Generalized top-spin analysis and new physics in e^+e^- collisions with beam polarization

B. Ananthanarayan,[1] Jayita Lahiri,[1] Monalisa Patra,[1] and Saurabh D. Rindani[2]

[1]*Centre for High Energy Physics, Indian Institute of Science, Bangalore 560 012, India*
[2]*Theoretical Physics Division, Physical Research Laboratory, Navrangpura, Ahmedabad 380 009, India*

A generalized top-spin analysis proposed some time ago in the context of the standard model and subsequently studied in varying contexts is now applied primarily to the case of $e^+e^- \rightarrow t\bar{t}$ with transversely polarized beams. This extends our recent work with new physics couplings of scalar (S) and tensor (T) types. We carry out a comprehensive analysis assuming only the electron beam to be transversely polarized, which is sufficient to probe these interactions, and also eliminates any azimuthal angular dependence due to the standard model or new physics of the vector (V) and axial-vector (A) type interactions. We then consider new physics of the general four-Fermi type of V and A type with both beams transversely polarized and discuss implications with longitudinal polarization as well. The generalized spin bases are all investigated in the presence of either longitudinal or transverse beam polarization to look for appreciable deviation from the SM prediction in case of the new physics. 90% confidence level limits are obtained on the interactions for the generalized spin bases with realistic integrated luminosity. In order to achieve this we present a general discussion based on helicity amplitudes and derive a general transformation matrix that enables us to treat the spin basis. We find that beamline basis combined with transverse polarization provides an excellent window of opportunity both for S, T and V, A new physics, followed by the off-diagonal basis. The helicity basis is shown to be the best in case of longitudinal polarization to look for new physics effects due to V and A.

I. INTRODUCTION

The International Linear Collider (ILC) [1] continues to be the foremost candidate for testing the standard model (SM) in the high energy frontier at high precision and is expected to be the successor to the LHC in this regime. The issue of beam polarization has been an important subject for theorists and experimentalists alike, and pioneering studies have been carried out to establish the importance of a physics program based on the availability of this [2]. There are several choices that face designers, including that of longitudinal or transverse beam polarization of one or both beams. In particular, if transverse beam polarization of only one of the beams is available, then any beyond the standard model (BSM) physics due to vector and axial-vector-like interactions will not be visible at linear order in the new physics. It is only BSM physics due to scalar and tensor-like interactions that would be visible at linear order due to simple considerations such as the chirality of interactions.

Our approach is based on the need to define a strong polarization program through a set of analytically accessible processes. At the planned ILC, $e^+e^- \rightarrow t\bar{t}$ is a process that will be studied at great precision to validate the SM and to look for deviations from it and is particularly suitable to meet this end. The process is of continued current interest, see, e.g., Refs. [3–13], and references therein. At a linear collider the top quarks are produced in a unique spin configuration. Since the top quark has a very short lifetime, the definite spin state in which the $t\bar{t}$ pair is produced is not spoiled by hadronization effects. Due to this the direction of the spin of the top quark is reflected in the angular distribution of its decay products. There are significant angular correlations between the decay products of the top quark and its spin and also between the decay products of the top quark and top antiquark. New physics effects, if present in the production or decay mechanism, will increase its sensitivity to spin correlation by modifying the angular distribution of the decay products [14–19].

The availability of beam polarization will significantly enhance the sensitivity to new physics (NP) provided the electron and positron beams have transverse polarization (TP) or longitudinal polarization (LP), each complementing the other, with distinct prospects of obtaining very high degree of polarization for both beams [2]. In this work, we will consider such a scenario to find any azimuthal angle (ϕ) dependence in $e^+e^- \rightarrow t\bar{t}$, where ϕ is the azimuthal angle of one of the final particle. The beam direction is taken as the z axis, and in the plane perpendicular to it, the polarization direction of the electron (or positron) is taken as the x axis. Note the approach here is complimentary to that taken in Ref. [20] where both beams were taken to be polarized. In order to really probe the extent to which the new physics can be probed using an analytical approach, we now extend our considerations to a general spin basis that was proposed by Parke and Shadmi (PS) [21] in the context of unpolarized and longitudinally polarized beams. It was argued that two specific choices of such a basis known as "beam-line" bases (BLB) and "off-diagonal" bases (ODB) could be advantageous as far as increasing sensitivity is concerned. The work was further extended by

Ref. [22] which shows that the above advantages for ODB will not change appreciably when QCD corrections are included. The impact of such bases in the context of anomalous couplings of the top quarks and in case of various new physics models has been studied extensively in the literature [23–25]. The main conclusion in those studies is that though the $t\bar{t}$ spin is most correlated in ODB compared to BLB and the helicity basis (HB) in SM, this enhanced spin correlation is not that beneficial for distinguishing the new physics effect in case of unpolarized or longitudinally polarized beams. To our knowledge the study of new physics along with the SM, has not been considered in the context of transverse polarization. In order to meet this objective we present a discussion on paving the way to these general spin bases in a formalism that employs known helicity amplitudes and a new method of introducing a transformation.

Whereas in reality, the top-quark spins are reconstructed only from the decays, our treatment which is analytical cannot account for decays since there is no clear cut closed form basis in which the transverse beam polarization can be accounted for. The inclusion of decays being an important tool for spin analysis has been considered in the literature [26,27]. However, a direct analysis of the top quark spin structure itself is an insightful exercise. The general spin basis considered here is a further diagnostic tool. By restricting ourselves to the $t\bar{t}$ final state without the decays also allows us to describe analytically the initial transverse beam polarization, since the kinematics continue to stay accessible. Thus, we have a consistent framework of inclusion of initial beams TP effects and general spin basis of final state top quark pairs. Nevertheless, the objective of our work is to establish that both CP-violating and CP-conserving BSM physics can be probed to linear order in an effective manner with one beam being polarized and advantages of a final-state spin analysis in a general basis.

We have employed in our study an effective field theory approach to look for physics beyond the standard model in a model independent manner. This is done by introducing higher dimensional operators consistent with the symmetries of SM namely gauge invariance and Lorentz invariance [28]. Since these operators are of higher dimensions, by dimensional analysis their coefficients have inverse powers of mass. The relevant higher dimensional operators, in the context of top pair production are listed in Ref. [29], and references therein. In this work we have considered the nonstandard interactions due to scalar and tensor type operators which cannot be probed at linear order unless TP is available [30], along with the vector and axial-vector type operators. These V, A operators can be probed at linear order with both unpolarized as well as longitudinal and transversely polarized beams. In this case both the beams have to have TP, to see their effect, unlike the scalar and tensor type operators whose effect can be seen with one or both the beams having TP. The question we pose is, how can the

effect of these operators be tested and how do they behave in different spin bases. The fact that spin reconstruction of the top quarks is now feasible, and has been tested in hadron colliders [31–33] motivates this work to look for these NP effects in different bases in the case of TP. Since we are looking for NP effects in the production mechanism, it is worth mentioning that, it has been recently pointed out that top polarization can be measured reliably from decay charged-lepton angular distributions without errors arising from the tbW couplings [34].

The scheme of this paper is as follows: In Sec. II we present our formalism. In Sec. III we recall for completeness the general spin basis. In Sec. IV we present the distribution in case of transverse polarization in terms of helicity amplitudes along with the transformation matrix that takes the helicity amplitudes from the helicity basis to any generic spin basis. In Sec. V we present the contribution of the BSM physics to the differential distribution and discuss the C, P, and T properties of the distribution. In Sec. VI we carry out a numerical analysis with some realistic choices of BSM parameters and demonstrate the advantages of the general basis, in case of both TP and LP. We also obtain 90% confidence level (C.L.) limits on the couplings provided no signal is observed for realistic beam polarization and integrated luminosity at the ILC. In Sec. VII we present a discussion and our conclusions.

II. TOP-QUARK SPIN CORRELATION AT LINEAR COLLIDERS

At future $e^+ e^-$ linear colliders, the spin of the top quark can be studied efficiently. This is due to the parity-violating interactions in top-quark production, which makes the produced tops naturally polarized. Furthermore, the polarization of the initial beams also helps in controlling the top polarization.

In $e^+ e^-$ collisions, $t\bar{t}$ is produced as follows:

$$e^+ e^- \rightarrow V^* \rightarrow t\bar{t}, \qquad V = \gamma, Z. \qquad (1)$$

The aim of this work is to look for the new physics effect in the top production and its effect in the spin correlation.

A. Four-Fermi operators

The fact that SM describes physics well up to the electroweak symmetry breaking scale, it can be viewed as a low energy theory, with the heavy fields being integrated out. Considering that new physics appears at a mass scale Λ, the Lagrangian can be written as an expansion in $1/\Lambda$:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{\text{SM}} + \frac{1}{\Lambda^2} \sum_i (\alpha_i \mathcal{O}_i + \text{H.c.}), \qquad (2)$$

where α's are the coefficients which parametrize nonstandard interactions (see Ref. [30], and references therein).

The operators generated at the tree level, which will directly contribute to the production process are as follows:

$$\mathcal{O}_{lq}^{(1)} = \frac{1}{2}(\bar{l}\gamma_\mu l)(\bar{q}\gamma^\mu q), \quad \mathcal{O}_{lq}^{(3)} = \frac{1}{2}(\bar{l}\gamma_\mu \tau^I l)(\bar{q}\gamma^\mu \tau^I q),$$

$$\mathcal{O}_{eu} = \frac{1}{2}(\bar{e}\gamma_\mu e)(\bar{u}\gamma^\mu u), \quad \mathcal{O}_{lu} = (\bar{l}u)(\bar{u}l),$$

$$\mathcal{O}_{qe} = (\bar{q}e)(\bar{e}q), \quad \mathcal{O}_{lq} = (\bar{l}e)\epsilon(\bar{q}u), \tag{3}$$

$$\mathcal{O}_{lq'} = (\bar{l}u)\epsilon(\bar{q}e),$$

where l, q denote, respectively, the left-handed electroweak $SU(2)$ lepton and quark doublets, and e and u denote $SU(2)$ singlet charged-lepton and up-quark right-handed fields. τ^I ($I = 1, 2, 3$) are the Pauli matrices, ϵ is the 2×2 antisymmetric matrix, $\epsilon_{12} = -\epsilon_{21} = 1$, and generation indices are suppressed.

The four-Fermi operators listed above, containing the information about physics beyond SM, after Fierz transformation take the form

$$\mathcal{L}^{4F} = \sum_{i,j=L,R}\left[S_{ij}(\bar{e}P_i e)(\bar{t}P_j t) + V'_{ij}(\bar{e}\gamma_\mu P_i e)(\bar{t}\gamma^\mu P_j t) \right.$$
$$\left. + T_{ij}\left(\bar{e}\frac{\sigma_{\mu\nu}}{\sqrt{2}}P_i e\right)\left(\bar{t}\frac{\sigma^{\mu\nu}}{\sqrt{2}}P_j t\right)\right], \tag{4}$$

with the coefficients satisfying the following constraints:

$$S \equiv S_{RR} = S^*_{LL}, \quad S_{LR} = S_{RL} = 0,$$
$$T \equiv T_{RR} = T^*_{LL}, \quad T_{LR} = T_{RL} = 0, \tag{5}$$
$$V' \equiv V'_{ij} = V'^*_{ij}.$$

In Eq. (4), $P_{L,R}$ are, respectively, the left- and right-chirality projection matrices and as shown in Ref. [29]; the S_{ij}, V'_{ij} and T_{ij} can be expressed in terms of the $\alpha's$ from the four-Fermi part of the Lagrangian Eq. (4). It can be checked that the Lagrangian is invariant under CP and T if S and T are real. In other words, nonzero values of Im S and/or Im T would imply CP violation. The V', A' terms are invariant under CP and T.

We have carried our analysis in terms of the helicity amplitudes given in the Appendix, which are same as Ref. [29], with the normalization factor taken care of.

III. SPIN BASES

The study of the polarized top quark requires a definite spin basis. It has been shown by Parke and Shadmi [21], that the degrees of spin correlations of top quarks in SM are quite distinct in different bases. The description of a generic spin basis by Parke and Shadmi [21] is with top spin states defined in the top-quark rest frame, with the spin-quantization direction $\hat{s}_t$, making an angle ξ^{PS} (we use the label PS to distinguish it from the angle we choose below) with the $\bar{t}$ measured in the direction of the incoming electron. The same definition holds for $\bar{t}$ spin states, with $\hat{s}_{\bar{t}}$, making an angle ξ^{PS} with the t momentum in the direction of the incoming positron in $\bar{t}$ rest frame. In our work ξ is the angle measured, from the direction opposite to the outgoing $\bar{t}$ in the direction of incoming electron, in t rest frame. A similar definition holds for the $\bar{t}$ rest frame, with ξ measured from the direction opposite to the outgoing t in the direction of incoming positron. For a polarized state $t_U\bar{t}_U$ refers to a top with spin along $\hat{s}_t$ and a top antiquark with spin along $\hat{s}_{\bar{t}}$. Analogous definitions hold for the polarized states $t_D\bar{t}_D$, $t_U\bar{t}_D$, and $t_D\bar{t}_U$.

With the above convention for ξ, three different bases are constructed:

(1) Helicity basis:

$$\cos\xi = 1, \tag{6}$$

with the top quark spin defined along its direction of motion. Moreover, in this basis, the polarized state $t_U\bar{t}_D$, will be the same as $t_R\bar{t}_L$ The partial analysis in this basis with both beams polarized and one of the top spins summed over was performed by us recently [20]. Here we are generalizing that study.

(2) Beamline basis:

$$\cos\xi = \frac{\cos\theta + \beta}{1 + \beta\cos\theta}, \tag{7}$$

where β is the top quark speed with $\beta = \sqrt{1 - 4m_t^2/s}$, and θ is the top scattering angle with respect to the electron beam. In this basis the top-quark spin axis is the electron direction in the top rest frame and the top antiquark spin axis is the positron direction in the antitop rest frame. It may be checked that in the ultra relativistic limit ($\beta = 1$), this basis reduces to the helicity basis.

(3) Off-diagonal basis: For this basis it can be shown in the SM, the production cross sections of the like spin states $t_U\bar{t}_U$ and $t_D\bar{t}_D$ vanish for the left- and right-handed electron beam for particular values of ξ_L and ξ_R, given by

$$\cos\xi_L = \frac{(\beta(V_{LL} - V_{LR}) + \cos\theta(V_{LL} + V_{LR}))}{\sqrt{(\beta(V_{LL} - V_{LR}) + \cos\theta(V_{LL} + V_{LR}))^2 + \frac{4m_t^2}{s}\sin^2\theta(V_{LL} + V_{LR})^2}}, \tag{8}$$

$$\cos\xi_R = \frac{(\beta(V_{RR} - V_{RL}) + \cos\theta(V_{RL} + V_{RR}))}{\sqrt{(\beta(V_{RR} - V_{RL}) + \cos\theta(V_{RL} + V_{RR}))^2 + \frac{4m_t^2}{s}\sin^2\theta(V_{RL} + V_{RR})^2}}, \tag{9}$$

where

$$V_{LL} = V_{LL}^\gamma + V_{LL}^Z, \qquad V_{LR} = V_{LR}^\gamma + V_{LR}^Z,$$
$$V_{RL} = V_{RL}^\gamma + V_{RL}^Z, \qquad V_{RR} = V_{RR}^\gamma + V_{RR}^Z, \tag{10}$$

and

$$V_{LL}^{\mathcal{V}} = (c_V^{e\mathcal{V}} + c_A^{e\mathcal{V}})(c_V^{t\mathcal{V}} + c_A^{t\mathcal{V}})/(s - M_{\mathcal{V}}^2),$$
$$V_{LR}^{\mathcal{V}} = (c_V^{e\mathcal{V}} + c_A^{e\mathcal{V}})(c_V^{t\mathcal{V}} - c_A^{t\mathcal{V}})/(s - M_{\mathcal{V}}^2),$$
$$V_{RL}^{\mathcal{V}} = (c_V^{e\mathcal{V}} - c_A^{e\mathcal{V}})(c_V^{t\mathcal{V}} + c_A^{t\mathcal{V}})/(s - M_{\mathcal{V}}^2), \tag{11}$$
$$V_{RR}^{\mathcal{V}} = (c_V^{e\mathcal{V}} - c_A^{e\mathcal{V}})(c_V^{t\mathcal{V}} - c_A^{t\mathcal{V}})/(s - M_{\mathcal{V}}^2),$$

with $\mathcal{V} = \gamma, Z$ and

$$c_V^{eZ} = (-1/2 + 2s_w^2)\Big/\Big(2\sqrt{1 - s_w^2}\sqrt{s_w^2}\Big),$$
$$c_A^{eZ} = -1\Big/\Big(4\sqrt{1 - s_w^2}\sqrt{s_w^2}\Big),$$
$$c_V^{tZ} = (1/2 - 4s_w^2/3)\Big/\Big(2\sqrt{1 - s_w^2}\sqrt{s_w^2}\Big), \tag{12}$$
$$c_A^{tZ} = 1\Big/\Big(4\sqrt{1 - s_w^2}\sqrt{s_w^2}\Big),$$

where $s_w^2 = \sin^2\theta_W = 0.231$, the Weinberg angle and $m_t = 172$ GeV, $m_Z = 90.1$ GeV is the mass of the top and Z boson, respectively.

There are two off-diagonal bases for the pair production considered here, one for $e_L^- e_R^+$ and the other for $e_R^- e_L^+$ scattering. For the $t\bar{t}$ production the two bases are almost coincident since the ratio V_{LL}/V_{LR} in Eq. (8) is approximately equal in sign and magnitude to the ratio V_{RR}/V_{RL} in Eq. (9). Therefore, for the rest of our numerical analyses we have used $\cos\xi_L$, henceforth written as $\cos\xi$ as the ODB. It is worth mentioning that ODB approaches the helicity basis for $\beta \to 1$. We would further like to mention that in our work we will call the ODB as the standard model off-diagonal basis (SMOD). Before embarking on to

BSM physics, we need to consider whether or not in the SM in the presence of TP this basis defined by Eqs. (8) and (9) continues to have the desirable property of $t_U\bar{t}_U$ and $t_D\bar{t}_D$ vanishing or not. An inspection of the defining condition given in Eq. (11) of PS and the distribution in the presence of TP shows that the property of SMOD where the final state with like spin configuration vanishes holds. Thus, in the presence of TP, the choice for $\cos\xi$ given by Eqs. (8) and (9) continues to be SMOD. Thus, any contribution to the cross section of $t_U\bar{t}_U$ and $t_D\bar{t}_D$ with and without TP is a signal of BSM physics.

Since for $\beta \to 1$, the BLB and SMOD approach the HB our definition of the convention for ξ gives a self-consistent set of bases.

IV. TRANSVERSE POLARIZATION AND GENERAL SPIN BASIS

The different spin bases described in the previous section have been investigated in case of SM [21]. It is found for the off-diagonal basis in polarized e^+e^- colliders not only do the like spin configurations vanish, but one spin configuration dominates the total cross section. The behavior of the bases for SM in the case of TP has not been addressed before. There has been a study to explore which spin basis is more suitable for studying new physics effects in top quark production [24,25], in the presence of LP. Here we consider the new physics effects described in Sec. II A, containing scalar and tensor type interactions along with nonstandard vector and axial-vector type of interactions. In earlier works [20,30], it was shown that scalar and tensor type operators cannot be probed at linear order unless TP is available. The cross section in the presence of TP, with new physics effects, is calculated from the helicity amplitudes from the expression [35], where contributions proportional to $|T_{LLIJ}|^2$, $|T_{RRIJ}|^2$, and $T_{RRIJ}^* T_{LLIJ}$ are discarded:

$$\frac{d\sigma(e^+e^- \to t_I\bar{t}_J)}{d\cos\theta d\phi} = |T_{RLIJ}|^2 + |T_{LRIJ}|^2 - \frac{1}{2}P_{e^-}^T P_{e^+}^T \mathrm{Re}\, e^{-2i\phi} T_{RLIJ}^* T_{LRIJ} + \frac{1}{2}P_{e^-}^T \mathrm{Re}\, e^{-i\phi}(T_{RLIJ}^* T_{LLIJ} + T_{RRIJ}^* T_{LRIJ})$$
$$- \frac{1}{2}P_{e^+}^T \mathrm{Re}\, e^{-i\phi}(T_{RLIJ}^* T_{RRIJ} + T_{LLIJ}^* T_{LRIJ}). \tag{13}$$

In the above equation, $P_{e^-}^T$ and $P_{e^+}^T$ are the degree of TP of the electron and positron, respectively. The direction of the electron polarization is fixed along the positive x axis, with the azimuthal angle of the polarization vector to be zero in the c.m. frame. Moreover, we have also considered the polarization vectors of the electron and the positron to be in the opposite direction.

The helicity amplitudes used for our analysis are defined in the Appendix, with the same order. In the expressions above, IJ denotes the different final-state spin combinations of UD, DU, UU, and DD. The above expression is in

the HB; therefore, in order to study the TP effect in other bases a rotation is performed on the spin of the top and antitop as described in the next subsection.

A. The transformation matrix

The amplitudes in a generic spin basis may be obtained from the amplitudes in the helicity basis by means of a transformation corresponding to a rotation of the t and $\bar{t}$ spin bases. We are giving here the expressions for a more general case, by considering different rotation angles ξ_t and $\xi_{\bar{t}}$ in the spin space of the t and $\bar{t}$. Thus, we can write

$$\begin{pmatrix} T'_{LRUU} \\ T'_{LRUD} \\ T'_{LRDU} \\ T'_{LRDD} \\ T'_{RLUU} \\ T'_{RLUD} \\ T'_{RLDU} \\ T'_{RLDD} \end{pmatrix} = R'(\xi_t, \xi_{\bar{t}}) \begin{pmatrix} T_{LRUU} \\ T_{LRUD} \\ T_{LRDU} \\ T_{LRDD} \\ T_{RLUU} \\ T_{RLUD} \\ T_{RLDU} \\ T_{RLDD} \end{pmatrix}, \qquad (14)$$

where the left-hand side corresponds to the helicity conserving amplitudes in a generic spin basis, and $LR \rightarrow LL$, $RL \rightarrow RR$ for the helicity violating amplitudes. The amplitudes T on the right-hand side are in the helicity basis, and $R'(\xi_t, \xi_{\bar{t}})$ is the transformation matrix corresponding to the parameters ξ_t and $\xi_{\bar{t}}$. $R'(\xi_t, \xi_{\bar{t}})$ operates on a column vector spanned by U and D in the helicity basis and is related to the 4×4 matrix $M(\xi_t, \xi_{\bar{t}})$ by

$$R'(\xi_t, \xi_{\bar{t}}) = \begin{pmatrix} M(\xi_t, \xi_{\bar{t}}) & 0 \\ 0 & M(\xi_t, \xi_{\bar{t}}) \end{pmatrix}, \qquad (15)$$

where $M(\xi_t, \xi_{\bar{t}})$ is the direct product of two rotation matrices. $R_t(\xi_t)$ and $R_{\bar{t}}(\xi_{\bar{t}})$ parametrize the effect of rotation in the spin space of t and $\bar{t}$, respectively. In a schematic notation, we have

$$R_t(\xi_t) = \begin{pmatrix} \cos\frac{\xi_t}{2} & -\sin\frac{\xi_t}{2} \\ \sin\frac{\xi_t}{2} & \cos\frac{\xi_t}{2} \end{pmatrix}, \qquad (16)$$

and analogously

$$R_{\bar{t}}(\xi_{\bar{t}}) = \begin{pmatrix} \cos\frac{\xi_{\bar{t}}}{2} & -\sin\frac{\xi_{\bar{t}}}{2} \\ \sin\frac{\xi_{\bar{t}}}{2} & \cos\frac{\xi_{\bar{t}}}{2} \end{pmatrix}. \qquad (17)$$

Thus, we have $M \equiv R_t(\xi_t) \otimes R_{\bar{t}}(\xi_{\bar{t}})$ given by

$$M(\xi_t, \xi_{\bar{t}}) = \begin{pmatrix} \cos\frac{\xi_t}{2}\cos\frac{\xi_{\bar{t}}}{2} & -\cos\frac{\xi_t}{2}\sin\frac{\xi_{\bar{t}}}{2} & -\sin\frac{\xi_t}{2}\cos\frac{\xi_{\bar{t}}}{2} & \sin\frac{\xi_t}{2}\sin\frac{\xi_{\bar{t}}}{2} \\ \cos\frac{\xi_t}{2}\sin\frac{\xi_{\bar{t}}}{2} & \cos\frac{\xi_t}{2}\cos\frac{\xi_{\bar{t}}}{2} & -\sin\frac{\xi_t}{2}\sin\frac{\xi_{\bar{t}}}{2} & -\sin\frac{\xi_t}{2}\cos\frac{\xi_{\bar{t}}}{2} \\ \sin\frac{\xi_t}{2}\cos\frac{\xi_{\bar{t}}}{2} & -\sin\frac{\xi_t}{2}\sin\frac{\xi_{\bar{t}}}{2} & \cos\frac{\xi_t}{2}\cos\frac{\xi_{\bar{t}}}{2} & -\cos\frac{\xi_t}{2}\sin\frac{\xi_{\bar{t}}}{2} \\ \sin\frac{\xi_t}{2}\sin\frac{\xi_{\bar{t}}}{2} & \sin\frac{\xi_t}{2}\cos\frac{\xi_{\bar{t}}}{2} & \cos\frac{\xi_t}{2}\sin\frac{\xi_{\bar{t}}}{2} & \cos\frac{\xi_t}{2}\cos\frac{\xi_{\bar{t}}}{2} \end{pmatrix}. \qquad (18)$$

However in the present work we take $\xi_t = \xi_{\bar{t}} = \xi$. Note, however, that we have the possibility of an even further generalization when $\xi_t \neq \xi_{\bar{t}}$, which is not studied here. The matrix defined in Eq. (18) has the property that when one goes to the generalized spin basis, e.g., the $(LRDU)$ amplitude in that basis gets an admixture from the $(LRUU)$, $(LRUD)$, and $(LRDD)$ of the helicity basis. A similar argument applies for the other helicity amplitudes in the generic spin basis. After taking $\cos\xi = 1$ for the helicity basis, the IJ indices in the original and rotated bases are (trivially) the same. The matrix reduces to identity matrix for $\cos\xi = 1$. The value of $\cos\xi$ for the different bases are defined in Eqs. (6)–(9). In case of SMOD with the choice of angle $\cos\xi_L$ the helicity amplitudes T'_{LRUU} and T'_{LRDD} vanish; whereas for the choice $\cos\xi_R$, the helicity amplitudes T'_{RLUU} and T'_{RLDD} vanish. Our choice of the angle ξ described in Sec. III thus makes transparent why the HB and the ODB are so-called as exemplified in the general derivation. We note that upon ϕ integration of Eq. (13) we obtain the results of PS [21] except of a factor of 2. Our result is larger by a factor of 2 and when we sum over all the helicities of the final state our result agrees with the SM prediction [30].

V. DISTRIBUTIONS AND THEIR PROPERTIES

We present here the distribution in the presence of TP, for different new physics of the type S, T, V, and A. The degree of polarization expected in the ILC is about 80% for the electron and 60% for the positrons [2]. In the distributions given below, $e^+e^- \rightarrow t_U \bar{t}_U$ or $t_D \bar{t}_D$ is defined as UU/DD and $e^+e^- \rightarrow t_U \bar{t}_D$ or $t_D \bar{t}_U$ is defined as UD/DU, and

$$A_L = V_{LL} + V_{LR}, \qquad A_R = V_{RL} + V_{RR}, \qquad B_L = V_{LL} - V_{LR}, \qquad B_R = V_{RL} - V_{RR},$$
$$B = \frac{\alpha}{2}\sqrt{\frac{3\beta}{s}}, \qquad A = \sqrt{\frac{3\beta}{64\pi^2 s}}. \qquad (19)$$

The differential cross section for the process $e^- e^+ \rightarrow t\bar{t}$, in presence of the new physics S and T in generic spin bases for different spin configurations is obtained by

$$\frac{d\sigma_{UU/DD}}{d\cos\theta d\phi} = \frac{d\sigma_{SM}}{d\cos\theta d\phi} + \frac{1}{8}ABs(\sin\phi\{\sqrt{s}\sin\xi[(A_L - A_R)\cos\theta + \beta(B_L + B_R)] + 2m_t(A_R - A_L)\cos\xi\sin\theta\}\{(P_{e^-}^T - P_{e^+}^T)$$

$$\times [\sqrt{s}\cos\xi(2\mathrm{Im}T\cos\theta - \beta\mathrm{Im}S) + 4m_t\mathrm{Im}T\sin\xi\sin\theta] \mp \sqrt{s}(P_{e^-}^T + P_{e^+}^T)(\mathrm{Im}S - 2\beta\mathrm{Im}T\cos\theta)\}$$

$$+ \cos\phi\{\sqrt{s}\sin\xi[(A_L + A_R)\cos\theta + \beta(B_L - B_R)] - 2m_t(A_L + A_R)\cos\xi\sin\theta\}\{(P_{e^-}^T + P_{e^+}^T)$$

$$\times [\sqrt{s}\cos\xi(2\mathrm{Re}T\cos\theta - \beta\mathrm{Re}S) + 4m_t\mathrm{Re}T\sin\xi\sin\theta] \pm \sqrt{s}(P_{e^+}^T - P_{e^-}^T)(\mathrm{Re}S - 2\beta\mathrm{Re}T\cos\theta)\}$$

$$+ 2(P_{e^-}^T + P_{e^+}^T)\{\sqrt{s}\sin\xi(A_L\cos\theta + \beta B_L) - 2A_L m_t\cos\xi\sin\theta\}\{\sqrt{s}[\cos\xi\cos\phi(\beta\mathrm{Re}S - 2\mathrm{Re}T\cos\theta)$$

$$\pm \sin\phi(\mathrm{Im}S - 2\beta\mathrm{Im}T\cos\theta)] - 4m_t\mathrm{Re}T\sin\theta\sin\xi\cos\phi\}), \tag{20}$$

where

$$\frac{d\sigma_{SM}}{d\cos\theta d\phi} = \frac{1}{16}B^2 s\left(s\sin^2\xi[(A_L^2 + A_R^2)\cos^2\theta + 2\beta\cos\theta(A_L B_L - A_R B_R) + \beta^2(B_L^2 + B_R^2)]\right.$$

$$- 4m_t\cos\xi\sin\theta(\sin\xi\sqrt{s}[(A_L^2 + A_R^2)\cos\theta + \beta(A_L B_L - A_R B_R)] - m_t\cos\xi\sin\theta(A_L^2 + A_R^2))$$

$$+ 2sP_{e^-}^T P_{e^+}^T\cos2\phi\left[2A_L\frac{m_t}{\sqrt{s}}\cos\xi\sin\theta - \sin\xi(\beta B_L + A_L\cos\theta)\right]$$

$$\left.\times\left[2A_R\frac{m_t}{\sqrt{s}}\cos\xi\sin\theta + \sin\xi(\beta B_R - A_R\cos\theta)\right]\right), \tag{21}$$

and

$$\frac{d\sigma_{UD/DU}}{d\cos\theta d\phi} = \frac{d\sigma_{SM}'}{d\cos\theta d\phi} + \frac{1}{8}ABs((P_{e^-}^T - P_{e^+}^T)\sin\phi\{4m_t\mathrm{Im}T\cos\xi\sin\theta + \sqrt{s}\sin\xi(\beta\mathrm{Im}S - 2\mathrm{Im}T\cos\theta)\}\{\sqrt{s}\cos\xi[(A_L - A_R)$$

$$\times\cos\theta + \beta(B_L + B_R)] \pm [A_L + A_R + \beta(B_L - B_R)\cos\theta] + 2m_t(A_L - A_R)\sin\xi\sin\theta\} + (P_{e^-}^T + P_{e^+}^T)$$

$$\times\cos\phi\{4m_t\mathrm{Re}T\cos\xi\sin\theta + \sqrt{s}\sin\xi(\beta\mathrm{Re}S - 2\mathrm{Re}T\cos\theta)\}\{\sqrt{s}\cos\xi[(A_L + A_R)\cos\theta + \beta(B_L - B_R)]$$

$$\pm [A_L - A_R + \beta(B_L + B_R)\cos\theta] + 2m_t(A_L + A_R)\sin\xi\sin\theta\} - 2(P_{e^-}^T + P_{e^+}^T)\cos\phi\{\sqrt{s}[\cos\xi(A_L\cos\theta + \beta B_L)$$

$$\pm A_L \pm \beta B_L\cos\theta] + 2A_L m_t\sin\theta\sin\xi\}\{\sqrt{s}\sin\xi(\beta\mathrm{Re}S - 2\mathrm{Re}T\cos\theta) + 4m_t\mathrm{Re}T\cos\xi\sin\theta\}), \tag{22}$$

where

$$\frac{d\sigma_{SM}'}{d\cos\theta d\phi} = \frac{1}{32}B^2 s\left(s\cos\xi\{2\cos\xi[(A_L^2 + A_R^2)\cos^2\theta + 2\beta\cos\theta(A_L B_L - A_R B_R) + \beta^2(B_L^2 + B_R^2)] + 4\cos\theta[\pm(A_L^2 - A_R^2)\right.$$

$$\pm \beta^2(B_L^2 - B_R^2)] \pm 4\beta(\cos^2\theta + 1)(A_L B_L + A_R B_R)\} + 2s[(A_L^2 + A_R^2) + 2\beta\cos\theta(A_L B_L - A_R B_R)$$

$$+ \beta^2\cos^2\theta(B_L^2 + B_R^2)] + 8m_t\sqrt{s}\sin\xi\sin\theta\{\cos\xi[(A_L^2 + A_R^2)\cos\theta + \beta(A_L B_L - A_R B_R)] \pm (A_L^2 - A_R^2)$$

$$+ \beta\cos\theta(A_L B_L + A_R B_R)\} + 8m_t^2(A_L^2 + A_R^2)\sin^2\xi\sin^2\theta + 4sP_{e^-}^T P_{e^+}^T\cos2\phi\left\{\pm\cos\xi(A_L\cos\theta + \beta B_L)\right.$$

$$\left.+ A_L + \beta B_L\cos\theta \pm 2A_L\frac{m_t}{\sqrt{s}}\sin\xi\sin\theta\right\}\left\{-(A_R \pm \beta B_R\cos\xi) + \cos\theta(\pm A_R\cos\xi + \beta B_R)\right.$$

$$\left.\left.\pm 2A_R\frac{m_t}{\sqrt{s}}\sin\xi\sin\theta\right\}\right). \tag{23}$$

The distribution in presence of the new physics of the vector and axial-vector type denoted as A_L', A_R', B_L', B_R', with the same form as A_L, A_R, B_L, B_R defined in Eq. (19) is obtained by

$$\frac{d\sigma_{UU/DD}}{d\cos\theta d\phi} = \frac{d\sigma_{SM}}{d\cos\theta d\phi} + \frac{d\sigma_{NP}}{d\cos\theta d\phi} + \frac{d\sigma_{NP}^{TP}}{d\cos\theta d\phi}. \tag{24}$$

The contributions to $A_{L,R}'$ and $B_{L,R}'$ enter as vertex corrections to the $t\bar{t}\gamma$ and $t\bar{t}Z$ vertices and from some additional gauge bosons due to some higher symmetry suppressed by the new physics scale. In the equation above $\frac{d\sigma_{SM}}{d\cos\theta d\phi}$ is the SM distribution given in Eq. (21), $\frac{d\sigma_{NP}}{d\cos\theta d\phi}$ is the distribution in the absence of TP in case of new physics given by

$$\frac{d\sigma_{NP}}{d\cos\theta d\phi} = \frac{1}{8}B^2 s(-2m_t\sqrt{s}\cos\xi\sin\xi\sin\theta\{\beta(A_L B'_L + A'_L B_L - A_R B'_R - A'_R B_R) + 2\cos\theta(A_L A'_L + A_R A'_R)\}$$
$$+ s\sin^2\xi\{\beta\cos\theta(A_L B'_L + A'_L B_L - A_R B'_R - A'_R B_R) + \cos^2\theta(A_L A'_L + A_R A'_R) + \beta^2(B_L B'_L + B_R B'_R)\}$$
$$+ 4m_t^2\cos^2\xi\sin^2\theta(A_L A'_L + A_R A'_R)), \tag{25}$$

and $\frac{d\sigma_{NP}^{TP}}{d\cos\theta d\phi}$ shows the distribution due to new physics in the presence of TP:

$$\frac{d\sigma_{NP}^{TP}}{d\cos\theta d\phi} = \frac{1}{8}B^2 s P_{e^-}^T P_{e^+}^T \cos 2\phi(-2m_t\sqrt{s}\cos\xi\sin\xi\sin\theta[\beta(-A_L B'_R - A'_L B_R + A_R B'_L + A'_R B_L) + 2\cos\theta(A_L A'_R$$
$$+ A'_L A_R)] + s\sin^2\xi[\beta\cos\theta(-A_L B'_R - A'_L B_R + A_R B'_L + A'_R B_L) + \cos^2\theta(A_L A'_R + A'_L A_R)$$
$$- \beta^2(B_L B'_R + B'_L B_R)] + 4m_t^2\cos^2\xi\sin^2\theta(A_L A'_R + A'_L A_R)). \tag{26}$$

Similarly,

$$\frac{d\sigma_{UD/DU}}{d\cos\theta d\phi} = \frac{d\sigma'_{SM}}{d\cos\theta d\phi} + \frac{d\sigma'_{NP}}{d\cos\theta d\phi} + \frac{d\sigma_{NP}^{TP'}}{d\cos\theta d\phi}, \tag{27}$$

where the same definition follows as in Eq. (24) with $\frac{d\sigma'_{SM}}{d\cos\theta d\phi}$ given by Eq. (21).

$$\frac{d\sigma'_{NP}}{d\cos\theta d\phi} = \frac{1}{16}B^2 s(\cos\xi\{\pm 4m_t\sqrt{s}\sin\xi\sin\theta[\beta(A_L B'_L + A'_L B_L - A_R B'_R - A'_R B_R) + 2\cos\theta(A_L A'_L + A_R A'_R)]$$
$$\pm 2s[2\cos\theta(A_L A'_L - A_R A'_R + \beta^2(B_L B'_L - B_R B'_R)) \pm \beta(\cos^2\theta + 1)(A_L B'_L + A'_L B_L + A_R B'_R + A'_R B_R)]\}$$
$$+ 2s\cos^2\xi\{\beta\cos\theta(A_L B'_L + A'_L B_L - A_R B'_R - A'_R B_R) + \cos^2\theta(A_L A'_L + A_R A'_R) + \beta^2(B_L B'_L + B_R B'_R)\}$$
$$\pm 4m_t\sqrt{s}\sin\xi\sin\theta\{\beta\cos\theta(A_L B'_L + A'_L B_L + A_R B'_R + A'_R B_R) + 2(A_L A'_L - A_R A'_R)\}$$
$$+ 2s\{\beta\cos\theta(A_L B'_L + A'_L B_L - A_R B'_R - A'_R B_R) + (A_L A'_L + A_R A'_R) + \beta^2\cos^2\theta(B_L B'_L + B_R B'_R)\}$$
$$+ 8m_t^2\sin^2\xi\sin^2\theta(A_L A'_L + A_R A'_R)), \tag{28}$$

and

$$\frac{d\sigma_{NP}^{TP'}}{d\cos\theta d\phi} = \frac{1}{8}B^2 s P_{e^-}^T P_{e^+}^T \cos 2\phi(2m_t\sqrt{s}\cos\xi\sin\xi\sin\theta[\beta(-A_L B'_R - A'_L B_R + A_R B'_L + A'_R B_L) + 2\cos\theta(A_L A'_R + A'_L A_R)]$$
$$+ s\cos^2\xi[\beta\cos\theta(-A_L B'_R - A'_L B_R + A_R B'_L + A'_R B_L) + \cos^2\theta(A_L A'_R + A'_L A_R) - \beta^2(B_L B'_R + B'_L B_R)]$$
$$\mp \beta\sin\theta(s\cos\xi\sin\theta \mp m_t\sqrt{s}\sin\xi\cos\theta)(A_L B'_R + A'_L B_R + A_R B'_L + A'_R B_L) + 4m_t^2\sin^2\xi\sin^2\theta(A_L A'_R + A'_L A_R)$$
$$+ s[\beta\cos\theta(A_L B'_R + A'_L B_R - A_R B'_L - A'_R B_L) - (A_L A'_R + A'_L A_R) + \beta^2\cos^2\theta(B_L B'_R + B'_L B_R)]). \tag{29}$$

It is interesting to examine the above distributions from the point of view of the C, P, and T properties of the interactions. As noted earlier, the only couplings which can lead to CP violation are ImS and ImT. If, as noted in Ref. [30], the t and $\bar{t}$ spins are not observed, the only CP-odd observable possible is $(\vec{p}_{e^-} - \vec{p}_{e^+}) \times (\vec{s}_{e^-} - \vec{s}_{e^+}) \cdot (\vec{p}_t - \vec{p}_{\bar{t}})$, which would get an expectation value from the terms in the distribution proportional to $(P_{e^-}^T + P_{e^+}^T)\sin\theta\cos\phi$. These terms can indeed be seen to be proportional to ImS or ImT in the sum of the distributions for the various t and $\bar{t}$ up and down spin projections. When the t and $\bar{t}$ spins are observed, there are more observables possible which are CP odd, which depend on these spins. These are $(\vec{p}_{e^-} - \vec{p}_{e^+}) \times (\vec{s}_{e^-} + \vec{s}_{e^+}) \cdot (\vec{s}_t - \vec{s}_{\bar{t}})$ and $(\vec{p}_{e^-} - \vec{p}_{e^+}) \times (\vec{s}_{e^-} - \vec{s}_{e^+}) \cdot (\vec{s}_t + \vec{s}_{\bar{t}})$.

We first take the simplest case of helicity basis distributions. Because of the dependence on the difference of t and $\bar{t}$ spins in the first of these observables, it would occur in the difference between the distributions for the UD and DU spin projections. It can be seen to be proportional to $P_{e^-}^T - P_{e^+}^T$. The second observable would occur in the sum of the distributions for the spin projections UU and DD, and would be proportional to $P_{e^-}^T + P_{e^+}^T$. In either case, the angular dependence of the CP-violating terms will be proportional to $\sin\phi$, occurring with the couplings ImS and ImT.

Let us now consider other spin bases. The amplitudes corresponding to each of these is obtained by transformation of the spin amplitudes by the matrix given of Eq. (18), with an appropriate value of ξ. Such a transformation, however, does not change the fact that the distributions, to linear order in the new physics couplings, contain the CP-violating couplings ImS and ImT with the same azimuthal dependence, viz., $\sin\phi$. Thus, in any spin basis, an

asymmetry (which we discuss below) that can isolate the $\sin\phi$ terms, will be a measure of CP violation. Also, the transformation to a different spin basis does not change the dependence on e^+ and e^- polarizations.

VI. NUMERICAL ANALYSIS

We consider the azimuthal distribution of the final state, in the presence of different types of nonstandard couplings. In case of the S and T type interactions apart from the azimuthal distributions different asymmetries are constructed to isolate their contributions. The results are presented for the three different bases considered here. The effect of LP is also considered in the presence of the V and A type interactions, on the total production cross section along with the fraction of the top quarks produced. An asymmetry is also considered to test these interactions for different cases of initial LP, to measure the angular correlation of $t\bar{t}$. We have also done an analysis to put bound on the various anomalous couplings considered here.

A. New physics due to S and T interactions

First, the scalar and the tensorial type of couplings are considered. We are considering the case with only one of the initial beams being transversely polarized. For our analysis, we have taken $P_{e^-}^T = 0.8$ and $P_{e^+}^T = 0$. This choice of beam polarization has the advantage of eliminating the ϕ contribution from SM, along with the contribution if any from the new physics due to V and A type of interactions. The ϕ distribution in the HB, BLB, and SMOD is shown in Fig. 1. For the purpose of illustration we have taken magnitude of the NP couplings to be of the order 10^{-6} GeV^{-2}, inspired by the sensitivity that was expected at ILC with realistic polarization and integrated luminosity [20]. In this work, the values of the couplings are chosen to be 0.5×10^{-6} GeV^{-2}, a choice for which we have checked that the linear approximation holds good.

We note that the figures given in this subsection are plotted for two cases, (a) Re$S = 0.5 \times 10^{-6}$ GeV^{-2}, keeping the values of other couplings to be zero, and
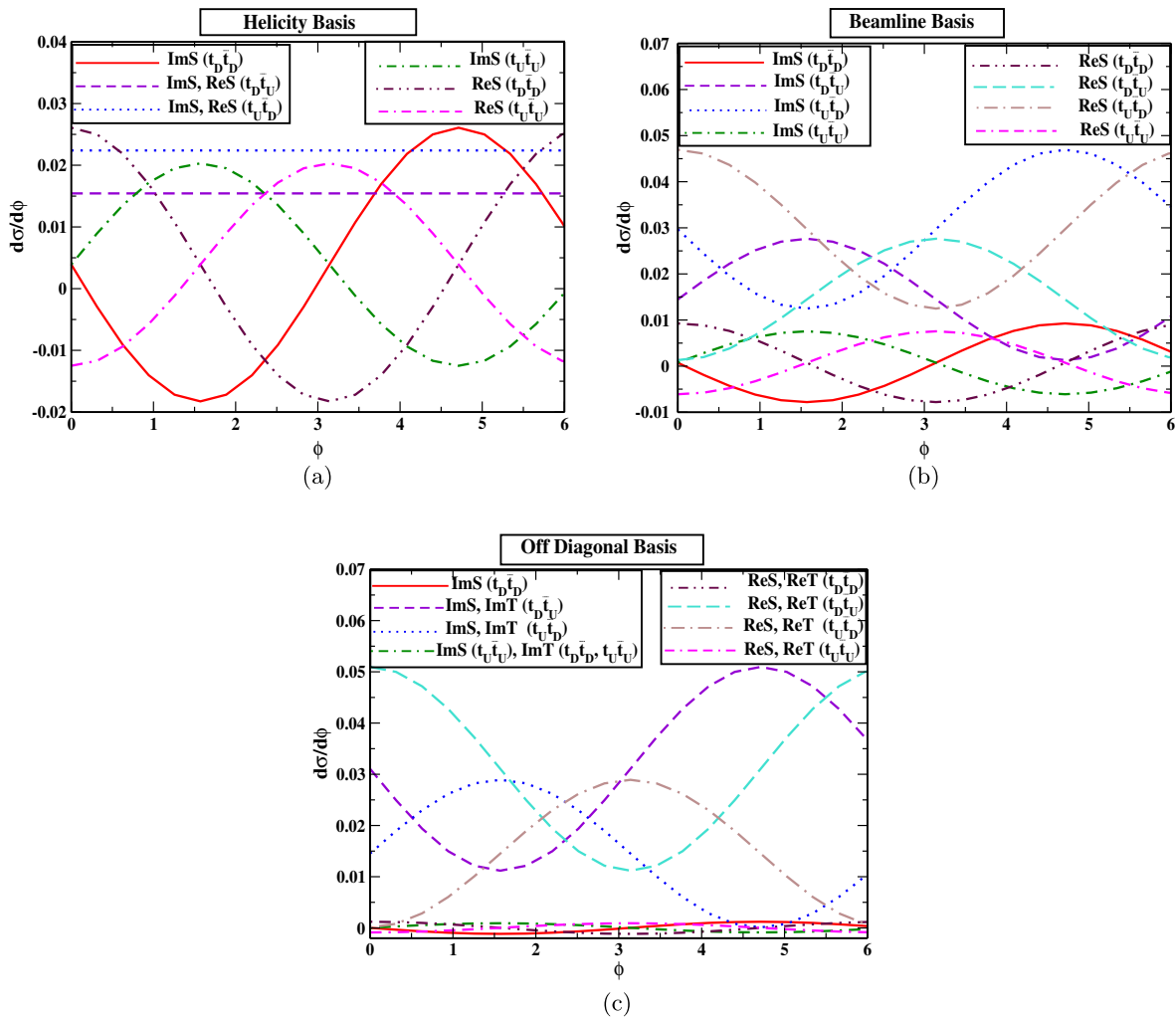

FIG. 1 (color online).   The azimuthal distribution of the top quark pairs at a 500 GeV linear collider in case of different bases considered here, with $P_{e^-}^T = 0.8$ and $P_{e^+}^T = 0$. The distribution in the presence of different anomalous couplings are considered, with the value of the respective coupling to be 0.5×10^{-6} GeV^{-2}, keeping the value of others zero.

TABLE I. The ϕ independent contribution coming from SM, in different bases with only one of the initial beams being transversely polarized. This constant term is present in all the distribution due to S and T interactions.

Spin configurations	Helicity basis	Beamline basis	SM off-diagonal basis
$t_D \bar{t}_D$	0.0078	0.0015	0.0000
$t_D \bar{t}_U$	0.0309	0.0289	0.0621
$t_U \bar{t}_D$	0.0448	0.0594	0.0290
$t_U \bar{t}_U$	0.0078	0.0015	0.0000

(b) Im$S = 0.5 \times 10^{-6}$ GeV^{-2}, with the other couplings equal to zero. The Fig. 1(a), shows that in the HB for the final-state spin configuration of the form $t_D\bar{t}_U$ and $t_U\bar{t}_D$, ImS and ReS has no ϕ dependence. There is no contribution to the ϕ dependence in this basis as S does not contribute to the helicity amplitudes in the $t_D\bar{t}_U$ and $t_U\bar{t}_D$ sector. However, in the other bases Figs. 1(b) and 1(c) due to the action of the transformation matrix Eq. (15), there

are nontrivial contributions. The azimuthal distributions due to the presence of T is almost the same as the distribution from S, so we do not show their distribution in the HB and BLB. The contribution due to T is shown for the off-diagonal basis, Fig. 1(c), with the contribution from either Re/ImT to be 0.5×10^{-6} GeV^{-2}, keeping the other couplings to be zero. In case of SMOD, the distribution from the same final-state spin configuration $t_U\bar{t}_U$ and $t_D\bar{t}_D$ is almost equal to zero, in the presence of S and T. This behavior is similar to the distribution in the presence of SM only. But the other final-state spin configurations show a measurable ϕ distribution contrary to SM behavior which is ϕ independent. For completion, we show in Table I, the contribution from SM in different bases, with different final-state spin configuration. We would like to point out that, provided in ILC only one of the beams is transversely polarized observation of modulation in any of the final-state spin configuration will be a signature of S and T interactions. This behavior holds for all the three bases discussed here. The azimuthal distribution considered
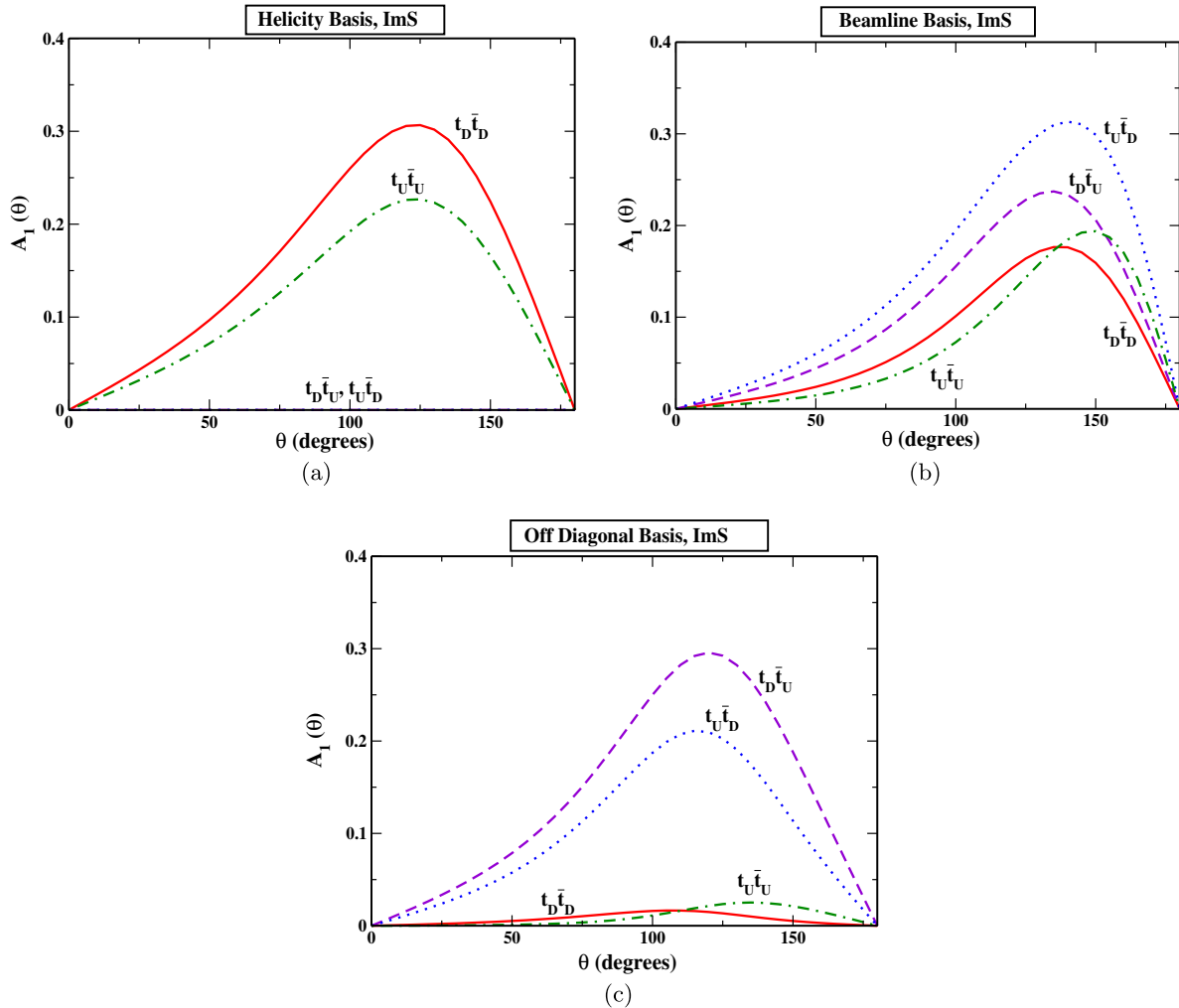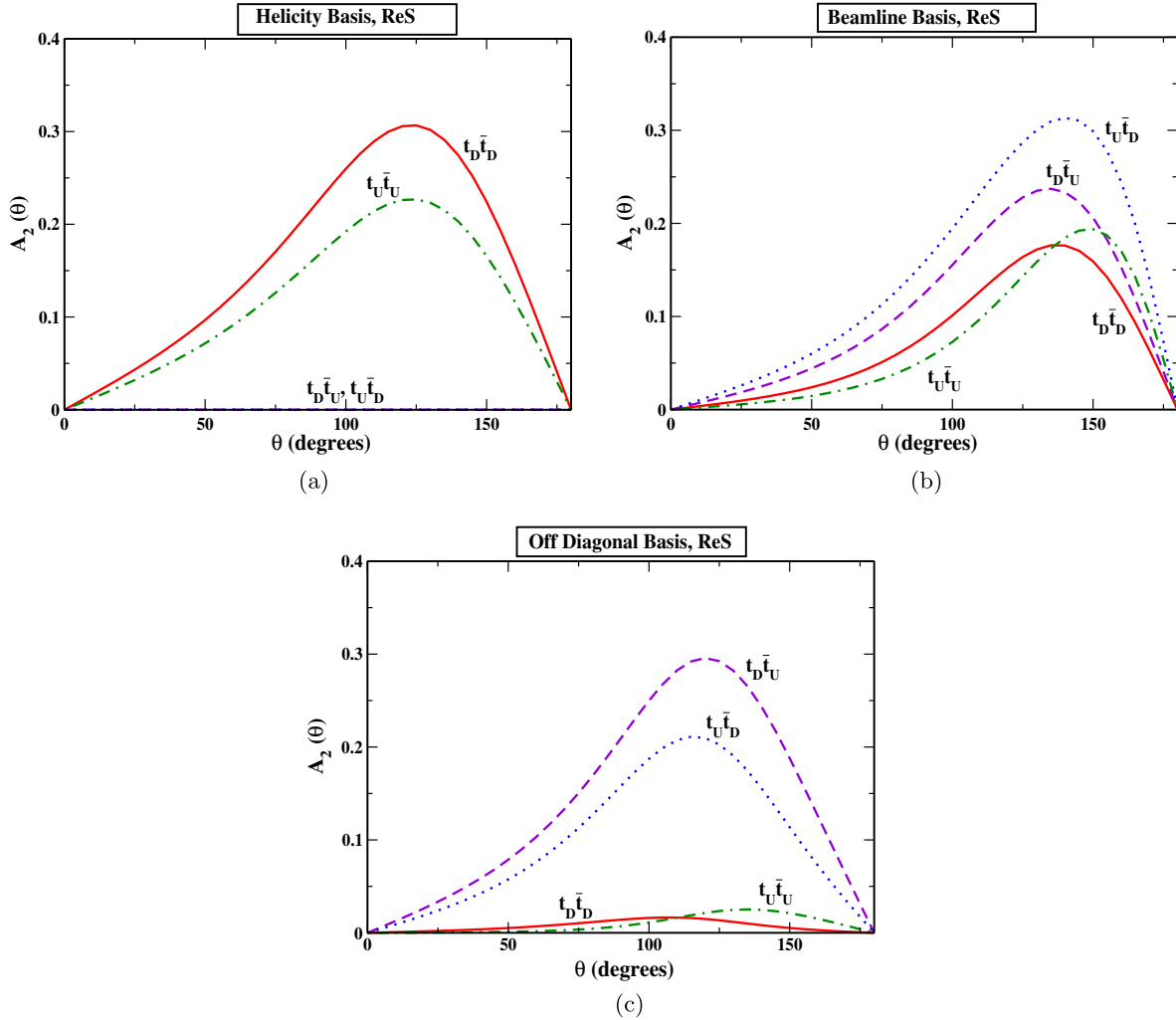

FIG. 2 (color online). $A_1(\theta)$ as a function of θ with $P_{e^-}^T = 0.8$, $P_{e^+}^T = 0$ at $\sqrt{s} = 500$ GeV in case of different bases for all combinations of final-state spin configurations with Im$S = 0.5 \times 10^{-6}$ GeV^{-2}, keeping the values of other couplings to be zero.

(a)

(b)

(c)

FIG. 3 (color online). $A_2(\theta)$ as a function of θ with $P^T_{e^-} = 0.8$, $P^T_{e^+} = 0$ at $\sqrt{s} = 500$ GeV in case of different bases for all combinations of final-state spin configurations with Re$S = 0.5 \times 10^{-6}$ GeV^{-2} and the values of other couplings are considered zero.

above has a supplement in the form of a constant term from SM. We, therefore, consider various asymmetries which isolate the contributions from S and T type of interactions only. The asymmetries considered here are those that were earlier studied in Ref. [20]. The asymmetries are schematically given by

$$A_1(\theta) = \frac{1}{\sigma^{SM}(\theta)}\left[\int_0^{\pi}\frac{d\sigma}{d\Omega}d\phi - \int_{\pi}^{2\pi}\frac{d\sigma}{d\Omega}d\phi\right], \quad (30)$$

$$A_2(\theta) = \frac{1}{\sigma^{SM}(\theta)}\left[\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{d\sigma}{d\Omega}d\phi - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\frac{d\sigma}{d\Omega}d\phi\right], \quad (31)$$

where dσ corresponds to a particular final state spin configurations as given in the Figures.

The asymmetry defined in Eq. (30) contains both ImS and ImT with zero contribution from the real part of the couplings, in case of all the final-state spin configurations. In view of the earlier discussion, this asymmetry isolates the CP-violating couplings. It may be noted, however, that

the initial state is not an eigenstate of CP, which would require the e^+ and e^- polarizations to be equal in magnitude. The asymmetry, thus, is not explicitly a purely CP-odd observable. However, since it depends only on the CP-violating couplings ImS and ImT, it is a measure of CP violation. Similarly, the other asymmetry defined in Eq. (31) receives contribution from ReS and ReT only. It is thus a measure of CP-conserving interactions.

For our calculations, we have only considered the contribution from ImS and ReS, with the value of T taken to be zero. In Figs. 2 and 3 we show the asymmetry $A_1(\theta)$ and $A_2(\theta)$ as a function of θ at $\sqrt{s} = 500$ GeV for all the bases. Depending on the bases involved, the new physics effect due to S and T contributes to the asymmetries. Let us now discuss what can be concluded from Figs. 2 and 3. From Fig. 2 it is seen that for the various final-state spin combinations the HB, BLB, and SMOD perform almost similarly as regards the sensitivity to ImS. The asymmetry for all the spin configurations is most significant in BLB. The same result holds for ImT. Figure 3 also shows that ReS

(a)

(b)
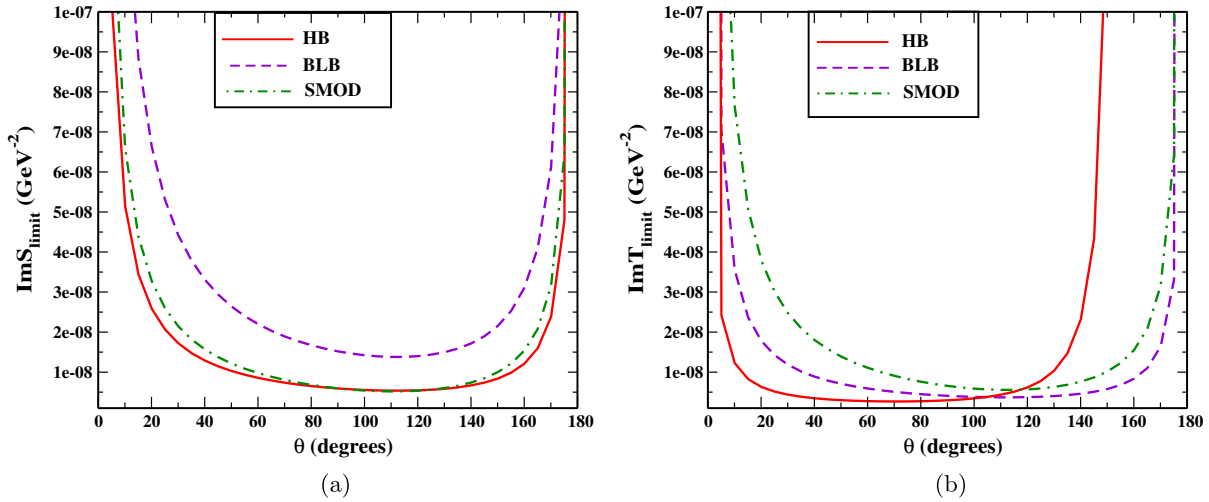
FIG. 4 (color online).   90% C.L. limit obtained on the couplings ImS and ImT from the modified asymmetry $A_1(\theta)$ at $\sqrt{s} = 500$ GeV with an integrated luminosity of 500 fb^{-1} for $P^T_{e^-} = 0.8$ and $P^T_{e^+} = 0$, plotted as a function of θ for different bases.

produces a similar signal as ImS. We note that the observ-ance of these asymmetries, in case of any of the beams being transversely polarized will be a signal of S and T type of physics. We further note that in the presence of TP the BLB is almost equally sensitive to effects from NP, for the different spin configurations of t and $\bar{t}$ compared to SMOD and HB, which are sensitive to only particular spin configurations. The largest asymmetries are all comparable in the three bases.

It is also possible to ask what are the 90% C.L. limits on S and T that can be obtained at the ILC. For this purpose, we consider an integrated luminosity of $\mathcal{L} = 500$ fb^{-1} at $\sqrt{s} = 500$ GeV and the same beam polarization. The sensitivity, characterized by the limit $\mathcal{C}_{\text{limit}}$ on a given coupling, is given by

$$\mathcal{C}_{\text{limit}} = \frac{1.64}{|A|\sqrt{N_{SM}}}, \qquad (32)$$

where $|A|$ is the asymmetry for unit value of the coupling and N_{SM} is the number of SM events. The asymmetries used are those given in Eqs. (30) and (31), However, we consider the case of resolving the t spin and sum over the $\bar{t}$ spin and take the difference as given below:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}\bigg|_{UD+UU} - \frac{d\sigma}{d\Omega}\bigg|_{DU+DD}. \qquad (33)$$

Similar analyses can be done by considering the spin of $\bar{t}$ and summing over the spin of t. This is analogous to the considerations of Ref. [20] for each of the spin bases. However, in Ref. [20] both beams were perfectly polarized and only the HB was considered. This is a continuation of that work to check the sensitivity of the different bases for obtaining limits on S and T. The asymmetries now are sensitive to the t polarization dependent part of the cross section and the number of events increases compared to the case when individual spin is measured. We present the

limits on the couplings ImS and ImT, for different bases in Fig. 4 for $P^T_{e^-} = 0.8$ and $P^T_{e^+} = 0$. The limits for ReS and ReT obtained from the modified asymmetry $A_2(\theta)$ are similar to those obtained for the imaginary parts of those couplings so we do not present the result here. Figure 4(a) shows that the best limit for ImS is obtained from the HB and SMOD. The best limit is around 5×10^{-9} GeV^{-2} and is obtained at $\theta = 110°$. The limit obtained from BLB is poorer by about an order of magnitude. Note that the limits obtained here depends on the degree of TP. In the ideal condition $P^T_{e^-}$, $P^T_{e^+} = 1$, the best limit is obtained from the HB and BLB for ImS and is around 3×10^{-9} GeV^{-2}. SMOD fares badly in this case, and the limit obtained is of the order 10^{-8} GeV^{-2}. Similarly for ImT, from Fig. 4(b) we see that the best limit is obtained from the HB and is 3×10^{-9} GeV^{-2}. The other bases behave simi-larly and give a limit of 5×10^{-9} GeV^{-2}. Here also the limits obtained from different bases are sensitive to the degree of TP. Therefore, an investigation in different bases with different degrees of TP is necessary to obtain limits on S and T type couplings.

B. New physics due to V and A interactions

The analytical form of the differential distribution in the presence of transverse polarization, due to vector and axial-vector type of nonstandard interactions is shown in Eqs. (24) and (27). These anomalous couplings are sensi-tive to longitudinal beam polarization, unlike the S and T interactions considered before. Moreover, their effect in the presence of transversely polarized beams can be seen only when both the beams are polarized. We will first study their effect in the presence of TP, and compare the devia-tion from the SM. The case of unpolarized and longitudinal beam polarization is considered later.

The azimuthal distribution in the presence of $A'_{L,R}$ along with the SM is shown in Fig. 5. The analyses are performed
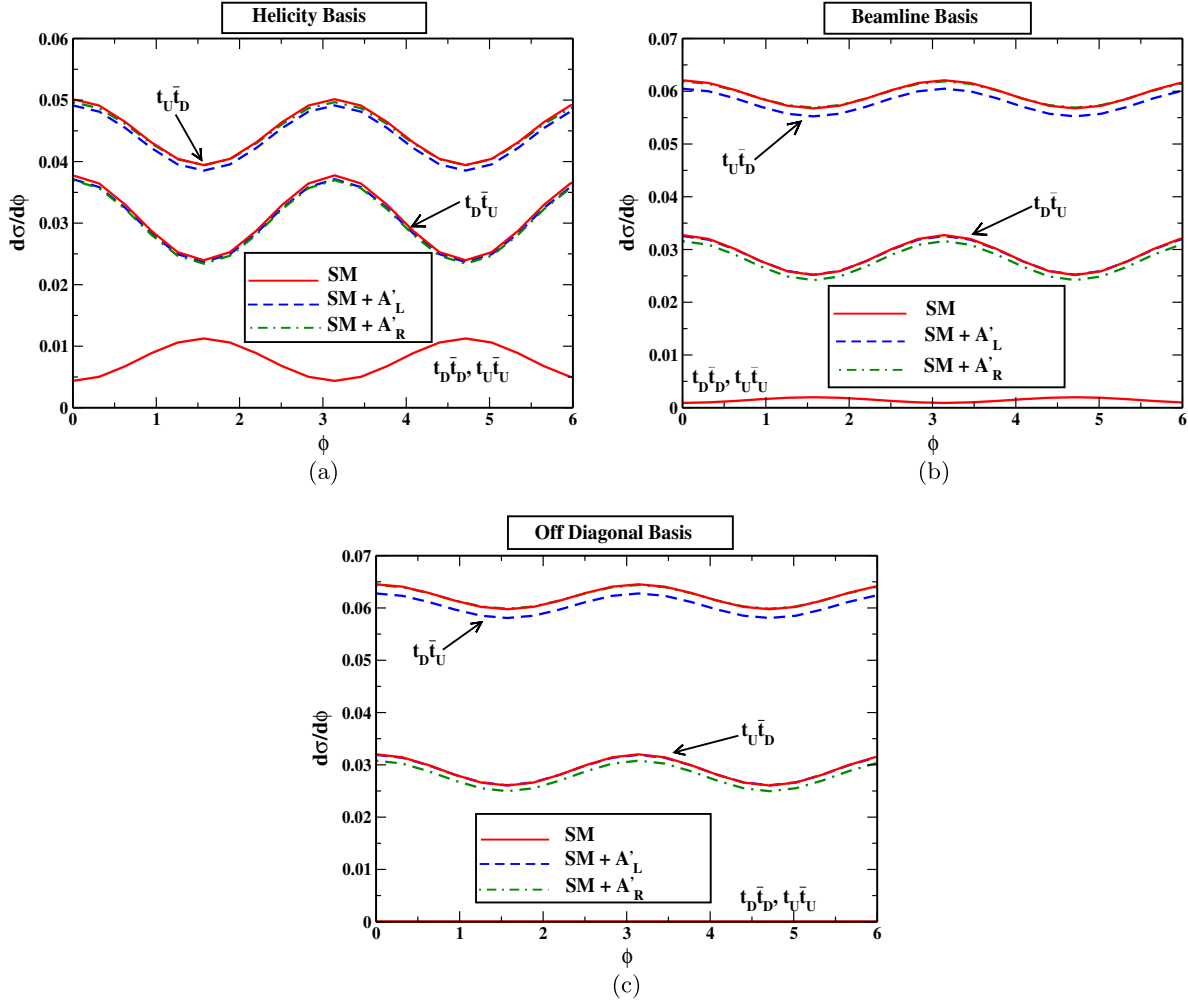
FIG. 5 (color online). The azimuthal distribution of the top quark pairs in different final state polarization at a 500 GeV linear collider for $P^T_{e^-} = 0.8$, $P^T_{e^+} = 0.6$. Different spin bases discussed in the paper are considered for SM and the case with either of the anomalous coupling $A'_{L,R} = 10^{-7}$ GeV^{-2} while keeping the value of others to be zero.

by taking each time one of the anomalous couplings to be 10^{-7} GeV^{-2}, while the others are kept at zero. We have checked the linear approximation holds for this choice of couplings. We are here following the spirit of Ref. [3] for the chosen value of the anomalous coupling. As in this analysis the contribution from $A'_{L,R}$ and $B'_{L,R}$ is considered at linear order only; therefore, the effect due to new physics on the total cross section is from its interference with the SM couplings. This is in contrast with Ref. [3], where they consider new physics to quadratic order. The deviation from SM in case of different bases for different couplings varies depending upon the nature of the couplings. We note that the couplings $B'_{L,R}$, are always accompanied by $\sin\xi$, in case of the final state polarization combination $t_U\bar{t}_U$ and $t_D\bar{t}_D$, Eq. (24). This reduces to zero in the HB, showing no effect in the azimuthal distribution.

We note from Fig. 5, that the azimuthal distribution for the $t_U\bar{t}_U$ and $t_D\bar{t}_D$ final-state spin combination, is always the same in case of all the bases. The behavior in case of

HB can be understood from the helicity amplitudes for the $t_U\bar{t}_U$ and $t_D\bar{t}_D$ final-state spin combination (see the Appendix) in the presence of V and A interactions. They are the same apart from a minus sign. In case of the other bases BLB and SMOD, the amplitudes for these final-spin configurations are obtained by the action of the transformation matrix Eqs. (15) and (18), resulting in the amplitudes which are also equal for $t_U\bar{t}_U$ and $t_D\bar{t}_D$ apart from a minus sign. Therefore, these particular spin configurations always show the same behavior in case of the three bases, for all the observables considered here.

C. Effect of longitudinal beam polarization

The new physics in the form of nonstandard interactions of V and A type, can also be studied with unpolarized and longitudinally polarized beams. For a complete analysis, we carry out a detailed study of the behavior due to these nonstandard interactions (V, A) in the presence of both unpolarized and longitudinally polarized beams. In case of
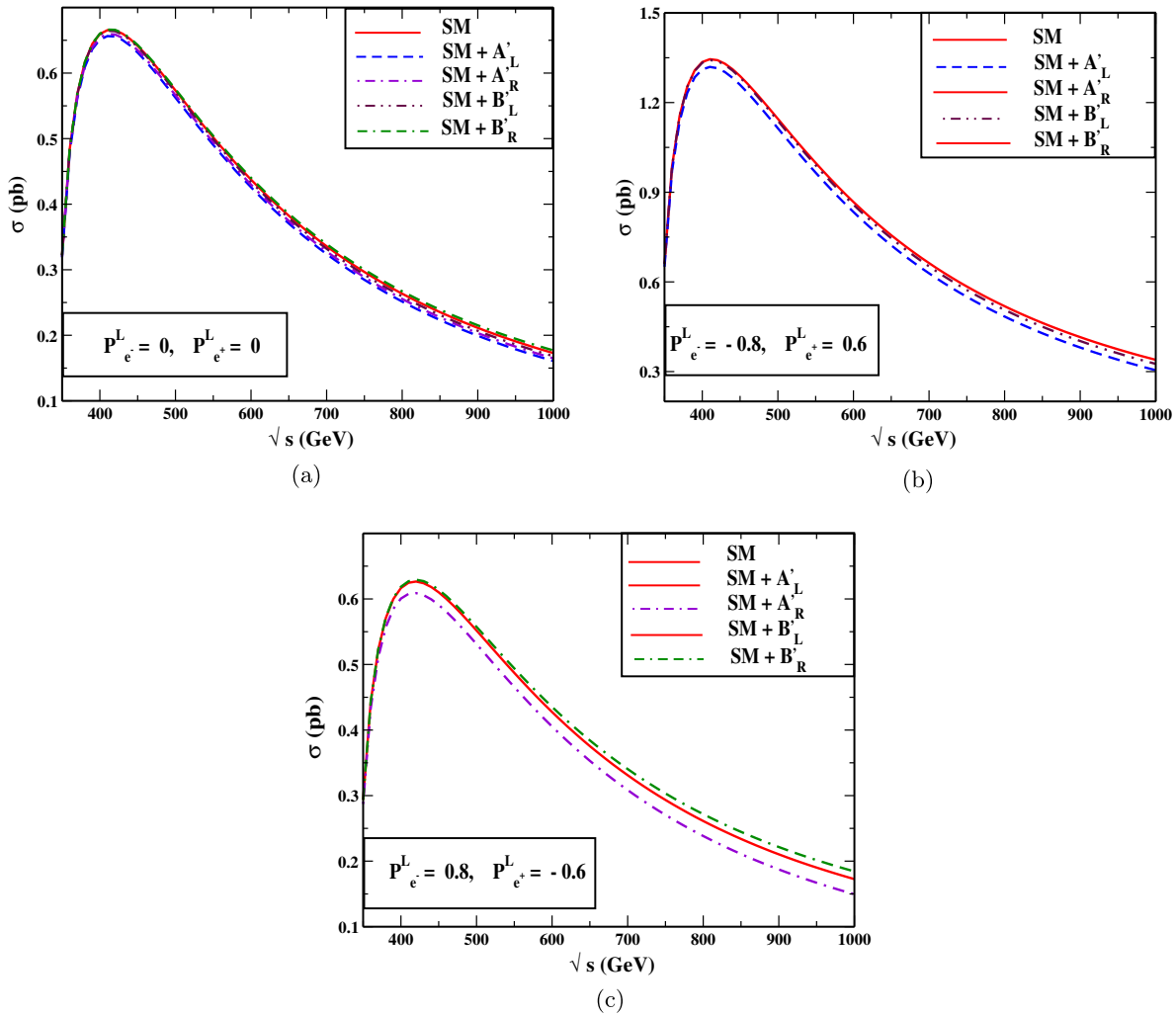
FIG. 6 (color online).   The total cross section in case of different beam polarizations for $t\bar{t}$ pair production as a function of center of mass energy $\sqrt{s}$ for SM and in the presence of different anomalous couplings each with a value of 10^{-7} GeV^{-2} contributing individually, keeping the values of other couplings to be zero.

LP, the differential cross section is obtained in a straight forward manner:

$$\frac{d\sigma(e^{+}e^{-} \rightarrow t_{I}\bar{t}_{J})}{d\cos\theta} = \frac{\pi}{2}((1 + P_{e^{-}}^{L})(1 - P_{e^{+}}^{L})|T_{RLIJ}|^{2} + (1 - P_{e^{-}}^{L})(1 + P_{e^{+}}^{L})|T_{LRIJ}|^{2}), \quad (34)$$

where $P_{e^{-}}^{L}$ and $P_{e^{+}}^{L}$ are the LP of the electron and positron beam, respectively. The amplitudes T_{RLIJ} and T_{LRIJ} are defined in the Appendix. We show in Fig. 6 the total unpolarized cross section along with the polarized one for different cases of beam polarization, as a function of $\sqrt{s}$ in case of $t\bar{t}$ pair production, for SM and in the presence of new physics. We have taken for the NP, the couplings with a value of 10^{-7} GeV^{-2} as in the case of TP. For the case of unpolarized beams [Fig. 6(a)], the deviation due to the anomalous couplings $B'_{L,R}$ from the SM is not much pronounced compared to $A'_{L,R}$. At $\sqrt{s} = 500$ GeV, the deviation due to $B'_{L,R}$ is about 0.4% from SM; whereas $A'_{L,R}$

produces 2% deviation. The implementation of beam polarization, with $P_{e^{-}}^{L} = -0.8$, $P_{e^{+}}^{L} = 0.6$, as shown in Fig. 6(b), increases the sensitivity to A'_{L} and B'_{L}, along with a increase in statistics. Figure 6(c) shows the polarization combination $P_{e^{-}}^{L} = 0.8$, $P_{e^{+}}^{L} = -0.6$, with an enhanced sensitivity to the anomalous couplings A'_{R} and B'_{R}.

The angular correlation between the $t\bar{t}$ in the case of the SM, is the best for the SMOD. We next describe an asymmetry where this angular correlation between the final state products can be observed. This was earlier considered in the case of hadron colliders [36,37], based upon the asymmetry in the number of like spin to unlike spin $t\bar{t}$ pairs produced:

$$A_{t\bar{t}} = \frac{(N_{UU} + N_{DD}) - (N_{DU} + N_{UD})}{N_{UU} + N_{DD} + N_{DU} + N_{UD}}. \quad (35)$$

N_{ij} ($i, j = U, D$) in Eq. (35) denotes the number of events for the top and antitop spin combinations t_{i} and $\bar{t}_{j}$.

TABLE II. The asymmetry $A_{t\bar{t}}$, Eq. (35) measuring the strength of the correlation in different spin bases for different beam polarizations in case of SM and other nonstandard interactions of V and A type at a centre of mass energy of 500 GeV.

Couplings	Spin bases	$P^L_{e^-} = 0.8, P^L_{e^+} = -0.6$	$P^L_{e^-} = -0.8, P^L_{e^+} = 0.6$	$P^L_{e^-} = 0, P^L_{e^+} = 0$
	HB	-0.6707	-0.6516	-0.6578
SM	BLB	-0.9608	-0.9247	-0.9364
	SMOD	-0.9940	-0.9999	-0.9980
	HB	-0.6707	-0.6525	-0.6586
SM + A'_L	BLB	-0.9610	-0.9270	-0.9383
	SMOD	-0.9940	-0.9999	-0.9980
	HB	-0.6727	-0.6516	-0.6583
SM + A'_R	BLB	-0.9638	0.9247	-0.9370
	SMOD	-0.9928	-0.9999	-0.9977
	HB	-0.6706	-0.6496	-0.6564
SM + B'_L	BLB	-0.9605	-0.9198	-0.9331
	SMOD	-0.9940	-0.9999	-0.9980
	HB	-0.6739	-0.6516	-0.6589
SM + B'_R	BLB	-0.9655	-0.9247	-0.9380
	SMOD	-0.9922	-0.9999	-0.9974

Table II shows correlations for different spin bases in the different cases of LP for SM along with new physics couplings of vector and axial-vector type. The correlation measured by the asymmetry $A_{t\bar{t}}$ is sensitive to the choice of the bases, along with the initial beam polarization. At a 500 GeV unpolarized linear collider, about 83% of the final state pairs have opposite helicity; whereas 17% have the same helicity. The correlation in Eq. (35) is seen to be about 66% from the results in Table II for the helicity basis. Although SMOD is the best basis for observing the correlation, it is not sensitive in distinguishing contributions from new physics. It can be seen from the Table, that for different cases of LP the HB and BLB are more sensitive to new physics couplings compared to SMOD.

For purpose of completeness, we obtain 90% C.L. limits on the V and A couplings for realistic beam polarizations and typical integrated luminosity. The limits obtained for the couplings in case of different bases for different beam polarization is shown in Table III. It can be seen from the Table that the limits obtained in this case are not very competitive. These interactions being similar to SM, it is very difficult to isolate their signatures unlike S and T. Therefore, it is very difficult to see the new physics signatures of V and A unless we go to higher centre of mass energy and higher luminosity.

We have also considered the fraction of $t\bar{t}$ pairs produced in different polarization states versus the e^+e^- c.m. energy in case of different spin bases. The presence of the anomalous couplings along with SM is considered, for different cases of initial LP. The fraction of $t\bar{t}$ pairs produced in a spin combination $s_t s_{\bar{t}}$ is defined as

$$\sigma_{\text{frac}} = \frac{\sigma(e^+e^- \to t_{s_t}\bar{t}_{s_{\bar{t}}})}{\sigma_{\text{tot}}}, \qquad (36)$$

TABLE III. 90% C.L. limit obtained on various coupling from the asymmetry $A_{t\bar{t}}$ Eq. (35), at $\sqrt{s} = 500$ GeV with an integrated luminosity of 500 fb^{-1} for different cases of LP.

| Couplings	Spin bases	$P^L_{e^-} = 0.8, P^L_{e^+} = -0.6$	$P^L_{e^-} = -0.8, P^L_{e^+} = 0.6$	$P^L_{e^-} = 0, P^L_{e^+} = 0$
	HB	0.0051 GeV^{-2}	0.0035 GeV^{-2}	0.0050 GeV^{-2}
A'_L	BLB	0.0037 GeV^{-2}	0.0026 GeV^{-2}	0.0036 GeV^{-2}
	SMOD	0.0031 GeV^{-2}	0.0022 GeV^{-2}	0.0031 GeV^{-2}
	HB	0.0040 GeV^{-2}	0.0034 GeV^{-2}	0.0050 GeV^{-2}
A'_R	BLB	0.0033 GeV^{-2}	0.0356 GeV^{-2}	0.0035 GeV^{-2}
	SMOD	0.0031 GeV^{-2}	0.0014 GeV^{-2}	0.0030 GeV^{-2}
	HB	0.0031 GeV^{-2}	0.0022 GeV^{-2}	0.0031 GeV^{-2}
B'_L	BLB	0.0018 GeV^{-2}	0.0012 GeV^{-2}	0.0017 GeV^{-2}
	SMOD	0.0031 GeV^{-2}	0.0022 GeV^{-2}	0.0031 GeV^{-2}
	HB	0.0031 GeV^{-2}	0.0022 GeV^{-2}	0.0031 GeV^{-2}
B'_R	BLB	0.0026 GeV^{-2}	0.0012 GeV^{-2}	0.0021 GeV^{-2}
	SMOD	0.0035 GeV^{-2}	0.0067 GeV^{-2}	0.0038 GeV^{-2}
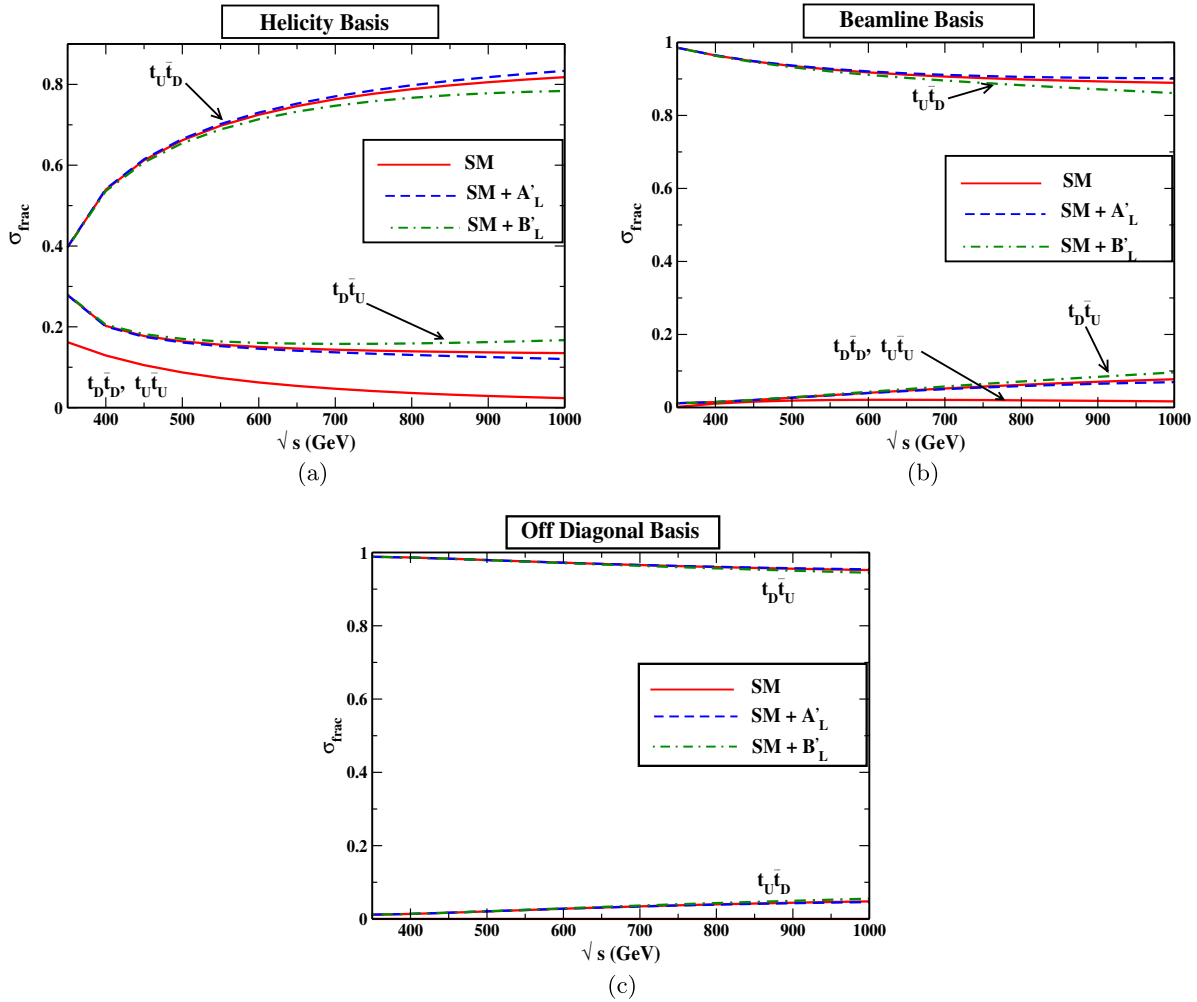
FIG. 7 (color online).   The fraction of the $t\bar{t}$ pair production as a function of center of mass energy $\sqrt{s}$ for SM and in the presence of A'_L, B'_L each with a value of 10^{-7} GeV^{-2} contributing individually, keeping the values of other couplings to be zero. The three bases discussed here are considered, with an initial beam polarization of $P^L_{e^-} = -0.8$, $P^L_{e^+} = 0.6$.

where σ_{tot} is the total cross section for unpolarized t, $\bar{t}$, with possible inclusion of anomalous couplings. The results are shown in Figs. 7 and 8, for realistic initial beam polarizations of $P^L_{e^-} = -0.8$, $P^L_{e^+} = 0.6$, and $P^L_{e^-} = 0.8$, $P^L_{e^+} = -0.6$.

In the SMOD shown in Figs. 7(c) and 8(c) the polarization of $t\bar{t}$ states compared to the BLB in Figs. 7(b) and 8(b) and the HB in Figs. 7(c) and 8(c) is more. With the left-handed electrons initial beam polarization, the spin configuration $t_D\bar{t}_U$ gives the dominant contribution in the SMOD; whereas for right handed initial beam polarization the dominant configuration comes from $t_U\bar{t}_D$. This behavior is similar to that observed in case of the asymmetry $A_{t\bar{t}}$ measuring the spin correlation in different bases. The effect of the new physics is most dominantly seen in the HB and the least in the SMOD. These additional effects are mainly from the interference term between the NP and SM. In the SMOD, since one of the final-state spin configuration is dominant, it forces the new physics contributing to

the interference term to be in the same spin configuration. The helicity basis on the other hand treats the final state spin configurations with opposite helicities almost equally allowing the new physics to freely interfere with the SM couplings, leading to deviations compared to the SM predictions. Thus, in case of LP or unpolarized beams both in the case of work [24] and in the present work the HB and BLB works better compared to SMOD, in the search for new physics.

VII. DISCUSSIONS AND CONCLUSIONS

The top quark due to its large mass compared to other quarks has been an interesting tool to look for significant deviations from the SM. Because of its short lifetime, the top quark decays before hadronization, with its spin surviving and showing its effect through the distribution of the decay products. The measurement of the spin can be done through the analysis of the decay products. At the ILC the $t\bar{t}$ pairs will be produced in large numbers, and will be an
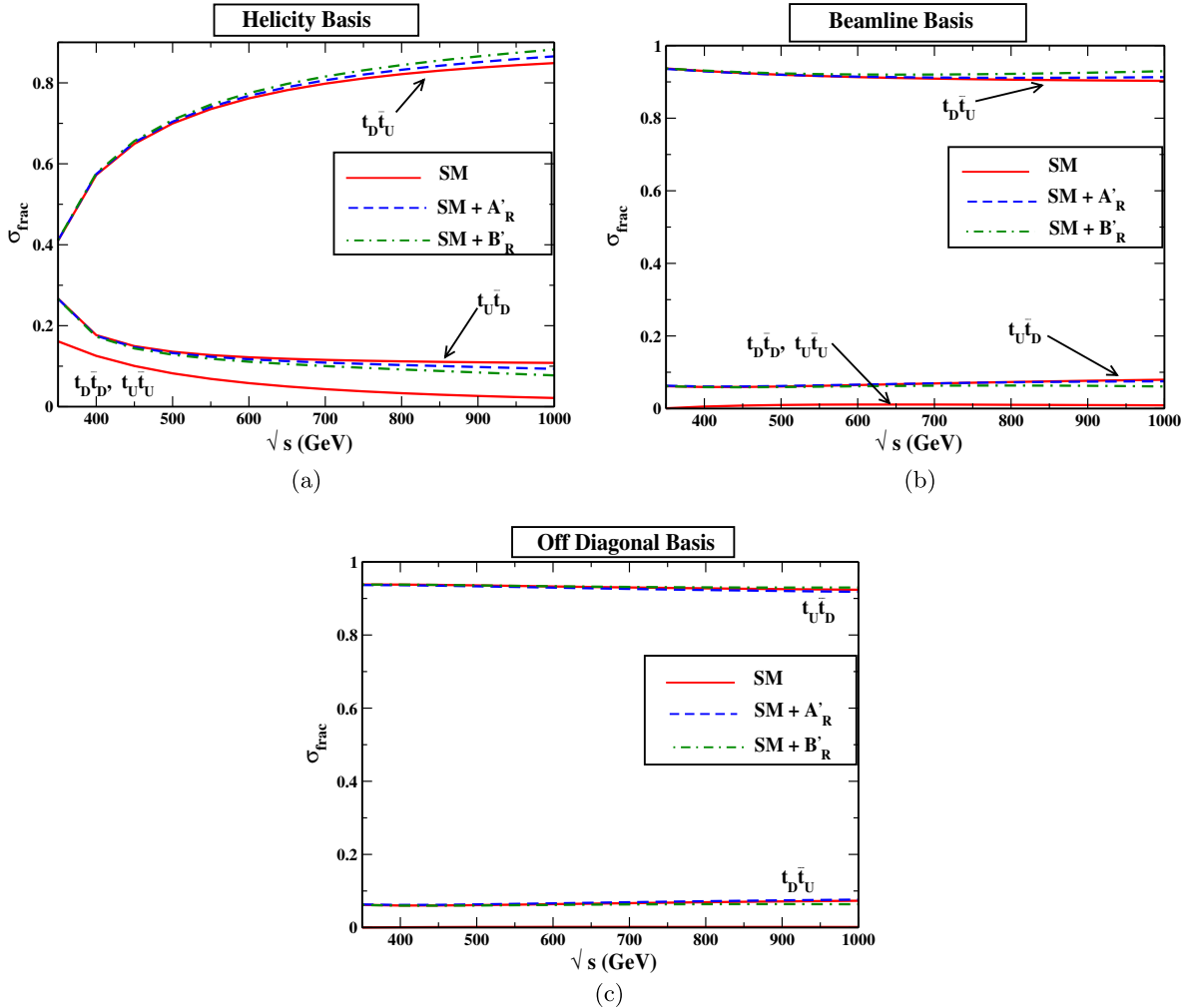
FIG. 8 (color online).   The fraction of the $t\bar{t}$ pair production as a function of center of mass energy $\sqrt{s}$ for SM and in the presence of A'_R, B'_R each with a value of 10^{-7} GeV^{-2} contributing individually, keeping the values of other couplings to be zero. The three bases discussed here are considered, with an initial beam polarization of $P^L_{e^-} = 0.8$, $P^L_{e^+} = -0.6$.

ideal tool to look for BSM effects in top quarks couplings. The degree of polarization can be changed by tuning the initial beams polarization. The spin correlation in top quark production is therefore an interesting issue in top quark physics.

PS [21] had suggested different spin bases to study spin correlations. We have presented a new and simpler derivation, which accounts the most general physics of S, P, V, A, and T types. The reason for these new studies is the realization that the number of like spin and unlike spin top quarks can be made significantly different by an appropriate choice of spin bases. Experimentally the $t\bar{t}$ spin correlation is measured by analyzing the angular distribution of the t and $\bar{t}$ decay products. As the top quark decays through left-handed weak force, it analyses its own polarization through its decay products. To make a practical use of the bases defined here, the direction of the charged lepton momentum in a top leptonic decay must be observed, giving an indication of the top polarization direction. The

angular distribution of the decay products in top quark decays, according to the polarization of the parent top quark is given by

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_i} = \frac{1}{2}(1 \pm A_i \cos\theta_i), \qquad (37)$$

where the $\pm$ sign in front of A_i is used for right-handed and left-handed quarks, respectively. The angle θ_i is the angle between the spin quantization axis and the momentum of the decay particle in top quark rest frame. A_i is defined as the spin analysing power coefficient equal to 1 for the charged lepton or the down type quark. The SMOD discussed in Ref. [21], has the feature that the production cross section of like spin states is almost negligible. With the initial polarization of left (right) handed electron beam, the spin configuration $t_U\bar{t}_D$ ($t_D\bar{t}_U$) gives the dominant cross section. Therefore, observation of a sizable event rate in the like spin states or a significant deviation from the

dominant cross section in this basis will account for new physics signals.

In this work we have looked for new physics signatures of the scalar (S) and tensor (T) type along with the vector (V) and axial-vector (A) type of interactions. The signatures of these S and T interactions can only be probed at linear order with TP of one or both the beams. We check the sensitivity of the bases discussed earlier, to S and T interactions, with one of the beams having TP. This eliminates the ϕ contribution from SM and other interactions of V and A type. Each of the bases bears a different signature. The BLB is found to be the most preferred basis in the presence of TP, as it receives contribution from all the final state spin configuration. We then consider some asymmetries, where the SM contribution vanish. Therefore, any sizable observation will confirm the signatures of S and T type of physics. Here, too, BLB is most sensitive to these NP effects, followed by SMOD and HB. Thus as discussed in Sec. V CP violation as a probe of new physics through non vanishing ImS and ImT, is most likely to show up in the BLB analysis. We have also used an asymmetry analogous to the one considered in Ref. [20] to obtain 90% C.L. on the couplings with realistic polarization and luminosity.

We have also looked for new physics in the form of V and A type of interactions, with all possible types of initial-beam polarizations (TP and LP). In case of TP, both the initial beams have to be polarized. The analysis in case of TP shows similar results as in S and T scenario, with the BLB being the most sensitive to NP, and the HB receiving the smallest amount of contributions from NP. For unpolarized and longitudinal polarized beams, we have studied the correlation asymmetry for the different bases and have quoted the results in Table II. The asymmetry for the purpose of 90% C.L. is not very competitive. In case of LP, the result is contrary to TP, with the HB receiving the significant amount of contribution from NP, and SMOD being the least sensitive. In the ILC, with the planned polarization program, BLB is the best in the presence of TP, followed by SMOD and HB, for the NP considered here. Similarly, HB is the best in presence of LP followed by BLB and SMOD for the study of NP.

While in the past we have taken the approach of eliminating the sensitivity with realistic degrees of polarization and integrated luminosity $\mathcal{L}$, here we take a complementary approach of assuming some "realistic" values for BSM couplings to study the size of the signal. This enables us to clearly establish that the sensitivity levels established in Ref. [20] can be significantly improved. Since a realistic study will necessarily involve detector simulation studies, our approach provides a clear analytical picture of the scale to which BSM physics in the sector considered here can be probed.

ACKNOWLEDGMENTS

APPENDIX: HELICITY AMPLITUDES

The helicity amplitudes for the process $e^+e^- \to t\bar{t}$ are defined below. They are the same as considered in Ref. [29], with the normalization factor taken care of. The amplitudes of e^-, e^+, t, and $\bar{t}$ are defined in the order T_{LRIJ}, where L denotes the left-handed electron beam e_L^-, R for right-handed positron beam e_R^+, and IJ denotes the different final-state combinations of $t\bar{t}$, i.e., DD, DU, UD, and UU. Similarly, T_{RLIJ} denotes the right-handed electron beam e_R^- and left-handed positron beam e_L^+.

For the helicity-conserving interactions, the amplitudes are as follows:

$$
\begin{aligned}
T_{LRUU} &= B_1 A_L m_t \sin\theta, \\
T_{LRUD} &= B_1 (EA_L + kB_L)(1 + \cos\theta), \\
T_{LRDU} &= -B_1 (EA_L - kB_L)(1 - \cos\theta), \\
T_{LRDD} &= -B_1 A_L m_t \sin\theta, \\
T_{RLUU} &= B_1 A_R m_t \sin\theta, \\
T_{RLUD} &= -B_1 (EA_R + kB_R)(1 - \cos\theta), \\
T_{RLDU} &= B_1 (EA_R - kB_R)(1 + \cos\theta), \\
T_{RLDD} &= -B_1 A_R m_t \sin\theta.
\end{aligned} \tag{A1}
$$

All the expressions above have the normalization factor B_1 defined as $i\sqrt{3\beta\alpha^2/4}$. E is the beam energy $\sqrt{s}/2$ and $k = E\beta$, where $\beta = \sqrt{1 - 4m_t^2/s}$. The amplitudes in the presence of vector and axial vector type four-Fermi operator effects, has the same form as those above. In our analyses, we have considered the effect of new physics only through its interference with the SM amplitudes.

Similarly, for the helicity-violating interactions, the amplitudes are

$$
\begin{aligned}
T_{LLUU} &= A_1((E + k)(S_{LL} - 2T_{LL}\cos\theta) - S_{LR}(E - k)), \\
T_{LLUD} &= 2A_1 T_{LL} m_t \sin\theta, \\
T_{LLDU} &= 2A_1 T_{LL} m_t \sin\theta, \\
T_{LLDD} &= A_1((E - k)(S_{LL} + 2T_{LL}\cos\theta) - S_{LR}(E + k)), \\
T_{RRUU} &= A_1((E - k)(S_{RR} + 2T_{RR}\cos\theta) - S_{RL}(E - k)), \\
T_{RRUD} &= -2A_1 T_{RR} m_t \sin\theta, \\
T_{RRDU} &= -2A_1 T_{RR} m_t \sin\theta, \\
T_{RRDD} &= A_1((E + k)(S_{RR} - 2T_{RR}\cos\theta) - S_{RL}(E - k)).
\end{aligned} \tag{A2}
$$

The normalization factor for the above amplitudes A_1 is defined as $i\sqrt{3\beta/64\pi^2}$. The expressions for the scalar S and tensor T operators are as follows:

$$S_{RR} = \text{Re}S + i\text{Im}S, \qquad S_{LL} = \text{Re}S - i\text{Im}S, \qquad T_{RR} = \text{Re}T + i\text{Im}T, \qquad T_{LL} = \text{Re}T - i\text{Im}T. \qquad (A3)$$

[1] G. Aarons *et al.* (ILC Collaboration), arXiv:0709.1893.
[2] G. Moortgat-Pick *et al.*, Phys. Rep. **460**, 131 (2008).
[3] B. Grzadkowski, Z. Hioki, and M. Szafranski, Phys. Rev. D **58**, 035002 (1998).
[4] B. Grzadkowski and Z. Hioki, Phys. Rev. D **61**, 014013 (1999).
[5] B. Grzadkowski and Z. Hioki, Phys. Lett. B **476**, 87 (2000).
[6] S. D. Rindani, arXiv:hep-ph/0202045.
[7] E. Devetak, A. Nomerotski, and M. Peskin, Phys. Rev. D **84**, 034029 (2011).
[8] J. A. Aguilar-Saavedra, Nucl. Phys. **B843**, 638 (2011); **B851**, 443(E) (2011).
[9] J. A. Aguilar-Saavedra, M. C. N. Fiolhais, and A. Onofre, J. High Energy Phys. 07 (2012) 180.
[10] C. Zhang, N. Greiner, and S. Willenbrock, Phys. Rev. D **86**, 014024 (2012).
[11] P. Doublet, F. Richard, R. Poschl, T. Frisson, and J. Rouene, arXiv:1202.6659.
[12] S. Groote, J. G. Korner, B. Melic, and S. Prelovsek, arXiv:1209.0547.
[13] S. Groote, J. G. Körner, B. Melić, and S. Prelovsek, Phys. Rev. D **83**, 054018 (2011).
[14] A. Brandenburg, M. Flesch, and P. Uwer, Czech. J. Phys. **50**, 51 (2000).
[15] S. D. Rindani, Pramana **54**, 791 (2000).
[16] B. Grzadkowski and Z. Hioki, arXiv:hep-ph/0003294.
[17] T. Huang, arXiv:hep-ph/0009047.
[18] K. Y. Lee, S. C. Park, H. S. Song, and C. Yu, Phys. Rev. D **63**, 094010 (2001).
[19] S. J. Parke, Nuovo Cimento **C035N3**, 111 (2012).
[20] B. Ananthanarayan, M. Patra, and S. D. Rindani, Phys. Rev. D **83**, 016010 (2011).
[21] S. J. Parke and Y. Shadmi, Phys. Lett. B **387**, 199 (1996).
[22] J. Kodaira, T. Nasuno, and S. J. Parke, Phys. Rev. D **59**, 014023 (1998).
[23] T. Nasuno, arXiv:hep-ph/9906252.
[24] Z. H. Lin, T. Han, T. Huang, J. X. Wang, and X. Zhang, Phys. Rev. D **65**, 014008 (2001).
[25] C. X. Yue and L. N. Wang, J. Phys. G **34**, 139 (2007).
[26] S. J. Parke, arXiv:1005.0347.
[27] Y. Sumino and S. Tsuno, Phys. Lett. B **633**, 715 (2006).
[28] W. Buchmuller and D. Wyler, Nucl. Phys. **B268**, 621 (1986).
[29] B. Grzadkowski, Acta Phys. Pol. B **27**, 921 (1996).
[30] B. Ananthanarayan and S. D. Rindani, Phys. Rev. D **70**, 036005 (2004).
[31] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. D **83**, 031104 (2011).
[32] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **108**, 032004 (2012).
[33] G. Aad *et al.* (ATLAS Collaboration), Phys. Rev. Lett. **108**, 212001 (2012).
[34] R. M. Godbole, S. D. Rindani, and R. K. Singh, J. High Energy Phys. 12 (2006) 021.
[35] K. i. Hikasa, Phys. Rev. D **33**, 3203 (1986).
[36] T. Head, Nuovo Cimento **C033N4**, 229 (2010).
[37] T. Stelzer and S. Willenbrock, Phys. Lett. B **374**, 169 (1996).