Final state interactions in single- and multiparticle inclusive cross sections for hadronic collisions

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We study the role of low momentum transfer (soft) interactions between high transverse momentum heavy particles and beam remnants (spectators) in hadronic collisions. Such final state interactions are power suppressed for single-particle inclusive cross sections whenever that particle is accompanied by a recoiling high-p_T partner whose momentum is not fixed. An example is the single-top inclusive cross section in top-pair production. Final state soft interactions in multiparticle inclusive cross sections, including transverse momentum distributions, however, produce leading-power corrections in the absence of hard recoiling radiation. Nonperturbative corrections due to scattering from spectators are generically suppressed by powers of Λ/p'_T, where Λ is a hadronic scale and p'_T is the *largest* transverse momentum of radiation recoiling against the particles whose momenta are observed.

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I. INTRODUCTION

At hadron colliders, final states with high transverse momentum and massive strongly interacting particles play an important role in the search for physics beyond the Standard Model but present special challenges to theory. Our best theoretical predictions apply when cross sections can be computed using conventional collinear factorization [1], with corrections suppressed by powers of a hard scale. Such factorization has been shown for the inclusive production of electroweak bosons [1–4] and single-particle inclusive (1PI) cross sections in hadronic collisions [5]. On the other hand, how far specific observables may be generalized while retaining factorizability with small corrections is not fully understood [6]. While proofs of factorization require a full treatment of soft and collinear radiation, one of the basic ingredients is the cancellation of the final state interactions of the observed particles. In this paper, we will concentrate on corrections due to final state interactions in single- and multiparticle inclusive cross sections in hadronic collisions.

We present below an analysis based on light-cone ordered perturbation theory of interactions between hard, final state partons and spectator partons (remnants) from initial state hadrons. We observe that the cancellation of final state interactions involving one or more observed final state particles requires that the cross section be insensitive to changes in the recoil momentum of unobserved particles. This analysis suggests an estimate of the residuals of the cancellation and under what circumstances we can anticipate significant and perhaps measurable effects. We will see that the cancellation of final state interactions (FSI) requires that untagged particles produced in the hard collision carry sufficient momentum transverse to the beam axis to absorb the recoil of momenta transferred by these interactions to the observed particles. For top or antitop 1PI cross sections in top-pair production, the untagged partner plays this role. Uncanceled FSI effects are suppressed by at least a single power of m_t, consistent with a recent estimate of nonperturbative string-breaking effects in Ref. [7]. Indeed, our analysis is inspired in part by the apparent mismatch between observed top-pair asymmetries [8] and Standard Model predictions based on factorized cross sections [9–12]. We observe, however, that this mismatch is primarily in terms of normalization and that the roughly linear dependence of the asymmetry on both pair invariant mass and rapidity difference is shared by resummed QCD predictions [10] and the data as reported, for example, in Ref. [13]. It has also been suggested that differences in normalization could be due to the choice of renormalization and factorization scales in the predictions [14]. Final state interactions seem therefore unlikely candidates for an explanation of top quark asymmetry measurements based on single-particle inclusive cross sections. The more general question of when such corrections may be important, however, is of independent interest.

We will argue that for double-particle inclusive cross sections, involving for example both members of a top pair, the cancellation of FSI requires summation over states with additional high transverse momentum radiation. We will see that it is the transverse momenta of this radiation, rather than the mass or transverse momenta of the top pair, that controls the scale of power corrections to the factorized forms of cross sections. At low enough values of recoiling transverse momentum, collinear factorization is no longer possible.

The need to generalize collinear factorization whenever recoiling momentum is small compared to the scale of the hard scattering is familiar from transverse momentum distributions in electroweak boson production at hadron colliders [15,16]. At low values of the Z or H transverse momentum distribution collinear factorization must be replaced by factorization in terms of transverse momentum-dependent parton distributions, with explicit demonstrations given in Refs. [4,17]. Transverse momentum-dependent parton distributions are closely related to the beam function formalism developed in Ref. [18]. Recently, however, it was shown by Collins and Qiu [19] and Rogers and Mulders [20] that transverse momentum-dependent parton distribution factorization is not possible for the double inclusive cross sections of strongly interacting particles when scattering from spectators is taken into account. References [19,20] proceeded by a careful analysis of low-order diagrams. Here we provide an all-orders analysis of the conditions necessary specifically for leading-power collinear factorization in single- and multiparticle inclusive corrections.

We will frame our discussion in the context of top-pair production at hadron colliders, for its intrinsic interest, treating final state interactions to all orders in perturbation theory. Our considerations are simple but realistic, however, and the conclusions have much wider applicability. A related observable is the same sign dimuon asymmetry [21] measured at the Tevatron [22].

In the following section, we review the classification of initial and final states according to light-cone ordered perturbation theory (LCOPT), to illustrate the origin of long-distance effects and to study final state and other long-distance corrections in a general context. The cancellation of final state interactions in single-particle inclusive cross sections is discussed in Sec. III, first at lowest order and then to all orders. We find a pattern for the cancellation of final state interactions that illustrates the role of recoiling final state partons and which leads us to conclude that corrections to leading-power factorized cross sections are suppressed by the top quark mass in this case. In contrast, we show in Sec. IV that the cancellation of final state interactions for two-particle inclusive cross sections depends on the presence of additional high-p_T radiation, which also sets the scale for power-suppressed corrections. We conclude with a discussion of our results, their possible extensions and phenomenological implications in Sec. V.

II. TOP-PAIR FINAL STATES IN LIGHT-CONE ORDERED PERTURBATION THEORY

As noted in the introduction, we will consider massive-pair inclusive cross sections of the form

$$H_A(p_A) + H_B(p_B) \to t(p_t) + \bar{t}(p_{\bar{t}}) + X, \qquad (1)$$

with top production in mind, although our reasoning is more general. To be specific, when we consider a single-particle inclusive cross section, we fix the top quark momentum and integrate over the antitop quark momentum. The top quark mass m_t already provides a hard scale, and to avoid multiple scales we imagine that the transverse momentum of each of the quarks in the pair is of order m_t. For the single-particle inclusive cross section, we then have a factorized expression [5]

$$\frac{d\sigma}{d^3 p_t} = \sum_{ab} f_{a/A} \otimes f_{b/B} \otimes \hat{\sigma}^{\text{part}}_{ab\to t+X}(p_t) + \mathcal{O}(\Lambda/m_t), \qquad (2)$$

where for this discussion we consider only top-pair, not single-top, processes. The factorized cross section is a convolution in momentum fractions of a perturbatively calculable hard function $\hat{\sigma}^{\text{part}}_{ab\to t+X}$ and nonperturbative but process-independent parton distribution functions $f_{c/C}$, for each parton c in hadron H_C. Corrections are present but are suppressed by the hard scale m_t, as we shall verify below. The arguments for factorization in single-particle inclusive cross sections of this sort were assembled in Ref. [5] for hadrons produced in fragmentation. The case of the top quark is actually simpler, because the top quark pair is produced at short distances, and there is no need of a nonperturbative fragmentation function. Nevertheless, the top quark and antiquark do undergo interactions before they decay, and one of our goals is to show how these effects cancel in (2). We will then go on to study multi-particle inclusive cross sections, starting with the two-particle case where both the top and antitop momenta are observed:

$$\frac{d\sigma}{d^3 p_t d^3 p_{\bar{t}}} = \sum_{ab} f_{a/A} \otimes f_{b/B} \otimes \hat{\sigma}^{\text{part}}_{ab\to t+\bar{t}+X}(p_t, p_{\bar{t}}) + C_{\text{2PI}}, \qquad (3)$$

and to estimate the size of corrections C_{2PI} associated with the final state interactions of the top pair.

A. The notation of LCOPT

To quantify the effect of final state interactions we use LCOPT [23], relying on much of the same algebraic analysis as applied to jet cross sections in Ref. [24]. Effectively, in LCOPT the integration over the minus (or plus) light-cone components,

$$k^{\pm} = \frac{1}{\sqrt{2}}(k^0 \pm k^3), \qquad (4)$$

is carried out for each line momentum.

The result of minus integrals is a sum over diagrams with x^+-ordered vertices, separating states with lines whose minus momenta are fixed by the mass-shell condition. The characteristic feature of LCOPT is that all lines move "forward" in x^+, with positive plus momenta only [25–27]. For the production of a top quark pair, final states will be those that include the pair, and by implication, initial states are those that do not. To present the resulting diagrams, it is convenient to introduce the notation

$$[k]^- \equiv \frac{m^2 + k_\perp^2}{2k^+}, \tag{5}$$

with m the mass and k the momentum.

In LCOPT, a single covariant diagram $\mathcal{G}_{\{p\}\to\{q\}}$ with four-dimensional loop integrals is rewritten as the sum of diagrams in which all vertices are ordered (in x^+) and in which plus momentum integrals extend over only that range in which all lines flow forward (in x^+). Such a diagram can then be written as a sum over vertex orderings, T. Each ordering prescribes a set of states s, consisting of lines that appear between two vertices that are neighbors in the ordering. We represent it as

$$\mathcal{G}_{\{p\}\to\{q\}} = \sum_{\text{orderings } T} \int \prod_{\text{loops}\{l\}} d^2 l_\perp dl^+ \prod_{\text{lines}\{k\}} \frac{\theta(k^+)}{2k^+} \times \prod_{\text{states}\{s\} \text{in } T} \frac{1}{P^- - s([k]) + i\epsilon} N(\{p\}, \{q\}, [k]), \tag{6}$$

where $P^- = \sum_a p_a^-$ is the total incoming minus momentum and where

$$s([k]) = \sum_{\text{lines}\{k\} \in \text{state } s} [k]^- \tag{7}$$

is the sum of all the on-shell minus momenta in a specific state, determined as in (5). For any given state, the sum may include a subset of incoming lines p_a^- and/or (with a negative sign) outgoing lines q_j^-. An overall momentum conservation delta function and other constants have been suppressed. The factors $\theta(k^+)$ ensure that plus momenta flow forward, that is, from earlier to later vertices in the ordered amplitude (and the opposite in the complex conjugate). The factor $N(\{p\}, \{q\}, [k])$ represents all overall momentum and constant factors. We shall assume that N is a polynomial in loop momenta, in which case it does not affect our reasoning below.

B. Choice of frame

To analyze final state interactions between remnants of the initial state hadrons and the produced pair, it will be convenient to treat the momenta of the incoming hadrons on an equal footing. This forces us to choose a frame in which the 3-direction that defines the light-cone momentum component k^-, over which we will integrate, is not in the direction of either of the incoming hadrons, p_A and p_B in Eq. (1). To be specific, we will choose a center of mass frame in which these momenta are in the positive and negative 1-direction, so that

$$\frac{1}{2}\sqrt{\frac{S}{2}} = p_A^\pm = p_B^\pm, \qquad \frac{\sqrt{S}}{2} = p_A^0 = p_B^0, \qquad p_A^1 = p_A^0, \qquad p_B^1 = -p_B^0. \tag{8}$$

With this choice of frame, we define, for any momentum k,

$$k_\perp = (k^1, k^2). \tag{9}$$

Thus, the incoming hadrons start with equal and opposite $\perp$ momentum. When we want to refer to momenta transverse to the beam direction we will use the notation k_T:

$$k_T = (k^2, k^3). \tag{10}$$

C. Cut diagram notation, initial and final states

To construct our cross sections for any given out state with momenta $\{q\}$ from an in state with momenta $\{p\} = \{p_A, p_B\}$, we take the absolute square of the sum of covariant diagrams $\mathcal{G}_{\{p\}\to\{q\}}$, as they contribute to the S matrix. We will use the term "out state" to refer to a specific contribution to the inclusive cross section, to distinguish it among the class of "final states," which will refer to any state that includes the produced pair of heavy quarks (or other high-p_T particles). We are thus treating the tops as perturbatively produced rather than as part of the parton distributions, and for definiteness we are neglecting single-top production, although our analysis applies as well to this case.

The basic, parton model contribution to the cross section for pair production is represented as in Fig. 1 in cut diagram notation, where the vertical dotted line (the "cut") identifies the out state. In the figure, a $t\bar{t}$ pair with momenta p_t and $p_{\bar{t}}$ emerges from a hard scattering in both the amplitude (to the left of the vertical line) and complex conjugate amplitude (to the right) in proton-(anti) proton scattering with in state momenta p_A and p_B and with

$$P^- = p_A^- + p_B^-. \tag{11}$$

As usual, the hard scattering is initiated by "active" partons, of flavors a and b, whose momenta are taken proportional to the momenta of the incoming hadrons:

$$\hat{q}^\mu = x_q p_H^\mu = x_q\left(p_H^+, \frac{p_{H,\perp}^2}{2p_H^+}, p_{H,\perp}\right), \qquad H = A, B, \tag{12}$$

where, as usual, $0 < x_q \leq 1$ and where generally we will use the notation $\hat{q}$ to refer to an on-shell momentum. In our

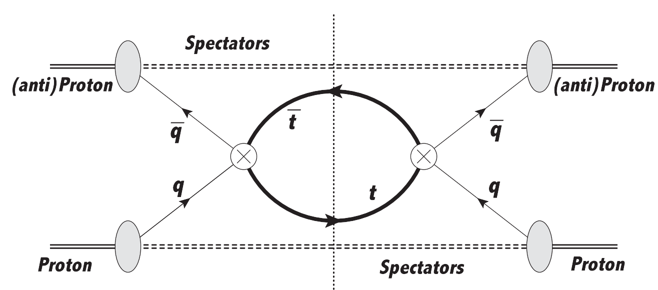

FIG. 1. Born cut diagram. The oval blobs are related to the parton distributions and the circle with a cross represents the hard scattering.

discussion below, we consider cut diagrams like Fig. 1 and its generalization to higher loops, Fig. 2, as LCOPT diagrams, in which all vertices are ordered. The ordering in the complex conjugate amplitude is opposite to that in the amplitude, so that the cut diagram as a whole describes forward scattering, with a sequence of states in the amplitude beginning with the in state of the process and culminating with the out state, followed by a sequence of states in the complex conjugate that take us back to the in state.

In Fig. 1 and below we have simplified our representation by denoting the collection of spectators for p_A and p_B by l and l', respectively, and showing them in the figure as double lines. Our arguments below will not depend on the x^+ ordering of the spectator interactions. Nonperturbative information, such as proton structure, is encoded in the initial state functions and in the distributions of spectators. In Fig. 1 there is no rescattering of the pair with spectators, and there is thus only a single final state, identical to the out state with the quark pair.

The generic form of higher-order cut diagrams that include soft final state interactions of the outgoing pair is illustrated by Fig. 2. In the case shown in the figure, there are four final states, as indicated by the vertical lines.[1]

Combining all cuts in a partonic c.m. frame, we can write the contribution to the cross section from an arbitrary region in momentum space, Π_{ab}, in which the hard scattering is initiated by parton a from A and b from B, as

$$
\begin{aligned}
&2p_t^+ \frac{d\sigma^{(\Pi_{ab})}_{AB\to t\bar{t}+X}}{d^3p_t} \\
&= \sum_{\text{orderings T of }\Pi_{ab}} \int \prod_{\text{loops}\{l\}} d^2l_\perp dl^+ \prod_{\text{lines}\{k\}} \frac{\theta(k^+)}{2k^+} \\
&\quad \times \int_0^1 dx\,\delta\left(x - \frac{x_a p_A^- + x_b p_B^-}{P^-}\right) I^{(T)*}_{ab/AB}(x, q'_a, q'_b, p_A, p_B) \\
&\quad \times \mathcal{F}^{(T)}_{ab}(x, x_a p_A, x_b p_B, p_t) I^{(T)}_{ab/AB}(x, q_a, q_b, p_A, p_B),
\end{aligned}
\tag{13}
$$

where now the sum over x^+ orderings and products over loops and lines refers to the entire cut diagram, including the final states. We have introduced the integration variable x to quantify the minus momentum available for the top pair and soft radiation in terms of the on-shell minus momenta of partons of momentum q_a and q_b, whose large momentum components are defined as in Eq. (12) above. Notice that the corresponding dependence in the complex conjugate amplitude is independent, although in the limit of zero final state momentum transfer to the pair, $q'_a = q_a$ and $q'_b = q_b$. Dependence on loop momenta $\{l\}$ is implicit. The function $I^{(T)}_{ab/AB}$ in Eq. (13) represents the effects of all initial states in the amplitude and $I^{(T)*}_{ab/AB}$ in the complex

[1]If a gluon momentum is collinear or hard, it becomes a part of the parton distribution functions and/or a participant in the hard scattering process.

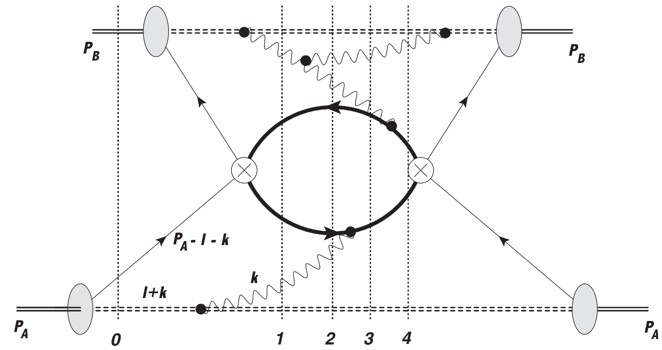

FIG. 2. Generic higher-order diagram. The oval blobs are related to the parton distributions and the round blob with a cross represents the hard scattering. The momenta refer to the discussion of the initial state labeled 0 in connection with Eq. (20).

conjugate amplitude. As noted above, initial states are precisely those states that do not include the top pair for the particular ordering T. The perturbative order of the I's will not play a role in our arguments on final states, nor do we have to assume that we have summed over the full set of states necessary to cancel nonfactoring initial state interactions [1–4].

The function $\mathcal{F}^{(T)}$ represents the product of denominators from the remaining, final states, which do include the quark pair, along with the momentum-conserving delta function associated with the out state. We shall also include in $\mathcal{F}$ the short-distance factors that describe the production of the top pair, which we denote by H in the amplitude (to the left of the cut) and H^* to the right. In LCOPT, these factors are given by denominators that are highly off-shell.

D. Leading regions, initial state jets and final state interactions

We wish to study the effects of final state interactions at leading power in the large scales of the problem, all of the order of the top mass. These contributions come from so-called "leading regions" [1], where in covariant perturbation theory, subsets of virtual lines are near the mass shell. These are regions (subspaces) where the integrands of loop momenta are singular and where momentum integrals are either pinched between coalescing singularities or forced to end points [24,28]. In LCOPT, of course, all lines are treated as on-shell, but the characterization of regions still holds. In the following, we will use extensively the logarithmic nature of (gauge invariant combinations of) integrals in gauge theory leading regions [1]. This implies that a cancellation in an integrand at the singular surface will suppress the integrand near the leading region, making its contribution finite.

In leading regions, a subdiagram of the full cut diagram has all loop momenta (including phase space loops) nearly parallel to the incoming hadron A, another to hadron B, and another subdiagram has all line momenta nearly zero.

These are referred to respectively as jet-A, jet-B and soft subdiagrams, which include the "spectator" lines of Figs. 1 and 2. Notice that lines of the out state appear in the jet and soft subdiagrams in general. Such a leading region contains a subspace of the total loop momentum and phase space at which all the jet and soft lines are exactly on shell. This subspace will sometimes be identified below as its corresponding "pinch surface" [24]. At the pinch surface, a line in the A or B jet takes on a momentum $\hat{q}$ that is exactly parallel to one of the incoming hadrons, $H = A, B$, as defined above in Eq. (12). Then for lines in the jet subdiagrams we expand in terms of $+$ and $\perp$ components:

$$q = \hat{q} + \delta q, \qquad \delta q \equiv (\delta q^+, 0^-, \delta q_\perp). \tag{14}$$

We emphasize again that in the frame we choose, both incoming hadrons are perpendicular to the spatial light-cone direction [see Eq. (8)]. In the sense of light-cone ordering, before the hard scattering the sum of all x_a in the A jet or x_b in the B jet is unity at the singular configuration. After the hard scattering, the fractional momenta of the remaining jet lines, the spectators, will add up to less than 1. We will use the term spectators below to refer to final state partons with transverse momenta at the hadronic scale. Partons with perturbative transverse momenta but still at small angles to the incoming momenta will be referred to as part of the forward jets. The spectators are part of the forward jets, but the jets also include perturbative radiation in general.

Near the singular surface, we can expand the on-shell minus momenta of jet lines in either jet relative to their values at the pinch surface,

$$[q]^- = \frac{(x_q p_A + \delta q)_\perp^2}{2(\hat{q}^+ + \delta q^+)} \equiv [\hat{q}]^- + \beta_q \cdot \delta q + \frac{1}{\hat{q}+} \delta q \cdot \overleftrightarrow{\gamma}_q \cdot \delta q + \cdots, \tag{15}$$

where we neglect terms beyond second order and where linear and quadratic terms are given explicitly by

$$\beta_q \cdot \delta q = \frac{p_{A,\perp} \cdot \delta q_\perp}{p_A^+} - \frac{[p_A]^-}{p_A^+} \delta q^+,$$
$$\delta q \cdot \overleftrightarrow{\gamma}_q \cdot \delta q = \frac{(\delta q_\perp)^2}{2} - \frac{(p_{A,\perp} \cdot \delta q_\perp)\delta q^+}{p_A^+} + \frac{[p_A]^-}{p_A^+}(\delta q^+)^2. \tag{16}$$

Equivalently, the components of the four-vector β_q (always zero in the plus entry) are given by

$$\beta_q^\mu = \frac{1}{\hat{q}^+}(0^+, \hat{q}^-, \hat{q}_\perp). \tag{17}$$

In fact, the combination $\beta_q \cdot \delta q$ is the on-shell value of the minus momentum for the linear eikonal propagator $1/(\hat{q} \cdot \delta q)$. Thus, the expansion in δq can be thought of as an expansion around the eikonal approximation for the heavy quarks [29]. The quadratic terms in this expansion are given by the nonzero elements of the matrix $(\overleftrightarrow{\gamma}_q)_{\mu\nu}$, defined as

$$(\overleftrightarrow{\gamma}_q)_{ij} = \frac{1}{2}\delta_{ij}, \qquad (\overleftrightarrow{\gamma}_q)_{++} = \frac{[\hat{q}]^-}{\hat{q}^+},$$
$$(\overleftrightarrow{\gamma}_q)_{i+} = (\overleftrightarrow{\gamma}_q)_{+i} = -\frac{\hat{q}_i}{2\hat{q}^+}. \tag{18}$$

We notice that all lines in jet H, $H = A, B$, for which $\hat{q} = x_q p_H$, have the same β_q and $\overleftrightarrow{\gamma}_q$:

$$\beta_q = \beta_{p_H} \equiv \beta_H, \qquad q \in J_H,$$
$$\overleftrightarrow{\gamma}_q = \overleftrightarrow{\gamma}_{p_H} \equiv \gamma_H, \qquad q \in J_H, \qquad H = A, B. \tag{19}$$

The components δq^+ and $\delta q_\perp$ along with the three components of soft loop momenta control the contribution from each leading region. In particular, the first-order term in the expansion of the momentum of a jet line depends only on the scaleless vector β_{p_H} and is independent of x_q, while the second-order terms depend on x_q only as an overall factor.

These results have immediate consequences for the initial state light-cone denominators. Consider, for example, the state 0 in Fig. 2, which consists of only two lines, the active and a spectator parton. The corresponding light-cone denominator is

$$p_A^- - [p_A - l - k]^- - [l + k]^- = -\frac{1}{x_l(1 - x_l)p_A^+}(\delta l + k) \cdot \overleftrightarrow{\gamma}_{p_A} \cdot (\delta l + k), \tag{20}$$

in which the linear terms cancel. In the general case, there is a similar contribution from the B jet when the state includes some of its lines, and indeed, there is no linear dependence on spectator momenta in initial state denominators. For this reason, the dependence of the initial state factors on the variable x in Eq. (13) (the total minus momentum flowing into the hard scattering) can be absorbed into overall factors like $x_l(1 - x_l)$ in denominators like Eq. (20). The resulting x dependence is hence smooth and in fact analytic. Now at any order, the logarithmic integrals associated with initial state singularities involve only soft momenta and the transverse momenta of spectator lines and are independent of the exact value of x. We thus expect smooth behavior in x to extend to all orders for initial states. This will play an important role in our arguments below. In the following section, we turn to a study of final state interactions.

III. CANCELLATION OF FINAL STATE INTERACTIONS IN SINGLE-PARTICLE INCLUSIVE CROSS SECTIONS

In this section we discuss the sum over the choice of out state among the final states of the cut diagram represented in Eq. (13). Taken together these factors may be represented in the general case of S final states as

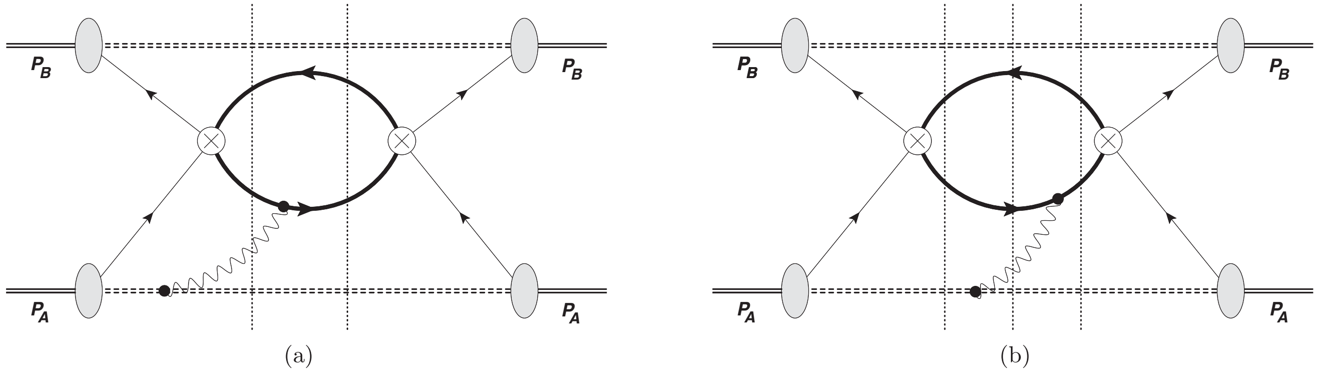

FIG. 3. The LCOPT diagrams with first-order final state interactions discussed in the text.

$$\mathcal{F}^{(T)} = \sum_{j=1}^{S} \int d^3 p_t^{(j)} \biggl(\prod_{i'=j+1}^{S} \frac{1}{P^- - s_{i'} - i\epsilon} \biggr) 2\pi\delta(P^- - s_j) \times \biggl(\prod_{i=1}^{j-1} \frac{1}{P^- - s_i + i\epsilon} \biggr) \delta^3(p_t - p_t^{(j)}) \times H_{ab}^*(x_a p_A, x_b p_B, p_t) H_{ab}(x_a p_A, x_b p_B, p_t), \quad (21)$$

where, again, s_j is the sum of on-shell minus momenta in the out state, and similarly for the final states in the amplitude and complex conjugate. The hard scattering functions H and H^* depend only weakly on soft momenta, and we can consider them as functions only of the on-shell active parton and the observed top momenta.

In our analysis of the nearly on-shell light-cone denominators, of course, we must keep track of the top quark momentum in each state. The factor $\delta^3(p_t - p_t^{(j)}) \equiv \delta(p_t^+ - p_t^{(j)+})\delta^2(p_{t,\perp} - p_{t,\perp}^{(j)})$ fixes the top quark momentum that appears in out state j to equal the prescribed value $\vec{p}_t$. For each choice of out state, j we treat the momentum of the top in that state, $p_t^{(j)}$, as a loop momentum that passes through the top and antitop lines and the hard scatterings only. The momenta of top lines in all other final states is then fixed by p_t and the sum of soft gluon emission and absorption. In the sum over j, we let each final state play the role of the out state in turn. Again, we suppress numerator factors, and because the choice of light-cone order T will be fixed for our argument, we will suppress it as well below.

A. Lowest order

To illustrate the mechanism of cancellation, we begin with final state hard parton (top)-spectator interactions in single-particle inclusive observables at first order. As we will show in the following subsection, the generalization of the proof of cancellation to all perturbative orders is straightforward.

First-order soft final state hard parton-spectator interactions are shown in Fig. 3. In these ordered diagrams, a single soft gluon is emitted from the spectators of line p_A and absorbed by the top quark. In addition to these two diagrams there are also their complex conjugates, exchanges between the t quark and the p_B-jet spectators, diagrams where t is exchanged with $\bar{t}$, and also diagrams where the gluon is emitted from an "active" line. The reasoning in these cases is equivalent.

In Fig. 3(a), the soft gluon is emitted from an initial state, in Fig. 3(b) from a final state. The former case has two final states, the latter three. We are interested in leading regions that involve soft gluon exchange, and we should note that the range in gluon momenta with leading-power behavior depends on the nature of the final state process. For example, in diagrams like those in Fig. 3 the wide-angle radiation of an on-shell gluon from a spectator with longitudinal momentum $l^- \sim xP$ and transverse momentum $\langle l_T \rangle$ (relative to the beam axis) is leading-power only for very soft momenta, $k^\pm \sim k_T \leq \langle l_T \rangle^2 / xP$ [30,31]. In contrast, an off-shell gluon exchange that mediates the elastic scattering of the top quark by a spectator is leading-power all the way to the scale of the spectator's transverse momentum, $k_T \sim \langle l_T \rangle$. Naturally such scattering processes have greater potential for phenomenological relevance. The arguments we give below cover both of these cases, however, and we will generally assume that soft gluon exchange involves momentum transfers up to the scale of the transverse momenta of spectators.

We begin with the simplest case of Fig. 3(a), which has only two final states. The sum over the two choices of out state ("cuts") in Fig. 3(a) is shown explicitly in Fig. 4, with assigned momenta for the pair. After using the three-dimensional momentum delta function to do the $p_t^{(j)}$ integral, the quantity $\mathcal{F}$ defined in Eq. (21) can be written, here with $S = 2$, as

$$\mathcal{F} = \biggl[2\pi\delta(D_2^{(2)}) \frac{1}{D_1^{(2)} + i\epsilon} + \frac{1}{D_2^{(1)} - i\epsilon} 2\pi\delta(D_1^{(1)}) \biggr] \times H_{ab}^*(x_a p_A, x_b p_B, p_t) H_{ab}(x_a p_A, x_b p_B, p_t), \quad (22)$$

where we denote by $D_i^{(j)}$ the minus momentum deficit [as in Eq. (21)] of final state i when j is the out state.

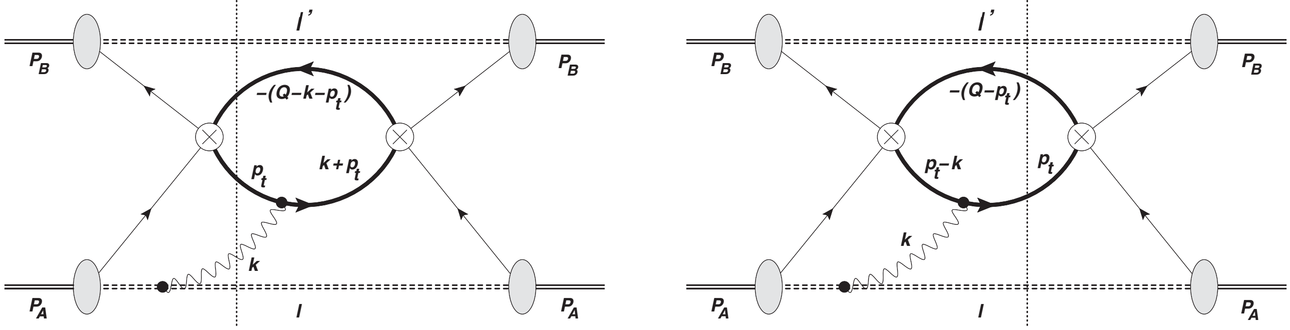

FIG. 4. The two cuts of the left diagram in Fig. 3. The relevant three-momenta are shown and $Q \equiv P_A + P_B - l - l'$.

When $i = j$, $D_j^{(j)}$ is the argument of the delta function in the corresponding term. For convenience, we define the total momentum flowing into the hard scattering in the amplitude by

$$Q \equiv p_A + p_B - l - l', \tag{23}$$

where l (l') represent all spectators at the pinch surface in the first (final) state in the amplitude after the hard scattering. We do not include in Q the momenta of soft lines like k in Fig. 4, which carry momenta between spectators and the top pair in the final state. In these terms, the functions $D_i^{(j)}$ for Eq. (22) can be written as

$$\begin{aligned}
D_1^{(1)} &= xP^- - [Q - p_t - k]^- - [p_t]^- + d_1 \\
&= xP^- - [Q - p_t]^- - [p_t]^- + \beta_{Q-p_t} \cdot k - \frac{1}{(Q - p_t)^+} k \cdot \overleftrightarrow{\gamma}_{Q-p_t} \cdot k + d_1 + \cdots, \\
D_1^{(2)} &= xP^- - [Q - p_t]^- - [p_t - k]^- + d_1 \\
&= xP^- - [Q - p_t]^- - [p_t]^- + \beta_{p_t} \cdot k - \frac{1}{p_t^+} k \cdot \overleftrightarrow{\gamma}_{p_t} \cdot k + d_1 + \cdots, \\
D_2^{(1)} &= xP^- - [Q - p_t - k]^- - [p_t + k]^- + d_2 \\
&= xP^- - [Q - p_t]^- - [p_t]^- + (\beta_{Q-p_t} - \beta_{p_t}) \cdot k + \frac{1}{(Q - p_t)^+} k \cdot \overleftrightarrow{\gamma}_{Q-p_t} \cdot k - \frac{1}{p_t^+} k \cdot \overleftrightarrow{\gamma}_{p_t} \cdot k + d_2 + \cdots, \\
D_2^{(2)} &= xP^- - [Q - p_t]^- - [p_t]^- + d_2.
\end{aligned} \tag{24}$$

In the second equalities for each of the first three $D_i^{(j)}$ we have expanded to second order in soft momentum k, following Eq. (15). For each denominator, we have added and subtracted the term xP^-, defined in Eq. (13), absorbing the term $-xP^-$, into d_1 and d_2, which depend only on the final state i and not on the choice of out state j. The d_i in Eq. (24) also depend on the details of the spectator and soft lines, whether or not connected directly to the top loop. Our arguments will not depend on their explicit form. To give an example, however, we can treat the double-dashed lines of Fig. 3 as single spectators, of momenta l and l', which gives

$$\begin{aligned}
d_1 &= (1 - x)P^- - [k]^- - [l]^- - [l']^-, \\
d_2 &= (1 - x)P^- - [l]^- - [l']^-.
\end{aligned} \tag{25}$$

We expand jet line momenta l and l' about the pinch surface, where $Q = x_a P_A + x_b P_B$ in Eq. (23). The expansion then follows Eq. (15), with

$$\begin{aligned}
[\hat{l}]^- &= (1 - x_a)p_A^-, \\
[\hat{l}']^- &= (1 - x_b)p_B^-,
\end{aligned} \tag{26}$$

where again x_h is the fractional momentum of the active parton from hadron $H = A, B$ at the pinch surface. To second order, as given in Eq. (15), we find

$$\begin{aligned}
d_1 &= -\frac{k_\perp^2}{2k^+} - \beta_{p_A} \cdot \delta l + \frac{\delta l \cdot \overleftrightarrow{\gamma}_{p_A} \cdot \delta l}{l^+} \\
&\quad - \beta_{p_B} \cdot \delta l' + \frac{\delta l' \cdot \overleftrightarrow{\gamma}_{p_A} \cdot \delta l'}{l'^+}, \\
d_2 &= -\beta_{p_A} \cdot \delta l + \frac{\delta l \cdot \overleftrightarrow{\gamma}_{p_A} \cdot \delta l}{l^+} \\
&\quad - \beta_{p_B} \cdot \delta l' + \frac{\delta l' \cdot \overleftrightarrow{\gamma}_{p_A} \cdot \delta l'}{l'^+}.
\end{aligned} \tag{27}$$

From Eqs. (23) and (26) we see that within each of the terms of (22), the large (that is, order m_t) terms cancel, and the uncut final state denominator is independent of x and is of order of the components of vector k^μ. Thus, the contributions from $\mathcal{F}$ to the corresponding cut diagrams are order $1/k$, where k will stand collectively for the terms $[k]^-$, $\beta_A \cdot l$, $\beta_B \cdot l'$. We will show that this $1/k$ behavior cancels after the sum over the two cuts.

To exhibit the cancellation of the singular, $1/k$ behavior just identified in final state interactions, we will apply the relation

$$2\pi\delta(y) = i\left(\frac{1}{y+i\epsilon} - \frac{1}{y-i\epsilon}\right) \tag{28}$$

to Eq. (22). This results in four terms, each a product of two propagators:

$$\begin{aligned} -i\mathcal{F} = \Big[&\frac{1}{D_2^{(2)}+i\epsilon}\times\frac{1}{D_1^{(2)}+i\epsilon} - \frac{1}{D_2^{(2)}-i\epsilon}\times\frac{1}{D_1^{(2)}+i\epsilon} \\ &+ \frac{1}{D_2^{(1)}-i\epsilon}\times\frac{1}{D_1^{(1)}+i\epsilon} - \frac{1}{D_2^{(1)}-i\epsilon}\times\frac{1}{D_1^{(1)}-i\epsilon}\Big] \\ &\times H^*_{ab}(x_a p_A, x_b p_B, p_t) H_{ab}(x_a p_A, x_b p_B, p_t). \end{aligned} \tag{29}$$

Of these four terms, the first and fourth have both $i\epsilon$ prescriptions the same in their denominators. These products of final state denominators therefore do not produce a pinch in the variable x, and since x dependence is otherwise analytic in the leading region we can deform the x contour away from points where the denominators would otherwise vanish. When x changes by any finite amount, it forces these denominators off-shell by an amount of order P^-, and their contributions can be absorbed in the hard scattering function.

In the remaining two terms of Eq. (29) there are denominators with opposite $i\epsilon$'s, so that in these terms the x integral is pinched in general. What we will now show is that singular behavior associated with these terms cancels. The mechanism of cancellation will be readily generalized to arbitrary order.

Neglecting the nonsingular terms we have

$$\begin{aligned} -i\mathcal{F} &= \Big[\frac{1}{D_2^{(1)}-i\epsilon}\times\frac{1}{D_1^{(1)}+i\epsilon} - \frac{1}{D_2^{(2)}-i\epsilon}\times\frac{1}{D_1^{(2)}+i\epsilon}\Big] H^*_{ab}(x_a p_A, x_b p_B, p_t) H_{ab}(x_a p_A, x_b p_B, p_t) \\ &= \Big[\frac{1}{D_2^{(2)}+(D_2^{(1)}-D_2^{(2)})-i\epsilon}\times\frac{1}{D_1^{(2)}+(D_1^{(1)}-D_1^{(2)})+i\epsilon} - \frac{1}{D_2^{(2)}-i\epsilon}\times\frac{1}{D_1^{(2)}+i\epsilon}\Big] \\ &\quad\times H^*_{ab}(x_a p_A, x_b p_B, p_t) H_{ab}(x_a p_A, x_b p_B, p_t), \end{aligned} \tag{30}$$

where the trivial rewriting of the second form shows manifestly that *if* $D_i^{(1)}$ equaled $D_i^{(2)}$ for $i = 1, 2$, the two terms would cancel identically. The differences, which are linear in the soft momentum k, can be read off from Eq. (24). In the notation of Eq. (15), we find

$$\begin{aligned} D_1^{(1)} - D_1^{(2)} &= -[p_t]^- - [Q-p_t-k]^- + [p_t-k]^- + [Q-p_t]^- \\ &= (\beta_{Q-p_t} - \beta_{p_t})\cdot k + \frac{k\cdot(\overleftrightarrow{\gamma}_{p_t})\cdot k}{2p_t^+} - \frac{k\cdot(\overleftrightarrow{\gamma}_{Q-p_t})\cdot k}{(Q-p_t)^+} + \cdots, \\ D_2^{(1)} - D_2^{(2)} &= -[p_t+k]^- - [Q-p_t-k]^- + [p_t]^- + [Q-p_t]^- \\ &= (\beta_{Q-p_t} - \beta_{p_t})\cdot k - \frac{k\cdot(\overleftrightarrow{\gamma}_{p_t})\cdot k}{2p_t^+} - \frac{k\cdot(\overleftrightarrow{\gamma}_{Q-p_t})\cdot k}{(Q-p_t)^+} + \cdots, \end{aligned} \tag{31}$$

both independent of x. To compensate for these differences we will again appeal to our observation above that the remainder of the diagram has a smooth dependence on the collective parton fraction x. Thus, up to corrections suppressed by $1/P^-$ we may perform a small shift $x \to x + \delta x$ in the first term in the second equality of Eq. (30), where

$$\delta x = \frac{(\beta_{Q-p_t} - \beta_{p_t})\cdot k}{P^-}, \tag{32}$$

which is power suppressed in the hard scale. We then find

$$\begin{aligned} -i\mathcal{F} = \Big[&\frac{1}{D_2^{(2)}+\hat\Delta_2^{(2)}-i\epsilon}\,\frac{1}{D_1^{(2)}+\hat\Delta_1^{(2)}+i\epsilon} \\ &- \frac{1}{D_2^{(2)}-i\epsilon}\,\frac{1}{D_1^{(2)}+i\epsilon}\Big] H^*_{ab}(x_a p_A, x_b p_B, p_t) \\ &\times H_{ab}(x_a p_A, x_b p_B, p_t), \end{aligned} \tag{33}$$

where $\hat\Delta_1^{(2)}$ and $\hat\Delta_2^{(2)}$ are both of order k^2. They can be read off from Eq. (31), and

$$\hat\Delta_1^{(2)} - \hat\Delta_2^{(2)} = 2\frac{k\cdot(\overleftrightarrow{\gamma}_{p_t})\cdot k}{p_t^+} + \cdots. \tag{34}$$

We conclude that because pinches of the x integral are found only in $\mathcal{F}$, the entire term integral is suppressed for fixed values of the soft gluon momentum k. We recall again that (gauge invariant) perturbative contributions to single-particle inclusive cross section are at worst logarithmically divergent [5]. Thus, cancellation of the leading power in k will lead to a finite integral. Under these circumstances, the integral over momentum k is dominated by $k \sim m_t$, and to fixed order in perturbation theory the gluon k, and every line connected to it can be absorbed into the hard scattering. The integration region $k \to 0$ appears only as the tail of a finite integral, and its contribution vanishes for $m_t \to 0$. We will return to this point in Sec. III C in connection with possible power corrections to these cross sections.

In summary, to lowest order in final state interactions we have seen that cancellation is manifest once we neglect terms that are quadratic in soft momenta compared to those that are linear in each of the light-cone denominators. Alternatively, if we expand the expression (33) for $\mathcal{F}$ in powers of $\hat{\Delta}_n^{(2)}/D_n^{(2)} \sim k, n = 1, 2$, the leading terms cancel and the integral is finite at the pinch surface.

B. Final state interactions at arbitrary order

We now generalize to arbitrary order the mechanism of cancellation found above at fixed order and with the minimum number of final states. For a generic diagram with S states, as the one illustrated by Fig. 2 and Eq. (22), the function $\mathcal{F}$ can be written as a sum over out states j:

$$\begin{aligned}
\mathcal{F} &= \sum_{j=1}^{S}\Big(\prod_{i'=j+1}^{S}\frac{1}{D_{i'}^{(j)} - i\epsilon}\Big)2\pi\delta(D_j^{(j)})\Big(\prod_{i=1}^{j-1}\frac{1}{D_i^{(j)} + i\epsilon}\Big) \\
&\quad \times H^*_{ab}(x_a p_A, x_b p_B, p_t)H_{ab}(x_a p_A, x_b p_B, p_t) \\
&= i\Big[\sum_{j=1}^{S}\Big(\prod_{i'=j+1}^{S}\frac{1}{D_{i'}^{(j)} - i\epsilon}\Big)\Big(\prod_{i=1}^{j}\frac{1}{D_i^{(j)} + i\epsilon}\Big) \\
&\quad - \sum_{j=1}^{S}\Big(\prod_{i'=j}^{S}\frac{1}{D_{i'}^{(j)} - i\epsilon}\Big)\Big(\prod_{i=1}^{j-1}\frac{1}{D_i^{(j)} + i\epsilon}\Big)\Big] \\
&\quad \times H^*_{ab}(x_a p_A, x_b p_B, p_t)H_{ab}(x_a p_A, x_b p_B, p_t). \qquad (35)
\end{aligned}$$

In the second equality we have used the delta function identity Eq. (28). Again we work with a fixed ordering and hence suppress the ordering label T.

As above, we will expand the denominators $D_i^{(j)}$ around an arbitrary pinch surface. Near the pinch surface, each final state i will contain a top quark, an antitop quark, lines parallel to p_A (the A jet), lines parallel to p_B (the B jet) and soft lines. There may also be light parton final state jets, but for the moment we neglect this possibility.

In the single-particle inclusive cross section, the top quark is fixed to momentum p_t when state $i = j$, the out state in Eq. (35). The top quark momentum in any other final state i then depends on the choice of out state j, because the flow of soft momenta must be adjusted as the choice of out state j changes. As above, we choose to sum over states at fixed on-shell momenta for all soft and jet lines, adjusting the flow of soft momenta only within the top quark loop. We denote the resulting top quark momentum for final state i with out state j by $p_{t,i}^{(j)}$, where $p_{t,j}^{(j)} = p_t$. Similarly, antitop momenta will be denoted by $p_{\bar{t},i}^{(j)}$.

In addition to the top pair for each final state i, the momenta of soft lines are denoted collectively by k, $k \in i$, and the momenta of lines in the A and B jets denoted collectively by $l = x_l p_A + \delta l$, $l \in i$ and $l' = x_{l'} p_B + \delta l'$, $l' \in i$, respectively.

In the notation just described, a generic final state denominator can be written as

$$D_i^{(j)} \equiv P^- - [p_{t,i}^{(j)}]^- - [p_{\bar{t},i}^{(j)}]^- - d_i(\{k\}, \{l\}, \{l'\}), \qquad (36)$$

where as above $P^- = p_A^- + p_B^-$ and where the function $d_i(\{k\}, \{l\}, \{l'\})$ contains the on-shell minus momenta of the soft lines and collinear spectator lines in final state i:

$$\begin{aligned}
d_i(\{k\}, \{l\}, \{l'\}) &= \sum_{k\in i}[k]^- + \sum_{l\in i}[x_l p_A + \delta l]^- \\
&\quad + \sum_{l'\in i}[x_{l'} p_B + \delta l']^-, \qquad (37)
\end{aligned}$$

which we can expand about the pinch surface, as in Eqs. (25) and (26):

$$\begin{aligned}
d_i(\{k\}, \{l\}, \{l'\}) &= d_i(\{k = 0\}, \{l = x_l P^-\}, \{l' = x_{l'} P^-\}) \\
&\quad + \hat{d}_i(\{k\}, \{\delta l\}, \{\delta l'\}) \\
&\equiv (1 - x)P^- + \hat{d}_i(\{k\}, \{\delta l\}, \{\delta l'\}). \qquad (38)
\end{aligned}$$

Using this expansion near the singular point in (36), we then have a direct generalization of Eq. (24):

$$D_i^{(j)} = xP^- - [p_{t,i}^{(j)}]^- - [p_{\bar{t},i}^{(j)}]^- - \hat{d}_i(\{k\}, \{\delta l\}, \{\delta l'\}). \qquad (39)$$

As in Eq. (24), the *only* dependence of $D_i^{(j)}$, Eq. (39), on the choice of out state is in the top quark and antiquark momenta $p_{t,i}^{(j)}$ and $p_{\bar{t},i}^{(j)}$. Also as in the previous subsection, we denote by k the collection of all soft gluon momenta k, and collinear δl and $\delta l'$. In this notation, all the $D_i^{(j)}$ are linear in k for $k \to 0$ as above.

For i the out state, that is, $i = j$ in our notation, $p_{t,j}^{(j)} = p_t$ is the momentum that defines the inclusive cross section. This is the direct generalization of the lowest-order procedure above and will lead directly to manifest cancellation of final state interactions, by considering the differences between denominators:

$$D_i^{(j+1)} - D_i^{(j)} = -[p_{t,i}^{(j+1)}]^- - [p_{\bar{t},i}^{(j+1)}]^- + [p_{t,i}^{(j)}]^- + [p_{\bar{t},i}^{(j)}]^-. \qquad (40)$$

We consider, then, the dependence of the top and antitop momenta in an arbitrary final state i on the choice of out state j.

Consider now final state $i \neq j$, which may be in the amplitude ($i < j$) or the complex conjugate ($i > j$). Given the momentum routing we have chosen, a top quark line momentum in either the amplitude or complex conjugate will differ from p_t by those soft momenta that are either emitted or absorbed by the top between state i and state j. We can then write the momentum of the top quark in state i that appears in Eq. (40) as

$$[p_{t,i}^{(j)}]^- = \left[p_t - \sum_{l=i}^{j-1} \sigma_l k_l\right]^- \quad \text{for } i \le j,$$
$$[p_{t,i}^{(j)}]^- = \left[p_t + \sum_{l=j}^{i-1} \sigma_l k_l\right]^- \quad \text{for } i > j, \tag{41}$$

where $\sigma_l = 1$ if k_l is absorbed by the top quark between state l and state $l+1$ and $\sigma_l = -1$ if k_l is emitted from the top quark between states l and $l+1$. When the states l and $l+1$ are separated by an interaction that does not involve the top quark, $\sigma_l = 0$. We note that in LCOPT the term ‘‘emitted’’ refers to a gluon whose momentum k_l flows forward from the vertex that separates states l and $l+1$ into state $l+1$, while ‘‘absorbed’’ implies that the gluon flows forward into the vertex from state l.

The situation for antitop momenta is similar, but in this case the momentum in state i equals a value that is independent of the choice of out state plus a correction due to the rerouting of soft momenta as we change j. All soft momenta that are emitted or absorbed from the top line after out state j are routed through the antitop line, while those that are emitted before j are not. The j-independent part of $p_{\bar{t},i}^{(j)}$ includes soft line momenta that attach directly to the antitop line, and dependence on all other soft and collinear loops that do not attach to the top quark, and which are held fixed as we change the choice of out state j. In summary, we denote the j-independent part of the antitop momenta by $\tilde{p}_{\bar{t},i}$, and in these terms we have

$$[p_{\bar{t},i}^{(j)}]^- = \left[\tilde{p}_{\bar{t},i} - \sum_{l\ge j} \sigma_l k_l\right]^- \quad \text{for all } i, \tag{42}$$

where again $\sigma_l = +1$ if k_l is absorbed by the top quark and $\sigma_l = -1$ if k_l is emitted from the top quark. We can now evaluate the change in light-cone denominators for a change in out state, Eq. (40).

We observe that $D_i^{(j+1)} \ne D_i^{(j)}$ only when $\sigma_j \ne 0$, that is, when the vertex that separates state $j+1$ from state j is the final interaction of the top quark before the out state. Otherwise the top and antitop momenta remain unchanged, and $D_i^{(j+1)} - D_i^{(j)} = 0$ identically for all i. In the case when the soft gluon-top quark interaction passes from the amplitude to the complex conjugate, we expand Eq. (40) using (15) to find

$$D_i^{(j+1)} - D_i^{(j)} = \sigma_j(\beta_{p_t} - \beta_{Q-p_t}) \cdot k_j + \hat{\Delta}_i^{(j)}, \tag{43}$$

where the terms linear in the soft momentum are independent of i. The remaining contributions $\hat{\Delta}_i^{(j)} \sim \mathcal{O}(k^2/m_t)$ are quadratic in all soft momenta connected to the top loop, generalizing the lowest order result in Eq. (34). The linear terms Eq. (43) are independent of the choice of final state i for any fixed out state j.

We now return to the analysis of the general final state factor in Eq. (35) and notice that as in Sec. III two terms, $j=1$ in the first sum of the expanded form and $j=S$ in the second, have all denominator poles on the same side of the x contour. These terms are suppressed by a power of the overall energy scale, P^-, by the same contour deformation argument for parameter x as above. Again following the argument of Sec. III we combine the remaining $2S-2$ terms into a sum of $S-1$ pairs with equal numbers of denominators with $+i\epsilon$ and equal numbers of denominators with $-i\epsilon$. To implement this step, we simply modify the summation variable $j \to j+1$ in the second sum of the second equality of Eq. (35) and then combine terms in the form of Eq. (30) to derive

$$-i\mathcal{F} = \sum_{j=1}^{S-1} \mathcal{F}^{(j)} + \cdots, \tag{44}$$

where we suppress terms in which the x integral can be deformed and where

$$\begin{aligned}-i\mathcal{F}^{(j)} = \Bigg[&\left(\prod_{i'=j+1}^{S} \frac{1}{D_{i'}^{(j)} - i\epsilon}\right)\left(\prod_{i=1}^{j} \frac{1}{D_i^{(j)} + i\epsilon}\right) \\ &- \left(\prod_{i'=j+1}^{S} \frac{1}{D_{i'}^{(j+1)} - i\epsilon}\right)\left(\prod_{i=1}^{j} \frac{1}{D_i^{(j+1)} + i\epsilon}\right)\Bigg] \\ &\times H_{ab}^*(x_a p_A, x_b p_B, p_t) H_{ab}(x_a p_A, x_b p_B, p_t).\end{aligned} \tag{45}$$

We now rewrite the differences in denominations found above in Eq. (43) as

$$D_i^{(j)} - D_i^{(j+1)} = P^- \delta x^{(j)} + \hat{\Delta}_i^{(j)}, \tag{46}$$

where

$$\delta x^{(j)} \equiv \frac{1}{P^-} \sigma_j (\beta_{p_t} \cdot k_j - \beta_{Q-p_t} \cdot k_j) \tag{47}$$

is independent of i. Now, using Eq. (47), we rewrite Eq. (45) as

$$\begin{aligned}\mathcal{F}^{(j)} = \Bigg[&\left(\prod_{i'=j+1}^{S} \frac{1}{D_{i'}^{(j+1)} + P^- \delta x^{(j)} + \hat{\Delta}_{i'}^{(j)} - i\epsilon}\right) \\ &\times \left(\prod_{i=1}^{j} \frac{1}{D_i^{(j+1)} + P^- \delta x^{(j)} + \hat{\Delta}_i^{(j)} + i\epsilon}\right) \\ &- \left(\prod_{i'=j+1}^{S} \frac{1}{D_{i'}^{(j+1)} - i\epsilon}\right)\left(\prod_{i=1}^{j} \frac{1}{D_i^{(j+1)} + i\epsilon}\right)\Bigg] \\ &\times H_{ab}^*(x_a p_A, x_b p_B, p_t) H_{ab}(x_a p_A, x_b p_B, p_t).\end{aligned} \tag{48}$$

As at lowest order, we can neglect the dependence of the hard functions on soft momenta, so that they give an overall factor. Then, as in Eq. (24), by a small shift $\delta x^{(j)}$ in the collective partonic fraction x applied to the first term in Eq. (48) one can *simultaneously* absorb the differences between all denominators into the smooth dependence of the remainder of the cross section, up to i-dependent corrections $\hat{\Delta}_i^{(j)}$, which are $\mathcal{O}(k^2)$ as discussed in Sec. III A. In turn, this implies that the terms inside the square brackets of Eqs. (45) and (48) are suppressed by the ratio

$$\frac{\hat{\Delta}_i^{(j)}}{D_i^{(j+1)}} \sim \frac{k}{m_t}, \tag{49}$$

where we have used that each denominator $D_i^{(j)}$ behaves linearly in soft momenta k when expanded around the pinch surface.

In summary, as in the example of Sec. III A, the exchange of soft gluons in the final state does not contribute at leading power to Eq. (13), the single-particle inclusive cross section for the top quark in top-pair production.

C. Power corrections

Because all divergences are logarithmic, the suppression we have just found is enough to eliminate long-distance behavior to any fixed order in perturbation theory from an arbitrary pinch surface involving final state interactions. Beyond fixed orders, however, soft gluons may be radiated by a spectator whose momentum is essentially nonperturbative. Even though order by order the contributions from soft gluon exchange are small, higher-order corrections associated, for example, with the running of the QCD coupling and its infrared Landau pole suggest that the perturbative series for the infrared region of momentum space diverges. Such reasoning, in fact, reconciles perturbation theory with the operator product expansion [32] and underlies the treatment of power corrections in deep-inelastic scattering [33] and event shapes in electron-positron annihilation [34].

For the hadronic scattering we are considering, this general reasoning suggests the presence of nonperturbative corrections associated with FSI. To estimate the nature of the power corrections in this case, we will follow Ref. [30] and interpret the finite remainder from the region of soft gluon exchange as an additive nonperturbative correction to leading-power factorization. To be specific, when the sum over final states produces a suppression of the form $\langle k\rangle/Q$ relative to leading-power behavior, with $\langle k\rangle \sim \Lambda \sim \Lambda_{\mathrm{QCD}}$ the size of the nonperturbative region and with Q the hard scale, we infer that in the full theory there is an additive correction to the leading-power factorized cross section of size Λ/Q. Again, for heavy quark production, the exchanged gluon may originate from a spectator whose transverse momentum relative to the beam is at a nonperturbative scale set by Λ_{QCD}. In the case at hand, we thus infer that nonperturbative corrections in Eq. (2) for single-particle annihilation cross sections due to FSI are indeed an expansion in powers of Λ/m_t. For practical considerations, such corrections are presumably negligibly small, given the size of the hard scale.

IV. MULTIPARTICLE CROSS SECTIONS AND CORRECTIONS TO FACTORIZATION

As we have seen, the cancellation of final state interactions for a single-particle inclusive top cross section depends on combining contributions with different antitop momenta. This was built into our argument by "routing" the soft momentum differently for different choices of out state, that is, by letting the antiquark recoil against final state momentum transfers due to the field of the spectators. In particular, we combine final states where $p_{\bar{t}} = Q - p_t$ and $p_{\bar{t}} = Q - p_t - k$. This rerouting works so long as the short-distance process that produces the top-antitop pair is insensitive to changes of order k in the antitop momentum. As we have just seen, corrections to this approximation are of order k/m_t in the single-particle inclusive case.

We can generalize our arguments to multiparticle cross sections, starting with two-particle inclusive (2PI). We begin by noting that in a 2PI cross section we hold fixed p_t *and* $p_{\bar{t}}$. It is then intuitively clear from our 1PI discussion that if we hold both the top and antitop momenta fixed, we need some other particle or particles to recoil against the soft radiation in order to cancel final state scatterings of both elements of the top pair. In the following we confirm this assertion in two steps: first, we show that the cancellation mechanism fails if there is no additional hard particle in the scattering process, and second, we identify the pattern of cancellation of final state interactions when such additional radiation is present. The same routing of soft momenta is necessary to fix the somewhat more inclusive pair transverse momentum distribution, so we expect the estimates of corrections that we derive below to apply in this case as well.

Consider a 2PI cross section with no additional hard radiation present (except the top and antitop), with Fig. 4 an example of such a configuration. Then, instead of routing the soft momentum k through the antiquark to maintain the same p_t for both out states of the diagram, we would have to route the soft momentum through lines labeled l and l' in the figure, in such a way that it appears in initial state light-cone denominators like Eq. (20). In these denominators, however, we cannot expand in k unless it is much smaller than δl, which is in general at a hadronic scale, precisely the scale of k, in fact. In this case, the final state interactions fail to cancel, for much the same reasons as for the example given in Ref. [19] in a polarized cross section. In our case, the effect is spin-independent and potentially leading-power, making it impossible to apply collinear factorization. In such a case, pair production without additional radiation, we expect leading-power corrections from final state interactions, as in the effects of rescattering found in Ref. [35] for the photoproduction of electron-positron pairs on nuclei. In the context of collider physics, restrictions on the phase space available for radiation lead as well to logarithmic corrections at leading power. For example, if the transverse momentum of radiation is limited to some value $\Lambda_{\rm QCD} \ll p_T \ll m_t$, then we will expect the cross section to be collinear factorizable up to corrections like $\Lambda_{\rm QCD}/p_T$. At the same time, logarithms like $\ln(m_t/p_T)$ will appear, and for many observables these logarithms have not yet been fully resummed or otherwise understood. The most striking example is perhaps the "superleading" logarithms [36], which result from a partial noncancellation that occurs when an upper limit is put on soft radiation in a finite region of phase space [37].

If we do not limit additional radiation, however, precisely the same arguments as in the previous section can be applied to show the cancellation of final state interactions in 2PI cross sections for top and antitop, so long there is an additional high-p_T jet (taken relative to the collision axis, not the LC axis above) as long as the cross section is inclusive enough in the momentum of the jet so that shifts by momenta of order k lead to states that are counted. In the following, we model the jet by a single parton of momentum $p_{\rm jet}$, a simplification that does not weaken the argument.

We start by assuming that $p_{\rm jet}\cdot p_c \sim m_t^2$ for the other partons c that take part in the hard scattering: $c = a, b, t, \bar{t}$. The production of the jet can be thought of as local, and exactly the same argument holds-we simply route the soft momentum k through the high-p_T jet rather than through the antitop. We then compute

$$D_i^{(j+1)} - D_i^{(j)} = -[p_{t,i}^{(j+1)}]^- - [p_{\bar{t},i}^{(j+1)}]^- - [p_{{\rm jet},i}^{(j+1)}]^- + [p_{t,i}^{(j)}]^- + [p_{\bar{t},i}^{(j)}]^- + [p_{{\rm jet},i}^{(j)}]^-, \qquad (50)$$

where $p_{{\rm jet},i}^{(j)}$ is the momentum of the additional final state parton recoiling against the quark pair in state i. As usual, the superscript (j) labels the out state. For this discussion, soft momenta that flow through the top *and* antitop lines are routed in such a way that the top and antitop momenta of the out state are fixed.

As above the differences can be compensated by a shift that is linear in the routing of soft momenta, which depends on j but which is the same for every i:

$$D_i^{(j)} - D_i^{(j+1)} = P^-\delta x^{(j)} + \hat{\Delta}_i^{(j)}. \qquad (51)$$

To derive $\delta x^{(j)}$, we recall the top momentum for each state i and find in (41)

$$[p_{t,i}^{(j+1)}]^- - [p_{t,i}^{(j)}]^- = -\sigma_j\beta_{p_t}\cdot k_j + \mathcal{O}(k^2), \qquad (52)$$

the same for each final state $i \neq j$. We shall not need the explicit form of the quadratic terms, but recall that they are an expansion in k/m_t, which we consider negligible for this discussion.

Since we fix both the top and antitop momenta, we introduce a similar notation for the antitop's momentum, whose dependence on soft momenta is now

$$[p_{\bar{t},i}^{(j)}]^- = \left[p_{\bar{t}} - \sum_{l=i}^{j-1}\bar{\sigma}_l k_l'\right]^- \quad \text{for } i \le j$$
$$= \left[p_{\bar{t}} + \sum_{l=j}^{i-1}\bar{\sigma}_l \bar{k}_l\right]^- \quad \text{for } i > j, \qquad (53)$$

where $\bar{\sigma}_l$ is defined by analogy to the top, positive for momentum flowing in to the antitop line, and negative for momentum flowing out. In this case, $p_{\bar{t},j}^{(j)} = p_{\bar{t}}$, the observed antitop momentum. Then we have again

$$[p_{\bar{t},i}^{(j)}]^- - [p_{\bar{t},i}^{(j)}]^- = -\bar{\sigma}_j\beta_{p_{\bar{t}}}\cdot\bar{k}_j + \mathcal{O}(k^2), \qquad (54)$$

independent of i, again neglecting terms explicitly suppressed by k/m_t.

The recoiling "jet" line plays the role that was played by the antitop in the 1PI discussion and absorbs soft momenta of both the top and the antitop:

$$[p_{{\rm jet},i}^{(j)}]^- = \left[\tilde{p}_{{\rm jet},i} - \sum_{l\ge j}\sigma_l k_l - \sum_{l\ge j}\bar{\sigma}_l\bar{k}_l\right]^-. \qquad (55)$$

Here $\tilde{p}_{{\rm jet},i}$ is defined by analogy to $\tilde{p}_{\bar{t},i}$ in Eq. (42) and is the momentum of the jet line in state i, not including soft momenta routed through the jet line from the final state interactions of the top and antitop quark after the cut (i.e., in the complex conjugate amplitude). The difference between jet momenta with out states $j+1$ and j is then given by the expansion

$$\begin{aligned}
&[p_{{\rm jet},i}^{(j+1)}]^- - [p_{{\rm jet},i}^{(j)}]^- \\
&= \beta_{\tilde{p}_{{\rm jet},i}}(\sigma_j k_j + \bar{\sigma}_j\bar{k}_j) \\
&\quad + \frac{2}{\tilde{p}_{{\rm jet},i}^+}(\sigma_j k_j + \bar{\sigma}_j\bar{k}_j)\cdot\overleftrightarrow{\gamma}_{\tilde{p}_{{\rm jet},i}}\cdot\left(\sum_{l\ge j}\sigma_l k_l + \sum_{l\ge j}\bar{\sigma}_l\bar{k}_l\right) \\
&\quad + \frac{1}{\tilde{p}_{{\rm jet},i}^+}(\sigma_j k_j + \bar{\sigma}_j\bar{k}_j)\cdot\overleftrightarrow{\gamma}_{\tilde{p}_{{\rm jet},i}}\cdot(\sigma_j k_j + \bar{\sigma}_j\bar{k}_j). \qquad (56)
\end{aligned}$$

Combining this expression with (52) and (54) to derive the difference of denominators in (50) and then expanding, we find

$$D_i^{(j+1)} - D_i^{(j)} = -\sigma_j(\beta_{\tilde{p}_{{\rm jet},i}} - \beta_{p_t})\cdot k_j - \bar{\sigma}_j(\beta_{\tilde{p}_{{\rm jet},i}} - \beta_{p_{\bar{t}}})\cdot\bar{k}_j + \hat{\Delta}_i^{(j)}, \qquad (57)$$

where now $\hat{\Delta}_i^{(j)}$ is an expansion in $k/p_{\rm jet}^+$ as well as k/m_t. Up to such quadratic terms we again find that an overall shift for each choice of j leads to cancellation of final state interactions just as for the 1PI case.

Such high-p_T jets, however, are absent in many, if not most, out states. When the recoiling momentum $p_{\rm jet}$ is much softer than the pair mass, or when it is far forward or backward, scales smaller than m_t can come into play. We can illustrate these corrections by considering the general form of the hard scattering when it involves an extra final state gluon, of momentum p'. In this case, the leading behavior is

$$H^*_{ab\to t\bar{t}g(p')}(x_a p_A, x_b p_B, p_t, p_{\bar{t}}, p') \times H_{ab\to t\bar{t}g(p')}(x_a p_A, x_b p_B, p_t, p_{\bar{t}}, p') = \sum_{c,d=a,b,t,\bar{t}} \frac{p_c \cdot p_d}{p_c \cdot p' p_d \cdot p'} H^*_{ab\to t\bar{t}}(x_a p_A, x_b p_B, p_t, p_{\bar{t}}) \times H_{ab\to t\bar{t}}(x_a p_A, x_b p_B, p_t, p_{\bar{t}}), \quad (58)$$

where to simplify this discussion of kinematic factors we suppress color dependence. In general, we would expect important contributions from $c, d = a, b$, the light partons that initiate the hard scattering, for which

$$\frac{p_a \cdot p_b}{p_a \cdot p' p_b \cdot p'} = \frac{1}{2}\frac{1}{p_T'^2}, \quad (59)$$

in terms of the squared transverse momentum of the gluon (jet) relative to the beam axis. Corrections then arise because p' depends on the choice of out state j. Expansions of the hard scattering are of the form

$$\frac{1}{(p' \pm k)_T^2} - \frac{1}{p_T'^2}, \quad (60)$$

considered as a series in k/p'_T. For an azimuthally symmetric cross section, we expect corrections to go as even powers of $1/p'_T$, but for spin-dependent cross sections, for example, odd powers can arise. Following the reasoning of Sec. III C, we infer that nonperturbative corrections to collinear factorization are suppressed by powers of Λ/p'_T, in terms of some hadronic scale Λ.

In summary, for two-particle inclusive and pair transverse momentum distributions, whenever the transverse momentum of recoiling radiation is much less than the pair mass, we have seen that nonperturbative corrections to collinear factorization are generally suppressed by powers of Λ/p'_T, where p'_T is the largest transverse momentum of an additional jet, rather than by powers of the top quark mass.

V. SUMMARY AND DISCUSSION

A. General considerations

We have studied FSI between hard partons produced in high-energy hadronic collisions and beam remnants from the initial state and have shown that FSI cancel for a large class of inclusive cross sections at leading power, to all orders in perturbation theory. Examples of processes for which these results are directly applicable include the forward-backward asymmetry of top quark pairs at the Tevatron and a variety of processes with jets or identified hadrons (light or heavy quark fragmentation).

We have given our derivation in the language of light-cone ordered perturbation theory. While the nature of our proof is somewhat technical, the physics underlying our findings is quite transparent: cancellation of FSI occurs when additional, unobserved, hard radiation is allowed in the final state. The role of this additional radiation is to recoil against the observed system of hard partons and thus effectively absorb the kinematic effects of rescattering between the final state hard system and the beam remnants.

We find that the details of the cancellation depend on the nature of the final state. For top-pair production and related processes, the size of corrections due to interactions with spectators is strongly suppressed for 1PI cross sections [5] but may be large for substantial portions of the total 2PI cross sections, when recoiling radiation is suppressed (because of jet vetoing, for example). Such corrections can appear both as higher-order perturbative corrections to a factorized cross section and as nonperturbative corrections suppressed by powers of perturbative scales. The particular scales, however, depend on the set of final states.

For single-particle inclusive observables, like single top in top-pair production, we find that the scales suppressing the final state interactions are the large scales in the problem, like p_t^+ and m_t. The arguments we have given are valid as long as p_t^+ and $p_{\bar{t}}^+ = (Q - p_t)^+$ are much larger than hadronic scales, including $\Lambda_{\rm QCD}$, which is clearly the case for the top quark.[2] In the massless limit, on the other hand, the validity of the expansions depends on the specific observable. For p_T distributions of massless partons, large transverse momenta are necessary to ensure a hard scattering. Then our considerations apply even in the massless case, so long as there is a hard particle recoiling against the observed particle.

We have found possible nonperturbative corrections to the leading power, factorized cross section of order $\Lambda_{\rm QCD}/q_T$, where again q_T is the maximum transverse momentum of additional perturbative radiation in a given set of final states. Note that even for $q_T \gg \Lambda_{\rm QCD}$ a lower choice of $\mu_{\rm fact} \leq q_T$ would also be necessary for the factorization scale in the leading-power term rather than the hardest scale in the problem (m_t in our case). The mechanism of cancellation that we have identified, however, applies to all of these cases.

Our considerations generalize a pattern identified in Refs. [19,20], showing how corrections associated with final state interactions can become important in inclusive cross sections with two observed particles. This is the case even as corrections to single-particle inclusive cross sections remain suppressed by the hardest scale of the final state [5]. Note that this means that calculations of charge asymmetries based on single-particle inclusive cross sections are stable against final state interactions but that limitations on the cross section, involving restrictions on quark pair momenta, or jet vetoes, can introduce sensitivity to nonperturbative scales.

[2]Recall that Q, defined in (23), is the total initial state momentum available to the partonic hard scattering.

We note that color and spin played no direct role in our analysis. The reason is that the FSI cancellation can be demonstrated by summing the cuts of one single (squared) diagram at a time. On the other hand, if one would like a computation or at least a better theoretical understanding of the remainder after the FSI cancellation, then theory-dependent group and spin factors will of course have to be taken into account. A toy model study of color reconnection effects in hadronic final states is described in Refs. [38,39] from a parton shower perspective. For photoproduction of jets, a related analysis [40] showed that the class of power corrections associated with soft gluon exchange in nuclei (collinear) factorizes into higher-twist nuclear matrix elements and is in this sense universal. For top-pair production in hadronic collisions, we cannot anticipate such a higher-twist factorization, precisely due to a mismatch in color factors associated with initial and final states, as emphasized in Refs. [19,20].

To close this brief discussion, we note some possible directions for extending the present work:

(1) A more detailed assessment of the remainders of the cancellation of FSI. Our arguments show that the remainder scales as the first power of $\langle l_T \rangle$. It is plausible, however, that in certain processes, the first power nonperturbative correction vanishes. This appears to be the case, for example, for inclusive Drell-Yan production [17,41]. Another interesting possibility, suggested by analogy to the QED analysis in Low's theorem [42–44], is that for top production the first nonleading power may be closely related to the derivative of the short-distance cross section. More work is needed to explore these possibilities. Clearly this is relevant since effects suppressed by the second power would likely not be experimentally accessible at high-energy hadron colliders while, as we discuss below, corrections suppressed by a single power of the hard scale might well be observable.

(2) Extension to other processes beyond top-pair production. In this work we focused on top-pair production because of its phenomenological relevance and simplified treatment of final state radiation. On the other hand, jet vetoes are very actively studied [45–49], notably in Higgs boson production. The effect of the jet veto is to introduce logarithms of the ratio of the large hard scale and the presumably much smaller veto scale. It is for this reason that, at the few-percent level, FSI will overlap with the effects in resummations performed for jet vetoes, which motivates the need for a better understanding of FSI.

(3) Extension to multiscale kinematics. We often refer to large p_T as the relevant hard scale in hadronic collisions, assuming we are reasonably inclusive in rapidities. However, in certain kinematic regions rapidities can be large and the right scale then will be a function of both p_T and y. Understanding scale setting in multiscale problems is an important open problem at present, with forward dijet production being a notable example. Reference [50] reports recent work in this direction. We believe that the analysis presented here offers an additional perspective on the complexity of this problem.

(4) Relation to factorization. Among the challenges in the analysis of perturbative corrections associated with limitations on final state radiation is the quantitative control over the role of radiation vetoes [36], which is certainly related to the mechanisms of factorization revisited recently in Ref. [6].
While a full discussion of these issues would take us beyond the scope of this paper, we may note that the factorization arguments presented in Ref. [16] take into account the mismatch, investigated in Ref. [6], between initial and final state singularities that appear in the momentum integrals of soft lines connecting the incoming jets. We have confirmed in this paper that final state interactions whose transverse momenta are below a scale set by the definition of a semi-inclusive cross section cancel. Such a cancellation is essential to the factorization mechanisms outlined in Ref. [16]. For partons with transverse momenta above this scale, we do not see an obvious reason to believe that arguments for collinear factorization apply. We also see no reason to suspect, however, that collinear factorization fails below this scale or that these noncanceling, infrared finite effects may not be organized into a perturbative hard-scattering function. In any case, there is certainly more to be learned about how our results relate analytically to the appearance of super-leading and nonglobal logarithms [37] and how the latter are related to the generalized factorization formulated in Ref. [6].

B. Phenomenological implications

Phenomenologically, the effect of FSI between hard partons and beam remnants may be relevant for observables where additional radiation is suppressed, i.e., more exclusive observables. Let us consider an observed hard state H that is accompanied by unobserved hard radiation. For example, H can be a single top in top-pair production or a color singlet state like an electroweak vector or Higgs boson. Then, the inclusive observable $H + X$ is not very sensitive to FSI, in the sense that subleading power corrections are suppressed by the largest hard scale in the problem. On the other hand, the exclusive contributions $H + nj$, $n = 0, 1, \ldots$ separately could be quite sensitive to FSI since, as we have argued, in such observables FSI are suppressed by the inverse veto scale, which is much lower. Thus, corrections could be as large as $\mathcal{O}(1/20) \sim 5\%$ for a typical cut of around 20 GeV and a nonperturbative scale of 1 GeV. While at first it might seem surprising that FSI can have different impact within the same reaction, the effects of final state interactions in each exclusive channel can cancel in the fully inclusive cross section.

It is interesting to consider our findings in the light of available experimental data. In a recent study, the ATLAS Collaboration [51] measured the gap fraction f in top-pair events, which is the ratio of the cross section subject to a veto Q_0 and the cross section without a veto:

$$f(Q_0) = \frac{\sigma(Q_0)}{\sigma} \leq 1. \quad (61)$$

The veto Q_0 can be as low as 20 GeV $\ll m_t \approx 173$ GeV, which is the "typical" hard scale in top production. Reference [51] compares the data with various fixed order calculations (of leading and next-to-leading order) interfaced to parton showers. At lower Q_0 there are substantial uncertainties in both the data and theory predictions. Nevertheless, assuming an eventual decrease in systematic uncertainties, a full comparison between theory and data should take into account the possibility of, for example, $1/Q_0$ corrections, in the light of our findings above. The analysis of measurements such as these may make possible direct access to final state interactions in hard processes.

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