Generalized scaling ansatz and minimal seesaw mechanism

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Generalized scaling in flavor neutrino masses M_{ij} ($i, j = e, \mu, \tau$) expressed in terms of θ_{SC} and the atmospheric neutrino mixing angle θ_{23} is defined by $M_{i\tau}/M_{i\mu} = -\kappa_i t_{23}$ ($i = e, \mu, \tau$) with $\kappa_e = 1$, $\kappa_\mu = B/A$ and $\kappa_\tau = 1/B$, where $t_{23} = \tan\theta_{23}$, $A = \cos^2\theta_{SC} + \sin^2\theta_{SC} t_{23}^4$ and $B = \cos^2\theta_{SC} - \sin^2\theta_{SC} t_{23}^2$. The generalized scaling ansatz predicts the vanishing reactor neutrino mixing angle $\theta_{13} = 0$. It is shown that the minimal seesaw mechanism naturally implements our scaling ansatz. There are textures satisfying the generalized scaling ansatz that yield vanishing baryon asymmetry of the Universe (BAU). Focusing on these textures, we discuss effects of $\theta_{13} \neq 0$ to evaluate a CP-violating Dirac phase δ and BAU and find that BAU is approximately controlled by the factor $\sin^2\theta_{13}\sin(2\delta - \phi)$, where ϕ stands for the CP-violating Majorana phase whose magnitude turns out to be at most 0.1.

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I. INTRODUCTION

The recent observation of the nonvanishing reactor mixing angle θ_{13} [1] opens a new window to clarify properties of CP violation in neutrino physics. CP violation occurs in neutrino oscillations [2] and in leptogenesis [3] based on the seesaw mechanism [4]. In the seesaw mechanism, neutrinos are almost Majorana particles generated by heavy Majorana neutrinos as heavy as $\mathcal{O}(10^{10})$ GeV and turn out to be extremely light so that they are compatible with experimental observations [5–9]. Effects of CP violation in the lighter Majorana neutrinos are characterized by phases of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) unitary matrix U_{PMNS} [10], which converts massive neutrinos $\nu_{1,2,3}$ into flavor neutrinos $\nu_{e,\mu,\tau}$. Three phases, one CP-violating Dirac phase δ and two CP-violating Majorana phases $\phi_{2,3}$, are involved in U_{PMNS} [2]. On the other hand, CP violation in leptogenesis is characterized by phases related to heavy Majorana neutrinos. These two types of CP violation are, in principle, independent of each other. However, they can be correlated if there are some constraints that reduce the number of degrees of freedom, which result in relating two different types of CP phases. It is known that the minimal seesaw mechanism utilizing two heavy neutrinos [11] involves three physical CP-violating phases in leptogenesis, which are equivalent to δ and $\phi_{2,3}$; therefore, CP violation in leptogenesis can be controlled by δ and $\phi_{2,3}$.

The observed $\sin^2\theta_{13}$ is found to be $\sin^2\theta_{13} \approx 0.025$ [12,13], close to $\sin^2\theta_{13} = 0$, which suggests a theoretical principle that $\sin^2\theta_{13}$ vanishes as the first approximation and that a certain perturbation induces nonvanishing $\sin^2\theta_{13}$. There have been various theoretical ideas that give $\sin^2\theta_{13} = 0$ [14–23]. Among others, the generalized scaling ansatz in flavor neutrino masses, which is an extended version of the scaling ansatz [22], is proposed to discuss a new aspect of neutrinos [23]. The generalized scaling is described by two angles, θ_{SC} and the atmospheric neutrino mixing angle θ_{23}, which provide the following scaling rule among Majorana flavor neutrino masses M_{ij} ($i, j = e, \mu, \tau$):

$$\frac{M_{i\tau}}{M_{i\mu}} = -\kappa_i t_{23} \quad (i = e, \mu, \tau), \tag{1}$$

where $t_{23} = \tan\theta_{23}$, $(\kappa_e, \kappa_\mu, \kappa_\tau) = (1, B/A, 1/B)$ and

$$A = \cos^2\theta_{SC} + \sin^2\theta_{SC} t_{23}^4, \quad B = \cos^2\theta_{SC} - \sin^2\theta_{SC} t_{23}^2. \tag{2}$$

It can be proved that Eq. (1) indices $\theta_{13} = 0$. The condition to obtain $\theta_{13} = 0$ consists of the following two relations [21]:

$$M_{e\tau} = -t_{23} M_{e\mu}, \tag{3}$$

$$M_{\tau\tau} = M_{\mu\mu} + \frac{1 - t_{23}^2}{t_{23}} M_{\mu\tau}, \tag{4}$$

where Eq. (1) turns out to satisfy these relations and the generalized scaling maintains $\theta_{13} = 0$. The angle θ_{SC} itself is defined from $M_{\mu\tau}/M_{\mu\mu} = -\kappa_\mu t_{23}$ to be [23]

$$\sin^2\theta_{SC} = \frac{c_{23}^2(M_{\mu\tau} + t_{23} M_{\mu\mu})}{(1 - t_{23}^2) M_{\mu\tau} + t_{23} M_{\mu\mu}}, \tag{5}$$

where $c_{23} = \cos\theta_{23}$.

In this article, we would like to demonstrate that the generalized scaling rule is naturally realized in the minimal seesaw mechanism and to discuss the creation of the baryon asymmetry of the Universe (BAU) via leptogenesis [24]. In Sec. II, some of the seesaw textures that satisfy the generalized scaling ansatz are found to yield the vanishing BAU. In these textures, it is expected that breaking effects of the generalized scaling ansatz initiate creating BAU and simultaneously inducing Dirac CP violation as a result of $\theta_{13} \neq 0$. In Sec. III, we describe leptogenesis based on the minimal seesaw mechanism and show theoretical

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arguments to make predictions on possible correlations between BAU and CP-violating phases. In Sec. IV, a numerical analysis is performed to estimate the sizes of BAU and of Dirac and Majorana CP violations, which will be compared with our theoretical predictions. The final section, Sec. V, is devoted to a summary.

II. SEESAW TEXTURES

The minimal seesaw mechanism introduces two heavy Majorana neutrinos $N_{1,2}$ into the standard model. We understand that a 2×2 heavy neutrino mass matrix M_R and a charged lepton mass matrix are transformed into diagonal and real ones. After the heavy neutrinos are decoupled, the minimal seesaw mechanism generates a symmetric 3×3 light neutrino mass matrix M_ν given by $M_\nu = -m_D M_R^{-1} m_D^T$, where m_D is a 3×2 Dirac neutrino mass matrix. We parametrize M_R by

$$M_R = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \quad (M_2 > M_1), \tag{6}$$

and m_D by

$$m_D = \begin{pmatrix} \sqrt{M_1}a_1 & \sqrt{M_2}b_1 \\ \sqrt{M_1}a_2 & \sqrt{M_2}b_2 \\ \sqrt{M_1}a_3 & \sqrt{M_2}b_3 \end{pmatrix}, \tag{7}$$

which results in

$$\begin{aligned} M_\nu &= \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix} \\ &= -\begin{pmatrix} a_1^2+b_1^2 & a_1a_2+b_1b_2 & a_1a_3+b_1b_3 \\ a_1a_2+b_1b_2 & a_2^2+b_2^2 & a_2a_3+b_2b_3 \\ a_1a_3+b_1b_3 & a_2a_3+b_2b_3 & a_3^2+b_3^2 \end{pmatrix}, \end{aligned} \tag{8}$$

where the minus sign in front of the mass matrix is discarded for later discussions. One of the masses of $\nu_{1,2,3}$ is required to vanish owing to $\det(M_\nu) = 0$.

To obtain the seesaw version of the generalized scaling ansatz, we describe the basic conditions on M_ν, Eqs. (3) and (4), in terms of seesaw mass parameters $a_{1,2,3}$ and $b_{1,2,3}$ and search their solutions in much the same way that Eq. (1) is derived. These conditions are readily converted into

$$(a_3 + t_{23}a_2)a_1 + (b_3 + t_{23}b_2)b_1 = 0, \tag{9}$$

$$(t_{23}a_3 - a_2)(a_3 + t_{23}a_2) + (t_{23}b_3 - b_2)(b_3 + t_{23}b_2) = 0. \tag{10}$$

The minimal seesaw mechanism that keeps θ_{13} vanished should satisfy Eqs. (9) and (10). The simpler solutions to Eqs. (9) and (10) can be either

(1) $a_3 + t_{23}a_2 = b_3 + t_{23}b_2 = 0$, leading to

$$m_D = \begin{pmatrix} \sqrt{M_1}a_1 & \sqrt{M_2}b_1 \\ \sqrt{M_1}a_2 & \sqrt{M_2}b_2 \\ \sqrt{M_1}(-t_{23}a_2) & \sqrt{M_2}(-t_{23}b_2) \end{pmatrix}, \tag{11}$$

or

(2) $a_1 = 0$, $t_{23}a_3 - a_2 = 0$ and $b_3 + t_{23}b_2 = 0$, leading to

$$m_D = \begin{pmatrix} 0 & \sqrt{M_2}b_1 \\ \sqrt{M_1}a_2 & \sqrt{M_2}b_2 \\ \sqrt{M_1}a_2/t_{23} & \sqrt{M_2}(-t_{23}b_2) \end{pmatrix}, \tag{12}$$

and a solution with $b_1 = 0$ is

$$m_D = \begin{pmatrix} \sqrt{M_1}a_1 & 0 \\ \sqrt{M_1}a_2 & \sqrt{M_2}b_2 \\ \sqrt{M_1}(-t_{23}a_2) & \sqrt{M_2}b_2/t_{23} \end{pmatrix}. \tag{13}$$

Although there are other solutions,[1] the above solutions suffice to show consistent results with the generalized scaling ansatz.

For Eq. (11), case 1, we find that

$$M_{e\tau} = -t_{23}(a_1a_2 + b_1b_2) = -t_{23}M_{e\mu}, \tag{14}$$

and

$$\begin{aligned} M_{\mu\mu} &= a_2^2 + b_2^2, \\ M_{\mu\tau} &= -t_{23}(a_2^2 + b_2^2), \\ M_{\tau\tau} &= t_{23}^2(a_2^2 + b_2^2), \end{aligned} \tag{15}$$

from which Eq. (5) leads to

$$\sin^2\theta_{\rm SC} = 0, \tag{16}$$

corresponding to the inverted mass hierarchy with $m_3 = 0$ [22]. On the other hand, for Eq. (12), case 2, we find that

$$M_{e\tau} = -t_{23}b_1b_2 = -t_{23}M_{e\mu}, \tag{17}$$

and

$$\begin{aligned} M_{\mu\mu} &= a_2^2 + b_2^2, \qquad M_{\mu\tau} = \frac{1}{t_{23}}a_2^2 - t_{23}b_2^2, \\ M_{\tau\tau} &= \frac{1}{t_{23}^2}a_2^2 + t_{23}^2b_2^2, \end{aligned} \tag{18}$$

from which Eq. (5) leads to

$$\sin^2\theta_{\rm SC} = \frac{(a_2/t_{23})^2}{(a_2/t_{23})^2 + (t_{23}b_2)^2}. \tag{19}$$

[1] A solution can be supplied by $b_1 = -t_{23}a_1$, $b_2 = t_{23}^2/a_3$ and $b_3 = a_2$ with $t_{23}^2 = 1$, which describes a μ-τ symmetric seesaw model [25].

Similarly, we obtain that

$$\sin^2\theta_{\rm SC} = \frac{(b_2/t_{23})^2}{(t_{23}a_2)^2 + (b_2/t_{23})^2}, \tag{20}$$

for Eq. (13). These definitions of $\sin^2\theta_{\rm SC}$, depending upon the types of seesaw textures, provide the seesaw version of Eq. (5).

Since the case with $m_3 = 0$ is described by Eq. (11), Eqs. (12) and (13) should describe the normal mass hierarchy with $m_1 = 0$. In other words, the inverted mass hierarchy with $m_3 = 0$ realized at $\theta_{13} \neq 0$ does not approach the ideal textures Eqs. (12) and (13) at $\theta_{13} = 0$. This is because, for $\theta_{13} \neq 0$, we obtain that, for an arbitrary parameter x,

$$M_{\mu\mu} + 2xM_{\mu\tau} + x^2 M_{\tau\tau} = (a_2 + xa_3)^2 + (b_2 + xb_3)^2, \tag{21}$$

as well as

$$\begin{aligned} M_{\mu\mu} &+ 2xM_{\mu\tau} + x^2 M_{\tau\tau} \\ &= (-c_{23}s_{12} - s_{23}c_{12}\tilde{s}_{13} + x(s_{23}s_{12} - c_{23}c_{12}\tilde{s}^*_{13}))^2\tilde{m}_1 \\ &\quad + (c_{23}c_{12} - s_{23}s_{12}\tilde{s}_{13} - x(s_{23}c_{12} + c_{23}s_{12}\tilde{s}^*_{13}))^2\tilde{m}_2, \end{aligned} \tag{22}$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ $(i, j = 1, 2, 3)$ for θ_{12} being the solar neutrino mixing angle, and $\tilde{s}_{13} = s_{13}e^{i\delta}$ and $\tilde{m}_a = m_a e^{-i\varphi_a}$ $(a = 1, 2, 3)$ for φ_a being Majorana phases.[2] At $x = 1/t_{23}$, Eq. (22) vanishes if $\theta_{13} = 0$; therefore, Eq. (21) vanishes as well. On the other hand, at $x = 1/t_{23}$, it is Eq. (11) that gives the vanishing of Eq. (21). At $\theta_{13} = 0$, Eq. (11) is, thus, derived. When $m_3 \neq 0$, such as in a seesaw mechanism with a 3×3 m_D, is taken, Eqs. (12) and (13) can describe the inverted mass hierarchy.

III. *CP* VIOLATION AND LEPTOGENESIS

Leptogenesis creates BAU whose estimate contains the factor $(m_D^\dagger m_D)_{12}$ [3] which turns out to be $a_1^* b_1 + a_2^* b_2 + a_3^* b_3$. It is found that BAU vanishes for the seesaw textures, Eqs. (12) and (13), which yield $a_1^* b_1 + a_2^* b_2 + a_3^* b_3 = 0$ [26]. If these textures of m_D are adopted, *CP* violation of leptogenesis and of the Dirac type for flavor neutrinos becomes active only if sources of $\theta_{13} \neq 0$ are present [27]. For the rest of the discussions, we focus our attention on these seesaw textures to discuss how the creation of BAU relates to *CP* violation for flavor neutrinos. We restrict ourselves to discussions based on Eq. (12), from which results from Eq. (13) can be obtained by the interchange of $a_{1,2,3} \leftrightarrow b_{1,2,3}$ unless otherwise specified.

To obtain $\theta_{13} \neq 0$ and the nonvanishing BAU, we include breaking terms of the generalized scaling ansatz, which are denoted by δa_3 and δb_3 to give

$$a_3 = \frac{a_2}{t_{23}} + \delta a_3, \qquad b_3 = -t_{23}b_2 + \delta b_3. \tag{23}$$

The angle $\theta_{\rm SC}$ is still defined by Eq. (5), and the $i = \tau$ part of the generalized scaling rule Eq. (1) gets broken according to

$$M_{\tau\tau} + \kappa_\tau t_{23} M_{\mu\tau} = (a_3 + t_{23}a_2)\delta a_3 + \left(b_3 - \frac{b_2}{t_{23}}\right)\delta b_3. \tag{24}$$

The nonvanishing BAU is generated owing to $a_1^* b_1 + a_2^* b_2 + a_3^* b_3 \neq 0$, which is calculated to be

$$a_1^* b_1 + a_2^* b_2 + a_3^* b_3 = \frac{1}{t_{23}} a_2^* \delta b_3 - t_{23}\delta a_3^* b_2 + \delta a_3^* \delta b_3. \tag{25}$$

On the other hand, the *CP*-violating Dirac phase δ, which is contained in a specific version of $U_{\rm PMNS}$ defined by the Particle Data Group [28], is estimated to be [29]

$$\begin{aligned} \delta = \arg\Big[&\left(\frac{1}{t_{23}} M^*_{\mu\tau} + M^*_{\mu\mu} + \delta M^*_{\tau\tau}\right)\delta M_{e\tau} \\ &+ M_{ee}\delta M^*_{e\tau} - t_{23}M_{e\mu}\delta M_{\tau\tau}\Big], \end{aligned} \tag{26}$$

for the normal mass hierarchy, where $\delta M_{e\tau}$ and $\delta M_{\tau\tau}$ calculated from

$$\begin{aligned} \delta M_{e\tau} &= M_{e\tau} + t_{23}M_{e\mu}, \\ \delta M_{\tau\tau} &= M_{\tau\tau} - \left(M_{\mu\mu} + \frac{1 - t_{23}^2}{t_{23}} M_{\mu\tau}\right), \end{aligned} \tag{27}$$

turn out to be

$$\begin{aligned} \delta M_{e\tau} &= b_1 \delta b_3, \\ \delta M_{\tau\tau} &= \frac{1 + t_{23}^2}{t_{23}}(a_2\delta a_3 - b_2\delta b_3) + (\delta a_3)^2 + (\delta b_3)^2. \end{aligned} \tag{28}$$

At the same time, θ_{13} is calculated by the following formula [30]:

$$\tan 2\theta_{13} = \frac{2c_{23}\delta M_{e\tau}}{(s_{23}^2 M_{\mu\mu} + c_{23}^2 M_{\tau\tau} + 2s_{23}c_{23}M_{\mu\tau})e^{i\delta} - M_{ee}e^{-i\delta}}. \tag{29}$$

We can further approximate Eqs. (25), (26), and (29) to see that the breaking δb_3 is a main source to start creating BAU and inducing the nonvanishing δ and θ_{13}. The normal mass hierarchy demands that $|M_{\mu\mu,\mu\tau,\tau\tau}| \gg |M_{ee,e\mu,e\tau}|$, which is equivalent to $|a_{2,3}^2| \gg |b_{1,2,3}^2|$. For the region where second order terms with respect to δa_3 and δb_3 are safely neglected, we obtain

$$a_1^* b_1 + a_2^* b_2 + a_3^* b_3 \approx \frac{1}{t_{23}} a_2^* \delta b_3, \tag{30}$$

[2] The *CP*-violating Majorana phase ϕ is defined by $\phi = \varphi_3 - \varphi_2$ for $m_1 = 0$ and $\phi = \varphi_2 - \varphi_1$ for $m_3 = 0$.

$$\delta \approx \arg(a_2^{*2} b_1 \delta b_3), \tag{31}$$

as well as

$$\tan 2\theta_{13} \approx 2c_{23}s_{23}^2 \left| \frac{b_1 \delta b_3}{a_2^2} \right|, \tag{32}$$

for $\sin^2\theta_{SC} \approx 1$. Therefore, we understand that the main source of CP violations and $\theta_{13} \neq 0$ is the single breaking term δb_3. It should be noted that, for the $b_1 = 0$ texture,

$$a_1^* b_1 + a_2^* b_2 + a_3^* b_3 \approx \frac{1}{t_{23}} b_2 \delta a_3^* \tag{33}$$

is derived in place of Eq. (30).

We describe various seesaw parameters to estimate BAU from leptogenesis based on the minimal seesaw mechanism. The recipes to calculate BAU are given as follows [31,32]:

(1) The heavy neutrinos are taken to satisfy the hierarchical mass pattern of $M_1 \ll M_2$, where the CP asymmetry from N_2 is washed out.

(2) The CP asymmetry from the decay of N_1 is given by the flavor-dependent ε^α ($\alpha = e, \mu, \tau$):

$$\varepsilon^\alpha = \frac{1}{8\pi v^2} \frac{\mathrm{Im}[(m_D^\dagger)_{1\alpha}(m_D)_{\alpha 2}(m_D^\dagger m_D)_{12}]}{(m_D^\dagger m_D)_{11}} \cdot f\left(\frac{M_2}{M_1}\right), \tag{34}$$

where $v \simeq 174$ GeV and

$$f(x) = x\left[1 - (1+x^2)\ln\left(\frac{1+x^2}{x^2}\right) + \frac{1}{1-x^2}\right], \tag{35}$$

leading to $f(x) \approx -3/2x$, for $x \gg 1$, which is the present case.

(3) The washout effect on ε^α is controlled by $\eta(m_{1\mathrm{eff}}^\alpha)$, which takes the form

$$\eta(x) = \left(\frac{8.25 \times 10^{-3}\ \mathrm{eV}}{x} + \left(\frac{x}{2 \times 10^{-4}\ \mathrm{eV}}\right)^{1.16}\right)^{-1}, \tag{36}$$

where

$$m_{1\mathrm{eff}}^\alpha = \frac{(m_D^\dagger)_{1\alpha}(m_D)_{\alpha 1}}{M_1} \tag{37}$$

represents an effective mass.

(4) For $10^9 \lesssim M_1 \lesssim 10^{12}$ to be taken as our adopted range of M_1, the created lepton asymmetry Y_L, which becomes flavor dependent, is calculated to be

$$Y_L \approx \frac{1}{g_*}\frac{12}{37}\left[(\varepsilon^e + \varepsilon^\mu)\eta\left(\frac{417}{589}(|a_1|^2 + |a_2|^2)\right) + \varepsilon^\tau \eta\left(\frac{390}{589}|a_3|^2\right)\right], \tag{38}$$

where g_* is the effective thermodynamical number of the relativistic degree of freedom that is estimated to be 106.75 for the standard model at a cosmic temperature greater than 300 GeV and

$$\begin{aligned}
\varepsilon^e &= -\frac{3M_1}{16\pi v^2}\frac{\mathrm{Im}[a_1^* b_1(a_1^* b_1 + a_2^* b_2 + a_3^* b_3)]}{|a_1|^2 + |a_2|^2 + |a_3|^2}, \\
\varepsilon^\mu &= -\frac{3M_1}{16\pi v^2}\frac{\mathrm{Im}[a_2^* b_2(a_1^* b_1 + a_2^* b_2 + a_3^* b_3)]}{|a_1|^2 + |a_2|^2 + |a_3|^2}, \\
\varepsilon^\tau &= -\frac{3M_1}{16\pi v^2}\frac{\mathrm{Im}[a_3^* b_3(a_1^* b_1 + a_2^* b_2 + a_3^* b_3)]}{|a_1|^2 + |a_2|^2 + |a_3|^2}.
\end{aligned} \tag{39}$$

The obtained Y_L is related to the baryon asymmetry Y_B: $Y_B \approx -0.54 Y_L$, and the final baryon-photon ratio $\eta_B (= (n_B - n_{\bar{B}})/n_\gamma)$ is estimated to be $\eta_B = 7.04 Y_B$.

To make theoretical predictions on η_B, let us choose the flavor-independent estimation of η_B, where η_B is proportional to $\mathrm{Im}[(a_1^* b_1 + a_2^* b_2 + a_3^* b_3)^2]$. To see the dependence of η_B on δ and ϕ, we evaluate δb_3 appearing in Eq. (30). For the normal mass hierarchy, using the relations

$$\begin{aligned}
&a_2 \approx \sigma_{\mu\mu}\sqrt{M_{\mu\mu}}, \quad a_3 \approx \sigma_{\tau\tau}\sqrt{M_{\tau\tau}} \quad b_1 = \sigma_{ee}\sqrt{M_{ee}}, \\
&b_2 = \frac{M_{e\mu}}{\sigma_{ee}\sqrt{M_{ee}}}, \quad b_3 = \frac{M_{e\tau}}{\sigma_{ee}\sqrt{M_{ee}}},
\end{aligned} \tag{40}$$

where $\sigma_{ee,\mu\mu,\tau\tau} = \pm 1$ and $\sigma_{\tau\tau} = \sigma_{\mu\mu}$ is required for δa_3 to vanish at $\theta_{13} = 0$, we find that Eq. (28) yields

$$\delta b_3 \approx \frac{\sigma_{ee} s_{13} c_{13}}{s_{12} c_{23}} \frac{e^{i\delta}\tilde{m}_3}{\sqrt{\tilde{m}_2}}, \tag{41}$$

from $\delta M_{e\tau}$ expressed in terms of $m_{2,3}$ [29]. As a result,

$$a_1^* b_1 + a_2^* b_2 + a_3^* b_3 \approx \sigma_{ee}\sigma_{\mu\mu}\frac{s_{13}c_{13}}{s_{12}}\sqrt{\frac{m_3}{m_2}} m_3 e^{i(\delta - \frac{\phi}{2})} \tag{42}$$

is derived from Eq. (30). Since $\eta_B \propto \mathrm{Im}[(a_1^* b_1 + a_2^* b_2 + a_3^* b_3)^2]$, we reach $\eta_B \propto \sin^2\theta_{13}\sin(2\delta - \phi)$, which is the relation for the $a_1 = 0$ texture. On the other hand, for the $b_1 = 0$ texture, we similarly find that $\eta_B \propto -\sin^2\theta_{13}\sin(2\delta - \phi)$ from Eq. (33). Including M_1, we conclude that

$$\eta_B \propto \xi_B M_1 \sin^2\theta_{13}\sin(2\delta - \phi) \tag{43}$$

serves as a good prediction of η_B, where $\xi_B = 1$ ($\xi_B = -1$) for the $a_1 = 0$ ($b_1 = 0$) texture.

We also derive the relation between δ and ϕ from the $i = \mu$ part of Eq. (1) equivalent to Eq. (5), which can be rephrased in terms of $m_{2,3}$ as follows:

$$\begin{aligned}
&(As_{23}c_{13}c_{23}c_{13} + Bt_{23}s_{23}^2 c_{13}^2)m_3 e^{-i\phi} \\
&\quad = [A(c_{23}c_{12} - s_{23}s_{12}\tilde{s}_{13}^*)(s_{23}c_{12} + c_{23}s_{12}\tilde{s}_{13}^*) \\
&\qquad - Bt_{23}(c_{23}c_{12} - s_{23}s_{12}\tilde{s}_{13}^*)^2]m_2.
\end{aligned} \tag{44}$$

Therefore, we find that $\phi = 0$ at $\theta_{13} = 0$. For $\theta_{13} \neq 0$, the right-hand side of Eq. (44) can be approximated to be $(1 - t_{23}t_{12}\tilde{s}_{13}^*)t_{23}^3 c_{12}^2 m_2$, from which we derive

$$\tan\phi \approx -t_{23}t_{12}s_{13}\sin\delta, \tag{45}$$

numerically leading to $\tan\phi \approx -0.1\sin\delta$ for the observed data. We expect that the magnitude of ϕ is at most 0.1.

Since we would like to discuss effects of the CP-violating Dirac phase δ on the creation of Y_L, we may consider the renormalization effects that modify the magnitude of δ when δ is promoted to Y_L. It has been discussed that the renormalization effect is rather insignificant for neutrinos in the normal mass hierarchy [33], where we reside now.

IV. NUMERICAL ANALYSIS

We perform a numerical calculation of η_B by adopting the following parameters obtained from neutrino oscillations [12]:

$$\Delta m^2_{21}[10^{-5}\ \mathrm{eV}^2] = 7.62 \pm 0.19, \quad \Delta m^2_{31}[10^{-3}\ \mathrm{eV}^2] = 2.55^{+0.06}_{-0.09}, \tag{46}$$

$$\sin^2\theta_{12} = 0.320^{+0.016}_{-0.017}, \quad \sin^2\theta_{23} = 0.427^{+0.034}_{-0.027}, \quad \sin^2\theta_{13} = 0.0246^{+0.0029}_{-0.0028}, \tag{47}$$

where $\Delta m^2_{ij} = m^2_i - m^2_j$ for m_i specifying a mass of ν_i ($i = 1, 2, 3$). There is another similar analysis that has reported the slightly smaller values of $\sin^2\theta_{23} = 0.365$–$0.410$ [13]. The created η_B should be consistent with the WMAP observed data [34] of

$$\eta_B = (6.2 \pm 0.15) \times 10^{-10}. \tag{48}$$

To study the dependence of η_B on θ_{13}, δ and ϕ, we use $Y_B^{(0)}$ as an appropriately normalized Y_B/M_1, which becomes

$$Y_B^{(0)} \propto \xi_B \sin^2\theta_{13}\sin(2\delta - \phi). \tag{49}$$

We have searched acceptable parameter regions by changing M_1 up to 10^{12} GeV for the $a_1 = 0$ texture and up to 5×10^{11} GeV for the $b_1 = 0$ texture to see how BAU is created and how BAU depends on θ_{13} and δ. The neutrino masses, mixing angles and η_B are constrained by their experimental data, Eqs. (46)–(48), unless they are specified. Our theoretical predictions, which have been obtained by using certain approximations, are to be compared with numerical results obtained without such approximations. The results of our numerical analysis are shown in Figs. 1–9:

(1) In Fig. 1(a) for $\sin^2\theta_{13} \le 0.04$, the angle θ_{SC} should satisfy $0.87 \lesssim \sin^2\theta_{SC} \lesssim 0.99$ to cover the observed range of Δm^2_{21}.

(2) Figure 1(b) for $\sin^2\theta_{13} \le 0.04$ shows to what extent the general scaling rule of $M_{\tau\tau}/M_{\mu\tau} = -\kappa_\tau t_{23}$ is satisfied, and this scaling rule turns out to be satisfied within 70%.

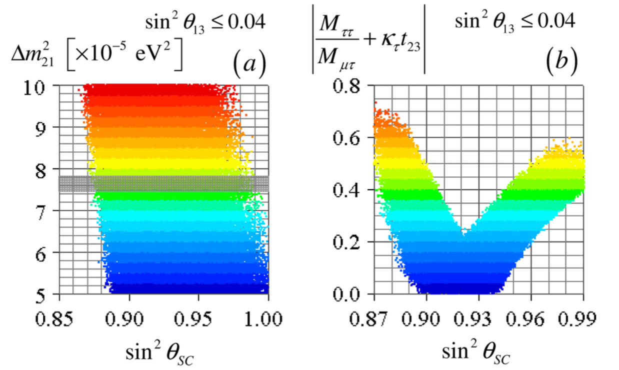

FIG. 1 (color online). Δm^2_{21} as a function of $\sin^2\theta_{SC}$, where the grey rectangle denotes the experimentally allowed region of Δm^2_{21}, and (b) $|(M_{\tau\tau}/M_{\mu\tau}) + \kappa_\tau t_{23}|$ as a function of $\sin^2\theta_{SC}$, where $\sin^2\theta_{SC}$ is restricted to reproduce the observed Δm^2_{21}, neither of which depends on the texture type.

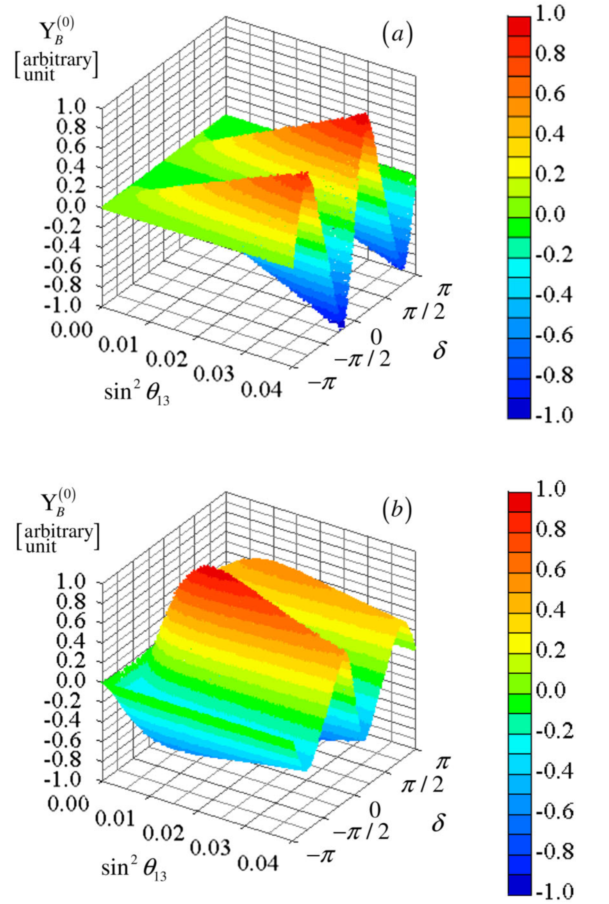

FIG. 2 (color online). $Y_B^{(0)}$, the appropriately normalized Y_B/M_1, as a function of $\sin^2\theta_{13}$ and δ for (a) the $a_1 = 0$ texture and (b) the $b_1 = 0$ texture.

(3) Figure 2 shows how $Y_B^{(0)}$, the appropriately normalized Y_B/M_1, evolves with $\sin^2\theta_{13}$ and δ:

(a) The feature that $Y_B^{(0)}$ increases as $\sin^2\theta_{13}$ increases appears for both textures, although it

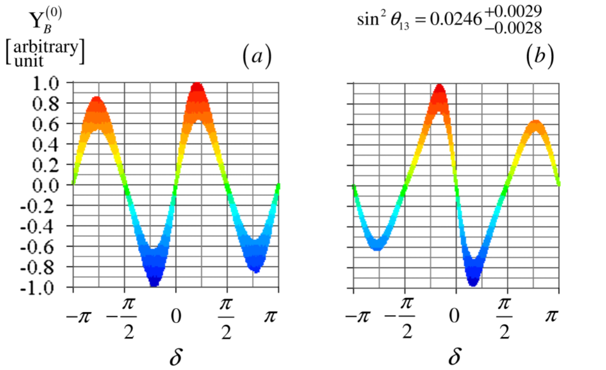

FIG. 3 (color online). $Y_B^{(0)}$, the appropriately normalized Y_B/M_1, as a function of δ for (a) the $a_1 = 0$ texture and (b) the $b_1 = 0$ texture.

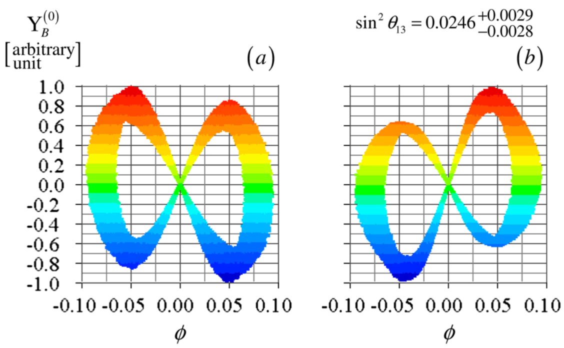

FIG. 4 (color online). The same as in Fig. 3 but for ϕ.

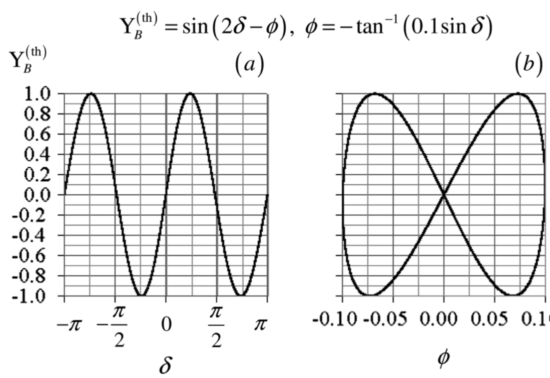

FIG. 5. $Y_B^{(\mathrm{th})} = \sin(2\delta - \phi)$ with $\phi = -\tan^{-1}(0.1\sin\delta)$ as a function of (a) δ and (b) ϕ for the $a_1 = 0$ texture, where figures for the $b_1 = 0$ texture are obtained by plotting $Y_B^{(\mathrm{th})} = -\sin(2\delta - \phi)$ with $\phi = -\tan^{-1}(0.1\sin\delta)$.

starts decreasing around $\sin^2\theta_{13} \approx 0.015$ for the $b_1 = 0$ texture, and the predicted proportionality of $Y_B^{(0)}$ to $\sin^2\theta_{13}$ in Eq. (49) is more visible for the $a_1 = 0$ texture.

(b) The oscillating $Y_B^{(0)}$ with δ is compatible with the prediction of Eq. (49), which is also compared with the results of Figs. 3 and 4.

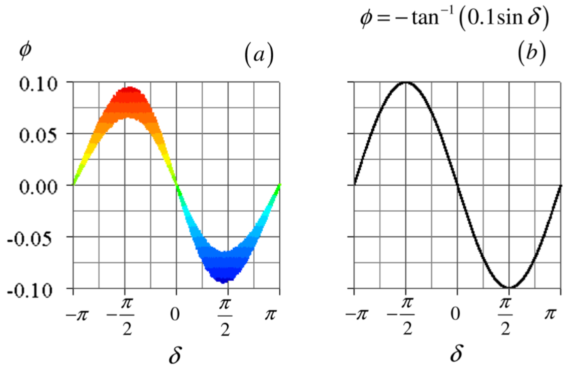

FIG. 6 (color online). ϕ as a function of δ for (a) numerical calculation of ϕ and (b) $\phi = -\tan^{-1}(0.1\sin\delta)$.

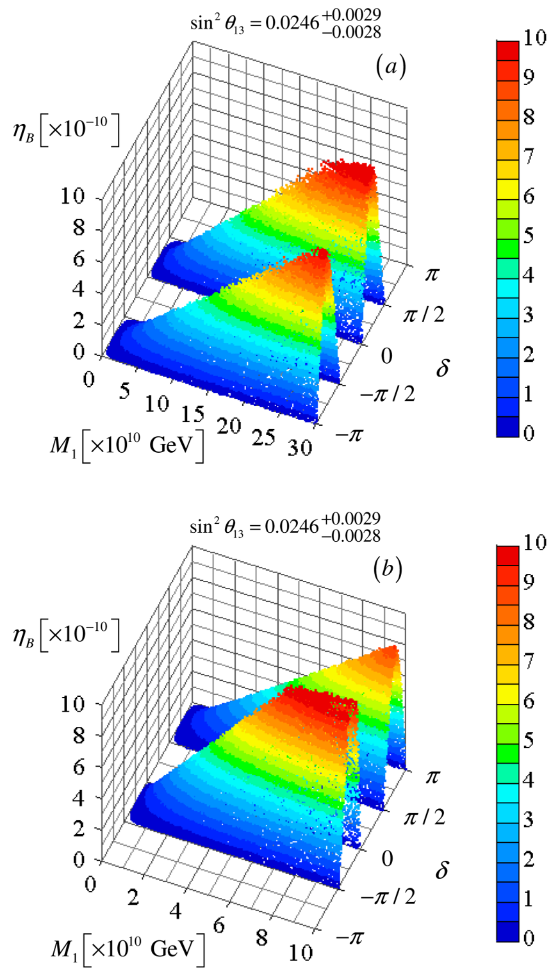

FIG. 7 (color online). η_B as a function of M_1 and δ for (a) the $a_1 = 0$ texture and (b) the $b_1 = 0$ texture.

(c) Leptogenesis starts creating BAU if $0 \lesssim \delta \lesssim \pi/2 \pmod{\pi}$ for the $a_1 = 0$ texture and if $-\pi/2 \lesssim \delta \lesssim 0 \pmod{\pi}$ for the $b_1 = 0$ texture.

(4) When θ_{13} is restricted to the observed values, Figs. 3 and 4 can be used to examine the oscillating

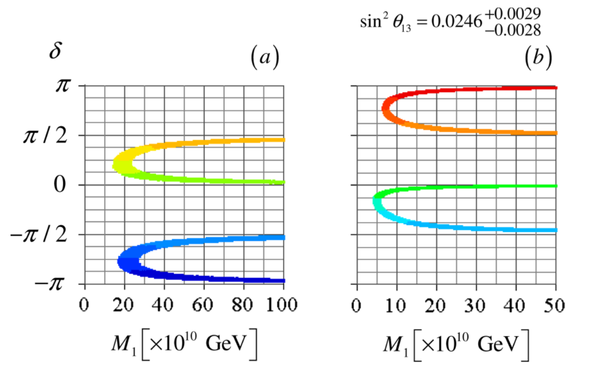

FIG. 8 (color online). δ as a function of M_1 to reproduce the observed η_B for (a) the $a_1 = 0$ texture and (b) the $b_1 = 0$ texture.

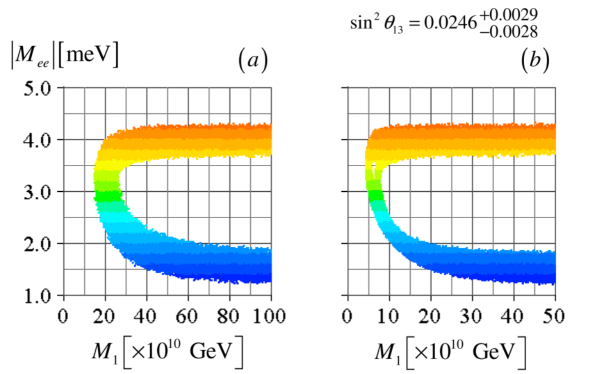

FIG. 9 (color online). The same as in Fig. 8 but for $|M_{ee}|$.

behavior found in Fig. 2. The gross features of the oscillating behavior of $Y_B^{(0)}$ in Fig. 3 and of the Lissajous-like behavior of $Y_B^{(0)}$ in Fig. 4 can be accounted for by the prediction of Eq. (49) as long as $\tan\phi \approx -0.1\sin\delta$ from Eq. (45) is used to calculate ϕ. Namely, two graphs of $Y_B^{(\mathrm{th})} = \sin(2\delta - \phi)$ with $\tan\phi = -0.1\sin\delta$ plotted in Fig. 5 for the $a_1 = 0$ texture depict similar shapes to those in Figs. 3(a) and 4(a). For the $b_1 = 0$ texture, $Y_B^{(\mathrm{th})} = -\sin(2\delta - \phi)$ with $\tan\phi = -0.1\sin\delta$ similarly accounts for the behavior of $Y_B^{(0)}$.

(5) Figure 6 compares the result of the calculated ϕ as a function of δ with our prediction of $\phi = -\tan^{-1}(0.1\sin\delta)$, where we understand that our prediction plotted in Fig. 6(b) can simulate ϕ well.

(6) The minimum value of M_1 to reproduce the observed η_B can be determined by Fig. 7 and, more explicitly, by Fig. 8 to be
   (a) 1.5×10^{11} GeV (1.8×10^{11} GeV) if $0 < \delta < \pi/2$ ($-\pi < \delta < -\pi/2$) for the $a_1 = 0$ texture;
   (b) 6.5×10^{10} GeV (4.5×10^{10} GeV) if $\pi/2 < \delta < \pi$ ($-\pi/2 < \delta < 0$) for the $b_1 = 0$ texture.

Since BAU inversely depends on $|a_1^2| + |a_2^2| + |a_3^2|$, a larger amount of η_B is expected for $|a_{1,2,3}^2| \ll |b_{2,3}^2|$ corresponding to the $b_1 = 0$ texture that certainly allows M_1 to take smaller values as in (b).

(7) When η_B and θ_{13} are consistent with the observed values, the correlation of δ with M_1 is shown in Fig. 8, where its behavior can also be explained by the prediction of $\eta_B \propto \xi_B M_1 \sin^2\theta_{13}\sin(2\delta - \phi)$ with $\tan\phi \approx -0.1\sin\delta$.
   (a) For the $a_1 = 0$ texture ($\xi_B = 1$), δ tends to approach 0 and $\pi/2$ as M_1 increases, and this behavior is consistent with the prediction of the factor $M_1\sin(2\delta - \phi)$ with $|\phi| \lesssim 0.1$, which tends to stay at the appropriate value corresponding to the observed η_B and which requires that $|\sin(2\delta - \phi)|$ gets smaller as M_1 gets larger and either $\delta \approx 0$ or $\delta \approx \pi/2$ (mod π) is a target value.
   (b) For the $b_1 = 0$ texture ($\xi_B = -1$), the same reasoning leads to the behavior that δ tends to approach $\pi/2$ and π (mod π) as M_1 increases.
   (c) Near the threshold to start creating the observed η_B, δ points to the value such that $|\sin(2\delta - \phi)|$ with $|\phi| \lesssim 0.1$ is nearly maximal and $\delta \approx \pi/4$ (mod π) is selected for the $a_1 = 0$ texture as can be read off from Fig. 8(a), while $\delta \approx -\pi/4$ (mod π) is selected for the $b_1 = 0$ texture as in Fig. 8(b).

(8) $|M_{ee}|$ to be measured by neutrinoless double beta decay [35] is computed to show Fig. 9 when η_B and θ_{13} are consistent with the observed values, which describes the correlation of $|M_{ee}|$ with M_1. We find that
   (a) 1.2 meV $\lesssim |M_{ee}| \lesssim$ 4.0 meV;
   (b) $|M_{ee}|$ starting around 3 meV increases up to around 4 meV or decreases down to around 1.5 meV as M_1 increases.

The behavior of $|M_{ee}|$ is consistent with the known estimation of $|M_{ee}| = |c_{13}^2 s_{12}^2 m_2 e^{i(\phi - 2\delta)} + s_{13}^2 m_3|$ once the constraint that $M_1\sin 2\delta$ is nearly constant is taken into account. For the $a_1 = 0$ texture, at $\delta = 0$, $\delta = \pi/4$ and $\delta = \pi/2$ selected as the key values in list 6, the scales of 4.3 meV, 3.3 meV (around the threshold) and 1.7 meV can be, respectively, calculated for $\phi = 0$. Then, we expect that $|M_{ee}|$ starting around 3.3 meV increases toward 4.3 meV or decreases toward 1.7 meV as shown in Fig. 9(a). The same explanation is possible for the $b_1 = 0$ texture in Fig. 9(b).

V. SUMMARY

We have found minimal seesaw models compatible with the generalized scaling ansatz of Eq. (1). The angle θ_{SC} is determined to be $\sin^2\theta_{\mathrm{SC}} = c_{23}^2(M_{\mu\tau} + t_{23}M_{\mu\mu})/[(1 - t_{23}^2)M_{\mu\tau} + t_{23}M_{\mu\mu}]$, which satisfies $0.87 \lesssim \sin^2\theta_{\mathrm{SC}} \lesssim 0.99$, where Δm_{21}^2 can stay in the observed range. The first seesaw texture is described by

$$m_D = \begin{pmatrix} \sqrt{M_1}a_1 & \sqrt{M_2}b_1 \\ \sqrt{M_1}a_2 & \sqrt{M_2}b_2 \\ \sqrt{M_1}(-t_{23}a_2+\delta a_3) & \sqrt{M_2}(-t_{23}b_2+\delta b_3) \end{pmatrix}, \quad (50)$$

where $\sin^2\theta_{SC} = 0$ is derived. The second seesaw texture consists of

$$m_D = \begin{pmatrix} 0 & \sqrt{M_2}b_1 \\ \sqrt{M_1}a_2 & \sqrt{M_2}b_2 \\ \sqrt{M_1}(a_2/t_{23}+\delta a_3) & \sqrt{M_2}(-t_{23}b_2+\delta b_3) \end{pmatrix}, \quad (51)$$

where $\sin^2\theta_{SC} = (a_2/t_{23})^2/[(a_2/t_{23})^2 + (t_{23}b_2)^2]$, and

$$m_D = \begin{pmatrix} \sqrt{M_1}a_1 & 0 \\ \sqrt{M_1}a_2 & \sqrt{M_2}b_2 \\ \sqrt{M_1}(-t_{23}a_2+\delta a_3) & \sqrt{M_2}(b_2/t_{23}+\delta b_3) \end{pmatrix}, \quad (52)$$

where $\sin^2\theta_{SC} = (b_2/t_{23})^2/[(t_{23}a_2)^2 + (b_2/t_{23})^2]$. However, the second textures in the inverted mass hierarchy with $m_3 = 0$ cannot be connected to those at $\theta_{13} = 0$ but are connected to the first texture with either $a_1 = 0$ or $b_1 = 0$. Therefore, the second textures should yield the normal mass hierarchy.

It is demonstrated that BAU vanishes for the second textures in the exact scaling limit. For the $a_1 = 0$ texture, the onset of CP violations and $\theta_{13} \neq 0$ is signaled by the nonvanishing δb_3. Namely, BAU depends on $a_2^*\delta b_3$, the CP-violating Dirac phase is approximated to be $\arg(a_2^{*2}b_1\delta b_3)$, and θ_{13} is evaluated to give $\tan 2\theta_{13} \approx 2c_{23}s_{23}^2|b_1\delta b_3/a_2^2|$. A similar conclusion is derived for the $b_1 = 0$ texture.

Our main prediction is $\eta_B \propto M_1 \sin^2\theta_{13}\sin(2\delta - \phi)$ for the $a_1 = 0$ texture and $\eta_B \propto -M_1\sin^2\theta_{13}\sin(2\delta - \phi)$ for the $b_1 = 0$ texture, together with $\tan\phi \approx -t_{23}t_{12}s_{13}\sin\delta$. The features of η_B found in the numerical analysis based on flavor-dependent leptogenesis turn out to be consistent with our predictions based on the simplified flavor-independent one:

(1) The proportionality of η_B to $\sin^2\theta_{13}$ shows up and is more visible for the $a_1 = 0$ texture.
(2) When θ_{13} is constrained to be the observed value, the oscillating η_B with δ and ϕ is well observed for both textures and is consistent with the prediction once the relation of $\tan\phi \approx -0.1\sin\delta$ is included.

Leptogenesis starts creating a sufficient amount of BAU compatible with the WMAP observation if

(1) $M_1 \gtrsim 1.5 \times 10^{11}$ GeV with $0 < \delta < \pi/2$ and $M_1 \gtrsim 1.8 \times 10^{11}$ GeV with $-\pi < \delta < -\pi/2$ for the $a_1 = 0$ texture;
(2) $M_1 \gtrsim 6.5 \times 10^{10}$ GeV with $\pi/2 < \delta < \pi$ and $M_1 \gtrsim 4.5 \times 10^{10}$ GeV with $-\pi/2 < \delta < 0$ for the $b_1 = 0$ texture.

The M_1 dependence of δ to reproduce the observed η_B is determined by the constraint that $M_1\sin(2\delta - \phi)$ is nearly constant. For M_1 to initiate leptogenesis, $|\sin(2\delta - \phi)| \approx 1$ with $|\phi| \lesssim 0.1$ is required and δ is predicted to be near $\pi/4$ (mod π). On the other hand, for larger M_1, $\sin(2\delta - \phi) \approx 0$ is required to lead to $\delta \approx 0$, $\pi/2$ (mod π). These two values are smoothly connected to $\delta \approx \pi/4$ (mod π) in the intermediate range of M_1. For $|M_{ee}|$, it is found that 1.2 meV $\lesssim |M_{ee}| \lesssim 4.0$ meV. The M_1 dependence of $|M_{ee}|$, which is a function of $2\delta - \phi$, is understood in a similar way.

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