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$\Delta F = 1$ constraints on composite Higgs models with left-right parity

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We analyze the bounds on the spectrum of composite Higgs models that come from flavor observables, by means of simple two-site effective Lagrangians, which incorporate a custodial symmetry and a left-right parity, and which could also be adopted in further phenomenological studies on composite Higgs models. We derive, in particular, an important constraint on the masses of the (t_L, b_L) partners, which does not depend on the flavor structure of the sector beyond the Standard Model. This bound is obtained from the "infrared" contribution to $b \to s\gamma$ induced by the flavor-conserving effective vertex $Wt_R b_R$. We find that the presence of a custodial symmetry can play a role in protecting this effective coupling and, as a consequence, in attenuating the constraint, which, however, remains of the order of 1 TeV. In addition to this bound, we calculate the constraints from the "ultraviolet" contribution to $b \to s\gamma$, induced by loops of heavy fermions, and to ϵ'/ϵ_K.

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I. INTRODUCTION

A possible solution to the hierarchy problem is based on an analogy with the pion mass stabilization in QCD: the Higgs, similarly to the pion, might be a composite state, generated by a new strong dynamics; as such, its mass is not sensitive to radiative corrections above the compositeness scale, assumed to be of the order of the TeV scale. A further protection, that allows the Higgs to be naturally lighter than the other resonances, exists if the composite Higgs is also the pseudo-Goldstone boson of a spontaneously broken global symmetry [1]. A pseudo-Goldstone boson Higgs is expected to be light and, as such, in agreement with the indication from the LEP electroweak precision data. In this project we will reconsider the bounds on the spectrum of composite Higgs models (CHMs) that come from flavor observables, with a special focus on $b \to s\gamma$. Instead of considering a full theory, we will work in an effective description valid at low energy. In particular, we will refer to a "two-site" (TS) description [2,3], where two sectors that comprise the Higgs—the weakly coupled sector of the elementary fields and the composite sector—are linearly coupled to each other through mass-mixing terms [4]. After diagonalization, the elementary/composite basis rotates to the mass eigenstate one, made up of Standard Model (SM) and heavy states that are admixtures of elementary and composite modes. Heavier particles have larger degrees of compositeness: heavy SM particles, like the top, are more composite, while the light ones are almost elementary. In order for composite Higgs models to be compatible with LEP precision data, the presence of a custodial symmetry in the composite sector is strongly suggested to avoid large corrections to the ρ parameter. The absence of large flavor-changing neutral currents is achieved instead by a sort of Glashow-Iliopoulos-Maiani mechanism that naturally emerges when the connection between the elementary and the strong sector proceeds via linear couplings [5]. In the absence of a symmetry protection, the LEP data also point toward a small degree of compositeness of the left-handed bottom quark (small corrections to $Z\bar{b}_L b_L$) and, by gauge invariance, of the left-handed top quark as well. This implies that, in order to obtain a heavy enough top quark, it is necessary to have an almost fully composite right-handed top quark. It has been shown, however, that the corrections to $Z\bar{b}_L b_L$ can be suppressed if the custodial symmetry of the strong sector includes a left-right parity [6]. This can allow for a smaller right-handed top compositeness. In order to study the phenomenology at energies lower than the compositeness scale, we derive two different models which incorporate a custodial symmetry and a left-right parity. We label these models TS5 and TS10. They describe the low-energy regime of the minimal composite Higgs models (MCHMs) defined in Refs. [7,8], in the limit in which only the leading terms in an expansion in powers of the Higgs field are retained.[1] In MCHMs, the Higgs arises as the pseudo-Goldstone boson associated to the $SO(5) \to O(4)$ breaking in the composite sector; where $O(4)$ includes $SO(4) \sim SU(2)_L \times SU(2)_R$, as well as a parity P_{LR} which exchanges $SU(2)_L$ with $SU(2)_R$. Composite fermions can be embedded in a $5 = (2,2) + (1,1)$ representation of $SO(5)$ in the TS5 model and in a $10 = (2,2) + (1,3) + (3,1)$ in the TS10. TS5 and TS10 extend the two-site description of Refs. [2,3] to consider five and ten $SO(5)$ representations for composite fermions. In particular, the TS5 model extends the two-site model of Ref. [3] to include the composite fermions needed to give mass to the bottom quark.

[1]See Ref. [9] for two- and three-site effective theories where the full Higgs nonlinearities are included.

We find two important bounds on the masses of the heavy fermions which do not depend on the flavor structure of the sector beyond the SM (BSM). The first comes from the measurement of the $Zb_L\bar{b}_L$ coupling, that we already mentioned and that can be suppressed assuming a P_{LR} symmetry. The second is obtained from the infrared (IR) contribution to $b \to s\gamma$ induced by the flavor-conserving effective vertex Wt_Rb_R. In composite Higgs models there are two classes of effects that lead to a shift of the $b \to s\gamma$ decaying rate compared to the SM prediction: Loops of heavy fermion resonances from the strong sector give an ultraviolet (UV) local contribution; they generate, at the compositeness scale, the flavor-violating dipole operators $\mathcal{O}_7$ and $\mathcal{O}_7'$, which define the effective Hamiltonian for the $b \to s\gamma$ decay. The virtual exchange of heavy resonances also generates the effective $V + A$ interaction of the W boson and the SM quarks, Wt_Rb_R, which in turn leads to a shift to $b \to s\gamma$ via a loop of SM particles. This latter IR contribution is enhanced by a chiral factor m_t/m_b and, since in this case the flavor violation comes entirely from the SM $V - A$ current, $\bar{t}_L\gamma^\mu s_L$, it gives a minimal flavor-violating (MFV) lower bound on the heavy fermion masses. We also discuss the role of a parity P_C, which is a subgroup of the custodial $SU(2)_V$, to protect the effective coupling Wb_Rt_R.

In general, stronger bounds can be obtained from the UV CHM contribution to $b \to s\gamma$ and from ϵ'/ϵ_K [10]; however, these latter bounds are model dependent and in principle could be loosened by acting on the New Physics (NP) flavor structure (see, for example, Ref. [11]). The bound from the IR contribution to $b \to s\gamma$, on the other hand, is robust, since it is a MFV effect.

The paper is organized as follows: In Sec. II we introduce our two-site models; in Sec. III we discuss the bound from $b \to s\gamma$. We first calculate the MFV bounds from the infrared contribution in a generic CHM, by naive dimensional analysis (NDA), and in the specific TS5 and TS10 models; we then proceed to calculate the non-MFV constraints from $b \to s\gamma$ and from ϵ'/ϵ_K. We draw our conclusions in Sec. IV.

II. EFFECTIVE THEORIES FOR COMPOSITE HIGGS MODELS

The idea behind composite Higgs models is that the electroweak symmetry breaking may be triggered by a new strong dynamics, in analogy with the chiral symmetry breaking in QCD. In these theories a new strong sector couples to a weakly coupled sector, which coincides with that of the Standard Model without the Higgs. The Higgs, as the pion in QCD, is a composite state coming from the latter strong dynamics. Its composite nature allows for a solution to the hierarchy problem. Indeed, its mass is not sensitive to radiative corrections above the compositeness scale, assumed to be of the order of the TeV scale. The electroweak-symmetry-breaking (EWSB) is transmitted to SM fermions by means of linear couplings [4] (generated by some UV physics at the UV scale Λ_{UV}) between elementary fermions ψ and composite fermions:

$$\Delta\mathcal{L} = \lambda\bar{\psi}\mathcal{O} + \mathrm{H.c.} \tag{1}$$

This way to communicate the EWSB can give a natural explanation of the hierarchies in the quark masses (through RG evolution of the elementary/composite couplings λ_i) and avoid the tension which occurs when trying to generate large enough quark masses and, at the same time, suppress FCNC processes.[2] As a consequence of linear couplings, a scenario of *Partial Compositeness* of the SM particles emerges. At energies below the compositeness scale, a composite operator $\mathcal{O}$ can excite from the vacuum a tower of composite fermions of increasing mass. Linear couplings [Eq. (1)] thus turn into mass-mixing terms between elementary fermions and towers of composite fermions χ_n:

$$\langle 0|\mathcal{O}|\chi_n\rangle = \Delta_n, \qquad \mathcal{L}_{\mathrm{mix}} = \sum_n \Delta_n(\bar{\psi}\chi_n + \mathrm{H.c.}), \tag{2}$$

$$\mathcal{L} = \mathcal{L}_{\mathrm{el}} + \mathcal{L}_{\mathrm{com}} + \mathcal{L}_{\mathrm{mix}}. \tag{3}$$

Because of the mass-mixing terms, the physical eigenstates, made up of SM and (new) heavy states, are an admixture of elementary and composite modes.

The low-energy phenomenology of such theories can be exhaustively studied, and calculation can be made easier, by considering a truncation of each tower of composite fermions to the first resonance, while other heavy states are neglected [2]. For example, the effective Lagrangian describing one elementary chiral field ψ_L and its composite partner χ is

$$\Delta\mathcal{L} = \bar{\psi}_L i\not{\partial}\psi_L + \bar{\chi}(i\not{\partial} - m_*)\chi - \Delta_L\bar{\psi}_L\chi_R + \mathrm{H.c.} \tag{4}$$

We can rotate the fermions from the elementary/composite basis to the mass eigenstate one, the light(SM)/heavy basis, according to

$$\tan\varphi_L = \frac{\Delta_L}{m_*}\begin{cases}|\mathrm{light}\rangle = \cos\varphi_L|\psi_L\rangle - \sin\varphi_L|\chi_L\rangle \\ |\mathrm{heavy}\rangle = \sin\varphi_L|\psi_L\rangle + \cos\varphi_L|\chi_L\rangle\end{cases}. \tag{5}$$

Our eigenstate fields are thus a heavy fermion of mass $m = \sqrt{m_*^2 + \Delta_L^2}$ and a light fermion, to be identified with the SM field, that will acquire a mass after the EWSB. These fields, as we see, are superpositions of elementary and composite states. The angle φ_L parametrizes the degree of compositeness of the physical fields. In particular, the SM fermion has a $\sin\varphi_L \equiv \frac{\Delta_L}{\sqrt{m_*^2+\Delta_L^2}}$ degree of compositeness (and a $\cos\varphi_L \equiv \frac{m_*}{\sqrt{m_*^2+\Delta_L^2}}$ degree of elementarity); the mass-mixing parameter Δ_L can be naturally much smaller

[2]Tension that, instead, affects Technicolor and Extended Technicolor models.

than the mass m_* of the composite fermion[3]; therefore, SM fermions are in general mostly elementary with a small degree of compositeness, while heavy fermions are mostly composite with a small degree of elementarity. We have a similar rotation, with angle φ_R, in the case of right-handed fermions. SM fermions acquire a mass after the EWSB; since the origin of this breaking resides, by assumption, in the composite sector (the Higgs is a fully composite state), the SM fermion mass arises from the composite part of left-handed and right-handed SM fields:

$$m_\psi = Y_* \frac{v}{\sqrt{2}} \sin\varphi_L \sin\varphi_R, \tag{6}$$

where Y_* is a Yukawa coupling among composites, from which the SM Yukawa $y = Y_* \sin\varphi_L \sin\varphi_R$ originates. In the following we will assume that the strong sector is flavor anarchic, so that there is no large hierarchy between elements within each matrix Y_*, and the hierarchy in the masses and mixings of the SM quarks comes entirely from the hierarchy in the elementary/composite mixing angles (such an "anarchic scenario" has been extensively studied in the framework of 5D warped models; see Refs. [5,12–15]). From Eq. (6) we can see that heavier SM particles have larger degrees of compositeness: heavy SM particles, like the top, have to be quite composite, while the light ones are almost elementary.

Experimental data give hints on the type of the new strong dynamics responsible for the EWSB. The LEP precision data suggest the presence of a custodial symmetry in the composite sector to avoid large corrections to the ρ parameter. In order to protect ρ (or equivalently the Peskin-Takeuchi T parameter) the composite sector must respect, minimally, a global symmetry:

$$SU(2)_L \times SU(2)_R \times U(1)_X,$$

where $SU(2)_L \times SU(2)_R$ is broken to the diagonal $SU(2)_V$ after the EWSB; the unbroken $SU(2)_V$ invariance acts as a custodial symmetry so that $\rho = 1$ at tree level.

The SM electroweak group $SU(2)_L \times U(1)_Y$ can be embedded into $SU(2)_L \times SU(2)_R \times U(1)_X$, so that hypercharge is realized as $Y = T^3_R + X$. The composite Higgs transforms as a bidoublet (2, 2) under $SU(2)_L \times SU(2)_R$, $\mathcal{H} \equiv (H, H^c)$, where H is the composite Higgs doublet and $H^c = i\sigma^2 H^*$ is its conjugate. The $\mathcal{H}$ vacuum expectation value breaks the $SU(2)_L \times SU(2)_R \times U(1)_X$ group down to $SU(2)_V \times U(1)_X$ and leads to the EWSB. Therefore, we have the following relation among charges:

$$Q = T^3_L + T^3_R + X = T^3_L + Y. \tag{7}$$

This scheme can also result from models where the Higgs arises as the pseudo-Goldstone boson associated to a $SO(5) \to SO(4) \sim SU(2)_L \times SU(2)_R$ breaking in the composite sector; or to a $SO(5) \to O(4)$ breaking, where $O(4)$ includes $SO(4) \sim SU(2)_L \times SU(2)_R$ as well as a parity P_{LR} which exchanges $SU(2)_L$ with $SU(2)_R$. This enhanced custodial symmetry can suppress the corrections to the coupling $Z\bar{b}_L b_L$, which are strongly constrained by LEP data [6].

A. P_{LR} and P_C symmetries

In MCHMs [7], the Higgs arises as the pseudo-Goldstone boson associated to the $SO(5) \to O(4)$ breaking in the composite sector, where the enhanced custodial symmetry $O(4)$ includes $SO(4) \sim SU(2)_L \times SU(2)_R$ as well as a parity P_{LR}, which exchanges $SU(2)_L$ with $SU(2)_R$. As shown in Ref. [6], this P_{LR} parity, as well as the P_C symmetry, a subgroup of the custodial $O(4)$, can protect the coupling $Z\bar{b}_L b_L$ against large corrections from the composite sector. Each composite operator has a definite left and right isospin quantum number, $T_{L,R}$, and a third component, $T^3_{L,R}$. We can also univocally assign to each SM field definite quantum numbers, $T_{L,R}$, $T^3_{L,R}$, corresponding to those of the composite operator to which it couples. P_{LR} and P_C are symmetries of the composite sector, P_{LR} exchanges $SU(2)_L$ with $SU(2)_R$, and P_C is the subgroup of $SU(2)_V$ that transforms $|T_L, T_R; T^3_L, T^3_R\rangle \to |T_L, T_R; -T^3_L, -T^3_R\rangle$. [$SO(3)$ vectors transform with $P_C = \mathrm{diag}(1, -1, -1)$]. For P_{LR} (P_C) to also be a symmetry of the interacting terms between SM fields and composite operators, $\Delta\mathcal{L} = \lambda\bar{\psi}\mathcal{O} + \mathrm{H.c.}$, the SM fields ψ have to be eigenstates of P_{LR} (P_C). This implies

$$T_L = T_R \qquad (T^3_L = T^3_R) \qquad (P_{LR}\,\text{invariance}), \tag{8}$$

$$T^3_L = T^3_R = 0 \qquad (P_C\,\text{invariance}). \tag{9}$$

If the above formulas hold, we can see that the coupling $Z\psi\bar{\psi}$,

$$g_\psi = \frac{g}{\cos\theta_W}(Q^3_L - Q\sin^2\theta_W), \tag{10}$$

is protected against large corrections. Indeed, the electric charge Q is conserved, and the charge of the $SU(2)_L$ third component, Q^3_L, is conserved by custodial invariance plus P_{LR} symmetry and by P_C symmetry. By custodial $U(1)_V$ invariance, $\delta Q^3_V = \delta Q^3_R + \delta Q^3_L = 0$; if there is also a P_{LR} invariance, $\delta Q^3_R = \delta Q^3_L$, and therefore $\delta Q^3_L = 0$. The same conservation, $\delta Q^3_L = 0$, is obtained by P_C invariance: the SM W^3_L has an odd parity under P_C, $W^3_L \to -W^3_L$; if ψ is a P_C eigenstate it must have $T^3_L = T^3_R = 0$; then the current $\bar{\psi}\gamma^\mu\psi$ is even under P_C, and it cannot couple to W^3_L, which is odd.

We will show (Sec.) that the P_C symmetry can also protect in a similar way the effective coupling Wt_Rb_R and, as a consequence, it can be responsible for an attenuation of the bound on heavy fermion masses, coming from the process $b \to s\gamma$.

[3]As a result of RG evolution above the compositeness scale. The smallness of Δ parameters also allows for a sort of Glashow-Iliopoulos-Maiani mechanism that suppresses large flavor-changing neutral currents [5].

In what follows, we present the two-site models, TS5 and TS10, which incorporate a custodial symmetry and a P_{LR} parity.[4]

B. TS5

In the TS5 model, we consider composite fermions filling the following $SO(4)\times U(1)_X \sim SU(2)_L \times SU(2)_R \times U(1)_X$ representations:

$$\mathcal{Q}=\begin{bmatrix} T & T_{5/3} \\ B & T_{2/3}\end{bmatrix}=(2,2)_{2/3}, \quad \tilde{T}=(1,1)_{2/3},$$
$$\mathcal{Q}'_{-1/3}=\begin{bmatrix} B_{-1/3} & T' \\ B_{-4/3} & B'\end{bmatrix}=(2,2)_{-1/3}, \quad \tilde{B}=(1,1)_{-1/3}, \tag{11}$$

and the composite Higgs in

$$\mathcal{H}=\begin{bmatrix} \phi_0^\dagger & \phi^+ \\ -\phi^- & \phi_0\end{bmatrix}=(2,2)_0. \tag{12}$$

The $SO(4)$ multiplets of composite fermions can be embedded into the fundamentals $\mathbf{5_{2/3(-1/3)}}$ of $SO(5)\times U(1)_X$, that decompose as $\mathbf{5_{2/3(-1/3)}}=\mathbf{(2,2)_{2/3(-1/3)}}\oplus \mathbf{(1,1)_{2/3(-1/3)}}$ under $SU(2)_L\times SU(2)_R\times U(1)_X$. [See Ref. [17] for a study of the same representations in a two-site description of $SO(5)$]. We are thus introducing two classes of composite fermions: those filling a $\mathbf{5_{2/3}}$ representation, with X charge $X=2/3$, and those in a $\mathbf{5_{-1/3}}$, with $X=-1/3$. We want to consider, indeed, the possibility that the SM quark doublet (t_L, b_L) couples to two different BSM operators, $\mathcal{Q}_{2/3}$ and $\mathcal{Q}'_{-1/3}$: the first responsible for generating the top mass, the second for generating the bottom mass. (t_L, b_L) is linearly coupled to (T, B) through a mass-mixing term we call Δ_{L1}, and to (T', B') through a mass-mixing term Δ_{L2}. t_R and b_R couple respectively to $\tilde{T}$ through a mass-mixing term Δ_{R1}, and to $\tilde{B}$ through a mass-mixing term Δ_{R2}. The fermionic Lagrangian reads, in the elementary/composite basis:

$$\begin{aligned}\mathcal{L} &= \bar{q}^i_L i\not{\partial} q^i_L + \bar{u}^i_R i\not{\partial} u^i_R + \bar{d}^i_R i\not{\partial} d^i_R + \mathrm{Tr}\{\bar{\mathcal{Q}}(i\not{\partial} - M_{Q*})\mathcal{Q}\} \\ &\quad + \bar{\tilde{T}}(i\not{\partial} - M_{\tilde{T}*})\tilde{T} + Y_{*U}\mathrm{Tr}\{\bar{\mathcal{Q}}\mathcal{H}\}\tilde{T} \\ &\quad + \mathrm{Tr}\{\bar{\mathcal{Q}}'(i\not{\partial} - M_{Q'*})\mathcal{Q}'\} + \bar{\tilde{B}}(i\not{\partial} - M_{\tilde{B}*})\tilde{B} \\ &\quad + Y_{*D}\mathrm{Tr}\{\bar{\mathcal{Q}}'\mathcal{H}\}\tilde{B} - \Delta_{L1}\bar{q}^3_L(T,B) - \Delta_{R1}\bar{t}_R\tilde{T} \\ &\quad - \Delta_{L2}\bar{q}^3_L(T',B') - \Delta_{R2}\bar{b}_R\tilde{B} + \mathrm{H.c.},\end{aligned} \tag{13}$$

where the superscript i runs over the three SM families ($i=1, 2, 3$), with $q^3_L \equiv (t_L, b_L)$, $u^3 \equiv t_R$, $d^3 \equiv b_R$. By construction, the elementary fields couple to the composite ones only through the mass-mixing terms, shown in the last row of Eq. (13). This implies that the SM Yukawa couplings arise only through the coupling of the Higgs to the composite fermions and their mixings to the elementary fermions. We further assume that the strong sector is flavor anarchic, so that the hierarchy in the masses and mixings of the SM quarks comes from the hierarchy in the mixing parameters $\Delta^i_{L,R}$. In this case the mixing parameters of the light elementary quarks can be safely neglected, and one can focus on just the third generation of composite fermions.[5]

As a consequence of the elementary/composite mass mixings, the top and the bottom masses arise, after the EWSB, from the Yukawa terms in the Lagrangian [Eq. (13)], $Y_{*U}\mathrm{Tr}\{\bar{\mathcal{Q}}\mathcal{H}\}\tilde{T}$ and $Y_{*D}\mathrm{Tr}\{\bar{\mathcal{Q}}'\mathcal{H}\}\tilde{B}$. The top mass will be proportional to $\Delta_{L1}\Delta_{R1}$, and the bottom mass to $\Delta_{L2}\Delta_{R2}$. The small ratio between the bottom and the top quark masses can be thus obtained both for $\Delta_{L2} \ll \Delta_{L1}$ ($\Delta_{R2}\sim\Delta_{R1}$) and for $\Delta_{R2}\ll\Delta_{R1}$ ($\Delta_{L2}\sim\Delta_{L1}$).

For t_R, b_R and their excited states, the rotation from the elementary/composite basis to the mass eigenstate one, the SM/heavy basis, is given by

$$\tan\varphi_R=\frac{\Delta_{R1}}{M_{\tilde{T}*}}, \quad s_R\equiv\sin\varphi_R, \quad c_R\equiv\cos\varphi_R,$$
$$\tan\varphi_{bR}=\frac{\Delta_{R2}}{M_{\tilde{B}*}}, \quad s_{bR}\equiv\sin\varphi_{bR}, \quad c_{bR}\equiv\cos\varphi_{bR},$$
$$\begin{cases} t_R = c_R t^{\rm el}_R - s_R\tilde{T}^{\rm com}_R \\ \tilde{T}_R = s_R t^{\rm el}_R + c_R\tilde{T}^{\rm com}_R\end{cases}, \quad \begin{cases} b_R = c_{bR} b^{\rm el}_R - s_{bR}\tilde{B}^{\rm com}_R \\ \tilde{B}_R = s_{bR} b^{\rm el}_R + c_{bR}\tilde{B}^{\rm com}_R\end{cases}. \tag{14}$$

Here $s_R(s_{bR})$ defines the degree of compositeness, $\xi_{tR}(\xi_{bR})$, of $t_R(b_R)$; $c_R(c_{bR})$ defines that of $\tilde{T}(\tilde{B})$, $\xi_{\tilde{D}}$. We will diagonalize analytically the mixing among q^3_L and the corresponding excited states by requiring the simplifying assumption $\Delta_{L2}\ll\Delta_{L1}$, that can naturally follow, for example, from the RG flow in the full theory [8]. The first two generations of elementary quarks do not need a field rotation from the elementary/composite basis to the mass eigenstate basis, since they do not mix with the composite fermions and can thus be directly identified with the corresponding SM states.

We can see that in this model t_R and b_R are both P_C and P_{LR} eigenstates, since they couple to $SU(2)_L\times SU(2)_R$ singlets $[T_L(\tilde{T},\tilde{B}) = T_R(\tilde{T},\tilde{B}),\ T^3_L(\tilde{T},\tilde{B}) = T^3_R(\tilde{T},\tilde{B}) = 0]$. Instead, t_L is a P_{LR} eigenstate only in the limit ($\Delta_{L1}=0$) in which it decouples from T $[T^3_L(T)\neq T^3_R(T)]$. Similarly, b_L is a P_{LR} eigenstate only for $\Delta_{L2}=0$, in which case it decouples from B' $[T^3_L(B')\neq T^3_R(B')]$.

[4]The TS5 model has been already briefly described in Ref. [16], where it was adopted to study the phenomenology of heavy colored vectors at the LHC.

[5]In fact, once produced, heavy fermions of the first two generations will also decay mostly to tops and bottoms, since flavor-changing transitions are not suppressed in the strong sector, while the couplings to the light SM quarks are extremely small; see the discussion in Ref. [2].

So far we have made field rotations to the mass eigenstate basis before the EWSB. After the EWSB, the SM top and bottom quarks acquire a mass, and the heavy masses get corrections of order $(\frac{Y_* v}{\sqrt{2} m_*})^2$. In the following, we assume $x \equiv (\frac{Y_* v}{\sqrt{2} m_*}) \ll 1$ and compute all quantities at leading order in x.

1. $\Delta_{L2} \ll \Delta_{L1}$

In this case, since $\Delta_{L2} \ll \Delta_{L1}$, b_L is, approximately, a P_{LR} eigenstate, so approximately we have a custodial symmetry protection to $Zb_L\bar{b}_L$.

The small ratio between the bottom and the top quark masses is obtained for $\Delta_{L2} \ll \Delta_{L1}$ ($\Delta_{R2} \sim \Delta_{R1}$); we have

$$m_t = \frac{v}{\sqrt{2}} Y_{*U} s_1 s_R, \tag{15}$$

$$m_b = \frac{v}{\sqrt{2}} Y_{*D} s_2 s_{bR}, \tag{16}$$

where $s_1 = \sin\varphi_{L1} = \frac{\Delta_{L1}}{\sqrt{M_{Q*}^2 + \Delta_{L1}^2}}$ defines the (t_L, b_L) degree of compositeness ξ_{qL}, and s_2 is a rotation angle proportional to Δ_{L2}; $s_2 = \frac{\Delta_{L2}}{M_{Q'*}} \cos\varphi_{L1}$.

The physical masses of the heavy fermions read

$$\begin{cases} M_{\tilde{T}} = \sqrt{M_{\tilde{T}*}^2 + \Delta_{R1}^2} \\ M_{\tilde{B}} = \sqrt{M_{\tilde{B}*}^2 + \Delta_{R2}^2} \\ M_T = M_B = \sqrt{M_{Q*}^2 + \Delta_{L1}^2} \\ M_{T5/3} = M_{T2/3} = M_{Q*} = M_T c_1 \\ M_{T'} = M_{B'} = \sqrt{M_{Q'*}^2 + \Delta_{L2}^2} \simeq M_{Q'*} \\ M_{B-1/3} = M_{B-4/3} = M_{Q'*} \end{cases}, \tag{17}$$

where $c_1 \equiv \cos\varphi_{L1}$ is the degree of compositeness, ξ_D, of the $SU(2)_L$ doublet $D = (T, B)$. Details can be found in Appendix A, Section A 1.

In order for the strong sector to respect the custodial invariance, as we have shown, composite fermions have to fill multiplets of $SU(2)_L \times SU(2)_R \times U(1)_X$. As a consequence, the heavy partner of the SM doublet $q_L^3 = (t_L, b_L)$, $D = (T, B)$ ($= 2_{1/6}$ under the SM electroweak group), is embedded in a larger multiplet, the bidoublet $\mathcal{Q}_{2/3} = (2, 2)_{2/3}$, that includes another doublet of heavy fermions, $(T_{5/3}, T_{2/3})(= 2_{7/6})$. The heavy fermions $T_{5/3}$ and $T_{2/3}$ in this latter doublet are called *custodians*. They share the same multiplet as the heavy partners of q_L^3, but they do not mix directly with the SM fermions. This implies that their masses tend to zero in the limit in which t_L becomes fully composite. (See, for example, the discussion in Ref. [18]). This can be seen from Eq. (17): $M_{T5/3(2/3)}$ is zero for $c_1 = 0$, i.e., for a fully composite t_L ($s_1 = 1$).

C. TS10

In TS10, we consider composite fermions embedded into a $\mathbf{10_{2/3}}$ representation of $SO(5) \times U(1)_X$, that decomposes as $\mathbf{10_{2/3}} = \mathbf{(2,2)_{2/3}} \oplus \mathbf{(1,3)_{2/3}} \oplus \mathbf{(3,1)_{2/3}}$ under $SU(2)_L \times SU(2)_R \times U(1)_X$. Therefore, we refer to this field content in the composite sector:

$$\mathcal{Q}_{2/3} = \begin{bmatrix} T & T_{5/3} \\ B & T_{2/3} \end{bmatrix} = (2,2)_{2/3},$$

$$\tilde{\mathcal{Q}}_{2/3} = \begin{pmatrix} \tilde{T}_{5/3} \\ \tilde{T} \\ \tilde{B} \end{pmatrix} = (1,3)_{2/3},$$

$$\tilde{\mathcal{Q}}'_{2/3} = \begin{pmatrix} \tilde{T}'_{5/3} \\ \tilde{T}' \\ \tilde{B}' \end{pmatrix} = (3,1)_{2/3},$$

$$\mathcal{H} = \begin{bmatrix} \phi_0^\dagger & \phi^+ \\ -\phi^- & \phi_0 \end{bmatrix} = (2,2)_0, \tag{18}$$

and to the following fermionic Lagrangian in the elementary/composite basis:

$$\begin{aligned} \mathcal{L} = {} & \bar{q}_L^3 i\not{\partial} q_L^3 + \bar{t}_R i\not{\partial} t_R + \bar{b}_R i\not{\partial} b_R + \mathrm{Tr}\{\bar{\mathcal{Q}}(i\not{\partial} - M_{Q*})\mathcal{Q}\} \\ & + \mathrm{Tr}\{\bar{\tilde{\mathcal{Q}}}(i\not{\partial} - M_{\tilde{Q}*})\tilde{\mathcal{Q}}\} + \mathrm{Tr}\{\bar{\tilde{\mathcal{Q}}}'(i\not{\partial} - M_{\tilde{Q}*})\tilde{\mathcal{Q}}'\} \\ & + Y_* \mathrm{Tr}\{\mathcal{H}\bar{\mathcal{Q}}\tilde{\mathcal{Q}}'\} + Y_* \mathrm{Tr}\{\bar{\mathcal{Q}}\mathcal{H}\tilde{\mathcal{Q}}\} - \Delta_{L1}\bar{q}_L^3(T, B) \\ & - \Delta_{R1}\bar{t}_R\tilde{T} - \Delta_{R2}\bar{b}_R\tilde{B} + \mathrm{H.c.} \end{aligned} \tag{19}$$

We have the following expressions for the top and bottom masses:

$$m_t = \frac{v}{2} Y_* s_1 s_R, \qquad m_b = \frac{v}{\sqrt{2}} Y_* s_1 s_{bR}, \tag{20}$$

and for the heavy fermion physical masses:

$$\begin{cases} M_{\tilde{T}} = \sqrt{M_{\tilde{Q}*}^2 + \Delta_{R1}^2} \\ M_{\tilde{B}} = \sqrt{M_{\tilde{Q}*}^2 + \Delta_{R2}^2} = M_{\tilde{T}} c_R / c_{bR} \simeq M_{\tilde{T}} c_R \\ M_{\tilde{T}5/3} = M_{\tilde{T}'5/3} = M_{\tilde{T}'} = M_{\tilde{B}'} = M_{\tilde{T}} c_R \\ M_T = M_B = \sqrt{M_{Q*}^2 + \Delta_{L1}^2} \\ M_{T2/3} = M_{T5/3} = M_T c_1 \end{cases}. \tag{21}$$

More details can be found in Appendix A, Section A 2.

Besides the custodians $T_{5/3}$ and $T_{2/3}$, which are light in the case of a composite q_L^3, $\tilde{T}_{5/3}$ and the fermions in the $\tilde{\mathcal{Q}}'_{2/3}$ triplet become light for a t_R with a large degree of compositeness. ($\tilde{B}$ also becomes light in this case.) In this model, both t_R and b_R are not P_{LR} eigenstates, and only t_R is a P_C eigenstate, as a consequence of the couplings to $\tilde{\mathcal{Q}}$ $[T_L(\tilde{T}, \tilde{B}) \neq T_R(\tilde{T}, \tilde{B})]$; in particular, b_R is not a P_C eigenstate, since $T_R^3(\tilde{B}) \neq 0$. b_L is exactly a P_{LR} eigenstate.

D. $Zb_L\bar{b}_L$ in the TS models

Shifts in the Z coupling to b_L, g_{Lb} have been extensively studied in the literature. See, for example, the studies in the context of Randall-Sundrum models [19] and in two-site descriptions [20]. The shifts arise after the EWSB because of electroweak mixings among b_L and heavy fermions. There is also a contribution from the mixing among neutral gauge bosons; however, this mixing is of the order $(\frac{v}{M_*})^2 \ll 1$, where M_* stands for the heavy neutral boson mass, and we will neglect it in what follows.

In two-site models without P_{LR} symmetry, there is no custodial symmetry protection to $Zb_L\bar{b}_L$, and so the shift on g_{Lb} is large. Naive dimensional analysis [21] gives the following (see, for example, Refs. [22,23]):

$$\frac{\delta g_{Lb}}{g_{Lb}} \sim \frac{m_t^2}{M_{Q*}^2 s_R^2} \sim \frac{Y_*^2 v^2 s_1^2}{M_{Q*}^2}. \tag{22}$$

This formula has been obtained by approximating $q^2 = M_Z^2 \simeq 0$. At $q^2 = M_Z^2$, the shift receives $O(\frac{M_Z^2}{M_{Q*}^2})$ corrections:

$$\frac{\delta g_{Lb}}{g_{Lb}} \sim \frac{M_Z^2 s_1^2}{M_{Q*}^2} \sim \left(\frac{v^2 Y_*^2 s_1^2}{M_{Q*}^2}\right)\frac{g^2}{Y_*^2}. \tag{23}$$

When compared to Eq. (22), there is a suppression $(\frac{g}{Y_*})^2$ (see, for example, Ref. [24]), so we will neglect it in the following.

LEP and SLD experiments fix an upper bound of 0.25% for the (positive) shift in the g_{Lb} from its SM value. Therefore, from Eq. (22), we derive the following bound for the heavy fermion mass in models without custodial symmetry protection to $Zb_L\bar{b}_L$:

$$M_{Q*} \gtrsim (3.2)\frac{1}{s_R}\ \text{TeV}. \tag{24}$$

In order to respect this limit without requiring too-large heavy fermion masses that would contrast with naturalness arguments, it is necessary to have a quite composite right-handed top (i.e., a not-small s_R). On the contrary, in models with custodial symmetry protection to $Zb_L\bar{b}_L$, there is no such restriction for the t_R degree of compositeness, and bounds are weaker than the one in Eq. (24). Indeed, in the TS5 model with $\Delta_{L2} \ll \Delta_{L1}$, where we have approximately a custodial symmetry protection to $Zb_L\bar{b}_L$ (the breaking is proportional to Δ_{L2} and is thus small), we obtain

$$\frac{\delta g_{Lb}}{g_{Lb}} = \left(\frac{Y_* v}{\sqrt{2}}\right)^2\left(\frac{s_2 c_{bR}}{\sqrt{2}M_{\tilde{B}}}\right)^2[T_L^3(\tilde{B}) - T_L^3(b_L)]$$
$$= \frac{1}{2}\frac{m_b^2}{M_{Q*}^2}\frac{c_{bR}^4}{s_{bR}^2} \simeq \frac{1}{2}\frac{m_t^2}{M_{Q*}^2}\frac{s_2^2}{s_R^2}. \tag{25}$$

As expected, the shift is proportional to s_2^2 (i.e., it is proportional to Δ_{L2}^2, the size of the custodial symmetry breaking), and it is small (notice that is also smaller than the effect at nonzero momentum). In the TS10 model, we obtain, again, a small shift:

$$\frac{\delta g_{Lb}}{g_{Lb}} = \left(\frac{Y_* v}{\sqrt{2}}\right)^2\frac{s_1^2}{M_{Q*}^2}[c_{bR}^4(T_L^3(\tilde{B}) - T_L^3(b_L))$$
$$+ (T_L^3(B') - T_L^3(b_L))]$$
$$= -\frac{m_b^2}{M_{Q*}^2}\frac{2 - s_{bR}^2}{2} \simeq -\frac{m_b^2}{M_{Q*}^2}. \tag{26}$$

Despite b_L being an exact P_{LR} eigenstate in the TS10 model, there is still a small modification that comes from the coupling of b_R that explicitly breaks P_{LR}. Notice that $\delta g_{Lb} = 0$, if we have $s_{bR} = 0$.

III. BOUNDS FROM FLAVOR OBSERVABLES

A. Constraint from the process $b \to s\gamma$

We define, following Ref. [25], the effective Hamiltonian for $b \to s\gamma$:

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}}V_{ts}^* V_{tb}[C_7(\mu_b)\mathcal{O}_7 + C_7'(\mu_b)\mathcal{O}_7'], \tag{27}$$

where $\mathcal{O}_7 = \frac{e}{8\pi^2}m_b\bar{b}\sigma^{\mu\nu}F_{\mu\nu}(1-\gamma_5)s$ and $\mathcal{O}_7' = \frac{e}{8\pi^2}m_b\bar{b}\sigma^{\mu\nu}F_{\mu\nu}(1+\gamma_5)s$.

In the SM, the W boson has a purely $V - A$ interaction to the fermions, and so the contribution to the $b \to s\gamma$ process has to proceed through mass insertions in the external legs (see Fig. 1). The Wilson coefficient C_7' is thus negligible, because of a suppression by a factor m_s/m_b in respect to the Wilson coefficient C_7 that, evaluated at the weak scale μ_w, is [25]

$$C_7^{SM}(\mu_w) = \frac{1}{2}\left[\frac{(8x_t^3 + 5x_t^2 - 7x_t)}{12(1-x_t)^3} - \frac{x_t^2(2-3x_t)}{2(1-x_t)^4}\ln(x_t)\right], \tag{28}$$

with $x_t = \frac{m_t^2}{M_W^2}$.

In composite Higgs models, there are two classes of effects that lead to a shift of the $b \to s\gamma$ decaying rate compared to the Standard Model prediction. The first comes from loops of heavy fermion resonances from the strong sector that generate the flavor-violating dipole operators $\mathcal{O}_7$, $\mathcal{O}_7'$ at the compositeness scale. We will refer to

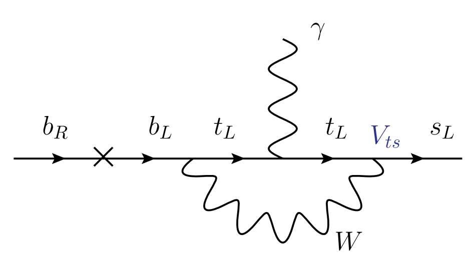

FIG. 1 (color online). One-loop infrared contribution to C_7 in the SM.

this as the UV contribution. The second contribution comes from the tree-level exchange of heavy resonances, which generates an effective $V + A$ interaction of the W boson and the SM quarks, which in turn leads to a shift to $b \to s\gamma$ via a loop of SM particles. This latter IR contribution is enhanced by a chiral factor m_t/m_b. Since in this case the flavor violation can come entirely from the SM $V - A$ current, it gives a quite model-independent lower bound on the heavy fermion masses.

By taking into account the experimental average value for the $b \to s\gamma$ branching ratio [26] and the theoretical calculation [27], we get, if the new physics contributions to C_7, $C_7^{\rm CH}$ and to C_7', $C_7'^{\rm CH}$ are considered separately, the following bounds (see Appendix B):

$$-0.098 \lesssim C_7^{\rm CH}(m_*) \lesssim 0.028, \tag{29}$$

$$|C_7'^{\rm CH}(m_*)| \lesssim 0.37, \tag{30}$$

where m_* denotes the mass of the heavy fermions in the loop (taking $m_* = 1$ TeV).

The infrared contribution to $b \to s\gamma$ from the composite Higgs model is at the weak scale μ_w instead of m_* (taking $\mu_W = M_W$); therefore, we have to account for a scaling factor

$$\begin{aligned} C_7^{\rm CH}(\mu_w) &= \left[\frac{\alpha_s(m_*)}{\alpha_s(m_t)}\right]^{16/21}\left[\frac{\alpha_s(m_t)}{\alpha_s(\mu_w)}\right]^{16/23} \\ C_7^{\rm CH}(m_*) &\approx 0.79 C_7^{\rm CH}(m_*). \end{aligned} \tag{31}$$

We get

$$-0.077 \lesssim C_7^{\rm CH}(\mu_w) \lesssim 0.023, \tag{32}$$

$$|C_7'^{\rm CH}(\mu_w)| \lesssim 0.29. \tag{33}$$

While the infrared contribution to C_7 involves a flavor-conserving operator and brings us to a MFV bound, the infrared contribution to C_7' as well as the ultraviolet contributions to C_7 and to C_7' involve flavor-violating operators. As a consequence, they will require some assumptions on the flavor structure of the NP sector.

We will now evaluate the bounds on heavy masses that come from the infrared contribution to C_7. We will first present estimates of such bounds in generic composite Higgs models, which can be obtained by NDA. Then we will calculate the bounds in the specific two-site models TS5 and TS10, introduced in Secs. II B and II C.

B. MFV bound from the infrared contribution to C_7

The infrared contribution to the process $b \to s\gamma$ is a one-loop contribution from the W boson accompanied by top quarks, where a mass insertion in the intermediate top quark states is allowed by the presence of a $(V + A)$ interaction of the W boson with the top and the bottom quarks (Fig. 2). This interaction originates from a term

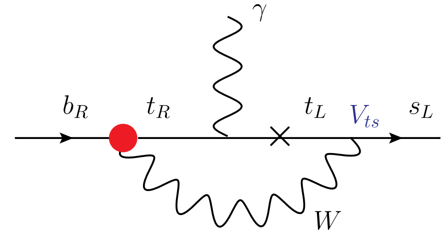

FIG. 2 (color online). One-loop infrared contribution to C_7. The red blob denotes the effective coupling $W t_R b_R$, generated from the composite sector.

$$\mathcal{L} \supset \mathcal{C}_R \mathcal{O}_R, \tag{34}$$

where $\mathcal{O}_R$ is the dimension-6 operator

$$\mathcal{O}_R \equiv H^{c\dagger} i D_\mu H \bar{t}_R \gamma^\mu b_R + \text{H.c.} \tag{35}$$

At low energy, after the EWSB, the interaction in Eq. (34) gives

$$\mathcal{L} \supset \frac{\mathcal{C}_R v^2}{2}\frac{g_2}{\sqrt{2}}\bar{b}_R \gamma^\mu t_R W^-_\mu. \tag{36}$$

This interaction gives a contribution to the Wilson coefficient C_7 in Eq. (27). We find

$$C_7^{\rm CH-IR}(\mu_w) = \frac{\mathcal{C}_R v^2}{2}\frac{m_t}{m_b} f_{RH}(x_t), \tag{37}$$

where $x_t = \frac{m_t^2}{M_W^2}$, and $f_{RH}(x_t)$ is the loop function [28]:

$$\begin{aligned} f_{RH}(x_t) = -\frac{1}{2}\bigg\{ & \frac{1}{(1-x_t)^3}\frac{2}{3}\bigg[-\frac{x_t^3}{2} - \frac{3}{2}x_t + 2 + 3x_t\log(x_t)\bigg] \\ & + \frac{1}{(1-x_t)^3}\bigg[-\frac{x_t^3}{2} + 6x_t^2 \\ & - \frac{15}{2}x_t + 2 - 3x_t^2\log(x_t)\bigg]\bigg\}. \end{aligned} \tag{38}$$

$f_{RH} = -0.777$ for $m_t = 174$ GeV and $M_W = 80.4$ GeV.

We point out that the bound on the CHM contributions to $b \to s\gamma$, $C_7^{\rm CH}$ in Eq. (32), can be directly translated into a bound on the effective vertex $W t_R b_R$, $v_R \equiv \frac{\mathcal{C}_R v^2}{2}$. By considering the bound in Eq. (32) and the relation in Eq. (37), we obtain

$$-0.0004 < v_R < 0.0013. \tag{39}$$

This bound from $b \to s\gamma$ can be compared with that from the measurement of the Wtb anomalous couplings at colliders. Reference [29] reports an expected bound of $-0.012 < v_R < 0.024$, that can be imposed by 14 TeV LHC measurements with 30 fb^{-1}. This latter can be obtained from studies on cross sections and top-decay observables (angular distributions and asymmetries) in the single top production at the LHC. Present searches for anomalous W couplings at the 7 TeV LHC [30] fix still mild bounds on v_R, $-0.34 < v_R < 0.39$, with 0.70 fb^{-1}.

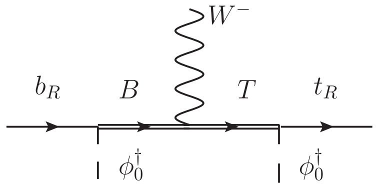

FIG. 3. The CHM contribution to the effective coupling Wt_Rb_R [at order $(\frac{Y_* v}{\sqrt{2}m_*})^2$].

We can see that the bound obtained from $b \to s\gamma$ is much stronger than that from the v_R measurement at the collider.

The CHM contribution to the effective coupling Wt_Rb_R is given by the exchange of heavy fermions that mix electroweakly with t_R and b_R (Fig. 3). At the order x^2, only the $SU(2)_L$ heavy doublets which are partners of (t_L, b_L) contribute to $\mathcal{C}_R$. This latter can be easily estimated by NDA [21]:

$$\mathcal{C}_R \sim \frac{Y_*^2 \xi_{bR} \xi_{tR} \xi_D^2}{M_D^2} \sim \frac{y_b y_t}{M_D^2} \frac{\xi_D^2}{\xi_{qL}^2}. \tag{40}$$

Equation (40) implies

$$C_7^{\mathrm{CH-IR}}(\mu_w) \sim \frac{m_t^2}{M_D^2} f_{RH}(x_t) \frac{\xi_D^2}{\xi_{qL}^2}. \tag{41}$$

Applying the condition in Eq. (32) to this infrared contribution, we get the estimated bound

$$\mathbf{M_D} \gtrsim \frac{\mathbf{1.0(0.54)}}{\xi_{\mathbf{qL}}} \,\mathbf{TeV}, \tag{42}$$

where the first number and the second number in parenthesis refer, respectively, to the cases of a positive and of a negative $C_7^{\mathrm{CH-IR}}$ contribution. Notice that in the case of a positive $C_7^{\mathrm{CH-IR}}$ contribution we obtain a stronger bound on M_D, since the constraint in Eq. (32) is asymmetric.

We find that a subgroup of the custodial symmetry $SU(2)_V$, the P_C parity, can give a suppression to the Wt_Rb_R coupling and, as a consequence, to the CHM infrared contribution to $b \to s\gamma$. The estimates we have just reported refer to generic composite Higgs models where there is not such P_C protection.

1. Protection by P_C parity

The P_C protection against the generation of the Wt_Rb_R vertex acts similarly to the P_{LR} and P_C protection against large corrections to the Zb_Lb_L coupling, which we have discussed in Sec. II A. P_C is a symmetry of the sector BSM, that is respected also by the interactions of t_R and b_R if these latter are P_C eigenstates. Since P_C acts as diag$(1, -1, -1)$ on $SO(3)$ vectors, the W is not a P_C eigenstate (the composite partners of W^1 and W^2 do not have the same P_C eigenvalue). In the case in which t_R and b_R are both P_C eigenstates, both the t_R and the b_R interactions must respect the P_C parity. Then the Wt_Rb_R vertex, which is P_C violating, since the W is not a P_C eigenstate, can arise only by paying for an additional factor that gives a suppression. By contrast, in models where t_R and b_R are not both P_C eigenstates—and, as such, their interactions do not have to respect the P_C parity—the Wt_Rb_R vertex can be generated without suppressions.

The TS5 falls into the class of models with P_C protection, since in the TS5 both t_R and b_R are P_C eigenstates. Considering the TS5 model, we can evaluate the suppression factor to Wt_Rb_R due to the P_C protection. We can find it in an easy way by promoting Δ_{L1} and Δ_{L2} to spurions, which enforce a $SU(2)_L \times SU(2)_R$ invariance:

$$-\Delta_{L1}\bar{q}_L^3(T, B) \to -\bar{q}_L^3 \mathcal{Q}_{2/3}\hat{\Delta}_{L1},$$
$$-\Delta_{L2}\bar{q}_L^3(T', B') \to -\bar{q}_L^3 \mathcal{Q}'_{-1/3}\hat{\Delta}_{L2},$$

where $\hat{\Delta}_{L1} = (\Delta_{L1}, 0) \equiv (1, 2)_{1/6}$ and $\hat{\Delta}_{L2} = (0, \Delta_{L2}) \equiv (1, 2)_{1/6}$. We can thus write the $\mathcal{O}_R$ operator [Eq. (35)] in the $[SU(2)_L \times SU(2)_R]$—invariant way:

$$\mathcal{O}_R = \frac{1}{f^2} \bar{q}_R^3 \hat{\Delta}_{L1} V_\mu \hat{\Delta}_{L2}^\dagger q_R^3 \gamma^\mu + \mathrm{H.c.}, \tag{43}$$

where f has the dimension of a mass $q_R^3 = (t_R, b_R) \equiv (1, 2)_{1/6}$, and $V_\mu \equiv H^{c\dagger} i D_\mu H$. Since P_C is a subgroup of the custodial $SU(2)_V$, the $[SU(2) \times SU(2)]$—invariant operator in Eq. (43) is also P_C invariant. We can notice that the P_C invariance has brought an additional factor $\frac{\Delta_{L1}\Delta_{L2}}{f^2}$ compared to Eq. (35).

Without P_C protection, the $D = (T, B)$ contribution to the Wt_Rb_R effective vertex in the TS5 model reads

$$s_R s_{bR} c_1^2 \left(\frac{Y_* v}{\sqrt{2}M_D}\right)^2 = \frac{m_b m_t}{M_D^2} \frac{c_1^2}{s_1^2};$$

the request for P_C invariance brings the additional factor $\frac{\Delta_{L1}\Delta_{L2}}{f^2}$. For $f^2 = M_{Q*} M_{Q'*}$, we obtain

$$\left(\frac{Y_* v}{\sqrt{2}M_D}\right)^2 s_R s_{bR} \frac{c_1 \Delta_{L1}}{M_{Q*}} \frac{c_1 \Delta_{L2}}{M_{Q'*}} = \left(\frac{Y_* v}{\sqrt{2}M_D}\right)^2 s_R s_{bR} s_1 s_2 = \frac{m_b m_t}{M_D^2}, \tag{44}$$

that is a suppression by a factor $s_1^2/c_1^2 \equiv \xi_{qL}^2/\xi_D^2$.

We can thus return to the estimated bounds on M_D from $C_7^{\mathrm{CH-IR}}$ in Eq. (42), and consider the case in which there is a P_C protection to the t_R and b_R interactions. In such a case the $\mathcal{C}_R$ contribution becomes

$$\mathcal{C}_R \sim \frac{y_b y_t}{M_D^2} \quad (\text{with } P_C), \tag{45}$$

which implies

$$C_7^{\mathrm{CH-IR}}(\mu_w) \sim \frac{m_t^2}{M_D^2} f_{RH}(x_t) \quad (\text{with } P_C) \tag{46}$$

and an estimated bound:

$$\mathbf{M_D \gtrsim 1.0(0.54)\,TeV} \quad (\text{with } P_C). \tag{47}$$

We will now calculate the bounds on M_D from $C_7^{\mathrm{CH-IR}}$ in the specific TS5 and TS10 models. As already discussed, the TS5 belongs to the class of models with P_C protection. The TS10, instead, falls in the class of models without P_C protection, because in the TS10, b_R is not a P_C eigenstate. We thus expect that the bound in the TS10 will receive an enhancement factor c_1/s_1 compared to that in the TS5.

In the TS5 model, we have a contribution to the $\mathcal{O}_R$ operator in Eq. (35), both from the doublet $D = (T, B)$ in the $X = 2/3$ representation and from the doublet $D' \equiv (T', B')$ in the $X = -1/3$. We find

$$\mathcal{C}_R^{\mathrm{TS5}} = -\frac{y_b y_t}{M_D^2}\left(1 + \frac{M_D^2}{M_{D'}^2}\right). \tag{48}$$

This implies

$$C_7^{\mathrm{CH-IR-TS5}}(\mu_w) = -\frac{m_t^2}{M_D^2} f_{RH}(x_t)\left(1 + \frac{M_D^2}{M_{D'}^2}\right). \tag{49}$$

Notice that the $\mathcal{C}_R^{\mathrm{TS5}}$ contribution is negative. This implies a positive contribution $C_7^{\mathrm{CH-IR-TS5}}$ (f_{RH} is negative). The condition in Eq. (32) is asymmetric and is stronger in the case of a positive $C_7^{\mathrm{CH-IR}}$. Applying this condition to the infrared contribution in Eq. (49), we get, for $r = \frac{M_D}{M_{D'}} = 1$, the following bound on the $D = (T, B)$ doublet mass:

$$\mathbf{M_D^{TS5} \gtrsim 1.4\ TeV}. \tag{50}$$

This bound becomes $M_D^{\mathrm{TS5}} \gtrsim 1.3(1.6)$ TeV, changing r to $r = 0.8(1.2)$. In the TS10 model, there is only one doublet, $D = (T, B)$, that gives a contribution to $\mathcal{C}_R$. We obtain

$$\mathcal{C}_R^{\mathrm{TS10}} = \frac{y_b y_t}{M_D^2}\frac{c_1^2}{s_1^2}, \tag{51}$$

which implies

$$C_7^{\mathrm{CH-IR-TS10}}(\mu_w) = \frac{m_t^2}{M_D^2} f_{RH}(x_t)\frac{c_1^2}{s_1^2}. \tag{52}$$

From the condition in Eq. (32) we get finally the bound

$$\mathbf{M_D^{TS10} \gtrsim (0.54)\frac{c_1}{s_1}\,TeV}. \tag{53}$$

Notice that, differently from the case of the TS5 contribution, $C_7^{\mathrm{CH-IR-TS10}}(\mu_w)$ is negative. As such, it is constrained less strongly by the condition in Eq. (32). As expected, we have found a c_1/s_1 enhancement of this bound, compared to Eq. (50).

We now proceed to evaluate the bounds from the C_7' contribution, and then those from the UV contributions. As we already pointed out, these are contributions that involve flavor-violating operators and require assumptions on the flavor structure of the NP sector. In what follows, we will consider the case of flavor anarchy of the composite Yukawa matrices. This scenario, we remember, assumes that there is no large hierarchy between elements within each matrix Y_* and that the quark mass hierarchy is completely explained by the elementary/composite mixing angles. We also set, for simplicity, $Y_{*U} = Y_{*D} = Y_*$.

C. Non-MFV constraints

1. Generational mixing

After the EWSB, the mass eigenstate basis is obtained, as in the SM, using unitary transformations: (D_L, D_R) and (U_L, U_R) for down- and up-type quarks, respectively. We will assume that the left rotation matrix has entries of the same order as those of the Cabibbo-Kobayashi-Maskawa matrix:

$$(D_L)_{ij} \sim (V_{\mathrm{CKM}})_{ij}. \tag{54}$$

The assumption of anarchical Y_* fixes the form of the rotation matrix D_R to be

$$(D_R)_{ij} \sim \left(\frac{m_i}{m_j}\right)\frac{1}{(D_L)_{ij}} \quad \text{for } i < j. \tag{55}$$

Considering the estimates of Eqs. (54) and (55), we can evaluate the generational mixing factors in the composite Higgs model contributions to C_7 (UV) and C_7'.

For the ultraviolet contribution to C_7', we consider the presence of a mass insertion that can generate the operator $\bar{b}_L\sigma^{\mu\nu}F_{\mu\nu}s_R$. This mass insertion brings to a factor $m_b(D_R)_{23} \sim \frac{m_s}{(D_L)_{23}} \sim \frac{m_s}{V_{ts}}$, where we have first used the estimate in Eq. (55) and then that in Eq. (54). The ultraviolet contribution to C_7 involves the operator $\bar{b}_R\sigma^{\mu\nu}F_{\mu\nu}s_L$, and we obtain from the mass insertion a generational mixing factor $m_b(D_L)_{23} \sim m_b V_{ts}$, where the last similitude follows from the assumption in Eq (54).

Evaluating, similarly, the generational mixing factor for the vertex Wt_Rs_R in $C_7'^{\mathrm{CH-IR}}$, one finds $(D_R)_{23} \sim \frac{m_s}{m_b(D_L)_{23}} \sim \frac{m_s}{m_b V_{ts}}$, making use, again, of the estimates of Eqs. (55) and (54). The flavor violation in $C_7^{\mathrm{CH-IR}}$ comes entirely from the SM vertex Wt_Ls_L, and it is accounted for by a factor V_{ts}. Therefore, we find that the composite Higgs model contribution to the Wilson coefficient C_7' is enhanced by a factor

$$\frac{m_s}{m_b V_{ts}^2} \sim 8 \tag{56}$$

compared to the contribution to C_7 both in the ultraviolet and in the infrared case.

2. Infrared contribution to C_7'

Taking into account the generational mixing factor in Eq. (56), the composite Higgs model contribution to the Wilson coefficient C_7' (in Fig. 4) is given by

$$C_7'^{\mathrm{CH-IR}}(\mu_w) = \frac{\mathcal{C}_R v^2}{2}\frac{m_s}{m_b V_{ts}^2}\frac{m_t}{m_b} f_{RH}(x_t). \tag{57}$$

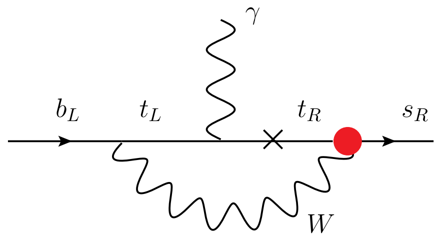

FIG. 4 (color online). One-loop infrared contribution to C_7'.

Considering the estimates for $\mathcal{C}_R$ in Eqs. (40) and (45), the condition on $C_7^{\prime\mathrm{CH-IR}}(\mu_w)$, Eq. (33), thus gives the estimated bounds

$$M_D \gtrsim 0.80\ \mathrm{TeV} \tag{58}$$

in models with P_C symmetry, and

$$M_D \gtrsim \frac{0.80}{\xi_{qL}}\ \mathrm{TeV} \tag{59}$$

in models without P_C symmetry.

Considering the specific TS5 and TS10 models, $C_7^{\prime\mathrm{CH-IR}}$ gives the bounds

$$M_D^{\mathrm{TS5}} \gtrsim 1.1\ \mathrm{TeV} \tag{60}$$

in the TS5, and

$$M_D^{\mathrm{TS10}} \gtrsim \frac{c_1}{s_1}(0.80)\ \mathrm{TeV} \tag{61}$$

in the TS10.

We can discuss how the bound on heavy masses can change in the case of a fully composite top: in the TS5 model, the bound on the doublet heavy fermion [Eq. (50)] does not depend on the top degree of compositeness (this remains almost true considering the full numerical calculation), and we obtain quite strong MFV bounds for both composite t_L and composite t_R. In the TS10 model, because of the P_C protection, we obtain strong bounds in the case of a fully composite t_R [Eq. (53)]. Reference [18] finds that corrections to S and T parameters give only weak constraints on a composite t_R (both in TS5 and in TS10). The IR contribution to $b \to s\gamma$, on the contrary, puts a quite strong constraint, especially in the TS10 model, on this limit case.

One can finally discuss the validity of our results, which have been obtained "analytically" [i.e., by considering an expansion in $x \equiv \frac{Y_* v}{\sqrt{2} m_*}$ and retaining only the $O(x)$ terms]. We find that the results from the numerical calculation of the bounds, obtained by diagonalizing numerically the fermionic mass matrices, do not differ more than $O(1)$ from those we have shown, which are obtained at order x in the assumption $x \ll 1$. This can also be found by considering that the exchange of relatively light custodians, that can give a contribution $\frac{Y_* v}{\sqrt{2} m_*^{\mathrm{CUST}}} > 1$ to the effective $Wt_R b_R$ vertex, has to be followed by the exchange of heavier composite fermions, that reduce the overall contribution. By definition, indeed, the custodians do not directly couple to SM fermions; therefore, their contribution to $Wt_R b_R$ is always accompanied by the exchange of heavier composite particles.

3. Ultraviolet contribution

In this case, the P_C parity does not influence the bounds, and we get contributions of the same size in the different models. The leading contribution comes from diagrams with heavy fermions and would-be Goldstone bosons in the loop[6] (Fig. 5):

$$C_7^{\mathrm{CH-UV}}, C_7^{\prime\mathrm{CH-}UV} \propto s_{Li} Y_{*ik} Y_{*kl} Y_{*lj} s_{Rj}. \tag{62}$$

The contribution of Eq. (62) is not aligned with the mass matrix $m_{dij} \sim s_{Li} Y_{*ij} s_{Rj}$; therefore, after the EWSB it remains nondiagonal in the flavor space.

Before going on to the specific TS5 and TS10 models, we can obtain estimated bounds from the UV contributions in generic composite Higgs models, by means of NDA. We obtain

$$C_7^{\mathrm{CH-UV}} \sim \frac{(Y_* v)^2}{M_D M_{\tilde{D}}} \xi_D \xi_{\tilde{D}}, \tag{63}$$

where $\tilde{D}$ denotes a heavy fermion which is a $SU(2)_L$ singlet, and

$$C_7^{\prime\mathrm{CH-UV}} \sim \frac{m_s}{m_b V_{ts}^2} \frac{(Y_* v)^2}{M_D M_{\tilde{D}}} \xi_D \xi_{\tilde{D}}, \tag{64}$$

where we have taken into account the generational mixing factor in Eq. (56). By comparing these results with those from the IR contributions in Eqs. (42) and (47), we see that the UV contribution gives a bound approximately Y_*/y_t ($\frac{Y_*}{y_t} \xi_{q_L}$, in the case of models without P_C protection) times stronger than the one from the IR contribution to C_7. Such UV bounds, however, are not as robust as the IR one, since they require, as we already pointed out, assumptions on the flavor structure of the BSM sector. In particular, we have estimated them in the scenario of flavor anarchy in the strong sector. Notice that in this anarchic scenario, much stronger bounds on the resonance masses, of the order of 20 TeV [13], come from ϵ_k.

In Ref. [22], the ultraviolet contribution to $b \to s\gamma$ in a two-site model without a P_{LR} protection to the t_R and b_R interactions is evaluated. In the following, we will describe in detail the contribution in the TS5 model, and we will report the results for TS10. We can calculate the $C_7^{\mathrm{CH-UV}}$ and $C_7^{\prime\mathrm{CH-UV}}$ ultraviolet contributions by considering the model-independent analysis of Ref. [22] and the generational mixing factor in Eq. (56). We get the following

[6]The contribution from heavy gluon and heavy fermion exchange is suppressed. Indeed, this contribution is approximately diagonal in the flavor space.

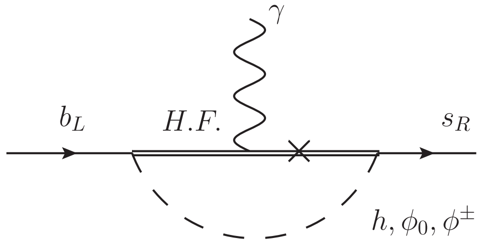

FIG. 5. One-loop CHM ultraviolet contribution to C_7'.

effective Hamiltonian for $b \to s\gamma$ with loops of heavy fermions and neutral would-be Goldstone bosons:

$$\mathcal{H}^{\rm eff}_{\rm neutral\ Higgs} = \frac{ie}{8\pi^2}\frac{(2\epsilon\cdot p)}{M_w^2}k_{\rm neutral}\Big[V_{ts}\bar{b}(1-\gamma_5)s + \frac{m_s}{m_b V_{ts}}\bar{b}(1+\gamma_5)s\Big], \qquad (65)$$

where

$$k_{\rm neutral} \approx \sum_{i=1}^{4}(|\alpha_1^{(i)}|^2 + |\alpha_2^{(i)}|^2)m_b\Big(\frac{1}{36}\Big)\frac{M_w^2}{m^2_{*(i)}} + \sum_{i=1}^{4}(\alpha_1^{(i)*}\alpha_2^{(i)})m_{*(i)}\Big(\frac{1}{6}\Big)\frac{M_w^2}{m^2_{*(i)}}, \qquad (66)$$

the index i runs over the four down-type heavy fermions of the model, $\mathbf{d}^{(i)} = \tilde{B}$, B', $B_{-1/3}$, B, and the $\alpha_1^{(i)}$, $\alpha_2^{(i)}$ coefficients are defined by the interactions

$$\mathcal{L} \supset \bar{\mathbf{d}}^{(i)}[\alpha_1^{(i)}(1+\gamma_5) + \alpha_2^{(i)}(1-\gamma_5)]bH + {\rm H.c.} \qquad (67)$$

After the EWSB, we find the following coefficients at $O(x)$:

$$\alpha_1^{(\tilde{B})} = \frac{Y_*^2 v}{2}s_{bR}\Big[\frac{1}{M_{B'}} + \frac{M_{B'} + c_{bR}M_{\tilde{B}}}{M_{B'}^2 - M_{\tilde{B}}^2}\Big], \qquad \alpha_2^{(\tilde{B})} = -\frac{Y_*}{2\sqrt{2}}s_2 c_{bR}, \qquad \alpha_1^{(B')} = \alpha_1^{(B_{-1/3})} = -\frac{Y_*}{2\sqrt{2}}s_{bR},$$
$$\alpha_2^{(B')} = \alpha_2^{(B_{-1/3})} = -\frac{Y_*^2 v}{4}s_2\Big[\frac{M_{B'}^2 M_{\tilde{B}} - s_{bR}^2 M_{\tilde{B}}^3 - c_{bR5}M_{B'}^3 + 2c_{bR}M_{B'}M_{\tilde{B}}^2}{M_{B'}M_{\tilde{B}}(M_{B'}^2 - M_{\tilde{B}}^2)}\Big]. \qquad (68)$$

The heavy fermion B gives a contribution of $O(s_2^2)$ to $k_{\rm neutral}$, and we neglect it.

Considering Eq. (66) and the coefficients in Eq. (68), and again neglecting $O(x^2)$ terms, we obtain

$$k_{\rm neutral} \approx -m_b M_W^2 Y_*^2 \frac{1}{8}\Big(\frac{c_{bR}}{M_{B'}M_{\tilde{B}}} - \frac{7}{18}\frac{s_{bR}^2}{M_{B'}^2}\Big). \qquad (69)$$

From this expression of $k_{\rm neutral}$, we obtain the following TS5 ultraviolet contributions to the Wilson coefficient of the effective Hamiltonian in Eq. (27):

$$C_7^{\rm CH-UV}(m_*) = \frac{1}{16}\frac{\sqrt{2}}{G_F}Y_*^2\Big(\frac{c_{bR}}{M_{B'}M_{\tilde{B}}} - \frac{7}{18}\frac{s_{bR}^2}{M_{B'}^2}\Big),$$
$$C_7^{\prime\rm CH-UV}(m_*) = \frac{1}{16}\frac{\sqrt{2}}{G_F}Y_*^2\Big(\frac{c_{bR}}{M_{B'}M_{\tilde{B}}} - \frac{7}{18}\frac{s_{bR}^2}{M_{B'}^2}\Big)\frac{m_s}{m_b V_{ts}^2}. \qquad (70)$$

Assuming s_{bR} is small, the above formulas become

$$C_7^{\rm CH-UV}(m_*) = \frac{1}{16}\frac{\sqrt{2}}{G_F}\frac{Y_*^2}{M_{B'}M_{\tilde{B}}},$$
$$C_7^{\prime\rm CH-UV}(m_*) = \frac{1}{16}\frac{\sqrt{2}}{G_F}\frac{Y_*^2}{M_{B'}M_{\tilde{B}}}\frac{m_s}{m_b V_{ts}^2}. \qquad (71)$$

Finally, the condition on $C_7^{\prime\rm CH-UV}$ in Eq. (30) gives the bound

$$\sqrt{M_{B'}M_{\tilde{B}}} \gtrsim (0.40)Y_*\ {\rm TeV}, \qquad (72)$$

where, for simplicity, we have set $s_{bR} = 0$. The condition in Eq. (29) on $C_7^{\rm CH-UV}$ gives a stronger bound,

$$\sqrt{M_{B'}M_{\tilde{B}}} \gtrsim (0.52)Y_*\ {\rm TeV}, \qquad (73)$$

if $C_7^{\rm CH-UV}(m_*)$ is a negative contribution.

There is also a contribution to $b \to s\gamma$ from diagrams with heavy fermions and charged Higgs in the loop. Following a similar procedure to the one used before (see Appendix C), we find, neglecting $O(x^2)$ terms,

$$k_{\rm charged} \approx m_b M_W^2 Y_*^2 \frac{5}{48}\frac{1}{M_{B'}M_{\tilde{B}}} + O(s_1^2) + O(s_{bR}^2). \qquad (74)$$

If we can neglect $O(s_1^2)$ and $O(s_{bR}^2)$ terms, $k_{\rm charged}$ gives a weaker bound than the one from $k_{\rm neutral}$. The full expression of $k_{\rm charged}$ can be found in Appendix D; here we have just reported, for simplicity, the result for small s_1 and s_{bR} angles.

In Fig. 6, we show the bound on the doublet mass M_T as a function of s_1 from the condition on $C_7^{\prime\rm CH-UV}$ for different values of the ratio $k = \frac{M_T}{M_{\tilde{T}}}$ between doublet and singlet masses, fixing $Y_* = 3$ (upper plot), and for different values of Y_*, fixing $k = 1$ (lower plot). We set $M_{\tilde{B}} = M_{\tilde{T}}$ and $M_{T'} = M_T$. These values are obtained by taking into account the strongest values between the neutral Higgs contribution and the charged Higgs one. We set $s_{bR} = s_1$.

4. Ultraviolet contribution in the TS10

For the TS10 model, applying the same procedure as for the case of TS5, we get

$$k_{\text{neutral}} = m_b M_W^2 Y_*^2 \frac{7M_T M_{T'}^2 s_1^2 - 18 M_{\tilde{B}} M_{\tilde{B}'}^2 \sqrt{1-s_1^2} + M_{\tilde{B}}^2 \left(7M_B s_1^2 - 18 M_{\tilde{B}'} \sqrt{1-s_1^2}\right)}{288 M_{\tilde{B}}^2 M_B M_{\tilde{B}'}^2} + O(s_{bR})$$
$$= -m_b M_W^2 Y_*^2 \frac{1}{16}\left(\frac{1}{M_B M_{\tilde{B}}} + \frac{1}{M_B M_{\tilde{B}'}}\right) + O(s_1^2) + O(s_{bR}), \quad (75)$$

$$k_{\text{charged}} = m_b M_W^2 Y_*^2 \left(\frac{5}{48}\frac{1}{M_B M_{\tilde{B}}} + \frac{5}{48}\frac{1}{M_B M_{\tilde{B}'}} + \frac{5}{96}\frac{s_R^2}{M_B^2}\right) + O(s_1^2) + O(s_{bR}^2). \quad (76)$$

If the left-handed bottom quark has a small degree of compositeness, we can neglect $O(s_1^2)$ (while s_{bR} is naturally very small in the TS10 model, in order to account for the ratio $m_b/m_t \ll 1$). The charged contribution, in this case, gives a stronger bound than the one from k_{neutral}:

$$\sqrt{M_B M_{\tilde{B}}} \gtrsim (0.58) Y_* \text{TeV}, \quad (77)$$

from the condition in Eq. (30) on $C_7^{\prime\text{CH-UV}}$. A stronger bound,

$$\sqrt{M_B M_{\tilde{B}}} \gtrsim (0.75) Y_* \text{TeV}, \quad (78)$$

comes from the condition in Eq. (30) on $C_7^{\text{CH-UV}}$, if this last contribution has a negative sign.

In Fig. 7, we show the bound on the doublet mass M_T as function of s_1 from the condition on $C_7^{\prime\text{CH-UV}}$ for different values of the ratio $k = \frac{M_T}{M_{\tilde{T}}}$ between the doublet and $\tilde{T}$ singlet masses, fixing $Y_* = 3$ (upper plot), and for different Y_* values, setting $k = \frac{M_T}{M_{\tilde{T}}} = 1$ (lower plot). The custodian singlet masses have the following relations with $M_{\tilde{T}}$: $M_{\tilde{B}} \simeq c_R M_{\tilde{T}}$, $M_{\tilde{B}'} = M_{\tilde{T}'} = c_R M_{\tilde{T}}$. All these bounds are obtained by taking into account the strongest values between the neutral Higgs contribution and the charged Higgs one.

We can see that in the TS10 model, the UV bounds are particularly strong in the case of fully composite t_R. This is an effect caused by the exchange of the custodians $\tilde{T}'$, $\tilde{B}'$ and of the $\tilde{B}$, that are light in the limit of a composite t_R.

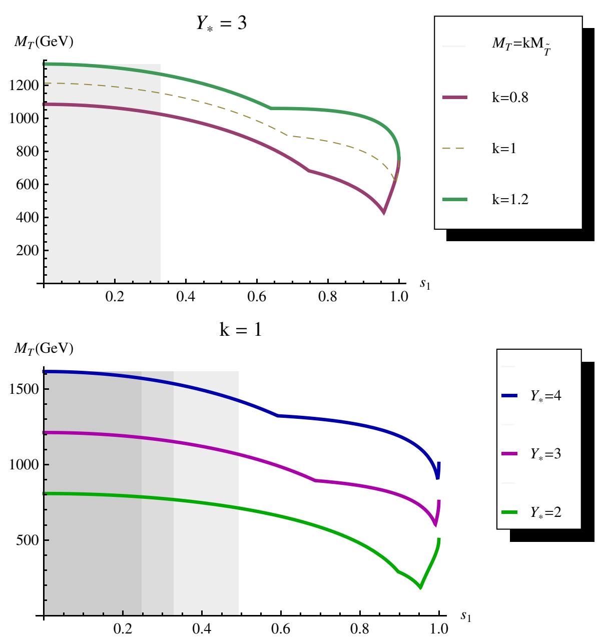

FIG. 6 (color online). Bounds from $C_7^{\prime\text{CH-UV}}$ in the TS5 model. Upper plot: Bounds for different values of $k = \frac{M_T}{M_{\tilde{T}}}$ and $Y_* = 3$. Lower plot: Bounds for different values of Y_* and $k = 1$. We set $M_{\tilde{B}} = M_{\tilde{T}}$ and $M_{T'} = M_T$. Also shown is the exclusion region for s_1, obtained from the condition $s_R = \frac{\sqrt{2}m_t}{Y_* v s_1} \leq 1$.

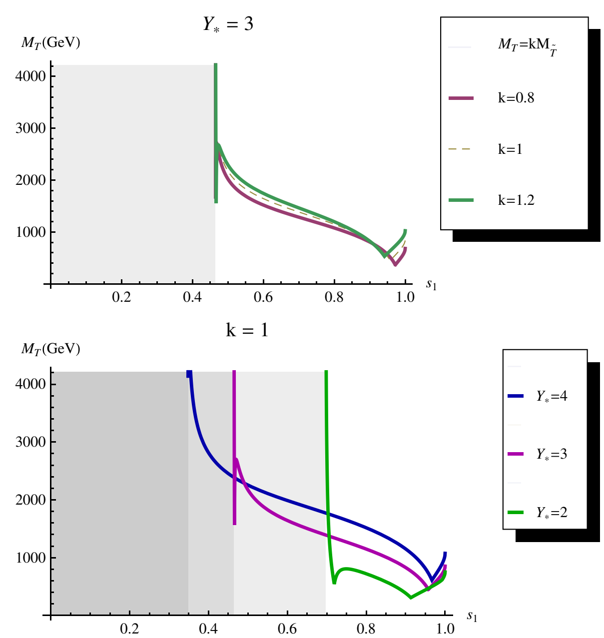

FIG. 7 (color online). Bounds from $C_7^{\prime\text{CH-UV}}$ in the TS10 model. Upper plot: Bounds for different values of $k = \frac{M_T}{M_{\tilde{T}}}$ ($M_{\tilde{B}} \simeq c_R M_{\tilde{T}}$, $M_{\tilde{B}'} = M_{\tilde{T}'} = c_R M_{\tilde{T}}$), fixing $Y_* = 3$. Lower plot: Bounds for different values of Y_*, fixing $k = 1$. We also show the exclusion region for s_1, obtained from the condition $s_R = \frac{2m_t}{Y_* v s_1} \leq 1$.

TABLE I. Estimated bounds from $b \to s\gamma$ in a generic composite Higgs model and in the specific TS5 and TS10 models at small elementary/composite mixing angles s_1 and s_{bR}. $\xi_{\psi/\chi}$ denotes the degree of compositeness of a SM/heavy fermion. In the specific TS5 and TS10 models, $\xi_{qL} \equiv s_1$, $\xi_D \equiv c_1$. $D = (T, B)$, and $\tilde{D}$ denotes a $SU(2)_L$ singlet heavy fermion. We highlight (in bold) the MFV bounds from $C_7^{\rm CH}$. For the estimated bounds from $C_7^{\rm CH}$ and for the bounds from $C_7^{\rm CH-UV}$, we indicate the values that can be obtained in the case of a positive (the first number) or a negative (the second number in parenthesis) contribution.

$C_7^{\rm CH-IR}(\mu_w)$	$\sim \frac{(y_t v)^2}{M_D^2}\xi_D^2$	w/P_C	
	ESTIMATED	TS5	
	$\mathbf{M_D \gtrsim 1.0(0.54)}$ **TeV**	$\mathbf{M_D \gtrsim 1.4}$ **TeV**	
MFV Bounds			
	$\sim \frac{(y_t v)^2}{M_D^2}(\frac{\xi_D}{\xi_{qL}})^2$	w/oP_C	
	ESTIMATED	TS10	
	$\mathbf{M_D \gtrsim 1.0(0.54)/\xi_{qL}}$ **TeV**	$\mathbf{M_D \gtrsim 0.54/s_1}$ **TeV**	
$C_7'^{\rm CH-IR}(\mu_w)$	$\sim \frac{(y_t v)^2}{M_D^2}\xi_D^2 \frac{m_s}{m_b V_{ts}^2}$	w/P_C	
	ESTIMATED	TS5	
	$M_D \gtrsim 0.80$ TeV	$M_D \gtrsim 1.1$ TeV	
	$\sim \frac{(y_t v)^2}{M_D^2}(\frac{\xi_D}{\xi_{qL}})^2 \frac{m_s}{m_b V_{ts}^2}$	w/oP_C	
	ESTIMATED	TS10	
	$M_D \gtrsim 0.80/\xi_{qL}$ TeV	$M_D \gtrsim 0.80/s_1$ TeV	
$C_7^{\rm CH-UV}(m_*)$		$\sim \frac{(Y_* v)^2}{M_D M_{\tilde{D}}}\xi_D \xi_{\tilde{D}}$	
	ESTIMATED	TS5	TS10
	$\sqrt{M_D M_{\tilde{D}}} \gtrsim 1.5(0.79) Y_*$ TeV	$\sqrt{M_D M_{\tilde{D}}} \gtrsim 0.52(0.28) Y_*$ TeV	$\sqrt{M_D M_{\tilde{B}}} \gtrsim 0.75(0.40) Y_*$ TeV
$C_7'^{\rm CH-UV}(m_*)$		$\sim \frac{(Y_* v)^2}{M_D M_{\tilde{D}}}\xi_D \xi_{\tilde{D}} \frac{m_s}{m_b V_{ts}^2}$	
	ESTIMATED	TS5	TS10
	$\sqrt{M_D M_{\tilde{D}}} \gtrsim (1.1) Y_*$ TeV	$\sqrt{M_D M_{\tilde{D}}} \gtrsim (0.40) Y_*$ TeV	$\sqrt{M_D M_{\tilde{B}}} \gtrsim (0.58) Y_*$ TeV

In particular, when t_R is fully composite ($s_R = 1$), $M_{\tilde{B}} (\simeq c_R M_{\tilde{T}})$ and $M_{\tilde{B}'} = M_{\tilde{T}'} (= c_R M_{\tilde{T}})$ vanish. This causes the divergence of the bounds for $s_R \to 1$. Such divergences can be seen in the curves in Fig. 7, when they approach the (grey) exclusion regions for s_1. (Indeed, the minimum value of s_1 allowed by the condition $s_R = \frac{2m_t}{Y_* v s_1} \leq 1$ is obviously obtained in the case $s_R = 1$.)

Table I summarizes our results. It shows the bounds on heavy fermion masses that can be obtained from the process $b \to s\gamma$. We report the estimated bounds in generic composite Higgs models (with or without P_C protection), which we have found by means of NDA, and the bounds in the specific two-site models TS5 and TS10. $\xi_{\psi/\chi}$ denotes the degree of compositeness of a SM/heavy fermion. In the specific TS5 and TS10 models, $\xi_{qL} \equiv s_1$, and $\xi_D \equiv c_1$. $D = (T, B)$, and $\tilde{D}$ denotes a $SU(2)_L$ singlet heavy fermion. For the estimated bounds from $C_7^{\rm CH}$ and for the bounds from $C_7^{\rm CH-UV}$, we indicate the values that can be obtained in the case of a positive (the first number) or a negative (the second number in parentheses) contribution.

D. Constraint from ϵ'/ϵ_K

The bound on the mass of the heavy fermions that comes from the direct CP-violating observable of the $K^0 \to 2\pi$ system, $Re(\epsilon'/\epsilon)$, can be even stronger in the assumption of anarchic Y_* than those obtained from $b \to s\gamma$, as already found in Ref. [10]. As we pointed out, however, it is a bound that strongly depends on the assumptions made on the flavor structure of the new physics sector.

As for the UV contribution to $b \to s\gamma$, the custodial symmetry does not influence the bound, and we obtain contributions of the same size in the different models. In what follows, we describe the bound in the TS5 and in the TS10. The contribution of New Physics can be parametrized at low energy by chromomagnetic operators:

$$\mathcal{O}_G = \bar{s}\sigma^{\mu\nu}T^a G^a_{\mu\nu}(1-\gamma_5)d,$$
$$\mathcal{O}'_G = \bar{s}\sigma^{\mu\nu}T^a G^a_{\mu\nu}(1+\gamma_5)d. \qquad (79)$$

As for the UV contribution to $b \to s\gamma$, the leading contribution to ϵ'/ϵ_K comes from diagrams with heavy fermions and Higgs in the loop, that generate the $\mathcal{O}_G$ and $\mathcal{O}'_G$ operators. (One-loop diagrams are the same as for the UV contribution to $b \to s\gamma$, Fig. 5, with the replacements $\gamma \to g$, $b \to s$ and $s \to d$.)

The related coefficients $\mathcal{C}_G$ and $\mathcal{C}'_G$, in analogy with $\mathcal{C}_7$ and $\mathcal{C}'_7$ of the UV contribution to $b \to s\gamma$, differ by a generational mixing factor that, in the assumption of anarchic Y_*, we estimate to be $\sim \frac{m_d}{m_s V_{us}^2}$. We consider only the generation mixing $(1-3) \times (2-3)$, via the third generation. In analogy with Eq. (65), we define

$$\mathcal{A}^{\text{eff-chromo}}_{\text{neutral Higgs}} = \frac{ig_s}{8\pi^2}\frac{(2\epsilon\cdot p)}{M_w^2}k^G_{\text{neutral}}\bigg[V_{us}\bar{s}(1-\gamma_5)d + \frac{m_d}{m_s V_{us}}\bar{s}(1+\gamma_5)d\bigg], \quad (80)$$

where

$$k^G_{\text{neutral}} \approx \sum_{i=1}^{4}(|\alpha_1^{(i)}|^2 + |\alpha_2^{(i)}|^2)m_s\bigg(-\frac{1}{12}\bigg)\frac{M_w^2}{m_{*(i)}^2} + \sum_{i=1}^{4}(\alpha_1^{(i)*}\alpha_2^{(i)})m_{*(i)}\bigg(-\frac{1}{2}\bigg)\frac{M_w^2}{m_{*(i)}^2}, \quad (81)$$

the index i runs over the four down-type heavy fermions of the model $\mathbf{d}^{(i)}$, and the $\alpha_1^{(i)}$, $\alpha_2^{(i)}$ coefficients are defined by the following interactions:

$$\mathcal{L} \supset \bar{\mathbf{d}}^{(i)}[\alpha_1^{(i)}(1+\gamma_5) + \alpha_2^{(i)}(1-\gamma_5)]bH + \text{H.c.} \quad (82)$$

After the EWSB, neglecting $O(x^2)$ terms, we find in the TS5 model

$$k^G_{\text{neutral}} = \frac{3}{8}m_s M_w^2\frac{Y_*^2}{M_{B'}M_{\tilde{B}}} + O(s_{sR}^2), \quad (83)$$

where s_{sR} defines the degree of compositeness of the right-handed strange quark and naturally has a small value. In the limit in which $s_{sR} = 0$, we obtain the same result also in the TS10 model.

We can thus calculate the $\mathcal{C}_G$ and $\mathcal{C}'_G$ contributions:

$$\mathcal{C}_G = -\frac{1}{16\pi^2}\frac{k^G_{\text{neutral}}}{M_w^2 m_s}V_{us}, \qquad \mathcal{C}'_G = \frac{m_d}{m_s V_{us}^2}\mathcal{C}_G. \quad (84)$$

Defining

$$\delta_{\epsilon'} = \frac{\text{Re}(\epsilon'/\epsilon)_{\text{CH}} - \text{Re}(\epsilon'/\epsilon)_{\text{SM}}}{\text{Re}(\epsilon'/\epsilon)_{\text{exp}}}, \quad (85)$$

we obtain

$$|\delta_{\epsilon'}| \approx (58\ \text{TeV})^2 B_G|\mathcal{C}_G - \mathcal{C}'_G| < 1, \quad (86)$$

where $\text{Re}(\epsilon'/\epsilon)_{\text{SM}}$ has been estimated as in Ref. [10], and B_G denotes the hadronic bag parameter, $\langle 2\pi_{I=0}|y_s\mathcal{O}_G|K^0\rangle$. We take $B_G = 1$,[7] and we take into account separately the contribution from $\mathcal{C}_G$ and $\mathcal{C}'_G$. In the limit $s_{sR} = 0$, we obtain from Eq. (86)

$$\sqrt{M_{B'}M_{\tilde{B}}} \gtrsim (1.3)Y_*\text{TeV}, \quad (87)$$

which is in agreement with the result in Ref. [10]. The contribution from the charged Higgs interactions gives weaker bounds than those from the neutral Higgs contribution.

[7]That corresponds to the estimate of the hadronic matrix element $\langle 2\pi_{I=0}|y_s\mathcal{O}_G|K^0\rangle$ in the chiral quark model and to the first order in the chiral expansion.

IV. CONCLUSIONS

Composite Higgs models are among the compelling scenarios for physics beyond the Standard Model that can give an explanation of the origin of the EWSB, and that are going to be tested at the LHC.

In this project, we have built simple two-site models, the TS5 and the TS10, which can represent the low-energy regime of minimal composite Higgs models with a custodial symmetry and a P_{LR} parity.

Working in these effective descriptions, we have reconsidered the bounds on the CHM spectrum implied by flavor observables. We have found in particular that the IR contribution to $b \to s\gamma$ induced by the flavor conserving effective vertex Wt_Rb_R implies a robust minimal flavor-violating bound on the mass (m_*) of the new heavy fermions. [To be more specific, on the heavy doublets, partners of $q_L = (t_L, b_L)$.] The relevance of shifts to Wt_Rb_R has been already pointed out in the literature (see, for example, Refs. [31,32]), even though its importance in setting a bound on heavy fermion masses was unestimated in previous studies. We have also shown how this bound can be stronger in the case of the absence of a symmetry (P_C) protection to the effective Wt_Rb_R vertex. In particular, we have found an estimated bound

$$m_* \gtrsim 1.0\ \text{TeV}$$

in models with P_C protection to the Wt_Rb_R vertex (where both t_R and b_R are P_C eigenstates), and a bound

$$m_* \gtrsim 1.0/\xi_{qL}\ \text{TeV},$$

where ξ_{qL} denotes the degree of compositeness of (t_L, b_L) in models without P_C protection. ξ_{qL} is naturally a small number; the bound could thus be very strong in these types of models. In the specific two-site models, the bounds we have found are

$$m_*^{\text{TS5}} \gtrsim 1.4\ \text{TeV}$$

in the TS5, and

$$m_*^{\text{TS10}} \gtrsim \frac{0.54}{\xi_{qL}}\ \text{TeV}$$

in the TS10.

Table I summarizes the results obtained for the bounds from $b \to s\gamma$. In addition to these bounds, we have calculated the constraints from the UV composite Higgs model contribution to $b \to s\gamma$. Figures 6 and 7 show the bounds in the TS5 and the TS10 models as functions of the t_L degree of compositeness. Our results have shown that these bounds can be stronger than those from the IR contribution, but they are model dependent; in particular, they strongly depend on the assumptions made about the flavor structure of the composite sector. We have obtained an estimated limit

$$m_* \gtrsim (0.52)Y_*\ \text{TeV}$$

in a specific NP flavor scenario (Y_* anarchic in the flavor space).

Even stronger bounds,

$$m_* \gtrsim (1.3) Y_* \text{ TeV},$$

can be obtained from ϵ'/ϵ_K, but again, they are model dependent and in principle could be loosened by acting on the NP flavor structure (as done, for example, in Ref. [11]). The lower IR bounds on m_* we have found from $b \to s\gamma$, on the contrary, are robust MFV bounds that cannot be evaded by assuming particular conditions on the structure of the strong sector.

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APPENDIX A: TWO-SITE MODELS

1. TS5

Fermions rotate from the elementary/composite basis to the "physical" light(SM)/heavy basis as [neglecting $O(\Delta_{L2}^2)$ terms]

$$\tan\varphi_{L1} = \frac{\Delta_{L1}}{M_{Q*}} \equiv \frac{s_1}{c_1}, \quad s_1 \equiv \sin\varphi_{L1}, \quad c_1 \equiv \cos\varphi_{L1}, \quad s_2 = \frac{\Delta_{L2}}{M_{Q'*}}\cos\varphi_{L1}, \quad s_3 = \frac{\Delta_{L2} M_{Q'*}}{\Delta_{L1}^2 + M_{Q*}^2 - M_{Q'*}^2}\sin\varphi_{L1},$$

$$\begin{cases} t_L = c_1 t_L^{\rm el} - s_1 T_L^{\rm com} - s_2 T_L^{\prime\rm com} \\ T_L = s_1 t_L^{\rm el} + c_1 T_L^{\rm com} + s_3 T_L^{\prime\rm com} \\ T_L' = (s_2 c_1 - s_1 s_3) t_L^{\rm el} - (s_1 s_2 + c_1 s_3) T_L^{\rm com} + T_L^{\prime\rm com} \end{cases}, \quad \begin{cases} b_L = c_1 b_L^{\rm el} - s_1 B_L^{\rm com} - s_2 B_L^{\prime\rm com} \\ B_L = s_1 b_L^{\rm el} + c_1 B_L^{\rm com} + s_3 B_L^{\prime\rm com} \\ B_L' = (s_2 c_1 - s_1 s_3) b_L^{\rm el} - (c_1 s_3 + s_1 s_2) B_L^{\rm com} + B_L^{\prime\rm com} \end{cases}, \tag{A1}$$

$$s_4 = \Delta_{L2} \frac{\Delta_{L1}}{\Delta_{L1}^2 + M_{Q*}^2 - M_{Q'*}^2}, \qquad \begin{cases} T_R = T_R^{\rm com} + s_4 T_R^{\prime\rm com} \\ T_R' = T_R^{\prime\rm com} - s_4 T_R^{\rm com} \end{cases}, \qquad \begin{cases} B_R = B_R^{\rm com} + s_4 B_R^{\prime\rm com} \\ B_R' = B_R^{\prime\rm com} - s_4 B_R^{\rm com} \end{cases}, \tag{A2}$$

$$\tan\varphi_R = \frac{\Delta_{R1}}{M_{\tilde{T}*}}, \qquad s_R \equiv \sin\varphi_R, \qquad c_R \equiv \cos\varphi_R, \qquad \tan\varphi_{bR} = \frac{\Delta_{R2}}{M_{\tilde{B}*}}, \qquad s_{bR} \equiv \sin\varphi_{bR}, \qquad c_{bR} \equiv \cos\varphi_{bR},$$
$$\begin{cases} t_R = c_R t_R^{\rm el} - s_R \tilde{T}_R^{\rm com} \\ \tilde{T}_R = s_R t_R^{\rm el} + c_R \tilde{T}_R^{\rm com} \end{cases}, \qquad \begin{cases} b_R = c_{bR} b_R^{\rm el} - s_{bR} \tilde{B}_R^{\rm com} \\ \tilde{B}_R = s_{bR} b_R^{\rm el} + c_{bR} \tilde{B}_R^{\rm com} \end{cases}. \tag{A3}$$

Physical heavy fermion masses are related to the bare ones according to

$$\begin{cases} M_{\tilde{T}} = \sqrt{M_{\tilde{T}*}^2 + \Delta_{R1}^2} = \frac{M_{\tilde{T}*}}{c_R} \\ M_{\tilde{B}} = \sqrt{M_{\tilde{B}*}^2 + \Delta_{R2}^2} = \frac{M_{\tilde{B}*}}{c_{bR}} \\ M_T = M_B = \sqrt{M_{Q*}^2 + \Delta_{L1}^2} = \frac{M_{Q*}}{c_1} \\ M_{T5/3} = M_{T2/3} = M_{Q*} \\ M_{T'} = M_{B'} = \sqrt{M_{Q'*}^2 + \Delta_{L2}^2} \simeq M_{Q'*} = M_{B-1/3} = M_{B-4/3} \end{cases}. \tag{A4}$$

In the elementary/composite basis, the Yukawa Lagrangian reads

$$\begin{aligned} \mathcal{L}^{\rm YUK} &= Y_{*U} {\rm Tr}\{\bar{\mathcal{Q}} \mathcal{H}\} \tilde{T} + Y_{*D} {\rm Tr}\{\bar{\mathcal{Q}}' \mathcal{H}\} \tilde{B} + \text{H.c.} \\ &= Y_{*U}\{\bar{T}\phi_0^\dagger \tilde{T} + \bar{T}_{2/3}\phi_0 \tilde{T} + \bar{T}_{5/3}\phi^+ \tilde{T} - \bar{B}\phi^- \tilde{T}\} + Y_{*D}\{\bar{B}_{-1/3}\phi_0^\dagger \tilde{B} + \bar{B}'\phi_0 \tilde{B} + \bar{T}'\phi^+ \tilde{B} - \bar{B}_{-4/3}\phi^- \tilde{B}\} + \text{H.c.} \end{aligned} \tag{A5}$$

After field rotation to the mass eigenstate basis, before EWSB, $\mathcal{L}^{\rm YUK}$ reads as in Eq. (A10).

After the EWSB, the top and bottom masses arise as

$$m_t = \frac{v}{\sqrt{2}} Y_{*U} s_1 s_R, \tag{A6}$$

$$m_b = \frac{v}{\sqrt{2}} Y_{*D} s_2 s_{bR}. \tag{A7}$$

We also have electroweak mixings among fermions. The fermionic mass matrices for up and down states read as follows, in the basis $(\bar{t}_L \bar{\tilde{T}}_L \bar{T}_{2/3L} \bar{T}_L \bar{T}'_L)$ $(t_R \tilde{T}_R T_{2/3R} T_R T'_R)$ for the up sector, and in the basis $(\bar{b}_L \bar{\tilde{B}}_L \bar{B}'_L \bar{B}_{-1/3L} \bar{B}_L)$ $(b_R \tilde{B}_R B'_R B_{-1/3R} B_R)$ for the down-type fermions:

$$\mathcal{M}_{up} = \begin{pmatrix} m_t & -Y_{*U}\frac{v}{\sqrt{2}} s_1 c_R & 0 & 0 & 0 \\ 0 & M_{\tilde{T}} & Y_{*U}\frac{v}{\sqrt{2}} & Y_{*U}\frac{v}{\sqrt{2}} & -s_4 Y_{*U}\frac{v}{\sqrt{2}} \\ -Y_{*U}\frac{v}{\sqrt{2}} s_R & Y_{*U}\frac{v}{\sqrt{2}} c_R & M_{T2/3} & 0 & 0 \\ -Y_{*U}\frac{v}{\sqrt{2}} c_1 s_R & Y_{*U}\frac{v}{\sqrt{2}} c_1 c_R & 0 & M_T & 0 \\ Y_{*U}\frac{v}{\sqrt{2}}(s_1 s_2 + c_1 s_3) s_R & -Y_{*U}\frac{v}{\sqrt{2}}(s_1 s_2 + c_1 s_3) c_R & 0 & 0 & M_{T'} \end{pmatrix}, \tag{A8}$$

$$\mathcal{M}_{\rm down} = \begin{pmatrix} m_b & -Y_{*D}\frac{v}{\sqrt{2}} s_2 c_{bR} & 0 & 0 & 0 \\ 0 & M_{\tilde{B}} & Y_{*D}\frac{v}{\sqrt{2}} & Y_{*D}\frac{v}{\sqrt{2}} & Y_{*D}\frac{v}{\sqrt{2}} s_4 \\ -Y_{*D}\frac{v}{\sqrt{2}} s_{bR} & Y_{*D}\frac{v}{\sqrt{2}} c_{bR} & M_{B'} & 0 & 0 \\ -Y_{*D}\frac{v}{\sqrt{2}} s_{bR} & Y_{*D}\frac{v}{\sqrt{2}} c_{bR} & 0 & M_{B-1/3} & 0 \\ -Y_{*D}\frac{v}{\sqrt{2}} s_3 s_{bR} & Y_{*D}\frac{v}{\sqrt{2}} s_3 c_{bR} & 0 & 0 & M_B \end{pmatrix}, \tag{A9}$$

$$\begin{aligned} \mathcal{L}^{\rm YUK} = {} & Y_{*U} c_1 c_R (\bar{T}_L \phi_0^\dagger \tilde{T}_R - \bar{B}_L \phi^- \tilde{T}_R) + Y_{*U} c_R (\bar{T}_{2/3L} \phi_0 \tilde{T}_R + \bar{T}_{5/3L} \phi^+ \tilde{T}_R) - Y_{*U}(s_1 s_2 + c_1 s_3) c_R (\bar{T}'_L \phi_0^\dagger \tilde{T}_R - \bar{B}'_L \phi^- \tilde{T}_R) \\ & - Y_{*U} s_1 c_R (\bar{t}_L \phi_0^\dagger \tilde{T}_R - \bar{b}_L \phi^- \tilde{T}_R) - Y_{*U} s_R (\bar{T}_{2/3L} \phi_0 t_R + \bar{T}_{5/3L} \phi^+ t_R) + Y_{*U}(s_1 s_2 + c_1 s_3) s_R (\bar{T}'_L \phi_0^\dagger t_R - \bar{B}'_L \phi^- t_R) \\ & - Y_{*U} c_1 s_R (\bar{T}_L \phi_0^\dagger t_R - \bar{B}_L \phi^- t_R) + Y_{*U} s_1 s_R (\bar{t}_L \phi_0^\dagger t_R - \bar{b}_L \phi^- t_R) + Y_{*U}(\bar{T}_R \phi_0^\dagger \tilde{T}_L - \bar{B}_R \phi^- \tilde{T}_L) \\ & + Y_{*U}(\bar{T}_{2/3R} \phi_0 \tilde{T}_L + \bar{T}_{5/3R} \phi^+ \tilde{T}_L) - Y_{*U} s_4 (\bar{T}'_R \phi_0^\dagger \tilde{T}_L - \bar{B}'_R \phi^- \tilde{T}_L) + Y_{*D} c_{bR} (\bar{B}_{-1/3L} \phi_0^\dagger \tilde{B}_R - \bar{B}_{-4/3L} \phi^- \tilde{B}_R) \\ & + Y_{*D} c_{bR} (\bar{B}'_L \phi_0 \tilde{B}_R + \bar{T}'_L \phi^+ \tilde{B}_R) - Y_{*D} s_{bR} (\bar{B}_{-1/3L} \phi_0^\dagger b_R - \bar{B}_{-4/3L} \phi^- b_R) - Y_{*D} s_{bR} (\bar{B}'_L \phi_0 b_R + \bar{T}'_L \phi^+ b_R) \\ & - Y_{*D} s_2 c_{bR} (\bar{b}_L \phi_0 \tilde{B}_R + \bar{t}_L \phi^+ \tilde{B}_R) + Y_{*D} s_2 s_{bR} (\bar{b}_L \phi_0 b_R + \bar{t}_L \phi^+ b_R) - Y_{*D} s_3 s_{bR} (\bar{B}_L \phi_0 b_R + \bar{T}_L \phi^+ b_R) \\ & + Y_{*D} s_3 c_{bR} (\bar{B}_L \phi_0 \tilde{B}_R + \bar{T}_L \phi^+ \tilde{B}_R) + Y_{*D}(\bar{B}'_R \phi_0 \tilde{B}_L + \bar{T}'_R \phi^+ \tilde{B}_L) + Y_{*U}(\bar{B}_{-1/3R} \phi_0^\dagger \tilde{B}_L - \bar{B}_{-4/3R} \phi^- \tilde{B}_L) \\ & + Y_{*D} s_4 (\bar{B}_R \phi_0 \tilde{B}_L + \bar{T}_R \phi^+ \tilde{B}_L) + \text{H.c.} \end{aligned} \tag{A10}$$

2. TS10

Fermions rotate from the elementary/composite basis to the "physical" light(SM)/heavy basis as

$$\tan\varphi_{L1} = \frac{\Delta_{L1}}{M_{Q*}} \equiv \frac{s_1}{c_1}, \qquad \begin{cases} t_L = c_1 t_L^{\rm el} - s_1 T_L^{\rm com} \\ T_L = s_1 t_L^{\rm el} + c_1 T_L^{\rm com} \end{cases}, \qquad \begin{cases} b_L = c_1 b_L^{\rm el} - s_1 B_L^{\rm com} \\ B_L = s_1 b_L^{\rm el} + c_1 B_L^{\rm com} \end{cases}, \tag{A11}$$

$$\tan\varphi_R = \frac{\Delta_{R1}}{M_{\tilde{Q}*}}, \qquad s_R \equiv \sin\varphi_R, \qquad c_R \equiv \cos\varphi_R, \qquad \tan\varphi_{bR} = \frac{\Delta_{R2}}{M_{\tilde{Q}*}}, \qquad s_{bR} \equiv \sin\varphi_{bR}, \qquad c_{bR} \equiv \cos\varphi_{bR},$$

$$\begin{cases} t_R = c_R t_R^{\rm el} - s_R \tilde{T}_R^{\rm com} \\ \tilde{T}_R = s_R t_R^{\rm el} + c_R \tilde{T}_R^{\rm com} \end{cases}, \qquad \begin{cases} b_R = c_{bR} b_R^{\rm el} - s_{bR} \tilde{B}_R^{\rm com} \\ \tilde{B}_R = s_{bR} b_R^{\rm el} + c_{bR} \tilde{B}_R^{\rm com} \end{cases}. \tag{A12}$$

Physical heavy fermion masses are related to the bare ones as

$$\begin{cases} M_{\tilde{T}} = \sqrt{M_{\tilde{Q}*}^2 + \Delta_{R1}^2} = \frac{M_{\tilde{Q}*}}{c_R} \\ M_{\tilde{B}} = \sqrt{M_{\tilde{Q}*}^2 + \Delta_{R2}^2} = \frac{M_{\tilde{Q}*}}{c_{bR}} \\ M_{\tilde{T}5/3} = M_{\tilde{T}'5/3} = M_{\tilde{T}'} = M_{\tilde{B}'} = M_{\tilde{Q}*} \\ M_T = M_B = \sqrt{M_{Q*}^2 + \Delta_{L1}^2} = \frac{M_{Q*}}{c_1} \\ M_{T2/3} = M_{T5/3} = M_{Q*} \end{cases} . \tag{A13}$$

In the elementary/composite basis, the Yukawa Lagrangian reads

$$\mathcal{L}^{\rm YUK} = +Y_* {\rm Tr}\{\mathcal{H}\bar{\mathcal{Q}}\tilde{\mathcal{Q}}'\} + Y_* {\rm Tr}\{\bar{\mathcal{Q}}\mathcal{H}\tilde{\mathcal{Q}}\}. \tag{A14}$$

After field rotation to the mass eigenstate basis, before EWSB, $\mathcal{L}^{\rm YUK}$ reads as in Eq. (A19).

After EWSB, the top and bottom masses arise as

$$m_t = \frac{v}{2} Y_* s_1 s_R, \tag{A15}$$

$$m_b = \frac{v}{\sqrt{2}} Y_* s_1 s_{bR}. \tag{A16}$$

The fermionic mass matrices for up and down states read as follows, in the basis $(\bar{t}_L \bar{\tilde{T}}_L \bar{T}_{2/3L} \bar{T}_L \bar{\tilde{T}}'_L)$ $(t_R \tilde{T}_R T_{2/3R} T_R \tilde{T}'_R)$ for the up sector, and in the basis $(\bar{b}_L \bar{\tilde{B}}_L \bar{\tilde{B}}'_L \bar{B}_L)$ $(b_R \tilde{B}_R \tilde{B}'_R B_R)$ for the down-type fermions:

$$\mathcal{M}_{\rm up}^{\rm TS10} = Y_* \frac{v}{2} \begin{pmatrix} \frac{m_t}{Y_* \frac{v}{2}} & -s_1 c_R & 0 & 0 & -s_1 \\ 0 & \frac{M_{\tilde{T}}}{Y_* \frac{v}{2}} & -1 & 1 & 0 \\ s_R & -c_R & \frac{M_{T2/3}}{Y_* \frac{v}{2}} & 0 & -1 \\ -c_1 s_R & c_1 c_R & 0 & \frac{M_T}{Y_* \frac{v}{2}} & c_1 \\ 0 & 0 & -1 & 1 & \frac{M_{\tilde{T}'}}{Y_* \frac{v}{2}} \end{pmatrix}, \tag{A17}$$

$$\mathcal{M}_{\rm down}^{\rm TS10} = Y_* \frac{v}{\sqrt{2}} \begin{pmatrix} \frac{m_b}{Y_* \frac{v}{\sqrt{2}}} & -s_1 c_{bR} & -s_1 & 0 \\ 0 & \frac{M_{\tilde{B}}}{Y_* \frac{v}{\sqrt{2}}} & 0 & 1 \\ 0 & 0 & \frac{M_{\tilde{B}'}}{Y_* \frac{v}{\sqrt{2}}} & 1 \\ -c_1 s_{bR} & c_1 c_{bR} & c_1 & \frac{M_B}{Y_* \frac{v}{\sqrt{2}}} \end{pmatrix}, \tag{A18}$$

$$\begin{aligned} \mathcal{L}^{\rm YUK} = {} & Y_* c_1 c_R \frac{1}{\sqrt{2}} (\bar{T}_L \phi_0^\dagger \tilde{T}_R - \bar{B}_L \phi^- \tilde{T}_R) - Y_* c_R \frac{1}{\sqrt{2}} (\bar{T}_{2/3L} \phi_0 \tilde{T}_R + \bar{T}_{5/3L} \phi^+ \tilde{T}_R) - Y_* s_1 c_R \frac{1}{\sqrt{2}} (\bar{t}_L \phi_0^\dagger \tilde{T}_R - \bar{b}_L \phi^- \tilde{T}_R) \\ & + Y_* s_1 s_R \frac{1}{\sqrt{2}} (\bar{t}_L \phi_0^\dagger t_R - \bar{b}_L \phi^- t_R) + Y_* s_R \frac{1}{\sqrt{2}} (\bar{T}_{2/3L} \phi_0 t_R + \bar{T}_{5/3L} \phi^+ t_R) - Y_* c_1 s_R \frac{1}{\sqrt{2}} (\bar{T}_L \phi_0^\dagger t_R - \bar{B}_L \phi^- t_R) \\ & + Y_* \frac{1}{\sqrt{2}} (\bar{T}_R \phi_0^\dagger \tilde{T}_L - \bar{B}_R \phi^- \tilde{T}_L) - Y_* \frac{1}{\sqrt{2}} (\bar{T}_{2/3R} \phi_0 \tilde{T}_L + \bar{T}_{5/3R} \phi^+ \tilde{T}_L) + Y_* (\bar{T}_{5/3L} \phi_0^\dagger \tilde{T}_{5/3R} - \bar{T}_{2/3L} \phi^- \tilde{T}_{5/3R}) \\ & + Y_* (\bar{T}_{5/3R} \phi_0^\dagger \tilde{T}_{5/3L} - \bar{T}_{2/3R} \phi^- \tilde{T}_{5/3L}) - Y_* s_1 c_{bR} (\bar{b}_L \phi_0 \tilde{B}_R + \bar{t}_L \phi^+ \tilde{B}_R) + Y_* s_1 s_{bR} (\bar{b}_L \phi_0 b_R + \bar{t}_L \phi^+ b_R) \\ & - Y_* c_1 s_{bR} (\bar{B}_L \phi_0 b_R + \bar{T}_L \phi^+ b_R) + Y_* c_1 c_{bR} (\bar{B}_L \phi_0 \tilde{B}_R + \bar{T}_L \phi^+ \tilde{B}_R) + Y_* (\bar{B}_R \phi_0 \tilde{B}_L + \bar{T}_R \phi^+ \tilde{B}_L) \\ & + Y_* (\bar{B}_R \phi_0^\dagger \tilde{B}'_L + Y_* \bar{T}_{2/3R} \phi^+ \tilde{B}'_L) Y_* \frac{1}{\sqrt{2}} (\bar{T}_R \phi_0^\dagger \tilde{T}'_L + \bar{B}_R \phi^- \tilde{T}'_L) - Y_* \frac{1}{\sqrt{2}} (\bar{T}_{2/3R} \phi_0 \tilde{T}'_L - \bar{T}_{5/3R} \phi^+ \tilde{T}'_L) \\ & + Y_* c_1 \frac{1}{\sqrt{2}} (\bar{T}_L \phi_0^\dagger \tilde{T}'_R + \bar{B}_L \phi^- \tilde{T}'_R) - Y_* \frac{1}{\sqrt{2}} (\bar{T}_{2/3L} \phi_0^\dagger \tilde{T}'_R - \bar{T}_{5/3L} \phi^+ \tilde{T}'_R) - Y_* s_1 \frac{1}{\sqrt{2}} (\bar{t}_L \phi_0^\dagger \tilde{T}'_R + \bar{b}_L \phi^- \tilde{T}'_R) \\ & + Y_* (\bar{T}_{5/3R} \phi_0 \tilde{T}'_{5/3L} - \bar{T}_R \phi^- \tilde{T}'_{5/3L}). + Y_* c_1 (\bar{B}_L \phi_0^\dagger \tilde{B}'_R - \bar{T}_L \phi^- \tilde{T}'_{5/3R}) - Y_* s_1 (\bar{b}_L \phi_0^\dagger \tilde{B}'_R - \bar{t}_L \phi^- \tilde{T}'_{5/3R}) \\ & + Y_* \bar{T}_{2/3L} \phi^+ \tilde{B}'_R + Y_* \bar{T}_{5/3L} \phi_0 \tilde{T}'_{5/3R} + {\rm H.c.} \end{aligned} \tag{A19}$$

APPENDIX B: BOUND DERIVATION

The SM prediction and the experimental measurement [26] of the $b \to s\gamma$ branching ratio are, respectively,

$$\mathrm{BR}_{\rm th} = (315 \pm 23)10^{-6}, \tag{B1}$$

$$\mathrm{BR}_{\rm ex} = (355 \pm 24 \pm 9)10^{-6}. \tag{B2}$$

The $b \to s\gamma$ decay rate is

$$\begin{aligned} \Gamma_{\rm tot} & \propto |\mathcal{C}_7(\mu_b)|^2 + |\mathcal{C}'_7(\mu_b)|^2 \\ & \approx |\mathcal{C}_7^{\rm SM}(\mu_b) + \mathcal{C}_7^{NP}(\mu_b)|^2 + |\mathcal{C}_7^{\prime\rm NP}(\mu_b)|^2. \end{aligned} \tag{B3}$$

If we consider only the $\mathcal{C}_7$ contribution, we obtain

$$\frac{\Gamma_{\rm tot}}{\Gamma_{\rm SM}} = 1 + 2\frac{{\rm Re}(\mathcal{C}_7^{\rm SM}(\mu_b)^* \mathcal{C}_7^{\rm NP}(\mu_b))}{|\mathcal{C}_7^{SM}(\mu_b)|^2} + O(\Delta\mathcal{C}_7^2). \tag{B4}$$

For $\mu_b = 5$ GeV, $\mu_W = M_W$, $\alpha_S = 0.118$, the SM contribution to C_7 at the scale μ_b reads [25]

$$C_7^{\rm SM}(\mu_b) = 0.695 C_7^{\rm SM}(\mu_W) + 0.086 C_8^{\rm SM}(\mu_W) - 0.158 C_2^{\rm SM}(\mu_W) = -0.300. \quad \text{(B5)}$$

The scaling factor of the NP contribution to C_7 from the scale μ_W to the scale μ_b is

$$\mathcal{C}_7^{\rm NP}(\mu_b) = \left(\frac{\alpha_S(\mu_W)}{\alpha_S(\mu_b)}\right)^{\frac{16}{23}} \mathcal{C}_7^{\rm NP}(\mu_w) = 0.695 \mathcal{C}_7^{\rm NP}(\mu_w). \quad \text{(B6)}$$

By considering all the previous equations, we obtain at 95% C.L.

$$-0.0775 < \mathcal{C}_7^{\rm NP}(\mu_w) < 0.0226.$$

The scaling factor of the NP contribution to C_7 from the scale $m_* = 1$ TeV to the scale μ_W is

$$\mathcal{C}_7^{\rm NP}(\mu_W) = \left(\frac{\alpha_S(m_*)}{\alpha_S(m_t)}\right)^{\frac{16}{21}} \left(\frac{\alpha_S(m_t)}{\alpha_S(\mu_W)}\right)^{\frac{16}{23}} \simeq 0.79 \mathcal{C}_7^{\rm NP}(m_*), \quad \text{(B7)}$$

and we obtain at 95% C.L.

$$-0.0978 < \mathcal{C}_7^{\rm NP}(m_*) < 0.0284.$$

If we consider only the $\mathcal{C}_7'$ contribution, we obtain

$$\frac{\Gamma_{\rm tot}}{\Gamma_{\rm SM}} \simeq 1 + \frac{|\mathcal{C}_7'^{\rm NP}(\mu_b)|^2}{|\mathcal{C}_7^{\rm SM}(\mu_b)|^2}. \quad \text{(B8)}$$

We have

$$C_7'(\mu_b) \simeq C_7'^{\rm NP}(\mu_b) = \left(\frac{\alpha_S(m_*)}{\alpha_S(m_t)}\right)^{\frac{16}{21}} \left(\frac{\alpha_S(m_t)}{\alpha_S(\mu_b)}\right)^{\frac{16}{23}} \mathcal{C}_7'^{\rm NP}(m_*) \simeq 0.55 \mathcal{C}_7'^{\rm NP}(m_*). \quad \text{(B9)}$$

By considering Eqs. (B1), (B2), and (B7)–(B9), we obtain at 95% C.L.

$$|\mathcal{C}_7'^{\rm NP}(\mu_w)| < 0.294, \qquad |\mathcal{C}_7'^{\rm NP}(m_*)| < 0.372.$$

APPENDIX C: CHARGED HIGGS ULTRAVIOLET CONTRIBUTION TO $b \to s\gamma$ IN THE TS5

$$\mathcal{H}^{\rm eff}_{\rm charged\ Higgs} = \frac{ie}{8\pi^2} \frac{(2\epsilon \cdot p)}{M_w^2} k_{\rm charged} \left[V_{ts} \bar{b}(1-\gamma_5)s + \frac{m_s}{m_b V_{ts}} \bar{b}(1+\gamma_5)s \right], \quad \text{(C1)}$$

where

$$k_{\rm charged} \approx \sum_{i=1}^{4} (|\alpha_1^{(i)}|^2 + |\alpha_2^{(i)}|^2) m_b \left(-\frac{2}{9}\right) \frac{M_w^2}{m_{*(i)}^2} + \sum_{i=1}^{4} (\alpha_1^{(i)*} \alpha_2^{(i)}) m_{*(i)} \left(-\frac{5}{6}\right) \frac{M_w^2}{m_{*(i)}^2}, \quad \text{(C2)}$$

the index i runs over the four up-type heavy fermions of the model $\mathbf{u}^{(i)}$, $m_{*(i)}$ denotes the physical mass of the $\mathbf{u}^{(i)}$ heavy fermion, and the $\alpha_1^{(i)}$, $\alpha_2^{(i)}$ coefficients derive from the following interactions:

$$\mathcal{L} \supset \bar{\mathbf{u}}^{(i)} [\alpha_1^{(i)} (1+\gamma_5) + \alpha_2^{(i)} (1-\gamma_5)] b H^+ + \text{H.c.} \quad \text{(C3)}$$

After the EWSB, we diagonalize the up-type quarks mass matrix of Eq. (A8) and the down-type one of Eq. (A9) perturbatively in $x \equiv (\frac{Y_* v}{\sqrt{2} m_*})$, neglecting $O(x^2)$. We find the following coefficients:

$$\alpha_1^{(\tilde{T})} = v Y_*^2 s_1 s_2 s_{bR} \frac{M_{\tilde{T}}^2 M_{\tilde{T}}^3 + M_{T'}^2 M_{\tilde{T}}^3 - M_{\tilde{T}}^5 + c_R M_T^3 M_{T'}^2 c_1}{4 M_T M_{\tilde{T}} (M_T^2 - M_{\tilde{T}}^2)(-M_{T'}^2 + M_{\tilde{T}}^2) c_1}, \qquad \alpha_2^{(\tilde{T})} = \frac{Y_* s_1 c_R}{2\sqrt{2}}, \qquad \alpha_1^{(T)} = \frac{Y_* s_1 s_2 s_{bR} M_{T'}^2}{2\sqrt{2} c_1 (M_{T'}^2 - M_T^2)},$$

$$\alpha_2^{(T)} = \frac{v Y_*^2 s_1}{4} \left(\frac{c_R M_{\tilde{T}} + c_1 c_R^2 M_T}{M_T^2 - M_{\tilde{T}}^2} + \frac{c_1 s_R^2}{M_T} \right), \qquad \alpha_1^{(T')} = -\frac{s_{bR} Y_*}{2\sqrt{2}},$$

$$\alpha_2^{(T')} = \frac{Y_*^2 v s_2}{4} \frac{s_1^2 c_R M_{\tilde{B}} M_{\tilde{T}} M_T M_{T'}^2 + s_1^2 c_1 c_R^2 M_{\tilde{B}} M_T^2 M_{T'}^2 + c_1 (M_{T'}^2 - M_{\tilde{T}}^2)(M_{T'}^3 c_{bR} + M_T^2 (s_1^2 s_R^2 M_{\tilde{B}} - c_{bR} M_{T'}))}{c_1 M_{\tilde{B}} M_{T'} (M_{T'}^2 - M_T^2)(M_{T'}^2 - M_{\tilde{T}}^2)}. \quad \text{(C4)}$$

The heavy fermion $T_{2/3}$ gives a contribution of $O(x^2)$ to $k_{\rm charged}$, and we can neglect it. Considering Eq. (C2) and the coefficients in Eq. (C4), and again neglecting $O(x^2)$ terms, we obtain

$$k_{\rm charged} = -m_b M_W^2 Y_*^2 \frac{-15 M_T^2 M_{T'} M_{\tilde{T}}^2 \sqrt{1 - s_{bR}^2} + M_{\tilde{B}} (15 M_{T'}^2 M_{\tilde{T}}^2 s_1^2 s_R^2 + M_T^2 (11 M_{T'}^2 s_1^2 (-1 + s_R^2) + M_{\tilde{T}}^2 (4 s_{bR}^2 + 15 s_1^2 s_R^2)))}{144 M_{\tilde{B}} M_T^2 M_{T'}^2 M_{\tilde{T}}^2}$$

and

$$k_{\rm charged} \approx m_b M_W^2 Y_*^2 \frac{5}{48} \frac{1}{M_{B'} M_{\tilde{B}}} + O(s_1^2) + O(s_{bR}^2), \quad \text{(C5)}$$

if we can neglect $O(s_1^2)$.

APPENDIX D: ULTRAVIOLET CONTRIBUTION

Summing up, we find the following in the TS5 model:

$$k_{\text{neutral}} = -m_b M_W^2 Y_*^2 \frac{1}{8}\left(\frac{c_{bR}}{M_{B'}M_{\tilde{B}}} - \frac{7}{18}\frac{s_{bR}^2}{M_{B'}^2}\right) = -m_b M_W^2 Y_*^2 \frac{1}{8}\frac{1}{M_{B'}M_{\tilde{B}}} + O(s_{bR}^2)$$

$$k_{\text{charged}} = m_b M_W^2 Y_*^2 \frac{5}{48}\frac{1}{M_{B'}M_{\tilde{B}}} + O(s_1^2) + O(s_{bR}^2)$$

and in the TS10 model:

$$k_{\text{neutral}} = m_b M_W^2 Y_*^2 \frac{7M_T M_{T'}^2 s_1^2 - 18 M_{\tilde{B}} M_{\tilde{B}'}^2 \sqrt{1-s_1^2} + M_{\tilde{B}}^2 (7 M_B s_1^2 - 18 M_{\tilde{B}'}\sqrt{1-s_1^2})}{288 M_{\tilde{B}}^2 M_B M_{\tilde{B}'}^2} + O(s_{bR})$$

$$= -m_b M_W^2 Y_*^2 \frac{1}{16}\left(\frac{1}{M_B M_{\tilde{B}}} + \frac{1}{M_B M_{\tilde{B}'}}\right) + O(s_1^2) + O(s_{bR}),$$

$$k_{\text{charged}} = m_b M_W^2 Y_*^2 \left(\frac{5}{48}\frac{1}{M_B M_{\tilde{B}}} + \frac{5}{48}\frac{1}{M_B M_{\tilde{B}'}} + \frac{5}{96}\frac{s_R^2}{M_{\tilde{B}}^2}\right) + O(s_1^2) + O(s_{bR}^2).$$

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