Low-lying scalars in an extended linear σ model

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We formulate an extended linear σ model of a quarkonia nonet and a tetraquark nonet as well as a complex isosinglet (glueball) by virtue of chiral symmetry $SU_L(3) \times SU_R(3)$ and $U_A(1)$ symmetry. In the linear realization formalism, we study the mass spectra and components of the low-lying scalars and pseudoscalars in this model. The mass matrices for physical states are obtained and the glueball candidates are examined. We find that the model can accommodate the mass spectra of low-lying states quite well. Our fits indicate that the most glueball-like scalar should be 2 GeV or higher while the glueball pseudoscalar is $\eta(1756)$. We also examine the parameter region where the lightest isoscalar $f_0(600)$ can be the glueball and quarkonia dominant but find such a parameter region may be confronted with the problem of the unbounded vacuum from below.

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I. INTRODUCTION

The pseudoscalar, vector, and axial-vector as well as tensor mesons of light quarks have been well understood in the naive quark model in terms of the chiral symmetry. Despite its success, the naive quark model cannot explain the scalar meson sector, which has the same quantum numbers as the vacuum. There are about 19 states which have twice more than the expected $\bar{q}q$ nonet as in vector and tensor sectors. Moreover, the masses and decay patterns of these low-lying scalars are different from the expectation of the naive quark model. To understand the nature of these scalars has been the focus of recent studies; e.g., see Refs. [1–5] and references therein.

Among the low-lying scalar mesons, the lightest scalar $f_0(600)$ or σ attracts much interest. It is widely believed that $f_0(600)$ is like the Higgs boson which plays a crucial role in the spontaneous chiral symmetry breaking. Confirmation of the existence of the elusive $f_0(600)$ from $\pi\pi$ scattering processes resolves a controversy that lasted for more than a few decades [2,6]. The πK scattering [7] and analysis from D decay $D^+ \to K^-\pi^+\pi^-$ [8] revealed that κ should also exist. BES II also found such a κ-like structure in J/Ψ decays [9]. Combined with the well determined sharp resonances [i.e., isoscalar $f_0(980)$ and isotriplet $a(980)$ from $\pi\pi$, $\pi\eta$, as well as KK scattering processes], it is now accepted in literature that these low-lying scalar mesons (say, less than 1 GeV) can be cast into a chiral nonet. The next important issue is the nature of this nonet.

There are a couple of viewpoints on the nature of this nonet. For example, the tetraquark model [10] can explain the mass hierarchy and decay pattern of this nonet quite successfully and is further supported from other experimental data, like the photon-photon collision data, which prefers the tetraquark interpretation for the lowest scalar meson nonet [11] [where it is demonstrated that $f_0(980)$ should be a tetraquark dominant state with great details]. An alternative interpretation is that this nonet is a bound state of the meson-meson molecule [12]. In any way, this nonet challenges a self-consistent interpretation in the naive quark model.

Nonetheless, agreement on the nature of this nonet has not been achieved yet. For example, recently by studying $\pi\pi$ and $\gamma\gamma$ scattering, it was found that this particle could have a sizable fraction of glueballs [13,14]. The K-matrix analysis [15] suggested that $f_0(600)/\sigma$ should be a glueball dominant state while $f_0(980)$ should be a mixture of tetraquark and $q\bar{q}$. A recent pole analysis with Padé approximation suggests $f_0(980)$ might be more like a molecular state [16]. The nature of this nonet is also a focus of lattice study [17]. For example, the physical state of σ and κ can have sizable tetraquark components, as demonstrated in a recent lattice simulation [18].

The great success of chiral symmetry in understanding the nature of the lightest pseudoscalars motivates us to extended the linear σ models in Refs. [19,20] to study the nature of these low-lying isoscalars and pseudoscalars (say, scalars less than 2 GeV). The historic review on scalars above 1 GeV but below 2 GeV can be found in Ref. [1]. Such models may shed some light on the nature of light isoscalars, especially on the issue of mixing among glueballs, quarkonia, and tetraquarks. One interesting question that can be addressed by such models is which of these low-lying isoscalar and pseudoscalar states are more glueball-like. The results from lattice simulations suggest $f_0(1500)$ or $f_0(1700)$ could be a glueball rich isoscalar while $\eta(1489)$ can be a glueball rich pseudoscalar [21].

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Since the 0^{++} and 0^{-+} glueball states can significantly mix with quarkonia and tetraquark states of the same quantum numbers, it is necessary to include these states and a glueball state in an extended linear σ model.

In this work, we extend our previous work [19] by including a nonet to accommodate the tetraquark states and by focusing on the mixing of quarkonia and tetraquarks. Comparing with the systematic work shown in Refs. [20,22] and the references therein, we extend the linear σ model by including a complex singlet field as a pure glueball field and introduce a determinant interaction term [23] instead of the logarithmic term to solve the $U_A(1)$ problem as in Ref. [24]. The motivation for constructing an extended linear sigma model consisting of effective quarkonia, tetraquark, and glueball fields comes from the physical considerations that scalar condensates are allowed by the QCD vacuum. So in principle, apart from quarkonia and tetraquark condensates, scalar glueball condensates should also be present in the QCD vacuum and need to be considered while studying the vacuum excitation. Now, the physical spectrum of mesons arises out of the mixing between the bare effective fields. The mixing pattern followed in our framework comes from the consideration that mesons having identical external quantum numbers can mix even if they have different internal flavor structures. As a result, isospin $\frac{1}{2}$ and 1 states have quarkonia and tetraquark components, whereas the isosinglet mesons are composed of all three effective fields. We attempt to address issues such as which states in the pseudoscalar and isoscalar sectors are most glueball-like. Our model predicts that the isoscalar glueball should be heavier than 2.0 GeV when the pseudoscalar $\eta(1726)$ is the best glueball candidate. The lowest isoscalar $f_0(600)$ is found to be a quarkonia dominant state with a considerable tetraquark component.

The rest of the paper is organized as follows. In Sec. II, we introduce the extended linear σ model and derive the vacuum conditions for the condensates of quarkonia and glueball fields. In Sec. III, we describe our analysis strategy for how to fix most of the free parameters in our model by taking into account experimental data. In Sec. IV, we present our main results of global fit and predictions. In Sec. V we close our study with discussions and conclusions. An Appendix is provided to show the mass matrices of isotriplet, isodoublet, and isosinglet states.

II. THE EXTENDED LINEAR σ MODEL

The extended linear σ model can be systematically formulated under the quark symmetry group $U_R(3)\times U_L(3)$ or alternatively $U_V(3)\times U_A(3)$. This bigger symmetry group can be further decomposed into $SU_R(3)\times SU_L(3)\times U_V(1)\times U_A(1)$ or $SU_V(3)\times SU_A(3)\times U_V(1)\times U_A(1)$. The $U_V(1)$ group can be related to the baryon number or the electric charge conservation laws and is always respected at low energy hadronic processes. Thus, in our model, the most relevant symmetry group is $SU_V(3)\times SU_A(3)\times U_A(1)$. To construct a Lagrangian in terms of this symmetry, we include three types of chiral fields: a 3×3 matrix field Φ which denotes the quarkonia states, a 3×3 matrix field Φ' which denotes the tetraquark states, and a complex field Y which denotes the pure glueball states and is a chiral singlet. The transformation properties of these fields under the chiral symmetry are defined as follows:

$$\Phi\rightarrow U_L\Phi U_R^{\dagger}, \qquad \Phi'\rightarrow U_L\Phi' U_R^{\dagger}, \tag{1}$$

where $U_{L,R}$ are group elements of the $SU_L(3)\times SU_R(3)$ symmetry. Under the $U_A(1)$ transformation, each field is changed by a global phase factor as defined below:

$$\Phi\rightarrow e^{2i\theta_A}\Phi, \quad \Phi'\rightarrow e^{-4i\theta_A}\Phi', \quad Y\rightarrow e^{-6i\theta_A}Y. \tag{2}$$

Following the convention of the linear sigma model, we express the quarkonia fields, the tetraquark fields, and the glueball fields as

$$\begin{aligned}\Phi &= T_a\phi_a = T_a(\sigma_a + i\pi_a),\\ \Phi' &= T_a{\phi'}_a = T_a({\sigma'}_a + i\pi'_a),\\ Y &= \frac{1}{\sqrt{2}}(y_1 + iy_2),\end{aligned} \tag{3}$$

where matrices $T_a = \frac{\Lambda_a}{2}$ are the generators of $U(3)$ and Λ_a are the Gell-Mann matrices with $\Lambda_0 = \sqrt{\frac{2}{3}}1_{3\times 3}$. Fields, σ_a and σ'_a, π_a and π'_a, and y_1 and y_2 denote quarkonia, tetraquark, and glueball bare states in the chiral basis, respectively.

Up to the mass dimension $O(p^4)$ (we assume that they are the most important operators to determine the nature of light scalars of ground states), the Lagrangian of our model can include two parts: the symmetry invariant part $\mathcal{L}_{\mathcal{S}}$ and the symmetry breaking one $\mathcal{L}_{\mathcal{SB}}$:

$$\mathcal{L} = \mathcal{L}_{\mathcal{S}} + \mathcal{L}_{\mathcal{SB}}. \tag{4}$$

The symmetry invariant part includes those terms which respect $SU_L(3)\times SU_R(3)$ symmetry as well as $U_A(1)$ symmetry:

$$\begin{aligned}\mathcal{L}_{\mathcal{S}} = {}& \mathrm{Tr}(\partial_\mu\Phi\partial^\mu\Phi^\dagger) + \mathrm{Tr}(\partial_\mu\Phi'\partial^\mu\Phi^{\dagger\prime}) + \partial_\mu Y\partial^\mu Y^\star - {m_\Phi}^2\mathrm{Tr}(\Phi^\dagger\Phi) - {m_{\Phi'}}^2\mathrm{Tr}(\Phi^{\dagger\prime}\Phi') - {m_Y}^2 YY^\star - \lambda_1\mathrm{Tr}(\Phi^\dagger\Phi\Phi^\dagger\Phi)\\ & - {\lambda_1}'\mathrm{Tr}(\Phi^{\dagger\prime}\Phi'\Phi^{\dagger\prime}\Phi') - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi\Phi^{\dagger\prime}\Phi') - \lambda_Y(YY^\star)^2 - [\lambda_3\epsilon_{abc}\epsilon^{\mathrm{def}}{\Phi_d}^a{\phi_e}^b{\phi_f}^{\prime c} + \mathrm{H.c.}] + [kY\mathrm{Det}(\Phi) + \mathrm{H.c.}].\end{aligned} \tag{5}$$

The symmetry breaking part includes the following terms:

$$\mathcal{L}_{\mathcal{SB}} = [\mathrm{Tr}(B.\Phi) + \mathrm{H.c.}] + [\mathrm{Tr}(B'.\Phi') + \mathrm{H.c.}] + (D.Y + \mathrm{H.c.}) - [\lambda_m \mathrm{Tr}(\Phi\Phi^{\dagger\prime}) + \mathrm{H.c.}]. \quad (6)$$

As evident from the symmetry breaking part of the Lagrangian, the first two terms violate the global flavor symmetry and all the terms in $\mathcal{L}_{\mathcal{SB}}$ violate the $U_A(1)$ symmetry. However, note that under $U_A(1)$ the first two terms transform differently.

The guiding principle for choosing the interaction terms between the quarkonia and tetraquark matrix fields is the naive dimensional analysis. For the terms which break the symmetries, we use the naive dimensional analysis and only take into account terms up to $O(p^2)$. To construct the symmetric terms in the Lagrangian we closely follow Ref. [24] where the choice of terms are limited by the number of internal quark plus antiquark lines at the vertex, which is set to 8. This particular rule, like in Ref. [24], dictates the choice of terms in our work. This restriction is relaxed for only two $O(p^4)$ terms: the tetraquark self-interaction term with coupling constant $\lambda_1{}'$ and the tetraquark and quarkonia interaction term proportional to λ_2. The reason to include these two terms stems from the practical consideration of making sure our potential is bounded. Except for these two terms, all of the terms in our Lagrangian obey the rule stated above. The mixing term between quarkonia and tetraquark fields with the coupling constant λ_m is believed to be crucial in determining the mixing between quarkonia and tetraquarks [3] after taking into account the instanton effects.

Other symmetric terms like $\mathrm{Tr}[\Phi^\dagger\Phi]\mathrm{Tr}[\Phi^\dagger\Phi]$, $\mathrm{Tr}[\Phi^\dagger\Phi']\mathrm{Tr}[\Phi^{\dagger\prime}\Phi]$, $\mathrm{Tr}[\Phi^\dagger\Phi]\ YY^*$, etc. should be small due to the Okuboï-Zweigï-Lizuka suppression rule and are omitted here. Terms like $\epsilon_{abc}\epsilon^{def}\Phi_d{}^a\Phi_e{}'^b\Phi_f{}'^c$ and $\epsilon_{abc}\epsilon^{def}\Phi_d{}'^a\Phi_e{}'^b\Phi_f{}'^c$ are omitted since they don't respect $U_A(1)$ symmetry and are assumed to be much smaller than those terms listed in Eq. (6). The term like $Y^*\mathrm{Tr}(\Phi\Phi^{\dagger\prime})$ apparently respects the chiral symmetry and the $U_A(1)$ symmetry but is neglected here since we want to keep the terms of the Lagrangian minimal in this study. Its effects can be studied and published elsewhere.

The mixing between quarkonia and tetraquark fields is determined by quadratic, cubic, and quartic interaction terms in the Lagrangian, while the glueball and quarkonia interaction is introduced through the instanton determinant term. Our choice is motivated from the observation of 't Hooft, who introduced the coupling between a scalar spurion field to the determinant of the quarkonia field in order to solve the $U_A(1)$ problem in a spontaneous symmetry breaking fashion. Furthermore, the study of lattice QCD and sum rules reveal that the instanton effect plays an important role in shaping the properties of the glueball ground state. Therefore, it is necessary to include such a term in the Lagrangian, as argued in Ref. [19].

At this point we would like to briefly discuss the pattern of symmetry breaking in our work. As evident from the Lagrangian and mentioned above, both chiral symmetry $SU_L(3) \times SU_R(3)$ and $U_A(1)$ symmetry are explicitly broken by the terms in $\mathcal{L}_{\mathcal{SB}}$. The 3×3 matrices B and B' responsible for the breaking of the symmetry can be parametrized as follows:

$$B(B') = T_a b_a (T_a b_a{}'). \quad (7)$$

Since the vacuum expectation values of the quarkonia and tetraquark fields can carry those quantum numbers which are allowed by the QCD vacuum, only $a = (0, 3, 8)$ fields are allowed. The choice of these external fields $b_a(b_a{}')$ control the nature and extent of the symmetry breaking.

For example,

(i) if $b_0(b_0{}') \neq 0$ and $b_3(b_3{}') = b_8(b_8{}') = 0$, then $SU_A(3)$ and $U_A(1)$ symmetries are explicitly broken, but all of the quark masses are equal.

(ii) if $b_0(b_0{}') \neq 0$, $b_3(b_3{}') = 0$ and $b_8(b_8{}') \neq 0$, then along with the broken $SU_A(3)$ and $U_A(1)$ symmetries, $SU_V(3)$ is also explicitly broken to $SU_V(2)$. As a result $m_u = m_d \neq m_s$, where m_i is the quark mass of the ith flavor.

(iii) $b_i(b_i{}') \neq 0$, $i = 0, 3, 8$. In this case, all of the symmetries, viz., $SU_A(3)$, $U_A(1)$, and $SU_V(3)$, are completely and explicitly broken. Consequently, $m_u \neq m_d \neq m_s$.

Out of these three scenarios, we will only consider the second case in this study. This is reasonable considering the up and down quark masses are nearly equal to each other and thereby indicating $SU_V(2)$ is a good (approximate) symmetry. The remnant $SU_V(2)$ isospin symmetry allows us to represent two condensates each for quarkonia and tetraquark fields as v_0, v_8 and v'_0, v'_8, respectively, while the gluonic condensate in our theory is labeled as v_y.

Expanding fields around these vacuum expectation values, we get the expression of tree level potential $V(v_0, v_8, v'_0, v'_8, v_y)$ which should be stable under the variation of condensates, i.e.,

$$\frac{\partial V(v_i, v'_i, v_y)}{\partial(v_i, v'_i, v_y)} = 0, \qquad i = 0, 8. \quad (8)$$

The explicit expressions for each equation can be worked out straightforwardly and are given below:

$$\begin{aligned}\frac{\partial V}{\partial v_0} = {}& b_0 + \frac{1}{4\sqrt{3}}(2v_0{}^2 - v_8{}^2)v_y k - v_0 m_\Phi{}^2 \\ & - \left(\frac{v_0{}^3}{3} + v_0 v_8{}^2 - \frac{v_8{}^3}{3\sqrt{2}}\right)\lambda_1 \\ & - \frac{1}{3}\left(v_8 v_0{}' v_8{}' - \frac{v_8 v_8{}'^2}{2\sqrt{2}} + \frac{v_0 v_0{}'^2}{2} + \frac{v_0 v_8{}'^2}{2}\right)\lambda_2 \\ & - \sqrt{\frac{2}{3}}(2v_0 v_0{}' - v_8 v_8{}')\lambda_3 - \frac{v_0{}'}{2}\lambda_m, \end{aligned} \quad (9)$$

$$\frac{\partial V}{\partial v_8} = b_8 - \frac{1}{2\sqrt{3}} v_8\left(v_0 + \frac{v_8}{\sqrt{2}}\right) v_y k - v_8 m_\Phi{}^2 - \left(v_0{}^2 v_8 - \frac{v_0 v_8{}^2}{\sqrt{2}} + \frac{v_8{}^3}{2}\right)\lambda_1 - \left(\frac{v_8 v_0'^2}{6} - \frac{v_8 v_0' v_8'}{3\sqrt{2}} + \frac{v_8 v_8'^2}{4} + \frac{v_0 v_0' v_8'}{3} - \frac{v_0 v_8'^2}{6\sqrt{2}}\right)\lambda_2 + \sqrt{\frac{2}{3}}(v_8 v_0{}' + v_0 v_8{}' + \sqrt{2} v_8 v_8{}')\lambda_3 - \frac{v_8{}'}{2}\lambda_m, \tag{10}$$

$$\frac{\partial V}{\partial v_0{}'} = b_0{}' - v_0{}' m_{\Phi'}{}^2 - \frac{1}{3}\left(\frac{v_0{}^2 v_0{}'}{2} + \frac{v_8{}^2 v_0{}'}{2} + v_0 v_8 v_8{}' - \frac{v_8{}^2 v_8{}'}{2\sqrt{2}}\right)\lambda_2 - \frac{1}{\sqrt{3}}\left(\sqrt{2} v_0{}^2 - \frac{v_8{}^2}{\sqrt{2}}\right)\lambda_3 - \left(\frac{v_0'^3}{3} + v_0{}' v_8'^2 - \frac{v_8'^3}{3\sqrt{2}}\right)\lambda_1{}' - \frac{v_0}{2}\lambda_m, \tag{11}$$

$$\frac{\partial V}{\partial v_8{}'} = b_8{}' - v_8{}' m_{\Phi'}{}^2 + \left(\frac{v_8{}^2 v_0{}'}{6\sqrt{2}} - \frac{v_0{}^2 v_8{}'}{6} - \frac{v_8{}^2 v_8{}'}{4} - \frac{v_0 v_8 v_0{}'}{3} + \frac{v_0 v_8 v_8{}'}{3\sqrt{2}}\right)\lambda_2 + \frac{1}{\sqrt{3}}(\sqrt{2} v_0 v_8 + v_8{}^2)\lambda_3 - \left(v_0'^2 - \frac{v_0{}' v_8{}'}{\sqrt{2}} + \frac{v_8'^2}{2}\right) v_8{}' \lambda_1{}' - \frac{v_8}{2}\lambda_m, \tag{12}$$

$$\frac{\partial V}{\partial v_y} = \sqrt{2} D + \frac{1}{2\sqrt{3}}\left(\frac{v_0{}^3}{3} - \frac{v_0 v_8{}^2}{2} - \frac{v_8{}^3}{3\sqrt{2}}\right) k - v_y m_y{}^2 - v_y{}^3 \lambda_Y. \tag{13}$$

These five constraints are nonlinear in terms of condensates v_0, v_8 and v_0', v_8', v_y, but are linear in terms of couplings. To avoid solving nonlinear equations in our numerical analysis, we can choose a set of v_0, v_8, and v_0', v_8', v_y as input to solve couplings.

More precisely, in order to guarantee that our values of $\{v_0', v_8'\}$ are physically meaningful, we replace them with two positive quantities, i.e., $\{v_q', v_s'\}$, where subscripts q and s stand for the nonstrange and strange quark components, respectively. The relation between $\{v_0', v_8'\}$ and $\{v_q', v_s'\}$ can be found from Ref. [25] and is provided below as

$$v_0' = \frac{\sqrt{2}}{\sqrt{3}} v_q' + \frac{1}{\sqrt{3}} v_s', \qquad v_8' = \frac{1}{\sqrt{3}} v_q' - \frac{\sqrt{2}}{\sqrt{3}} v_s'. \tag{14}$$

The quarkonia condenstates v_0 and v_8 are solved out from the decay constants of the pion and kaon, which are given below:

$$f_\pi = \left(\frac{\sqrt{2}}{\sqrt{3}} v_0 + \frac{1}{\sqrt{3}} v_8\right)\cos\theta_\pi - \left(\frac{\sqrt{2}}{\sqrt{3}} v_0' + \frac{1}{\sqrt{3}} v_8'\right)\sin\theta_\pi, \tag{15}$$

$$f_K = \left(\frac{\sqrt{2}}{\sqrt{3}} v_0 - \frac{1}{\sqrt{12}} v_8\right)\cos\theta_K - \left(\frac{\sqrt{2}}{\sqrt{3}} v_0' - \frac{1}{\sqrt{12}} v_8'\right)\sin\theta_K. \tag{16}$$

These two relations between the decay constants and our model parameters can be found by constructing the Noether current and utilizing the partially conserved axial-vector current relations, as demonstrated in Ref. [26].

III. NUMERICAL ANALYSIS STRATEGY

Due to the unbroken $SU_V(2)$ isospin symmetry, physical scalar and pseudoscalar states can be categorized into three groups with isospin quantum numbers as $I = 1$ (triplet), $\frac{1}{2}$ (doublet), and 0, respectively. Only bare quarkonia, tetraquark, and glueball fields with the same isospin quantum number can mix with one another to form physical states. Moreover, there is no mixing between scalar and pseudoscalar fields. Thus, the chiral singlet glueball field can only mix with the isospin singlets of quarkonia and tetraquark fields.

Using these facts, the physical states below 2 GeV can be tabulated as given in Table I, where the isodoublet $\{K, K'\}$ is connected with the isodoublet $\{K^*, K^{*\prime}\}$ by charge conjugation. Also, a similar relation holds for $\{\kappa, \kappa'\}$ and $\{\kappa^*, \kappa^{*\prime}\}$.

For both isotriplet and isodoublet sectors, a 2×2 mixing matrix can be used to describe the mixing among quarkonia and tetraquark states. For the isosinglet scalar and pesudoscalar sectors, a 5×5 mixing matrix must be used. To extract the relevant mass matrices, we use the substitutions $\sigma_0 \to v_0 + \sigma_0$, $\sigma_8 \to v_8 + \sigma_8$, $\sigma_0' \to v_0' + \sigma_0'$, $\sigma_8' \to v_8' + \sigma_8'$, and $y_1 \to v_y + y_1$, while assuming that other fields have no vacuum expectation value. These mass matrices are provided in the Appendix.

TABLE I. The categorization of scalar and pseudoscalar states in terms of isospin quantum numbers. States in the same category can mix with one another.

Isospin	$I = 1$	$I = \frac{1}{2}$	$I = 0$
Pseudoscalars ($P = -1$)	$\{\pi, \pi'\}$	$\{K, K'\}$, $\{K^*, K^{*\prime}\}$	$\{\eta_1, \eta_2, \eta_3, \eta_4, \eta_5\}$
Scalars ($P = 1$)	$\{a, a'\}$	$\{\kappa, \kappa'\}$, $\{\kappa^*, \kappa^{*\prime}\}$	$\{f_1, f_2, f_3, f_4, f_5\}$

It is useful to count the total number of free parameters in our model. There are 15 free parameters, as shown in our extended linear σ model given in Eq. (4). Most of these parameters can be fixed by the input from the isotriplet and isodoublet sectors. The only unfixed parameters are related to the glueball sector. Meanwhile, five vacuum stability conditions further reduce the number of free parameters. Therefore, our model is overconstrained by experimental data except for the glueball sector. Below we describe how to fix our model parameters.

(i) The tetraquark vacuum condensates $\{v'_q, v'_s\}$ are treated as input and are assumed to be positive but smaller than 2 GeV.

(ii) To fix the parameters in the triplet and doublet sector, we use the physical masses and decay constants of $\{\pi, \pi'\}$ and $\{K, K'\}$ mesons, as shown in Table II. The mixing angles for isotriplets and isodoublets, which are labeled as θ_K and θ_π, are treated as input and are restricted to vary in the range $\{-\frac{\pi}{4}, \frac{\pi}{4}\}$ (since it is widely believed that pions and kaons are quarkonia states-we will address this issue in our discussions). Accordingly, four free parameters in our model are fixed while two other free parameters are traded off by two mixing angles. We also impose the constraints on the trace of the mass matrix of isotriplets a and a', i.e., $\mathrm{Tr}[M_a^2] = \sum(M_a^2)^{\mathrm{Exp}}$ and the trace of mass matrix of isodoublet κ and κ', i.e., $\mathrm{Tr}[M_\kappa^2] = \sum(M_\kappa^2)^{\mathrm{Exp}}$. With these constraints, we choose parameters $\{v_0, v_8, m_\Phi^2, (m'_\Phi)^2, \lambda_1, \lambda_2, \lambda_3, \lambda'_1, \lambda_m, kv_y\}$ as solved out from input of isotriplet and isodoublet sectors.

(iii) In order to further constrain the parameters related to the glueball sector, following the method in Ref. [22], we consider two broad conditions from the isoscalar pseudoscalar sector:

$$\mathrm{Tr}[M_\eta{}^2]_{\mathrm{Model}} = \mathrm{Tr}[M_\eta{}^2]_{\mathrm{Exp}}, \tag{17}$$

$$\mathrm{Det}[M_\eta{}^2]_{\mathrm{Model}} = \mathrm{Det}[M_\eta{}^2]_{\mathrm{Exp}}, \tag{18}$$

where M_η is the mass matrix for the isoscalar pseudoscalar states. So, two more free parameters are fixed and we choose $\{k, m_Y^2 + \lambda_Y v_Y^2\}$ as solved from these two constraints. Combined with the solution of kv_Y given in the previous step, the parameter v_Y is solved out.

(iv) The five vacuum stability conditions given in Eqs. (9)–(13) can further help to reduce free parameters in our model. In practice, we choose the five free parameters $\{b_0, b_8, b'_0, b'_8, D\}$ as solved from these five equations.

After these inputs, there is one free parameter not fixed, which is selected as the glueball mass m_Y^2. By using it as input, we can predict the masses of the lowest scalar and pseudoscalar, as well as their components.

Considering that there is a large uncertainty in the determination of π' mass, we choose to vary this mass within the range 1.2–1.4 GeV. We scan five mass values in a 50 MeV step within the above specified range starting with 1.2 GeV.

There are a huge number of possible solutions in our parameter space. We regard that the best solution for the parameter set is the one which closely reproduces the mass spectra of scalars close to the experimental measured values. The best fit solution is determined on the basis of smallness of the below defined two quantities: the first one is χ_1 as defined in [22]

$$\chi_1 = \sum_{i=1}^{13} \frac{|M_i{}^{\mathrm{theo}} - M_i{}^{\mathrm{exp}}|}{M_i{}^{\mathrm{exp}}}, \tag{19}$$

and we also consider the second one which is defined by the least χ^2 method, labeled as χ_2 below:

$$\chi_2 = \sum_{i=1}^{13} \frac{|M_i{}^{\mathrm{theo}} - M_i{}^{\mathrm{exp}}|^2}{(\delta M_i{}^{\mathrm{exp}})^2}, \tag{20}$$

where $M_i{}^{\mathrm{theo(exp)}}$ is the mass of the each member of the scalar or pesudoscalar family calculated from our model (experiment) and $\delta M_i{}^{\mathrm{exp}}$ is the experimental error for each mass. The sum takes into account five pseudoscalar masses, four scalar masses, the masses of two triplets a and a', and the masses of two doublets κ and κ'. Each M_i^{exp} used in this work is tabulated in Table III.

We also take into account the decay width of the lowest scalar $f_0(600) \to \pi\pi$ as a constraint. This decay width is computed at the tree level and those solutions which give the decay width between 0.35–0.9 GeV are regarded as reasonable.

Below we explain how we choose our best solution. For this we apply two types of minimum χ_1 and χ_2 analysis in two stages. Typically, χ_1 has a smaller value while χ_2 has a

TABLE II. The experimental masses and decay constants of a triplet and a doublet.

Fields	π	π'	f_π	K	K'	f_K
Mass (GeV)	0.14	1.20–1.40	0.13	0.49	1.46	0.15

TABLE III. The experimental mass spectra for triplets, doublets, and isoscalars are used to determine the best fit.

Fields	a	a'	κ	κ'	η_1	η_2	η_3	η_4	η_5	f_1^0	f_2^0	f_3^0	f_4^0	f_5^0
Mass (GeV)	0.98	1.47	0.80	1.43	0.55	0.96	1.30	1.48	1.76	0.4–1.2	0.98	1.2–1.5	1.505	1.72

TABLE IV. The values of parameters in our fit, where the best value of $m_{\pi'}$ is found to be $m_{\pi'} = 1.2$ GeV.

Parameter	Value	Parameter	Value
θ_π (radian)	−0.604	$\lambda_1{}'$	8.248
θ_K (radian)	−0.714	λ_2	76.428
v_0 (GeV)	0.074	λ_3 (GeV)	−0.738
v_8 (GeV)	−0.115	λ_Y	38.327
$v_0{}'$ (GeV)	0.203	k	−78.15
$v_8{}'$ (GeV)	0.126	λ_m (GeV^2)	−1.044
v_y (GeV)	−0.109	b_0 (GeV^3)	−0.085
$m_Y{}^2$ (GeV^2)	3.0	b_8 (GeV^3)	−0.161
$m_\Phi{}^2$ (GeV^2)	−0.025	$b_0{}'$ (GeV^3)	0.166
$m_{\Phi'}{}^2$ (GeV^2)	0.744	$b_8{}'$ (GeV^3)	0.18
λ_1	35.465	D (GeV^3)	−0.265

larger value due to the small experimental errors for some scalars, like the η_1 and η_2. Firstly, for a given mass value of $m_{\pi'}$ we choose the best fit solution for the mass spectra for all scalars. We generate more than 8 million random parameter sets of $\{\theta_\pi, \theta_K, v_q', v_s'\}$ to find the best fit solution which minimizes the χ_i. For each value of $m_\pi{}'$ and each set of $\{\theta_\pi, \theta_K, v_q', v_s'\}$, we treat the bare glueball mass m_Y^2 as a scanning parameter varying from -9 to 9. After considering the constraint, the vacuum must be bounded below, i.e., the coupling λ_Y must be positive; we read out the best fit value for m_Y^2. Then we determine the best fit for each scanned $m_{\pi'}$ from 1.2 to 1.4 GeV with 50 MeV intervals. The final best fit is chosen from these five best fits with the minimum χ_1/χ_2 value.

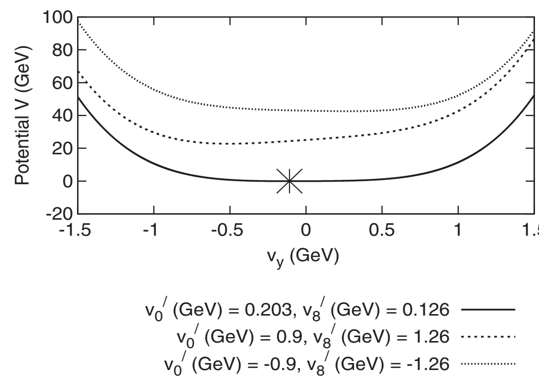

FIG. 1. The 1D plot of potential varying with v_Y. We show three sets of v_0' and v_8', where the first set of parameters corresponds to the best fit.

IV. RESULTS

To test our model, here we are going to demonstrate how well it can reproduce the mass spectra of mesons. Besides that, we are also interested to know the composition of these low-lying scalars.

The different χ_i definitions given in Eqs. (19) and (20) yield a similar best fit, presented in Table IV. We would like to highlight a few features from it. (1) In the absence of explicit symmetry breaking terms, it is the negative mass parameter $m_\Phi{}^2$ that triggers the spontaneous chiral symmetry breaking. (2) The sign of v_Y is correlated with the sign of k, and the sign of k is determined from the mass spectra of the pseudoscalar sector. (3) The couplings λ_1, λ_1', λ_Y are positive, which guarantees the potential is bounded from below. (4) The values of λ_1, λ_2, and λ_Y as well as k are large, which demonstrates the nonperturbative nature of the model. (5) The value of λ_m is found to be negative.

Our best fit result favors the case where the percentage of tetraquark components in the π' meson is about 67.7% and in the K' meson is about 57.2%. When comparing our result with the previous studies, we find that the tetraquark component of K' (1.46) in our result is quite low compared to 95% in Ref. [27] and 76% in Ref. [22]. It would be attributed to the effects of glueballs[1] or decay widths of these mesons will put a more stringent constraint on this percentage. For π' the percentage of the tetraquark component is in qualitative agreement with that of Ref. [22], where they found a tetraquark percentage of 85%.

One interesting issue is whether the best fit given in Table IV by solving Eqs. (9)–(13) can guarantee that our solution is the global minimum of the potential. The answer is affirmative. Obviously, the constraints from Eqs. (9)–(13) are crucial to guaranteeing our solutions are a local minimum. The linear and trilinear terms in the $\mathcal{L}_{\mathcal{SB}}$ can crucially split the eightfold degeneracy, as they do for the twofold degeneracy in the ϕ^4 theory. The global minimum of the potential can also be numerically examined by using the determined couplings and mass parameters as inputs to examine the shape of the potential in five

[1]Though glueballs do not directly mix with the other fields in the doublet sector, the parameters like gluonic condensates and the instanton coupling constants do contribute to the mass matrix in the doublet sector, as shown by kv_Y.

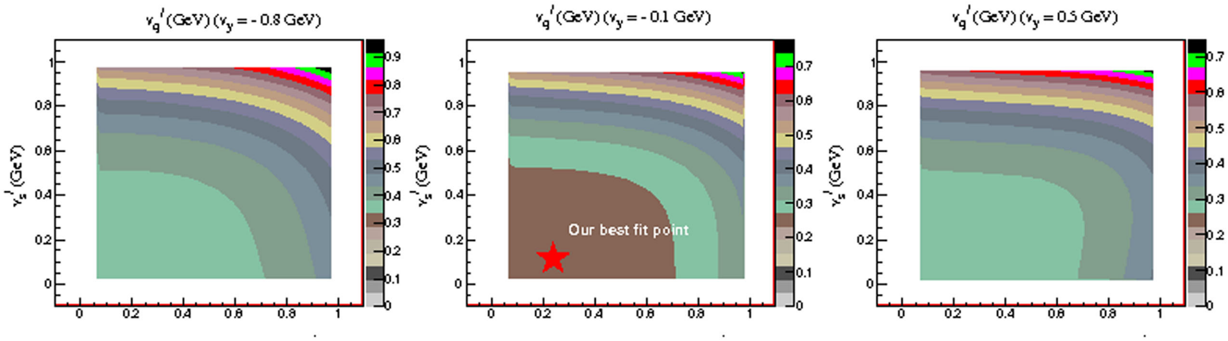

FIG. 2 (color online). The 2D contour in the v_q' and v_s' planes. We show three 2D contour plots with $v_Y = -0.8$, $v_Y = -0.1$, and $v_Y = 0.5$. The case $v_Y = -0.1$ is the best fit.

dimensional parameter space (i.e., $\{v_q, v_s, v_q', v_s', v_y\}$). For example, by choosing different sets of v_0' and v_8', we demonstrate that the shape of the potential varies with v_Y, as given in Fig. 1. By fixing v_q and v_s from physical conditions, we can examine the shape of the potential determined by (v_q', v_s', v_y), as demonstrated in Fig. 2. With these numerical self-consistency checks, it is found that our best fit point is indeed the global minimum of the potential.

With the parameter set given in Table IV, the predicted mass spectra and omponents for the triplet $\{a, a'\}$ and the doublet $\{\kappa, \kappa'\}$ are provided in Table V. It is found that the mass spectra of the triplet are close to their experimental values but those of the doublet deviate considerably. The a and κ' are more tetraquarklike while a' and κ are more quarkonialike.

The predicted mass spectra of pseudoscalars and scalars are shown in Tables VI and VII. We have a few comments

TABLE V. Mass spectra and components for the triplet and doublet sectors based on our fit, where the best value of $m_{\pi'}$ is found to be $m_{\pi'} = 1.2$ GeV.

π' mass (GeV)	Field	Our value (GeV)	Quarkonia (%)	Tetraquark (%)	Experimental value (GeV)
1.2	a	1.055	38.14	61.86	0.98
	a'	1.417	61.86	38.14	1.47
	κ	1.13	62.14	37.86	0.80
	κ'	1.186	37.86	62.14	1.43

TABLE VI. Mass spectra and components for the pseudoscalar mesons based on our fit, where the best value of $m_{\pi'}$ is found to be $m_{\pi'} = 1.2$ GeV.

π' mass (GeV)	$J^{PC} = 0^{-+}$	Our value (GeV)	Quarkonia (%)	Tetraquark (%)	Glueball (%)	Experimental value (GeV)
1.2	η_5	1.858	0.037	0.001	99.962	1.756 ± 0.009
	η_4	1.380	75.803	24.167	0.03	1.476 ± 0.004
	η_3	1.291	26.700	73.294	0.006	1.294 ± 0.004
	η_2	0.907	15.852	84.145	0.003	0.95766 ± 0.00024
	η_1	0.595	81.607	18.393	0.0	0.547853 ± 0.000024

TABLE VII. Mass spectra and components for the scalar mesons based on our fit, where the best value of $m_{\pi'}$ is found to be $m_{\pi'} = 1.2$ GeV.

π' mass (GeV)	$J^{PC} = 0^{++}$	Our value (GeV)	Quarkonia (%)	Tetraquark (%)	Glueball (%)	Experimental value (GeV)
1.2	${f_5}^0$	2.09	0.01	0.0	99.99	...
	${f_4}^0$	1.487	77.469	22.53	0.001	1.505 ± 0.006
	${f_3}^0$	1.347	22.177	77.82	0.003	1.2–1.5
	${f_2}^0$	1.124	21.561	78.439	0.0	0.980 ± 0.010
	${f_1}^0$	0.274	78.784	21.211	0.005	0.4–1.2

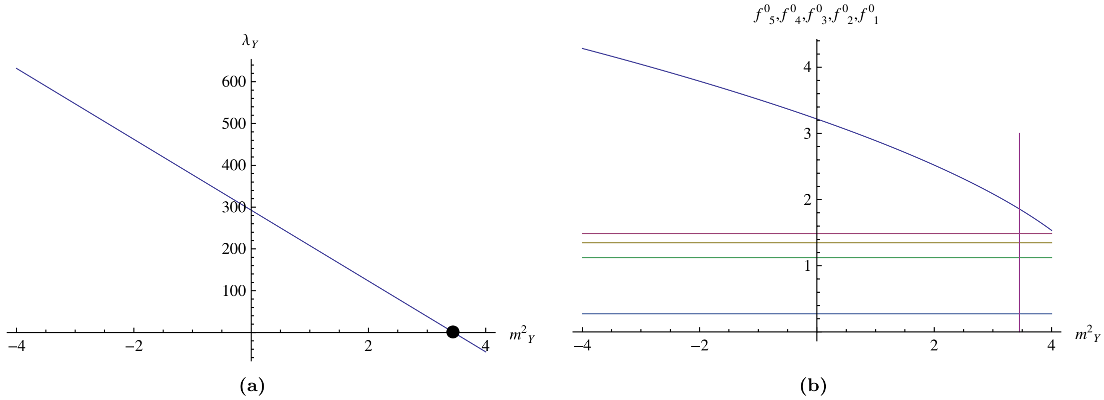

FIG. 3 (color online). (a) The dependence of λ_Y upon m_Y^2. A solid circle marker shows the point $\lambda_Y = 0$, which corresponds to $m_Y^2 = 3.452$ ($m_Y = 1.858$). (b) The dependence of mass of f_0 upon m_Y^2. A vertical line with m_Y^2 is drawn to read out the lowest mass $m_{f_0^5} = 1.86$.

in order. (1) The pseudoscalar mass spectra can fit experimental data better than the scalar mass spectra, but the mixing pattern of scalar and pseudoscalar is quite similar. (2) When the mass $m_{f_5^0} = 1.72$ GeV is used for our fit, we find a solution with $\lambda_Y < 0$. To guarantee the condition that the potential must be bound from below (i.e., $\lambda_Y > 0$), we have to keep $m_{f_5^0}$ out of our fit, which explains why in the definition of χ_i we only sum over the masses of four scalars. This condition can predict that the lightest scalar glueball should be around 2.0 GeV or so, as can be read from Fig. 3(b), while the lightest pseudoscalar glueball should be η_5. The mass splitting between these two glueball states is controlled by parameters v_Y and λ_Y and is found to be around 0.15 GeV. When compared with the lattice QCD prediction for the glueball bare mass reported in Ref. [28], where the mass is 1.611 GeV, our result $m_Y = 1.73$ GeV is slightly heavier than this prediction. When $m_Y = 1.611$ GeV is taken, then the predicted mass of the lightest glueball is $m_{f_5^0} = 2.29$ GeV. (3) The lightest scalar $f_1^0(600)$ is found to be 0.27 GeV or so and is a quarkonia dominant state.

In Fig. 3, we demonstrate the dependence of λ_Y and f^0 masses upon the free parameter m_Y^2, with the rest of the parameters given in Table IV. As shown in Fig. 3(a), when m_Y^2 is larger than 3.4 GeV^2, the λ_Y becomes negative. Then the potential of our model has to confront the problem of unbounded vacuum from below. In the allowed values of m_Y^2, the masses of f_i^0, $i = 1, 2, 3, 4$ are almost independent of its value, as demonstrated in Fig. 3(b). The upper bound of m_Y^2 is determined from the condition $\Gamma_{f_1^0} > 0.35$ GeV.

V. DISCUSSION AND CONCLUSION

In this work we develop a consistent model for the scalar mesons below 2 GeV and focus on the mixing effects on the mass spectra. In our model we have taken into account the quarkonia, tetraquark, and glueball scalar and pseudoscalar fields. Bare fields with the same quantum numbers are allowed to mix with one another to form the physical mesons. In this way our isospin triplet and doublet mesons are composed of quarkonia and tetraquark states and the isosinglet mesons are composed of all three chiral fields. We have presented our prediction from the model for the scalar mass spectra on the basis of two χ^2 methods and found that they yield similar results.

We also investigated candidates for the glueball dominant states in our model. What is more encouraging is that the determined value of the bare glueball mass, which is treated as a scanning parameter in our study, agrees quite well with the lattice result [28]. The consequence of the uncertainty in the bare glueball mass is also discussed in our work.

When the constraint for θ_π and θ_K to vary from $\{-\frac{\pi}{4}, \frac{\pi}{4}\}$ is changed to $\{-\frac{\pi}{2}, \frac{\pi}{2}\}$, we find solutions with $m_{\Phi'}^2 < 0$ and $m_\Phi^2 > 0$ which can accommodate data quite well but contradict the general belief that the pion and kaon are quarkonia states. It is also found that when the constraint for $\lambda_Y > 0$ is loosened, we can find solutions that the lightest scalar can be glueball dominant.

One should be cautious to interpret the results found in our study. The general qualitative understanding coming out of our study is that if the mixing of tetraquarks with glueballs is absent or negligible, then the heaviest scalar state is the glueball dominated meson. Since the bare glueball mass ($m_Y = 1.73$ GeV) is found to be heavy, after mixing with the two quark states, the physical glueball dominated meson is found to be even heavier. It should be interesting to investigate the mixing between tetraquark and glueball states and to examine how this mixing can affect our results.

It is enlightening to address the predictive power of our model. As evidence of the predictive power of our model, we would like to highlight the value of one particular parameter, namely, the bare glueball mass m_Y. The value of this parameter in our model is determined in a self-consistent way by following the best fit analysis. When we compare our best fit value 1.73 GeV for the bare glueball mass m_Y with the first principle lattice gauge theory result of 1.611 GeV, we find out that our result closely matches with the lattice data. This matching no doubt indicates the credibility of the predictive power of our model. Except for the prediction of mass spectra, especially the masses of glueball candidates, our model can also be used to predict decay properties of scalar mesons. Since free parameters in our model can be fixed from the global fit with limited inputs, the decay modes and branching fractions of all scalars and pseudoscalars can be determined as predictions. In particular, total and partial decay widths of glueball states can be predicted, which is helpful to determine which states found in experiments are more glueball-like. Furthermore, the cross sections of scalar and pseudoscalar scattering processes (say, $\pi\pi$, πK, $\pi\eta$, KK scatterings, etc.) can also be predicted in our model, which can be used to determine the production channels of glueball states at colliders. When loop effects of scattering amplitudes are taken into account, our model can also predict the poles of bound states of mesons, which are necessary in order to decide whether there are glueball signatures in data.

To simplify our study, we have assumed that operators in our Lagrangian are the most crucial ones to determine the mass spectra. We can extend the current study to those cases where the interaction terms between glueball and tetraquark fields are included. More than one choice is available to define the interaction between different fields; for example, our choice of the instanton induced term is different from that in Ref. [24]. It would be interesting to show the difference between these two different parametrizations. We included the decay width of the $f_0(600) \to \pi\pi$ to constrain our parameter space; to put a tight constraint on our parameter sets we can consider more decay widths of all scalars and pseudoscalars and even should include $\pi\pi$ and πK scattering data. In order to examine whether our model can accommodate all experimental data, a global fit by treating all free parameters on the same footing in our model is necessary. To extend our model by including the tetraquark field as demonstrated in Refs. [29,30] to an AdS/QCD framework is also straightforward. Following the previous study [25,31–33], we can extend our model to study the role played by tetraquark states in the chiral phase transition at finite temperature and finite chemical potential.

ACKNOWLEDGMENTS

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APPENDIX: EXPRESSIONS FOR THE SCALAR AND PSEUDOSCALAR MASS MATRIX AND DECAY WIDTH

Different elements of the scalar mass matrix are as follows:

$$(M_s{}^2)_{11} = m_\Phi{}^2 + (v_0^2 + v_8^2)\lambda_1 + \frac{1}{6}(v_0'^2 + v_8'^2)\lambda_2 + 2\sqrt{\frac{2}{3}}v_0'\lambda_3 - \frac{v_0 v_y}{\sqrt{3}}k, \tag{A1}$$

$$(M_s{}^2)_{22} = m_\Phi{}^2 + \left(v_0^2 - \sqrt{2}v_0 v_8 + \frac{3}{2}v_8^2\right)\lambda_1 + \left(\frac{1}{6}v_0'^2 - \frac{1}{3\sqrt{2}}v_0'v_8' + \frac{1}{4}v_8'^2\right)\lambda_2 - \frac{2}{\sqrt{3}}\left(\frac{v_0'}{\sqrt{2}} + v_8'\right)\lambda_3 + \frac{1}{\sqrt{3}}\left(\frac{v_0 v_y}{2} + \frac{v_8 v_y}{\sqrt{2}}\right)k, \tag{A2}$$

$$(M_s{}^2)_{33} = m_{\Phi'}{}^2 + (v_0'^2 + v_8'^2)\lambda_1' + \frac{1}{6}(v_0^2 + v_8^2)\lambda_2, \tag{A3}$$

$$(M_s{}^2)_{44} = m_{\Phi'}{}^2 + \left(v_0'^2 - \sqrt{2}v_0'v_8' + \frac{3v_8'^2}{2}\right)\lambda_1' + \left(\frac{v_0^2}{6} - \frac{v_0 v_8}{3\sqrt{2}} + \frac{v_8^2}{4}\right)\lambda_2, \tag{A4}$$

$$(M_s{}^2)_{55} = m_Y{}^2 + (3v_y{}^2)\lambda_Y, \tag{A5}$$

$$(M_s{}^2)_{12} = \frac{1}{2}\left[2\left(2v_0 v_8 - \frac{v_8^2}{\sqrt{2}}\right)\lambda_1 + 2\left(\frac{v_0'v_8'}{3} - \frac{v_8'^2}{6\sqrt{2}}\right)\lambda_2 - 2\left(\sqrt{\frac{2}{3}}v_8'\right)\lambda_3 + \left(\frac{v_8 v_y}{\sqrt{3}}\right)k\right] = (M_s{}^2)_{21}, \tag{A6}$$

$$(M_s{}^2)_{13} = \frac{1}{2}\left[\lambda_m + \frac{2}{3}\left(v_0 v_0' + v_8 v_8'\right)\lambda_2 + 4\sqrt{\frac{2}{3}}v_0\lambda_3\right] = (M_s{}^2)_{31}, \tag{A7}$$

$$(M_s{}^2)_{14} = \frac{1}{2}\left[\frac{2}{3}\left(v_8 v_0' + v_0 v_8' - \frac{v_8 v_8'}{\sqrt{2}}\right)\lambda_2 - 2\left(\sqrt{\frac{2}{3}}v_8\right)\lambda_3\right] = (M_s{}^2)_{41}, \tag{A8}$$

$$(M_s{}^2)_{15} = \frac{1}{2}\left[-\frac{1}{\sqrt{3}}\left(v_0^2 - \frac{v_8^2}{2}\right)k\right] = (M_s{}^2)_{51}, \tag{A9}$$

$$(M_s^2)_{23} = \frac{1}{2}\left[\frac{2}{3}\left(v_8 v_0' + v_0 v_8' - \frac{v_8 v_8'}{\sqrt{2}}\right)\lambda_2 - 2\sqrt{\frac{2}{3}} v_8 \lambda_3\right] = (M_s^2)_{32}, \tag{A10}$$

$$(M_s^2)_{24} = \frac{1}{2}\left[\lambda_m + 2\left(\frac{v_0 v_0'}{3} - \frac{v_8 v_0'}{3\sqrt{2}} - \frac{v_0 v_8'}{3\sqrt{2}} + \frac{v_8 v_8'}{2}\right)\lambda_2 - \frac{2\sqrt{2}}{\sqrt{3}}(v_0 + \sqrt{2} v_8)\lambda_3\right] = (M_s^2)_{42}, \tag{A11}$$

$$(M_s^2)_{25} = \frac{1}{2}\left[\frac{1}{\sqrt{3}}\left(v_0 v_8 + \frac{v_8^2}{\sqrt{2}}\right)k\right] = (M_s^2)_{52}, \tag{A12}$$

$$(M_s^2)_{34} = \frac{1}{2}\left[2\left(2 v_0' v_8' - \frac{v_8'^2}{\sqrt{2}}\right)\lambda_1' + \frac{2}{3}\left(v_0 v_8 - \frac{v_8^2}{2\sqrt{2}}\right)\lambda_2\right] = (M_s^2)_{43}, \tag{A13}$$

$$(M_s^2)_{35} = (M_s^2)_{53} = (M_s^2)_{45} = (M_s^2)_{54} = 0, \tag{A14}$$

where $M^2_{\sigma_0\sigma_0} = (M_s^2)_{11}$, $M^2_{\sigma_8\sigma_8} = (M_s^2)_{22}$, $M^2_{\sigma_0'\sigma_0'} = (M_s^2)_{33}$, $M^2_{\sigma_8'\sigma_8'} = (M_s^2)_{44}$, $M^2_{y_1y_1} = (M_s^2)_{55}$, $M^2_{\sigma_0\sigma_8} = (M_s^2)_{12}$, $M^2_{\sigma_0\sigma_0'} = (M_s^2)_{13}$, $M^2_{\sigma_0\sigma_8'} = (M_s^2)_{14}$, $M^2_{\sigma_0 y_1} = (M_s^2)_{15}$, $M^2_{\sigma_8\sigma_0'} = (M_s^2)_{23}$, $M^2_{\sigma_8\sigma_8'} = (M_s^2)_{24}$, $M^2_{\sigma_8 y_1} = (M_s^2)_{25}$, $M^2_{\sigma_0'\sigma_8'} = (M_s^2)_{34}$, $M^2_{\sigma_0' y_1} = (M_s^2)_{35}$, $M^2_{\sigma_8' y_1} = (M_s^2)_{45}$.

Different elements of the pseudoscalar mass matrix are as follows:

$$(M_\eta^2)_{11} = m_\Phi^2 + \frac{1}{3}(v_0^2 + v_8^2)\lambda_1 + \frac{1}{6}(v_0'^2 + v_8'^2)\lambda_2 - 2\left(\sqrt{\frac{2}{3}} v_0'\right)\lambda_3 + \left(\frac{v_0 v_y}{\sqrt{3}}\right)k, \tag{A15}$$

$$(M_\eta^2)_{22} = m_\Phi^2 + 2\left(\frac{v_0^2}{6} - \frac{v_0 v_8}{3\sqrt{2}} + \frac{v_8^2}{4}\right)\lambda_1 + \left(\frac{v_0'^2}{6} - \frac{v_0' v_8'}{3\sqrt{2}} + \frac{v_8'^2}{4}\right)\lambda_2 + \frac{2}{\sqrt{3}}\left(\frac{v_0'}{\sqrt{2}} + v_8'\right)\lambda_3 - \frac{1}{\sqrt{3}}\left(\frac{v_0 v_y}{2} + \frac{v_8 v_y}{\sqrt{2}}\right)k, \tag{A16}$$

$$(M_\eta^2)_{33} = m_{\Phi'}^2 + \frac{1}{3}(v_0'^2 + v_8'^2)\lambda_1' + \frac{1}{6}(v_0^2 + v_8^2)\lambda_2, \tag{A17}$$

$$(M_\eta^2)_{44} = m_{\Phi'}^2 + 2\left(\frac{v_0'^2}{6} - \frac{v_0' v_8'}{3\sqrt{2}} + \frac{v_8'^2}{4}\right)\lambda_1' + \left(\frac{v_0^2}{6} - \frac{v_0 v_8}{3\sqrt{2}} + \frac{v_8^2}{4}\right)\lambda_2, \tag{A18}$$

$$(M_\eta^2)_{55} = m_Y^2 + v_y^2 \lambda_Y, \tag{A19}$$

$$(M_\eta^2)_{12} = \frac{1}{2}\left[\frac{2}{3}\left(2 v_0 v_8 - \frac{v_8^2}{\sqrt{2}}\right)\lambda_1 + \frac{2}{3}\left(v_0' v_8' - \frac{v_8'^2}{2\sqrt{2}}\right)\lambda_2 + 2\left(\sqrt{\frac{2}{3}} v_8'\right)\lambda_3 - \left(\frac{v_8 v_y}{\sqrt{3}}\right)k\right] = (M_\eta^2)_{21}, \tag{A20}$$

$$(M_\eta^2)_{13} = \frac{1}{2}\left[\lambda_m - 4\sqrt{\frac{2}{3}} v_0 \lambda_3\right] = (M_\eta^2)_{31}, \tag{A21}$$

$$(M_\eta^2)_{14} = \frac{1}{2}\left[2\sqrt{\frac{2}{3}} v_8 \lambda_3\right] = (M_\eta^2)_{41}, \tag{A22}$$

$$(M_\eta^2)_{15} = \frac{1}{2}\left[\frac{1}{\sqrt{3}}\left(v_0^2 - \frac{v_8^2}{2\sqrt{3}}\right)k\right] = (M_\eta^2)_{51}, \tag{A23}$$

$$(M_\eta^2)_{23} = \frac{1}{2}\left[2\sqrt{\frac{2}{3}} v_8 \lambda_3\right] = (M_\eta^2)_{32}, \tag{A24}$$

$$(M_\eta^2)_{24} = \frac{1}{2}\left[\lambda_m + 2\left(\sqrt{\frac{2}{3}} v_0 + \frac{2}{\sqrt{3}} v_8\right)\lambda_3\right] = (M_\eta^2)_{42}, \tag{A25}$$

$$(M_\eta^2)_{25} = \frac{1}{2}\left[-\frac{1}{\sqrt{3}}\left(v_0 v_8 + \frac{v_8^2}{\sqrt{2}}\right)k\right] = (M_\eta^2)_{52}, \tag{A26}$$

$$(M_\eta^2)_{34} = \frac{1}{2}\left[\frac{2}{3}\left(2 v_0' v_8' - \frac{v_8'^2}{\sqrt{2}}\right)\lambda_1' + \frac{2}{3}\left(v_0 v_8 - \frac{v_8^2}{3\sqrt{2}}\right)\lambda_2\right] = (M_\eta^2)_{43}, \tag{A27}$$

$$(M_\eta^2)_{35} = (M_\eta^2)_{53} = (M_\eta^2)_{45} = (M_\eta^2)_{54} = 0, \tag{A28}$$

where $M^2_{\pi_0\pi_0} = (M_\eta^2)_{11}$ $M^2_{\pi_8\pi_8} = (M_\eta^2)_{22}$ $M^2_{\pi_0'\pi_0'} = (M_\eta^2)_{33}$ $M^2_{\pi_8'\pi_8'} = (M_\eta^2)_{44}$ $M^2_{y_2y_2} = (M_\eta^2)_{55}$ $M^2_{\pi_0\pi_8} = (M_\eta^2)_{12}$ $M^2_{\pi_0\pi_0'} = (M_\eta^2)_{13}$ $M^2_{\pi_0\pi_8'} = (M_\eta^2)_{14}$ $M^2_{\pi_0 y_1} = (M_\eta^2)_{15}$ $M^2_{\pi_8\pi_0'} = (M_\eta^2)_{23}$ $M^2_{\pi_8\pi_8'} = (M_\eta^2)_{24}$ $M^2_{\pi_8 y_1} = (M_\eta^2)_{25}$ $M^2_{\pi_0'\pi_8'} = (M_\eta^2)_{34}$ $M^2_{\pi_0' y_2} = (M_\eta^2)_{35}$ $M^2_{\pi_8' y_2} = (M_\eta^2)_{45}$.

For the decay constant, we have taken the following standard formula: corresponding to the interaction Lagrangian $\mathcal{L}_{\rm int} = G f_0 \pi_p \pi_p$ (the subscript p denotes the physical pion fields), the decay constant is given by

$$\Gamma = 3 s_f \frac{k_f}{8\pi m_{f_0}^2} |-iM|^2 \tag{A29}$$

where s_f is the symmetry factor, which in our case is $\frac{1}{2}$, and $k_f = \sqrt{\frac{m_{f_0}^2}{4} - m_{\pi_p}^2}$. At the tree level, $|-iM|^2 = G^2$.

We calculated this coupling constant from our bare Lagrangian, following the procedure presented in Ref. [34]. The explicit expression for the coupling constant is given below (where R_s stands for the rotation mass matrix for scalars):

$$
\begin{aligned}
g_{11} &= -\sqrt{2}(\sqrt{2}v_0 + v_8)\lambda_1 - \frac{v_y k}{2\sqrt{3}},\\
g_{12} &= -\frac{2}{3}\left(v_0{}' + \frac{v_8{}'}{\sqrt{2}}\right)\lambda_2 + 2\sqrt{\frac{2}{3}}\lambda_3,\\
g_{13} &= -\frac{1}{3}\left(v_0 + \frac{v_8}{\sqrt{2}}\right)\lambda_2, \quad g_{21} = -(\sqrt{2}v_0 + v_8)\lambda_1 + \frac{v_y k}{\sqrt{6}},\\
g_{22} &= -\frac{1}{3}(\sqrt{2}v_0{}' + v_8{}')\lambda_2 - \frac{4\lambda_3}{\sqrt{3}},\\
g_{23} &= -\frac{1}{3\sqrt{2}}\left(v_0 + \frac{v_8}{\sqrt{2}}\right)\lambda_2,\\
g_{31} &= -\frac{1}{3}\left(v_0{}' + \frac{v_8{}'}{\sqrt{2}}\right)\lambda_2 + \sqrt{\frac{2}{3}}\lambda_3,\\
g_{32} &= -\frac{2}{3}\left(v_0 + \frac{v_8}{\sqrt{2}}\right)\lambda_2, \quad g_{33} = -\sqrt{2}(\sqrt{2}v_0{}' + v_8{}')\lambda_1{}',\\
g_{41} &= -\frac{1}{3\sqrt{2}}\left(v_0{}' + \frac{v_8{}'}{\sqrt{2}}\right)\lambda_2 - \frac{2}{\sqrt{3}}\lambda_3,\\
g_{42} &= -\frac{1}{3}(\sqrt{2}v_0 + v_8)\lambda_2, \quad g_{43} = -(\sqrt{2}v_0{}' + v_8{}')\lambda_1',\\
g_{51} &= -\frac{1}{\sqrt{6}}\left(\frac{v_0}{\sqrt{2}} - v_8\right)k, \quad g_{52} = 0 = g_{53},\\
G_1 &= (R_s)_{51}[(R_{\pi\pi'})_{11}{}^2 g_{11} + (R_{\pi\pi'})_{11}(R_{\pi\pi'})_{12} g_{12}\\
&\quad + (R_{\pi\pi'})_{12}{}^2 g_{13}],\\
G_2 &= (R_s)_{52}[(R_{\pi\pi'})_{11}{}^2 g_{21} + (R_{\pi\pi'})_{11}(R_{\pi\pi'})_{12} g_{22}\\
&\quad + (R_{\pi\pi'})_{12}{}^2 g_{23}],\\
G_3 &= (R_s)_{53}[(R_{\pi\pi'})_{11}{}^2 g_{31} + (R_{\pi\pi'})_{11}(R_{\pi\pi'})_{12} g_{32}\\
&\quad + (R_{\pi\pi'})_{12}{}^2 g_{33}],\\
G_4 &= (R_s)_{54}[(R_{\pi\pi'})_{11}{}^2 g_{41} + (R_{\pi\pi'})_{11}(R_{\pi\pi'})_{12} g_{42}\\
&\quad + (R_{\pi\pi'})_{12}{}^2 g_{43}],\\
G_5 &= (R_s)_{55}[(R_{\pi\pi'})_{11}{}^2 g_{51} + (R_{\pi\pi'})_{11}(R_{\pi\pi'})_{12} g_{52}\\
&\quad + (R_{\pi\pi'})_{12}{}^2 g_{53}],\\
G &= G_1 + G_2 + G_3 + G_4 + G_5. \qquad \text{(A30)}
\end{aligned}
$$

The expressions of mass matrices for a-a' and π-π' mesons are given below:

$$(M_{aa'}{}^2)_{11} = m_\Phi{}^2 - A_{11}\lambda_1 - B_{11}\lambda_2 - C_{11}\lambda_3 - D_{11}v_y k, \qquad \text{(A31)}$$

$$(M_{aa'}{}^2)_{22} = m_{\Phi'}{}^2 - A_{22}\lambda_1{}' - B_{22}\lambda_2, \qquad \text{(A32)}$$

$$(M_{aa'}{}^2)_{12} = \frac{1}{2}[\lambda_m - A_{12}\lambda_2 - B_{12}\lambda_3], \qquad \text{(A33)}$$

$$(M_{\pi\pi'}{}^2)_{11} = m_\Phi{}^2 - \frac{1}{3}A_{11}\lambda_1 - B_{11}\lambda_2 + C_{11}\lambda_3 + D_{11}v_y k, \qquad \text{(A34)}$$

$$(M_{\pi\pi'}{}^2)_{22} = m_{\Phi'}{}^2 - \frac{1}{3}A_{22}\lambda_1{}' - B_{22}\lambda_2, \qquad \text{(A35)}$$

$$(M_{\pi\pi'}{}^2)_{12} = \frac{1}{2}[\lambda_m + B_{12}\lambda_3], \qquad \text{(A36)}$$

where

$$A_{11} = -v_0{}^2 - \sqrt{2}v_0 v_8 - \frac{v_8{}^2}{2}, \qquad \text{(A37)}$$

$$B_{11} = -\frac{v_0{}'^2}{6} - \frac{v_0{}' v_8{}'}{3\sqrt{2}} - \frac{v_8{}'^2}{12}, \qquad \text{(A38)}$$

$$C_{11} = \sqrt{\frac{2}{3}}v_0{}' - \frac{2}{\sqrt{3}}v_8{}', \qquad \text{(A39)}$$

$$D_{11} = -\frac{v_0}{2\sqrt{3}} + \frac{v_8}{\sqrt{6}}, \qquad \text{(A40)}$$

$$A_{22} = -v_0{}'^2 - \sqrt{2}v_0{}' v_8{}' - \frac{v_8{}'^2}{2}, \qquad \text{(A41)}$$

$$B_{22} = -\frac{v_0{}^2}{6} - \frac{v_0 v_8}{3\sqrt{2}} - \frac{v_8{}^2}{12}, \qquad \text{(A42)}$$

$$A_{12} = -\frac{2}{3}v_0 v_0{}' - \frac{\sqrt{2}}{3}v_8 v_0{}' - \frac{\sqrt{2}}{3}v_0 v_8{}' - \frac{1}{3}v_8 v_8{}', \qquad \text{(A43)}$$

$$B_{12} = 2\sqrt{\frac{2}{3}}v_0 - \frac{4}{\sqrt{3}}v_8. \qquad \text{(A44)}$$

The expressions of mass matrices for κ-κ' and K-K' mesons are given below:

$$(M_{\kappa\kappa'}{}^2)_{11} = m_\Phi{}^2 - E_{11}\lambda_1 - F_{11}\lambda_2 - G_{11}\lambda_3 - H_{11}v_y k, \qquad \text{(A45)}$$

$$(M_{\kappa\kappa'}{}^2)_{22} = m_{\Phi'}{}^2 - E_{22}\lambda_1{}' - F_{22}\lambda_2, \qquad \text{(A46)}$$

$$(M_{\kappa\kappa'}{}^2)_{12} = \frac{1}{2}[\lambda_m - E_{12}\lambda_2 - F_{12}\lambda_3], \qquad \text{(A47)}$$

$$(M_{KK'}{}^2)_{11} = m_\Phi{}^2 - I_{11}\lambda_1 - F_{11}\lambda_2 + G_{11}\lambda_3 + H_{11}v_y k, \qquad \text{(A48)}$$

$$(M_{KK'}{}^2)_{22} = m_{\Phi'}{}^2 - J_{22}\lambda_1{}' - F_{22}\lambda_2, \qquad \text{(A49)}$$

$$(M_{KK'}{}^2)_{12} = \frac{1}{2}[\lambda_m - K_{12}\lambda_2 + F_{12}\lambda_3], \quad (A50)$$

where

$$E_{11} = -v_0{}^2 + \frac{v_0 v_8}{\sqrt{2}} - \frac{v_8{}^2}{2}, \quad (A51)$$

$$F_{11} = -\frac{v_0{}'^2}{6} + \frac{v_0' v_8'}{6\sqrt{2}} - \frac{5v_8{}'^2}{24}, \quad (A52)$$

$$G_{11} = \sqrt{\frac{2}{3}}v_0' + \frac{v_8'}{\sqrt{3}}, \quad (A53)$$

$$H_{11} = -\frac{v_0}{2\sqrt{3}} - \frac{v_8}{2\sqrt{6}}, \quad (A54)$$

$$E_{22} = -v_0{}'^2 + \frac{v_0' v_8'}{\sqrt{2}} - \frac{v_8{}'^2}{2}, \quad (A55)$$

$$F_{22} = -\frac{v_0{}^2}{6} + \frac{v_0 v_8}{6\sqrt{2}} - \frac{5v_8{}^2}{24}, \quad (A56)$$

$$E_{12} = -\frac{2}{3}v_0 v_0' + \frac{v_8 v_0'}{3\sqrt{2}} + \frac{v_0 v_8'}{3\sqrt{2}} - \frac{1}{12}v_8 v_8', \quad (A57)$$

$$F_{12} = 2\sqrt{\frac{2}{3}}v_0 + \frac{2}{\sqrt{3}}v_8, \quad (A58)$$

$$I_{11} = -\frac{v_0{}^2}{3} + \frac{v_0 v_8}{3\sqrt{2}} - \frac{7v_8{}^2}{6}, \quad (A59)$$

$$J_{22} = -\frac{v_0{}'^2}{3} + \frac{v_0' v_8'}{3\sqrt{2}} - \frac{7v_8{}'^2}{6}, \quad (A60)$$

$$K_{12} = -\frac{3}{4}v_8 v_8'. \quad (A61)$$

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