned}\Pi_{44}^{\langle 8\rangle}(M_B^2) = &-\frac{m_c^2\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}{3\cdot 2^4\pi^2}\\ &\times\int_0^1 d\alpha\left[\frac{\alpha^2 - 2m_c^2}{M_B^2\alpha(1-\alpha)}\right]e^{-\frac{m_c^2}{M_B^2\alpha(1-\alpha)}}.\end{aligned} \tag{A9}$$

Finally, for the mixed term we have

$$\rho_{24}^{\langle\bar{q}q\rangle}(s) = -\frac{s\langle\bar{q}q\rangle^2}{3\cdot 2^3\pi^2}(1 + 2m_c^2/s)\sqrt{1 - 4m_c^2/s}, \tag{A10}$$

$$\Pi_{24}^{\langle\bar{q}Gq\rangle}(M_B^2) = -\frac{m_c^2\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}{3\cdot 2^3\pi^2}\int_0^1\frac{d\alpha}{\alpha}e^{-\frac{m_c^2}{M_B^2\alpha(1-\alpha)}}. \tag{A11}$$

In all these expressions we have used the following definitions:

$$F = (\alpha + \beta)m_c^2 - \alpha\beta s, \tag{A12}$$

$$H = m_c^2 - \alpha(1-\alpha)s, \tag{A13}$$

and the integration limits are:

$$\alpha_{\rm min} = \frac{1 - \sqrt{1 - 4m_c^2/s}}{2}, \tag{A14}$$

$$\alpha_{\rm max} = \frac{1 + \sqrt{1 - 4m_c^2/s}}{2}, \tag{A15}$$

$$\beta_{\rm min} = \frac{\alpha m_c^2}{(s\alpha - m_c^2)}. \tag{A16}$$

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