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Light scalars in semileptonic decays of heavy quarkonia

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We study the mechanism of production of the light scalar mesons in the $D_s^+ \to \pi^+\pi^- e^+\nu$ decays: $D_s^+ \to s\bar{s}e^+\nu \to [\sigma(600) + f_0(980)]e^+\nu \to \pi^+\pi^- e^+\nu$, and we compare it with the mechanism of production of the light pseudoscalar mesons in the $D_s^+ \to (\eta/\eta')e^+\nu$ decays: $D_s^+ \to s\bar{s}e^+\nu \to (\eta/\eta')e^+\nu$. We show that the $s\bar{s} \to \sigma(600)$ transition is negligibly small in comparison with the $s\bar{s} \to f_0(980)$ one. As for the $f_0(980)$ meson, the intensity of the $s\bar{s} \to f_0(980)$ transition makes near thirty percent from the intensity of the $s\bar{s} \to \eta_s$ ($\eta_s = s\bar{s}$) transition. So, the $D_s^+ \to \pi^+\pi^- e^+\nu$ decay supports the previous conclusions about a dominant role of the four-quark components in the $\sigma(600)$ and $f_0(980)$ mesons.

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At present, the nontrivial nature of the well-established light scalar resonances $f_0(980)$ and $a_0(980)$ is denied by very few people. As for the nonet as a whole, even a cursory look at PDG review [1] gives an idea of the four-quark structure of the light scalar meson nonet, $\sigma(600)$, $\kappa(800)$, $f_0(980)$, and $a_0(980)$, inverted in comparison with the classical P wave $q\bar{q}$ tensor meson nonet, $f_2(1270)$, $a_2(1320)$, $K_2^*(1420)$, $\phi_2'(1525)$. Really, while the scalar nonet cannot be treated as the P wave $q\bar{q}$ nonet in the naive quark model, it can be easy understood as the $q^2\bar{q}^2$ nonet, where σ has no strange quarks, κ has the s quark, f_0 and a_0 have the $s\bar{s}$ pair. Similar states were found by Jaffe in 1977 in the MIT bag [2].

By now, it is established also that the mechanisms of the $a_0(980)$, $f_0(980)$, and $\sigma(600)$ meson production in the ϕ radiative decays [3–8], in the photon-photon collisions [9,10], and in the $\pi\pi$ scattering [7,8] are the four-quark transitions and thus indicate to the four-quark structure of the light scalars [11].

In addition, the absence of the $J/\psi \to \gamma f_0(980)$, $a_0(980)\rho$, $f_0(980)\omega$ decays in contrast to the intensive the $J/\psi \to \gamma f_2(1270)$, $\gamma f_2'(1525)$, $a_2(1320)\rho$, $f_2(1270)\omega$ decays argues against the P wave $q\bar{q}$ structure of $a_0(980)$ and $f_0(980)$ also [12].

It is time to explore the light scalar mesons in the decays of heavy quarkonia [13–15]. The semileptonic decays are of prime interest because they have the clear mechanisms; see, for example, Fig. 1.

As Fig. 1 suggests, the $D_s^+ \to s\bar{s}e^+\nu \to [\sigma(600) + f_0(980)]e^+\nu \to \pi^+\pi^- e^+\nu$ decay is the perfect probe of the $s\bar{s}$ component in the $\sigma(600)$ and $f_0(980)$ states [13,14].

Below we study the mechanism of production of the light scalar mesons in the $D_s^+ \to \pi^+\pi^- e^+\nu$ decays: $D_s^+ \to s\bar{s}e^+\nu \to [\sigma(600) + f_0(980)]e^+\nu \to \pi^+\pi^- e^+\nu$, and we compare it with the mechanism of production of the light pseudoscalar mesons in the $D_s^+ \to (\eta/\eta')e^+\nu$ decays: $D_s^+ \to s\bar{s}e^+\nu \to (\eta/\eta')e^+\nu$, in a model of the Nambu-Jona-Lasinio type [16].

The amplitudes of the $D_s^+ \to P$ (pseudoscalar) $e^+\nu$ and $D_s^+ \to S$ (scalar) $e^+\nu$ decays have the form

$$M[D_s^+(p) \to P(p_1)W^+(q) \to P(p_1)e^+\nu] = \frac{G_F}{\sqrt{2}}V_{cs}V_\alpha L^\alpha,$$
$$M[D_s^+(p) \to S(p_1)W^+(q) \to S(p_1)e^+\nu] = \frac{G_F}{\sqrt{2}}V_{cs}A_\alpha L^\alpha, \tag{1}$$

where G_F is the Fermi constant, V_{cs} is the Cabibbo-Kobayashi-Maskawa matrix element,

$$\begin{aligned} V_\alpha &= f_+^P(q^2)(p+p_1)_\alpha + f_-^P(q^2)(p-p_1)_\alpha, \\ A_\alpha &= f_+^S(q^2)(p+p_1)_\alpha + f_-^S(q^2)(p-p_1)_\alpha, \\ L_\alpha &= \bar{\nu}\gamma_\alpha(1+\gamma_5)e, \qquad q = (p-p_1). \end{aligned} \tag{2}$$

The influence of the $f_-^P(q^2)$ and $f_-^S(q^2)$ form factors are negligible because of the small mass of the positron.

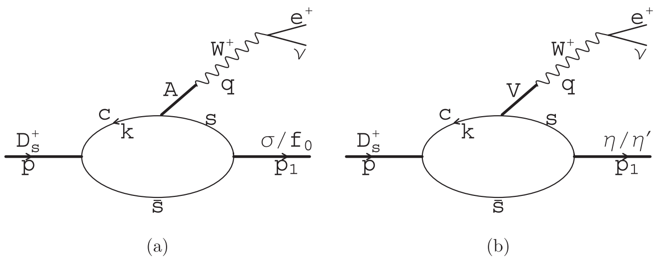

FIG. 1. Model of the $D_s^+ \to \sigma/f_0 e^+\nu$ and $D_s^+ \to (\eta/\eta')e^+\nu$ decays.

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The decay rates in the stable P and S states are

$$\begin{aligned}\frac{d\Gamma(D_s^+\to Pe^+\nu)}{dq^2}&=\frac{G_F^2|V_{cs}|^2}{24\pi^3}p_1^3(q^2)|f_+^P(q^2)|^2,\\ \frac{d\Gamma(D_s^+\to Se^+\nu)}{dq^2}&=\frac{G_F^2|V_{cs}|^2}{24\pi^3}p_1^3(q^2)|f_+^S(q^2)|^2,\\ p_1(q^2)&=\frac{\sqrt{m_{D_s^+}^4-2m_{D_s^+}^2(q^2+m_P^2)+(q^2-m_P^2)^2}}{2m_{D_s^+}},\quad\text{or}\\ p_1(q^2)&=\frac{\sqrt{m_{D_s^+}^4-2m_{D_s^+}^2(q^2+m_S^2)+(q^2-m_S^2)^2}}{2m_{D_s^+}}.\end{aligned}\tag{3}$$

For the $f_+^P(q^2)$ and $f_+^S(q^2)$ form factors, we use the vector dominance model

$$\begin{aligned}f_+^P(q^2)&=f_+^P(0)\frac{m_V^2}{m_V^2-q^2}=f_+^P(0)f_V(q^2),\\ f_+^S(q^2)&=f_+^S(0)\frac{m_A^2}{m_A^2-q^2}=f_+^S(0)f_A(q^2),\end{aligned}\tag{4}$$

where $V=D_s^*(2112)^\pm$, $A=D_{s1}(2460)^\pm$ [1].

Following Fig. 1, we write $f_+^P(0)$ and $f_+^S(0)$ in the form

$$f_+^P(0)=g_{D_s^+c\bar{s}}F_Pg_{s\bar{s}P},\qquad f_+^S(0)=g_{D_s^+c\bar{s}}F_Sg_{s\bar{s}S},\tag{5}$$

where $g_{D_s^+c\bar{s}}$ is the $D_s^+\to c\bar{s}$ coupling constant, $g_{s\bar{s}P}$ and $g_{s\bar{s}S}$ are the $s\bar{s}\to P$ and $s\bar{s}\to S$ coupling constants.

We know the structure of η and η':

$$\eta=\eta_q\cos\phi-\eta_s\sin\phi,\qquad \eta'=\eta_q\sin\phi+\eta_s\cos\phi,\tag{6}$$

where $\eta_q=(u\bar{u}+d\bar{d})/\sqrt{2}$ and $\eta_s=s\bar{s}$. The angle $\phi=\theta_i+\theta_P$, where θ_i is the ideal mixing angle with $\cos\theta_i=\sqrt{1/3}$ and $\sin\theta_i=\sqrt{2/3}$, i.e., $\theta_i=54.7°$, and θ_P is the angle between the flavor-singlet state η_1 and the flavor-octet state η_8.

So,

$$g_{s\bar{s}\eta}=-g_{s\bar{s}\eta_s}\sin\phi,\qquad g_{s\bar{s}\eta'}=g_{s\bar{s}\eta_s}\cos\phi.\tag{7}$$

The Particle Data Group [1] gives the θ_P band $-20°\lesssim\theta_P\lesssim-10°$ that gives us the opportunity to extract information about the $s\bar{s}\to\eta_s$ coupling constant $g_{s\bar{s}\eta_s}$ from experiment and to compare with the $s\bar{s}\to f_0$ coupling constant $g_{s\bar{s}f_0}$ extracted from experiment also. We consider the next set of θ_P:

$$\begin{aligned}\theta_P&=-11°:\ \eta=0.72\eta_0-0.69\eta_s,\\ \eta'&=0.69\eta_0+0.72\eta_s,\\ \theta_P&=-14°:\ \eta=0.76\eta_0-0.65\eta_s,\\ \eta'&=0.65\eta_0+0.76\eta_s,\\ \theta_P&=-18°:\ \eta=0.8\eta_0-0.6\eta_s,\\ \eta'&=0.6\eta_0+0.8\eta_s.\end{aligned}\tag{8}$$

The amplitude of the $D_s^+\to s\bar{s}e^+\nu\to[\sigma(600)+f_0(980)]e^+\nu\to\pi^+\pi^-e^+\nu$ decay is

$$\begin{aligned}M(D_s^+\to s\bar{s}e^+\nu\to\pi^+\pi^-e^+\nu)&=\frac{G_F}{\sqrt{2}}V_{cs}L^\alpha(p+p_1)_\alpha g_{D_s^+c\bar{s}}f_A(q^2)e^{i\delta_B^{\pi\pi}}\frac{1}{\Delta(m)}(F_\sigma g_{s\bar{s}\sigma}D_{f_0}(m)g_{\sigma\pi^+\pi^-}\\ &\quad+F_\sigma g_{s\bar{s}\sigma}\Pi_{\sigma f_0}(m)g_{f_0\pi^+\pi^-}+F_{f_0}g_{s\bar{s}f_0}\Pi_{f_0\sigma}(m)g_{\sigma\pi^+\pi^-}+F_{f_0}g_{s\bar{s}f_0}D_\sigma(m)g_{f_0\pi^+\pi^-}),\end{aligned}\tag{9}$$

where m is the invariant mass of the $\pi\pi$ system, $\Delta(m)=D_{f_0}(m)D_\sigma(m)-\Pi_{f_0\sigma}(m)\Pi_{\sigma f_0}(m)$, $D_\sigma(m)$ and $D_{f_0}(m)$ are the inverted propagators of the σ and f_0 mesons, and $\Pi_{\sigma f_0}(m)=\Pi_{f_0\sigma}(m)$ is the off-diagonal element of the polarization operator, which mixes the σ and f_0 mesons. All the details can be found in Refs. [7,8,10].

The double differential rate of the $D_s^+\to s\bar{s}e^+\nu\to[\sigma(600)+f_0(980)]e^+\nu\to\pi^+\pi^-e^+\nu$ decay is

$$\begin{aligned}\frac{d^2\Gamma(D_s^+\to\pi^+\pi^-e^+\nu)}{dq^2dm}&=\frac{G_F^2|V_{cs}|^2}{24\pi^3}g_{D_s^+c\bar{s}}^2|f_A(q^2)|^2p_1^3(q^2,m)\frac{1}{8\pi^2}m\rho_{\pi\pi}(m)\left|\frac{1}{\Delta(m)}\right|^2|F_\sigma g_{s\bar{s}\sigma}D_{f_0}(m)g_{\sigma\pi^+\pi^-}\\ &\quad+F_\sigma g_{s\bar{s}\sigma}\Pi_{\sigma f_0}(m)g_{f_0\pi^+\pi^-}+F_{f_0}g_{s\bar{s}f_0}\Pi_{f_0\sigma}(m)g_{\sigma\pi^+\pi^-}+F_{f_0}g_{s\bar{s}f_0}D_\sigma(m)g_{f_0\pi^+\pi^-}|^2,\end{aligned}\tag{10}$$

where $\rho_{\pi\pi}(m)=\sqrt{1-4m_\pi^2/m^2}$.

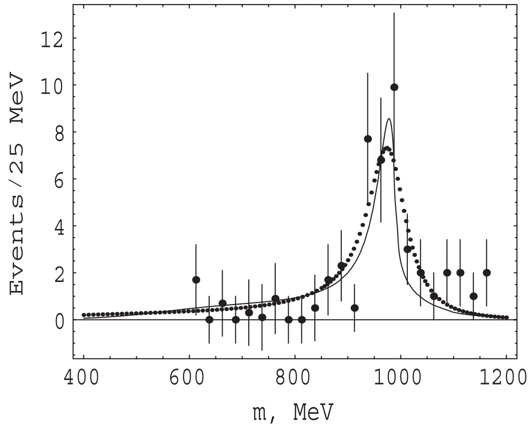

FIG. 2. The CLEO data [13] on the invariant $\pi^+\pi^-$ mass (m) distribution for $D_s^+ \to \pi^+\pi^- e^+\nu$ decay with the subtracted backgrounds, which are calculated in Ref. [13]. The dotted line is fit from Ref. [13], Fig. 9, corresponding to $\mathrm{BR}(D_s^+ \to f_0(980)e^+\nu)\mathrm{BR}(f_0(980) \to \pi + \pi^-) = (0.20 \pm 0.03 \pm 0.01)$. Our theoretical curve is the solid line.

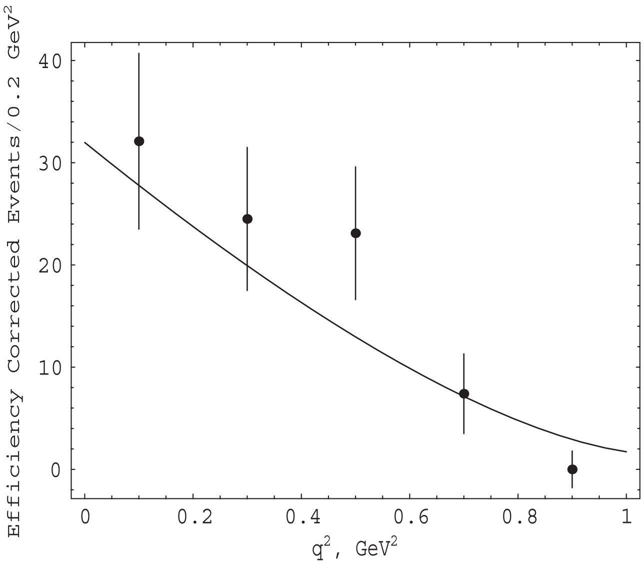

FIG. 3. The q^2 distribution for $\mathrm{BR}(D_s^+ \to f_0(980)e^+\nu)$. The axial-vector dominance model, see Eq. (4), describes the CLEO data [13] quite satisfactorily.

When $\Pi_{\sigma f_0}(m) = \Pi_{f_0\sigma}(m) = 0$ and $g_{s\bar{s}\sigma} = 0$:

$$\frac{d^2\Gamma(D_s^+ \to \pi^+\pi^- e^+\nu)}{dq^2dm} = \frac{G_F^2|V_{cs}|^2}{24\pi^3} g^2_{D_s^+ c\bar{s}} |f_A(q^2)|^2 p_1^3(q^2, m) \frac{2}{\pi} \frac{m^2\Gamma(f_0 \to \pi^+\pi^- m)}{|D_{f_0}(m)|^2}. \tag{11}$$

When fitting the CLEO [13], we use the parameters of the resonances obtained in Ref. [8] in the analysis of the $\pi\pi$ scattering and the $\phi \to \gamma(\sigma + f_0) \to \pi^0\pi^0$ decay. So the 44 events in Fig. 2 determine only one parameter $f_+^\sigma(0)/f_+^{fo}(0)$. In this case the Adler self-consistency condition [the Adler zero at m^2 near $(m_\pi^2)/2$] determines $f_+^\sigma(0)/f_+^{f_0}(0) = (F_\sigma g_{s\bar{s}\sigma})/(F_{f_0} g_{s\bar{s}f_0}) = 0.039$, 0.014, 0.055, 0.058, 0.032, 0.055 for six fits from Ref. [8]. So the intensity of the $\sigma(600)$ production is much less than the intensity of the $f_0(980)$ production $[(f_+^\sigma(0)/f_+^{f_0}(0))^2 \leq 0.003]$. That is, we find the direct evidence of decoupling of $\sigma(600)$ with the $s\bar{s}$ pair. As far as we know, this is truly a new result, which agrees well with the decoupling of $\sigma(600)$ with the $K\bar{K}$ states, obtained in Ref. [8] $g^2_{\sigma K^+K^-}/g^2_{\sigma\pi^+\pi^-} = 0.04$, 0.001, 0.01, 0.01, 0.003, 0.025 for six fits. The decoupling of $\sigma(600)$ with the $K\bar{K}$ states means also the decoupling of $\sigma(600)$ with $\sigma_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ because σ_q results in $g^2_{\sigma K^+K^-}/g^2_{\sigma\pi^+\pi^-} = 1/4$. Results of our analysis of the CLEO [13] data are shown in Table I and on Figs. 2 and 3. The parameters of the $\sigma(600)$ and $f_0(980)$ mesons are taken from fit 1 of Ref. [8], which describes the spectrum on Fig. 2 better than others $((F_\sigma g_{s\bar{s}\sigma})/(F_{f_0} g_{s\bar{s}f_0}) = 0.039$, $g^2_{\sigma K^+K^-}/g^2_{\sigma\pi^+\pi^-} = 0.04)$. So, the CLEO experiment gives new support in favor of the four-quark ($ud\bar{u}\bar{d}$) structure of the $\sigma(600)$ meson.

In the chirally symmetric model of the Nambu-Jona-Lasinio type the coupling constants of the pseudoscalar and scalar partners with quarks are equal to each other, i.e., $g_{s\bar{s}\eta_s} = g_{s\bar{s}f_{0s}}$, where $f_{0s} = s\bar{s}$. In approximation when the mass of the strange quark much less the mass of the charmed quark ($m_s/m_c \ll 1$) $F_{f_0} = F_{\eta'}$ [17] and we find from Table I (see the last line) that $g^2_{s\bar{s}f_0}/g^2_{s\bar{s}\eta_s} \approx 0.3$. So, the $f_{0s} = s\bar{s}$ part in the $f_0(980)$ wave function is near thirty percent. Taking into account the suppression of the $f_0(980)$ meson coupling with the $\pi\pi$ system,

TABLE I. Results of the analysis of the CLEO [13] data. All quantities are defined in the text.

$\mathrm{Br}(D_s^+ \to f_0e^+ \to \pi^+\pi^- e^+\nu) = 0.17\%$			
$(F_\sigma g_{s\bar{s}\sigma})/(F_{f_0} g_{s\bar{s}f_0})$	$(F^2_{f_0} g^2_{s\bar{s}f_0})/(F^2_\eta g^2_{s\bar{s}\eta})$	$(F^2_{f_0} g^2_{s\bar{s}f_0})/(F^2_{\eta'} g^2_{s\bar{s}\eta'})$	$(F^2_\eta g^2_{s\bar{s}\eta})/(F^2_{\eta'} g^2_{s\bar{s}\eta'})$
0.039	0.67	0.49	0.73
The $\eta - \eta'$ mixing			
θ_P	−11°	−14°	−18°
$(F^2_{f_0} g^2_{s\bar{s}f_0})/(F^2_\eta g^2_{s\bar{s}\eta_s})$	0.32	0.29	0.24
$(F^2_{f_0} g^2_{s\bar{s}f_0})/(F^2_{\eta'} g^2_{s\bar{s}\eta_s})$	0.27	0.28	0.31

$g^2_{f_0\pi^+\pi^-}/g^2_{f_0K^+K^-} = 0.154$; see fit 1 in the Table I of Ref. [8], one can conclude that the $f_{0q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ part in the $f_0(980)$ wave function is suppressed also. So, the CLEO experiment gives strong support in favor of the four-quark ($sd\bar{s}\,\bar{d}$) structure of the $f_0(980)$ meson, too.

Certainly, there is an extreme need in experiment on the $D_s^+ \to \pi^+\pi^- e^+\nu$ decay with high statistics.

Of great interest is the experimental search for the decays $D^0 \to d\bar{u}e^+\nu \to a_0^-(980)e^+\nu \to \pi^-\eta e^+\nu$ and $D^+ \to d\bar{d}e^+\nu \to a_0^0(980)e^+\nu \to \pi^0\eta e^+\nu$ (or the charge conjugate ones), which will give the information about the $a_q^- = d\bar{u}$ (or $a_q^+ = u\bar{d}$) and $a_q^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$ components in the $a_0^-(980)$ and a_0^0 wave functions, respectively.

No less interesting is also the search for the decays $D^+ \to d\bar{d}e^+\nu \to [\sigma(600) + f_0(980)]e^+\nu \to \pi^+\pi^- e^+\nu$ (or the charge conjugate ones), which will give the information about the $\sigma_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $f_{0q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ components in the $\sigma(600)$ and $f_0(980)$ wave functions, respectively.

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[11] In particular, the was shown that the ideal $q\bar{q}$ model prediction $g^2_{f_0(980)\gamma\gamma}:g^2_{a_0^0(980)\gamma\gamma} = 25:9$ is excluded by experiment in contrast to the similar prediction for the tensor states $f_2(1270)$ and $a_2(1320)$. We mean here the $f_0(980) = (u\bar{u} + d\bar{d})\sqrt{2}$ and $a_0^0(980) = (u\bar{u} - d\bar{d})\sqrt{2}$ case for equality of the masses: $m_{f_0} = m_{a_0^0}$.

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