Transport properties of anisotropically expanding quark-gluon plasma within a quasiparticle model

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(Received 24 February 2012; published 4 December 2012)

The bulk and shear viscosities (η and ζ) have been studied for quark-gluon plasma produced in relativistic heavy ion collisions within semiclassical transport theory, in a recently proposed quasiparticle model of (2 + 1)-flavor lattice QCD equation of state. These transport parameters have been found to be highly sensitive to the interactions present in hot QCD. Contributions to the transport coefficients from both the gluonic sector and the matter sector have been investigated. The matter sector is found to be significantly dominating over the gluonic sector in the cases of both η and ζ. The temperature dependences of the quantities $\zeta/\mathcal{S}$ and ζ/η indicate a sharply rising trend for the ζ, closer to the QCD transition temperature. Both η and ζ are shown to be equally significant for the temperatures that are accessible in the relativistic heavy ion collision experiments and hence play a crucial role in investigating the properties of the quark-gluon plasma.

DOI: 10.1103/PhysRevD.86.114008 PACS numbers: 25.75.−q, 24.85.+p, 05.20.Dd, 12.38.Mh

I. INTRODUCTION

The study of transport coefficients for hot QCD matter is an area of intense research since the discovery of a fluidlike picture of quark-gluon plasma (QGP) in the relativistic heavy ion collider (RHIC) at Brookhaven National Laboratory BNL [1]. The discovery of the QGP is attributed to the fact that at extreme energy density and temperature, ordinary nuclear matter goes through a transition to the QGP phase as predicted by the finite temperature quantum chromodynamics (QCD) (this transition is shown to be a crossover [2] at the vanishing baryon density).

To describe a fluid, shear and bulk viscosities (η and ζ, respectively) are very important physical quantities that characterize dissipative processes during hydrodynamic evolution. The former describes the entropy production due to the transformation of the shape of the hydrodynamic system at a constant volume, and the latter describes the entropy production at the constant rate of change of the volume of the system (hot fireball at the RHIC). Moreover, $\eta/\mathcal{S}$, and $\zeta/\mathcal{S}$ serve as the inputs while studying the hydrodynamic evolution of the fluid [3,4]. One can also couple hydrodynamics with the Boltzmann descriptions at the later stages after the collisions of heavy ions at the RHIC by maintaining the continuity of the entire stress-energy tensor and currents. The process could be translated in terms of the viscous modifications to the thermal distribution functions of particles. This leads to a smooth transition from the hydrodynamic regime where the mean free paths are short to a region where hydrodynamics is inapplicable and Boltzmann treatments seem to be justified [5]. Therefore, this opens a way to study the impact of transport coefficients of the QGP in various processes at the RHIC and the ongoing heavy ion experiments at the Large Hadron Collider (LHC), CERN (e.g., dilepton production, quarkonia physics, etc.). Regarding viscous corrections to the dilepton production rate at the RHIC, we refer the reader to Ref. [6]. The determinations of η and ζ have to be done separately from a microscopic theory—either from a transport equation [7] with appropriate force, collision, and source terms or equivalently from the field theoretic approach by employing the Green-Kubo formulas [8] (long wavelength behavior of the correlations among various components of the stress-energy tensor).

The QGP is strongly interacting at the RHIC [1], as inferred from the flow measurements, and strong jet quenching has been observed there. This observation is found to be consistent with the lattice simulations of the hot QCD equation of state (EOS) [9,10], which predicts a strongly interacting behavior even at temperatures that are of the order of a few T_c (the QCD transition temperature). The flow measurements suggest a very tiny value for the ratio of η to the entropy density, $\mathcal{S}$ ($\eta/\mathcal{S}$), for the QGP and the near-perfect fluid picture [11–14] (except near the QCD transition temperature, where $\zeta/\mathcal{S}$ is equally significant as $\eta/\mathcal{S}$ [15–18]).

Preliminary studies at the LHC [19–21] reconfirm the above-mentioned observations regarding the QGP. In heavy-ion collisions at the LHC, in addition to the elliptic flow obtained at the RHIC, there are other interesting flow patterns, viz., the dipolar and the triangular flow, which are sensitive to the initial collision geometry [22]. There have been recent interesting studies to understand them at LHC [19,23]. A more precise measurement of various flows and jet quenching at LHC is awaited. On the other hand, ζ has achieved considerable attention in the context of the QGP after the interesting reports on its rising value close to the QCD transition temperature [15]. Subsequently, the

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possible impact of the large bulk viscosity of the QGP at the RHIC have been studied by several authors. Song and Heinz [24] have studied the interplay of shear and bulk viscosities in the context of collective flow in heavy ion collisions. Their study revealed that one cannot simply ignore the bulk viscosity while modeling the QGP. In this context, there are other interesting studies in the literature [25–31]. The role of bulk viscosity in the freeze-out phenomenon has been offered in Refs. [21,32]. Effects of bulk viscosity in the hadronic phase, and in the hadron emission, have been studied in Ref. [33]. Interestingly, in the recent investigations, these transport coefficients are found to be very sensitive to the interactions [13,14] and the nature of the phase transition in QCD [34]. Another crucial aspect of ζ is its influence on the domain of applicability of hydrodynamics at the RHIC, viz., the phenomenon cavitation. This phenomenon has been addressed in detail in the context of the diverging value of ζ near the QCD transition temperature in Refs. [35,36]. Thus, the determinations of η and ζ for the QGP have multifaceted dimensions and significant impact on the variety of physical phenomena at the RHIC and the LHC. Subsequently, the cavitation in a particular string theory model ($N = 2^*$ SU(N) theory which is nonconformal and mass deformation of $N = 4$, SU(4) Yang-Mills) has been investigated by Klimek *et al.* in Ref. [37]. They have observed the absence of cavitation before phase transition is reached by investigating the flow equations in a $(1 + 1)$-dimensional boost-invariant setup, which is in contrast to the finding of Rajgopal and Tripuraneni [35] for hot QCD. They further argued that such a behavior is mainly due to the smaller value of ζ, a sharp rise in the relaxation time for such theories near the transition point, and perhaps the quantum corrections to η and ζ [38]. These studies might play a crucial role in understanding the behavior of strongly coupled QGP in the RHIC and the LHC.

The determinations of η and ζ have been performed adopting the viewpoint based on the inference drawn from the experimental results and the lattice QCD (the best known nonperturbative technique to address the QGP). Lattice QCD has indeed been very successful in studying the QGP thermodynamics. However, the computation of the transport coefficients in lattice QCD is a very nontrivial exercise, due to several uncertainties and inadequacy in their determination. Despite that, there are a few first results computed from lattice QCD for bulk and shear viscosities [39–42] that have observed a small value of $\eta/\mathcal{S}$ and large $\zeta/\mathcal{S}$ at the RHIC. A very recent interesting analysis [43] suggests that it is possible to compare the direct lattice results with the experiments at the RHIC. From such a comparison, the QCD transition temperature came out to be around 175 MeV. More refined lattice studies on η and ζ are awaited in the near future.

The work presented in this paper is an attempt (i) to achieve the temperature dependence of η and ζ (The gluonic as well as the matter sector contributions to these transport parameters have been obtained by combing a transport equation with a recently proposed quasiparticle model [44–46] of $(2 + 1)$-flavor lattice QCD EOS. A noteworthy point is that the matter sector has largely been ignored in the literature in this context.) and (ii) to understand the small $\eta/\mathcal{S}$ and large $\zeta/\mathcal{S}$ for the QGP for the temperatures closer to T_c. More precisely, inputs have been taken from the computations of η and ζ in the quasiparticle models [13,14,18,47,48] and combined with a transport theory determination of them in the presence of chromo-Weibel instabilities [12,49,50]. The present work is an extension of our recent work on η [13,14] and ζ [18] for the gluonic sector to the $(2 + 1)$-flavor QCD.

The paper is organized as follows. In Sec. II, we present the formalism to compute the η and ζ. The quasiparticle model and transport equation have also been discussed in brief in the same section. In Sec. III, we have presented the results on the temperature dependence of η and ζ in $(2 + 1)$-flavor lattice QCD and relevant physics. In Sec. IV, we have presented conclusions and future prospects for the present work.

II. DETERMINATION OF TRANSPORT COEFFICIENTS

There may be a variety of physical phenomena that lead to the viscous effects in the QGP (or in general any interacting system) [5]. Among them, our particular focus is on the viscous effects that get contributions from the the classical chromofields.

The idea adopted here is based on the mechanism earlier proposed in Refs. [12,50,51] to explain the small viscosity of a weakly coupled but expanding QGP. The mechanism in the context of the QGP is solely based on the particle transport processes in the turbulent plasmas [52] that are characterized by strongly excited random field modes in certain regimes of instability. They coherently scatter the charged particles and thus reduce the rate of momentum transport. This eventually leads to the suppression of the transport coefficients in plasmas. This phenomenon has been studied in both electromagnetic plasmas [53] and non-Abelian plasmas (QCD plasma) by Asakawa *et al.* [12,50] and further employed for the realistic QGP EOS in Refs. [13,14].

The condition for the spontaneous formation of turbulent fields can be achieved in electromagnetic plasmas with an anisotropic momentum distribution [54] of charged particles and in the QGP with an anisotropic distribution of thermal partons [55]. In the context of pure SU(3) gauge theory, this mechanism turns out to be successful in explaining the small shear viscosity of the QGP and the larger bulk viscosity for the temperatures accessible at the RHIC and the LHC [14,18]. Here, an extension has been made to the case of realistic EOS for the QGP by incorporating the effects from the matter sector (quark-antiquarks).

It will be seen later that the analysis leads to an interesting observation regarding the relative contribution of the gluonic and the matter sectors to the transport parameters. Before that, we present a brief description of the quasiparticle understanding of (2 + 1)-flavor lattice QCD that furnishes an appropriate modeling of the equilibrium state.

A. The quasiparticle description of hot QCD

The quasiparticle description of the hot QCD medium effects is not a new concept. There have been several attempts so far to understand the hot QCD medium effects in terms of noninteracting/weakly interacting quasipartons, viz., effective thermal mass models [56,57], effective mass models with temperature-dependent bag parameters to cure the problem of thermodynamic inconsistency [57], effective quasiparticles with gluon condensate [58], Polyakov loop models [59] (the Polyakov loop acts as effective fugacity), and the quasipartons with effective fugacities [44–46]. The last one that will be employed here, shown to be fundamentally distinct from all other mentioned models, is in the spirit of Landau's theory of Fermi liquids. Moreover, the model has been highly successful in interpreting the lattice QCD thermodynamics and bulk and transport properties of hot QCD matter and the QGP in relativistic heavy ion collisions.

In our quasiparticle description for (2 + 1)-flavor lattice QCD [46], we start with the ansatz that the lattice QCD EOS can be interpreted in terms of noninteracting quasipartons having effective fugacities that encode all the interaction effects. We denote them as the gluon-effective fugacity z_g and the quark-antiquark fugacity z_q. In this approach, the hot QCD medium is divided in to two sectors, viz., the effective gluonic sector and the matter sector (light quark sector and strange quark sector). The former refers to the contribution of gluonic action to the pressure which also involves contributions from the internal fermion lines. On the other hand, the latter involve interactions among quarks and antiquarks, as well as their interactions with gluons. The ansatz can be translated into the form of the equilibrium distribution functions $f_{\rm eq} \equiv \{f_g, f_q, f_s\}$ (This notation will be useful later while writing the transport equation in both the sectors in compact notations.) as follows,

$$f_g = \frac{z_g \exp(-\beta p)}{(1 - z_g \exp(-\beta p))}, \qquad f_q = \frac{z_q \exp(-\beta p)}{(1 + z_q \exp(-\beta p))},$$
$$f_s = \frac{z_q \exp(-\beta\sqrt{p^2 + m^2})}{(1 + z_q \exp(-\beta\sqrt{p^2 + m^2}))}, \tag{1}$$

where m denotes the mass of the strange quark, which we choose to be 0.1 GeV. The parameter $\beta = T^{-1}$ denotes the inverse of the temperature. Here, we are working in the units where the Boltzmann constant $K_B = 1$, $c = 1$, and $h/2\pi = 1$. The notation p is nothing but $p \equiv |\vec{p}|$.

We use the notation $\nu_g = 2(N_c^2 - 1)$ for gluonic degrees of freedom, $\nu_q = 2 \times 2 \times N_c \times 2$ for light quarks and $\nu_s = 2 \times 2 \times N_c \times 1$ for the strange quark for SU(N_c). Here, we are dealing with SU(3), so $N_c = 3$. Since the model is valid in the deconfined phase of QCD (beyond T_c), the mass contributions of the light quarks can be neglected as compared to the temperature. Therefore, in our model, we only consider the mass for the strange quarks.

The effective fugacity is not merely a temperature-dependent parameter that encodes the hot QCD medium effects. It is very interesting and physically significant. The physical significance is reflected in the modified dispersion relation in both the gluonic and the matter sectors. In this description, the effective fugacities modify the single quasiparton energy as follows,

$$\omega_g = p + T^2 \partial_T \ln(z_g) \qquad \omega_q = p + T^2 \partial_T \ln(z_q)$$
$$\omega_s = \sqrt{p^2 + m^2} + T^2 \partial_T \ln(z_q). \tag{2}$$

These dispersion relations can be explicated as follows. The single quasiparton energy not only depends upon its momentum but also gets contributions from the collective excitations of the quasipartons. The second term is like the gap in the energy spectrum due to the presence of quasiparticle excitations. This makes the model more in the spirit of the Landau theory of Fermi liquids. For a detailed discussion on the interpretation and physical significance of z_g and z_q, we refer the reader to our recent work [46]. Henceforth, we shall use "gluonic sector" in place of "effective gluonic sector" for the sake of ease. We shall now proceed to the determination of η and ζ in the presence of chromo-Weibel instabilities.

B. Chromo-Weibel instability and the anomalous transport

The determinations of η and ζ have been done in a multistep way. First, we need an appropriate modeling of distribution functions for the equilibrium state. Second, we need to set up an appropriate transport equation to determine the form of the perturbations to the distribution functions. These two steps eventually determine these transport coefficients. For the former step, we employ the quasiparticle model for the (2 + 1)-flavor lattice QCD EOS discussed earlier.

Both η and ζ have two contributions, the same as in the case of the shear viscosity in Ref. [12], the first is the Vlasov term which captures the long-range component of the interactions and the second is the collision term, which models the short-range component of the interaction. Here, we shall concentrate only on the former case. The determinations of shear and bulk viscosities from an appropriate collision term will be a matter of future investigations. Importantly, the analysis adopted here is based on the weak coupling limit in QCD; therefore, the results are shown beyond $1.2T_c$, assuming the validity of weak coupling results for the QGP there. Note that the interplay for anomalous and collisional components of η has been discussed in Refs. [12–14], and in the case of ζ for the pure gauge theory, a discussion has been presented regarding the interplay of the collisional [60–62] and anomalous components in Ref. [18]. It seems that at the conceptual level, all the observations in Ref. [18] regarding the interplay will remain valid here. Since we do not have results for the matter sector, we shall not offer a quantitative discussion on such an interplay here. There have been computations of transport parameters in the case of pure gauge theory based on the effective mass models within the relaxation time approximation [63]. The approach adopted and the physical setup are entirely distinct in the present case. It is to be noted that the gluonic component in all the quantities is denoted by sub/superscript g, for the light- quark components by q, and for the strange-quark components by s.

C. Determination of ζ and η

Let us first briefly outline the standard procedure of determining transport coefficients in transport theory [7,12]. The bulk and shear viscosities, ζ and η, of the QGP in terms of equilibrium parton distribution functions are obtained by comparing the kinetic theory definition of the stress tensor with the fluid dynamic definition of the viscous stress tensor.

In kinetic theory, the stress tensor is defined as

$$T^{\mu\nu} = \sum \int \frac{d^3\vec{p}}{(2\pi)^3\omega} p^\mu p^\nu f(\vec{p},\vec{r}), \tag{3}$$

where the sum is over all species (in the present case, gluons, light quarks, and strange quarks) including the internal degrees of freedom which is implicit in Eq. (3). The quantities $\omega \equiv \{\omega_g, \omega_q, \omega_s\}$ combined denote the quasiparticle dispersions, and $f(\vec{p},\vec{r})$ is the combined notation for the quasiparticle distribution functions.

This form of $T^{\mu\nu}$ does not capture the medium modifications encoded in the nontrivial dispersion relations, ω, and hence does not implement the thermodynamic consistency condition correctly. This is very crucial in its own merit and also needed to relate to the hydrodynamic definition of $T^{\mu\nu}$. In the present case, to obtain the correct expression of the energy density, one needs to modify the 4-momenta of the quasiparticles, which is not allowed in the model in view of the particular mathematical structure of the equilibrium distribution functions in Eq. (1). To cure the problem, the definition of $T^{\mu\nu}$ needs to be modified such that $u_\mu u_\nu T^{\mu\nu} = \epsilon$ (true energy density). This can be achieved by the revised definition of $T^{\mu\nu}$ in the case of our quasiparticle model with effective fugacities,

$$\begin{aligned} T^{\mu\nu} = \sum\bigg\{ & \int \frac{d^3\vec{p}}{(2\pi)^3\omega} p^\mu p^\nu f(\vec{p},\vec{r}) \\ & + \int \frac{d^3\vec{p}}{(2\pi)^3 E_p\omega}(\omega - E_p) p^\mu p^\nu f_0(\vec{p},\vec{r}) \\ & + \int \frac{d^3\vec{p}}{(2\pi)^3}(\omega - E_p) u^\mu u^\nu f_0(\vec{p},\vec{r})\bigg\}, \end{aligned} \tag{4}$$

where E_p denote the dispersions without medium modifications, $E_p = p$ for gluons and light quarks and $E_p = \sqrt{p^2+m^2}$ for the s-quarks, and antiquarks, respectively. Therefore, one can clearly realize the presence of the factors, $T^2\frac{d\ln(z_g)}{dT}$ and $T^2\frac{d\ln(z_q)}{dT}$ in the expression for $T^{\mu\nu}$. The second term in the right-hand side of Eq. (4) ensures the correct expression for the pressure, and the third term ensures the correct expression for the energy density, and hence the definition of $T^{\mu\nu}$ incorporates the thermodynamic consistency condition correctly. This issue is realized in a similar way in the effective mass quasiparticle models in Ref. [64], and accordingly the modified definition of $T^{\mu\nu}$ is employed which contains the temperature derivative of the effective mass.

On the other hand, in hydrodynamics the expression for the viscous stress tensor up to first order in the gradient expansion is given by

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu} - \Pi\Delta^{\mu\nu} + \pi^{\mu\nu}, \tag{5}$$

where u^μ is the fluid 4-velocity, $g^{\mu\nu}$ is the metric tensor, $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ is the orthogonal projector, Π is the bulk part of the stress tensor, and $\pi^{\mu\nu}$ is the shear stress. Here, ϵ is the energy density and P is the pressure of the fluid.

In the first-order (Navier-Stokes) approximation, the viscous (dissipative) parts of the stress-energy tensor in Eq. (5) can be obtained in the local rest frame of the fluid (LRF) as

$$\begin{aligned} \pi_{ij} &= -2\eta(\nabla u)_{ij} \\ (\nabla u)_{ij} &= \frac{\partial_i u_j + \partial_j u_i}{2} - \frac{1}{3}\delta_{ij}\partial_i u^j, \\ \Pi &= -\zeta\nabla\cdot\vec{u} \equiv \partial_k u^k, \end{aligned} \tag{6}$$

where $(\nabla u)_{ik}$ is the traceless, symmetrized velocity gradient, $\nabla\cdot\vec{u}$ is the divergence of the fluid velocity field, and η and ζ refer to the (η_g, η_q, η_s) and $(\zeta_g, \zeta_q, \zeta_s)$ (later we shall write them explicitly). In the LRF, $[u^\mu = (1,0,0,0)]$, $f_0 \equiv \{f_g, f_q, f_s\}$.

Next, to determine ζ an η, one writes the parton distribution functions as

$$f(\vec{p},\vec{r}) = \frac{1}{z_{g/q}^{-1}\exp(\beta u^\mu p_\mu + f_1(\vec{p},\vec{r})) \mp 1}. \tag{7}$$

Assuming that $f_1(\vec{p},\vec{r})$ is a small perturbation to the equilibrium distribution, we expand $f(\vec{p},\vec{r})$ and keeping only the linear order term in f_1, one obtains

$$f(\vec{p},\vec{r}) = f_0(p) + \delta f(\vec{p},\vec{r}) = f_0(p)(1 + f_1(\vec{p},\vec{r})(1 \pm f_0(p)), \qquad (8)$$

where $f_0 \equiv \{f_g, f_q, f_s\}$ and similarly $f_1 \equiv \{f_1^g, f_1^q, f_1^s\}$ in the LRF, and $p \equiv |\vec{p}|$ throughout the computations. The plus sign in the bracket is for gluons, and the minus sign is for fermions (q and s). Next, we shall consider these quantities explicitly in the gluonic and the matter sectors. As discussed in Refs. [12,14], ζ and η are determined by taking the following form of the perturbation f_1,

$$f_1^g(\vec{p},\vec{r}) = -\frac{1}{\omega_g T^2} p_i p_j (\Delta_{1g}(p)(\nabla u)_{ij} + \Delta_{2g}(\vec{p})(\nabla\cdot\vec{u})\delta_{ij})$$
$$f_1^q(\vec{p},\vec{r}) = -\frac{1}{\omega_q T^2} p_i p_j (\Delta_{1q}(p)(\nabla u)_{ij} + \Delta_{2q}(\vec{p})(\nabla\cdot\vec{u})\delta_{ij})$$
$$f_1^s(\vec{p},\vec{r}) = -\frac{1}{\omega_s T^2} p_i p_j (\Delta_{1s}(p)(\nabla u)_{ij} + \Delta_{2s}(\vec{p})(\nabla\cdot\vec{u})\delta_{ij}). \qquad (9)$$

Here, dimensionless functions $\Delta_{1g,1q,1s}(p)$, $\Delta_{2g,2q,2s}(\vec{p})$ measure the deviation from the equilibrium configuration. $\Delta_1(p)$, $\Delta_2(\vec{p})$ lead to η and ζ, respectively. Note that $\Delta_{1g,1q,1s}(p)$ is an isotropic function of the momentum in contrast to $\Delta_{2g,2q,2s}(\vec{p})$, which is an anisotropic in momentum $\vec{p}$. This is specifically associated with the structure of the Vlasov operator in the present case. In this case, we seek a solution of the effective transport equation for the bulk viscosity that satisfies the Landau-Lifshitz (LL) condition, $u_\mu \delta T^{\mu\nu} = 0$, to ensure that we have followed the description of Chakraborty and Kapusta [47], which has been discussed in Sec. II E.

Since ζ and η are Lorentz scalars; they may be evaluated conveniently in the LRF (in the LRF $f_0 \equiv f_{\rm eq}$). Considering the a boost invariant longitudinal flow, $\nabla\cdot\vec{u} = \frac{1}{\tau}$ and, $(\nabla u)_{ij} = \frac{1}{3\tau}\,{\rm diag}(-1,-1,2)$ in the LRF. In this case, the perturbations, $f_1(p)$ take the form

$$f_1^g(\vec{p}) = -\frac{\Delta_{1g}(p)}{\omega_g T^2 \tau}\left(p_z^2 - \frac{p^2}{3}\right) - \frac{\Delta_{2g}(\vec{p})}{\omega_g T^2 \tau} p^2$$
$$f_1^q(\vec{p}) = -\frac{\Delta_{1q}(p)}{\omega_q T^2 \tau}\left(p_z^2 - \frac{p^2}{3}\right) - \frac{\Delta_{2q}(\vec{p})}{\omega_q T^2 \tau} p^2, \qquad (10)$$
$$f_1^s(\vec{p}) = -\frac{\Delta_{1s}(p)}{\omega_s T^2 \tau}\left(p_z^2 - \frac{p^2}{3}\right) - \frac{\Delta_{2s}(\vec{p})}{\omega_s T^2 \tau} p^2,$$

where τ is the proper time ($\tau = \sqrt{t^2 - z^2}$). The shear viscosities are obtained in terms of entirely unknown functions $\Delta_{1g,1q,1s}(p)$ as

$$\eta_g = \frac{\nu_g}{15T^2}\int \frac{d^3\vec{p}}{8\pi^3}\frac{p^4}{\omega_g^2}\Delta_{1g}(p) f_g(1+f_g)$$
$$\eta_q = \frac{\nu_q}{15T^2}\int \frac{d^3\vec{p}}{8\pi^3}\frac{p^4}{\omega_q^2}\Delta_{1q}(p) f_q(1-f_q) \qquad (11)$$
$$\eta_s = \frac{\nu_s}{15T^2}\int \frac{d^3\vec{p}}{8\pi^3}\frac{p^4}{\omega_s^2}\Delta_{1s}(p) f_s(1-f_s).$$

The bulk viscosities are obtained in terms of the unknown functions $\Delta_{2g,2q,2s}(\vec{p})$,

$$\zeta_g = \frac{\nu_g}{3T^2}\int \frac{d^3\vec{p}}{8\pi^3}\frac{p^2}{\omega_g^2}(p^2 - 3c_s^2\omega_g^2)\Delta_{2g}(\vec{p}) f_g(1+f_g)$$
$$\zeta_q = \frac{\nu_q}{3T^2}\int \frac{d^3\vec{p}}{8\pi^3}\frac{p^2}{\omega_q^2}(p^2 - 3c_s^2\omega_q^2)\Delta_{2q}(\vec{p}) f_q(1-f_q)$$
$$\zeta_s = \frac{\nu_s}{3T^2}\int \frac{d^3\vec{p}}{8\pi^3}\frac{p^2}{\omega_s^2}(p^2 - 3c_s^2\omega_s^2)\Delta_{2s}(\vec{p}) f_s(1-f_s). \qquad (12)$$

Notice that while obtaining the expression for the bulk viscosity, we have exploited the LL condition for the stress-energy tensor. The factor $(-3c_s^2\omega^2)$ in the right-hand side of Eq. (12) is coming only because of that. The appearance of this factor is not so straightforward. To obtain it, one has to look for a particular solution of the transport equation for ζ so that the viscous stress tensor satisfies the LL condition. Such a solution is obtained by invoking the conservation laws and thermodynamic relations in quite a general way in Ref. [47] and is valid in the present case at the level of formalism (see Sec. II E). The modifications will appear only in terms on new equilibrium distribution functions, and the modified dispersion relations, ω. There is no such issue with the η since physically it is associated with the response with the change in the shape of the system at constant volume; on the other hand, ζ is linked with the volume expansion at a fixed shape. Here, c_s^2 is the speed of sound square extracted from the lattice data on $(2+1)$-flavor lattice QCD. The determination of $\Delta_{1g,1q,1s}(p)$ and $\eta_{g,q,s}$ can easily be done following Refs. [13,14], and $\Delta_{2g,2q,2s}(\vec{p})$ and $\zeta_{g,q,s,}$ following Ref. [18].

D. Determination of the perturbative Δ_1 and Δ_2

To obtain an analytic expression for the perturbations, $\Delta_{1,2}$, in our analysis, one needs to first set up the transport equation in the presence of turbulent color fields. This has been done in Refs. [12–14] in the recent past. Here, we only quote the linearized transport equation, with Vlasov-Dupree diffusive term, which arises after considering the ensemble average over turbulent color fields in the light cone frame. The transport equation thus obtained reads

$$v^\mu \frac{\partial}{\partial x^\mu} f_{\rm eq}(p) + \mathbf{V}_A f_1 f_{\rm eq}(p)(1 \pm f_{\rm eq}(p)) = 0, \qquad (13)$$

where ($f_{\rm eq} \equiv f_g, f_q, f_s$), and $v^\mu \equiv (1, \vec{v}_p)$, where $\vec{v}_p = \partial_{\vec{p}}\omega$ is the quasiparticle velocity. It is easy to realize that the quasiparticle model does not change the group velocity of the quasipartons. Note that Eq. (13) is written in the absence of the collision term and assuming the weak coupling approximation.

The mathematical structure of the Vlasov-Dupree operator is as follows,

$$\mathbf{V}_A = \frac{g^2 C_2}{2(N_c^2-1)\omega^2}\langle E^2 + B^2\rangle \tau_m \mathbf{L}^2, \tag{14}$$

where C_2 is the quadratic Casimir invariant for partons. For gluons, $C_2 = N_c$, and for quarks, $C_2 = (N_c^2-1)/2N_c$. Here, $\omega \equiv \{\omega_g, \omega_q, \omega_s\}$ denotes the quasiparton dispersions, and g^2 is the QCD coupling constant at finite temperature. The quantities E^a and B^a denote the chromofield strengths, where a is the SU(3) color index, and $\langle E^2 + B^2\rangle \equiv \langle E^a \cdot E^a + B^a \cdot B^a\rangle$. The bracket $\langle \cdots \rangle$ denotes the ensemble average over the color field configurations which are turbulent (grow in time with a time scale τ_m) as described in Ref. [12]. The anomalous transport coefficients in this approach are obtained by invoking the argument that soft color fields are turbulent. Their action on quasipartons can be described by considering the ensemble average over the color fields that leads to an effective force term in the linearized transport equation. The parameter τ_m is the time scale associated with instability in the field, and the operator $\mathbf{L}^2$ is

$$\mathbf{L}^2 = -(\vec{p}\times\partial_{\vec{p}})^2 + (\vec{p}\times\partial_{\vec{p}})|_z^2 \quad \equiv -(L^p)^2 + (L^p{}_z)^2. \tag{15}$$

Since $\mathbf{L}^2$ contains angular momentum operator L^p, it therefore gives a nonvanishing contribution while operating on an anisotropic function of $\vec{p}$. It will always lead to the vanishing contribution while operating on an isotropic function of $\vec{p}$. Following [14], the expression for the $\Delta_{1g}(p)$ is obtained as

$$\Delta_{1g}(p) = \frac{2(N_c^2-1)\omega_g^2 T}{3C_g g^2\langle E^2+B^2\rangle\tau_m}. \tag{16}$$

On the other hand, expressions for $\Delta_{1q,1s}$ are obtained as

$$\begin{aligned}\Delta_{1q}(p) &= \frac{2(N_c^2-1)\omega_q^2 T}{3C_f g^2\langle E^2+B^2\rangle\tau_m}\\ \Delta_{1s}(p) &= \frac{2(N_c^2-1)\omega_s^2 T}{3C_f g^2\langle E^2+B^2\rangle\tau_m}.\end{aligned} \tag{17}$$

Now, we write the transport equation containing only those terms that contribute to bulk viscosity ζ as

$$\begin{aligned}&\left(\frac{p^2}{3\omega^2} - c_s^2\right)\frac{\omega}{T}(\nabla\cdot\vec{u}) f_{\rm eq}(1\pm f_{\rm eq})\\ &\quad = \frac{g^2 C_2}{3(N_c^2-1)\omega^2}\langle E^2+B^2\rangle\tau_m \mathbf{L}^2 f_1(\vec{p},\vec{r}) f_{\rm eq}(1\pm f_{\rm eq}).\end{aligned} \tag{18}$$

Following [18], we can obtain the mathematical forms of the corresponding perturbations, Δ_2. We shall write down the expressions in the gluonic sector and matter sector separately to avoid any confusion. The expression for $\Delta_{2g}(p)$ is obtained as

$$\Delta_{2g}(\vec{p}) = \frac{4(N_c^2-1)T\omega_g^2}{N_c g^2\langle E^2+B^2\rangle\tau_m p^2}\left(\frac{p^2}{3} - c_s^2\omega_g^2\right)\ln\left(\frac{p_T}{\sqrt{6}T}\right). \tag{19}$$

On the other hand, the expressions for $\Delta_{2q,2s}$ are obtained as

$$\begin{aligned}\Delta_{2q}(\vec{p}) &= \frac{4(N_c^2-1)T\omega_q^2}{C_2 g^2\langle E^2+B^2\rangle\tau_m p^2}\left(\frac{p^2}{3} - c_s^2\omega_q^2\right)\ln\left(\frac{p_T}{\sqrt{6}T}\right)\\ \Delta_{2s}(\vec{p}) &= \frac{4(N_c^2-1)T\omega_s^2}{C_2 g^2\langle E^2+B^2\rangle\tau_m p^2}\left(\frac{p^2}{3} - c_s^2\omega_s^2\right)\ln\left(\frac{p_T}{\sqrt{6}T}\right).\end{aligned} \tag{20}$$

Next, we relate the denominator of Eqs. (17), (19), and (20), to the parton energy loss parameter $\hat{q} \equiv \hat{q}_g, \hat{q}_q$, via the relation [51],

$$\hat{q} = \frac{2g^2 C_2}{3(N_c^2-1)}\langle E^2+B^2\rangle\tau_m. \tag{21}$$

The relation of $\hat{q}$ with the transport parameters in the present analysis is attributed to the fact that radiative energy loss ($\hat{q}$ being a measure) depends on the rate of momentum exchange between the fast parton and the QCD medium. More precisely, $\hat{q}$ is assumed as a rate of growth of the transverse momentum fluctuations of a fast parton to an ensemble of turbulent color fields, expressed as in Eq. (21).

Now the gluonic contributions to η and ζ in terms of $\hat{q}$ can be rewritten as follows:

$$\begin{aligned}\eta_g &= \frac{T^6}{\hat{q}}\frac{64(N_c^2-1)}{3\pi^2}\mathrm{PolyLog}[6, z_g],\\ \zeta_g &= \frac{4(N_c^2-1)}{3T\pi^2\hat{q}}\iint p_T dp_T dp_z\left(\frac{p^2}{3} - c_s^2\omega_g^2\right)^2\\ &\quad\times\ln\left(\frac{p_T}{p_0}\right)\times f_g(1+f_g).\end{aligned} \tag{22}$$

On the other hand, quark-antiquark viscosities in the matter sector are obtained as

$$
\begin{aligned}
\eta_q &= \frac{64N_c^2\nu_q}{3\pi^2\hat{q}(N_c^2-1)}\{-\mathrm{PolyLog}[6,-z_q]\} \\
\eta_s &= \frac{64N_c^2\nu_s}{3\pi^2\hat{q}(N_c^2-1)}\bigg\{-\mathrm{PolyLog}[6,-z_q] \\
&\quad + \frac{\tilde{m}^2}{2}\mathrm{PolyLog}[5,-z_q]\bigg\} \\
\zeta_{q,s} &= \frac{N_c\nu_{q,s}}{3C_fT\pi^2\hat{q}}\iint p_T dp_T dp_z\bigg(\frac{p^2}{3}-c_s^2\omega_{q,s}^2\bigg)^2 \\
&\quad \times \ln\bigg(\frac{p_T}{p_0}\bigg)\times f_{q,s}(1-f_{q,s}). \qquad (23)
\end{aligned}
$$

Here $\tilde{m} \equiv m/T$ (mass of the strange quark scaled with temperature), and the *PolyLog* functions that appear in the expressions for $\eta_{g,q,s}$ are defined in terms of the series representation as

$$
\mathrm{Ploylog}[n,x] = \sum_{k=1}^{\infty}\frac{x^k}{k^n}, \qquad (24)
$$

where n is a positive integer, and the convergence of the series is ensured by the fact that $x \leq 1$. Moreover, $\mathrm{PolyLog}[n,1] \equiv \zeta(n)$ and also $\mathrm{PolyLog}[n,-1] \sim -\zeta(n)$.

Clearly from Eqs. (22) and (23), the various components of η and ζ have strong dependence on the hot QCD EOS through the parameters $z_{g,q}$ and their first-order derivatives with respect to temperature, the speed of sound c_s^2 and $\hat{q}$ (speed-of-sound dependence is only there in ζ). Therefore, before discussing the results for a particular lattice EOS utilized in this analysis, it is instructive to discuss the dependence of lattice EOS on η and ζ in view of the uncertainties in the height and width of the interaction measure (trace anomaly) computed in lattice QCD at finite temperature by different collaborations. The temperature dependence of z_g and z_q is mainly dependent on the temperature dependence of the interaction measure. The former is directly related to the contributions coming from the gluonic action and later depends on the interaction measure in the $(2+1)$-flavor QCD subtracting gluonic contribution. Therefore, they both carry effects of lattice artifacts and uncertainties from the beginning of their determination. The same is true for c_s^2, since it has strong dependence on the behavior of the interaction measure as a function of temperature. In fact, c_s^2 is related to the temperature derivative of the trace anomaly scaled with the energy density [65]. Therefore, it would be appropriate to compare the predictions on ζ and η based on the lattice data from various groups on the hot QCD EOS. However, this is beyond the scope of the present work, since we need lattice data from various lattice groups not only for the full $(2+1)$-flavor QCD but also for the contributions from the gluonic action to the EOS within the same lattice computational setup, which is not an easy task. Moreover, it is not possible to use the pure SU(3) EOS since it shows at the first-order transition, in contrast to crossover shown by $(2+1)$-flavor QCD at vanishing baryon density. Leaving aside the above comparison for the future, we only concentrate here on a particular set of lattice data [66]. A similar analysis with the more recent lattice data on the QGP EOS may possibly induce quantitative modifications to η and ζ through the temperature dependence of z_g and c_s^2. We strongly believe that the interesting physical observations regarding η and ζ will remain intact. The present analysis led us to strongly believe that there will a be a strong impact of temperature dependence of the interaction measure specifically on ζ and the ratio ζ/η for the temperatures closer to T_c.

Next, the components of η employing the ideal EOS for quarks and gluons [equivalently, the ideal form of their thermal distribution functions, which are nothing but the equilibrium distribution functions obtained by putting $z_{g,q} \equiv 1$ in Eq. (1)] can straightforwardly be obtained from Eqs. (22) and (23) by substituting $z_g \equiv 1$ and $z_q \equiv 1$. To denote these components, the superscript *Id* (stands for the ideal EOS) is used. We thus obtain

$$
\begin{aligned}
\eta_g^{Id} &= \frac{T^6}{\hat{q}}\frac{64(N_c^2-1)}{3\pi^2}\zeta(6), \\
\eta_q^{Id} &= \frac{T^6}{\hat{q}}\frac{64N_c^2\nu_q}{3\pi^2(N_c^2-1)}\times\frac{31}{32}\zeta(6)\}, \qquad (25) \\
\eta_s^{Id} &= \frac{T^6}{\hat{q}}\frac{64N_c^2\nu_s}{3\pi^2(N_c^2-1)}\bigg\{\frac{31}{32}\zeta(6)+\frac{\tilde{m}^2}{2}\times\frac{15}{16}\zeta(5)\bigg\}.
\end{aligned}
$$

Here, the following relations have been utilized: $\mathrm{PolyLog}[5,-1] = \frac{15}{16}\zeta(5)$ and $\mathrm{PolyLog}[6,1] \equiv \zeta(6) \equiv -\frac{32}{31}\mathrm{PolyLog}[6,-1]$. To appreciate the above expressions more, we can redo the whole analysis with $z_{g,q} \equiv 1$ and unmodified dispersion relations $\omega_{g,q} = p$ and $\omega_s = \sqrt{p^2+m^2}$; we shall end up with the ideal components of η displayed in Eq. (25). The expressions in Eq. (25) will be utilized in the next section while investigating the role of interactions.

E. Landau-Lifshitz condition and the bulk viscosity

Here, we shall briefly describe the LL condition to obtain the form of the expression for ζ given in Eq. (12). We shall argue below that the solution thus obtained follows the LL condition adopting a recent analysis of Charkobarty and Kapusta [47]. Inputs have also been taken from the recent work of Dusling and Schäfer [67] and Dusling and Teaney [64] regarding the viscous hydrodynamics.

Recall that the LL matching condition is a way to specify uniquely ϵ and u^μ in terms of the components of $T^{\mu\nu}$. In the LL convention,

$$
\epsilon = u^\mu u^\nu T^{\mu\nu} \qquad \epsilon u^\mu = u^\nu T^{\mu\nu}. \qquad (26)
$$

The other six independent components of $T^{\mu\nu}$ are obtained by a nonequilibrium viscous stress $\Pi^{\mu\nu} = \pi^{\mu\nu} - \Delta^{\mu\nu}\Pi$ that satisfies $u_\mu\Pi^{\mu\nu} = 0$. It is sufficient that this condition is satisfied in the LRF. This can be translated into the fact that the energy shift due to the nonequilibrium terms vanishes. Denoting this energy shift by $\delta\epsilon$, we obtain the following condition,

$$\delta\epsilon = 0 = \sum_a \int \frac{d^3\vec{p}}{8\pi^3}\omega\delta f, \tag{27}$$

where a sums over g, q, and s. As stated earlier, ω and δf are the combined notations for the nonequilibrium part of the distribution function for these three sectors. Here, we have considered the medium modified dispersion for the single-particle energy to implement the interaction correctly. This is also the same spirit as in the case of the effective mass quasiparticle models described in Ref. [67]. Such effects are encoded in the form of δf through Δ_1 and Δ_2 in the present case. This condition can straightforwardly be satisfied in the case of shear viscosity due to the specific form of $\pi^{\mu\nu}$. The nontrivialities are there in the bulk viscosity sector, which we discuss below.

Next, using Eqs. (18)–(20), we can write Eq. (27) in the presence of the bulk viscosity as

$$\delta\epsilon = \sum_a \int \frac{d^3\vec{p}}{8\pi^3}\omega^2\left(\frac{p^2}{3} - c_s^2\omega^2\right)\tilde{\Delta}_2 f_{\rm eq}(1 \pm f_{\rm eq}). \tag{28}$$

From the expression for Δ_2 in Eqs. (19) and (20), one can easily read off $\tilde{\Delta}_2$ as

$$\tilde{\Delta}_2 = \frac{4(N_c^2 - 1)\omega}{T\tau C_2 g^2\langle E^2 + B^2\rangle\tau_m}\ln\left(\frac{p_T}{\sqrt{6}T}\right). \tag{29}$$

Here, C_2 denotes the respective quadratic Casimir invariants of SU(N_c).

The energy shift in Eq. (28) will vanish if $\omega^2\tilde{\Delta}_2$ happens to be independent of ω and $\vec{p}$ [67], which is based on the definition of the speed of sound ($c_s^2 = \frac{\partial P}{\partial\epsilon}$ at constant $\mathcal{S}$). In this case, Eq. (28) will read

$$\sum_a \int \frac{d^3\vec{p}}{8\pi^3}(p^2 - 3c_s^2\omega^2) f_{\rm eq}(1 \pm f_{\rm eq}) = 0. \tag{30}$$

The above condition cannot be achieved with the ω dependence of $\tilde{\Delta}_2$ in the present case. It will be useful while obtaining the expression for ζ, invoking the LL condition below. In the case of collisional processes only, the quantity $\tilde{\Delta}_2$ is closely related to the relaxation time which is obtained in terms of the inverse of the transport cross section [67]. Clearly, our particular solution for ζ obtained by solving the effective transport equation does not satisfy the LL condition.

Next, we discuss how one gets a physically relevant solution based on this particular solution for ζ that satisfies the LL condition. To that end, we closely follow a recent analysis of Chakraborty and Kapusta [47]. Let us now define a quantity $A_a(\omega)$ for the computational convenience here as

$$A_a(\omega) = \frac{\omega}{3}(p^2 - 3c_s^2\omega^2)\tilde{\Delta}_2. \tag{31}$$

Recall that $\omega \equiv \{\omega_g, \omega_q, \omega_s\}$ and $f_{\rm eq} \equiv \{f_g, f_q, f_s\}$.

In this notation, bulk viscosity ζ will have the following expression (in terms of the particular solution),

$$\zeta = \frac{1}{3}\sum_a \int \frac{d^3\vec{p}}{8\pi^3\omega} p^2 f_{\rm eq}(1 \pm f_{\rm eq}) A_a(\omega). \tag{32}$$

Now following [47], we can consider a shift in $A_a(\omega)$ as $A_a(\omega) \to A'_a(\omega) = A_a(\omega) - b\omega$ in the absence of conserved charges and chemical potentials. This generates another set of solutions with coefficient b being arbitrary. This leads to the following expression for ζ,

$$\zeta = \frac{1}{3}\sum_a \int \frac{d^3\vec{p}}{8\pi^3\omega} p^2 f_{\rm eq}(1 \pm f_{\rm eq})(A_a(\omega) - b\omega). \tag{33}$$

Now to fix b, we demand that the new solution must satisfy the LL condition. This translates into the LL condition for the new solution using Eq. (30) as

$$\sum_a \int \frac{d^3\vec{p}}{8\pi^3}\omega(A_a(\omega) - b\omega) f_{\rm eq}(1 \pm f_{\rm eq}) = 0. \tag{34}$$

Now, recast Eq. (34) as

$$\begin{aligned}\sum_a \int \frac{d^3\vec{p}}{8\pi^3} 3bc_s^2\omega^2 f_{\rm eq}(1 \pm f_{\rm eq}) \\ = \sum_a \int \frac{d^3\vec{p}}{8\pi^3} 3c_s^2\omega A_a(\omega) f_{\rm eq}(1 \pm f_{\rm eq}).\end{aligned} \tag{35}$$

Using the condition given in Eq. (30), we obtain,

$$\begin{aligned}\sum_a \int \frac{d^3\vec{p}}{8\pi^3\omega} b\omega p^2 f_{\rm eq}(1 \pm f_{\rm eq}) \\ = \sum_a \int \frac{d^3\vec{p}}{8\pi^3} 3c_s^2\omega A_a(\omega) f_{\rm eq}(1 \pm f_{\rm eq}).\end{aligned} \tag{36}$$

Substituting Eq. (36) into Eq. (34), we obtain the bulk viscosity ζ,

$$\zeta = \frac{1}{3}\sum_a \int \frac{d^3\vec{p}}{8\pi^3\omega} f_{\rm eq}(1 \pm f_{\rm eq}) A_a(\omega)(p^2 - 3c_s^2\omega^2). \tag{37}$$

Now, writing ζ in the component forms in Eq. (37), we eventually reached the desired expressions for ζ which are quoted in Eq. (12). Let us now proceed to investigate the temperature dependence of η and ζ.

III. TEMPERATURE DEPENDENCE OF η AND ζ

The determinations of η and ζ in the gluonic and matter sector are incomplete unless we fix the temperature dependence of $\hat{q}$ in both the sectors. The determination of $\hat{q}$ has been presented in the various phenomenological studies [68], either based on the eikonal approximation or the higher twist approximation, at a particular value of the temperature. Here, we choose the $\hat{q}$ for gluons as $4.5\ \mathrm{GeV}^2/\mathrm{fm}$ and $2.0\ \mathrm{GeV}^2/\mathrm{fm}$ for quarks at $T = 0.4$ GeV [69] (this temperature, we denote as T_0) since $\hat{q}$ appears in the denominator in the expressions for η and ζ. Therefore, any set of values higher then those mentioned above will further decrease the values of η and ζ. At $T = T_0$, we can see that $\hat{q}_g = 2.25\hat{q}_q$. At this juncture, we do not know these parameters at all temperatures, so we assume this relation holds for all temperatures. This assumption is based on the definition of $\hat{q}$ in the leading order in hot QCD [70], where it is the same for both gluons and quarks except that of the quadratic Casimir factor. We shall utilize the relation $\hat{q}_g = 2.25\hat{q}_q$, while studying the temperature dependence of various quantities in the next subsections. The exact temperature dependence of $\hat{q}$, employing the quasiparticle description of hot QCD, is not known to us at the moment. This will be a matter for future investigation.

A. Relative contributions

In this section, discussions are mainly on (i) relative contributions of various components of η with their ideal counter parts and (ii) gluonic versus matter sectors for η and ζ, respectively.

Note that the shear and bulk viscosities in the (2 + 1) flavor can be obtained by summing of all the individual contributions of the quasipartons as

$$\eta = \eta_g + \eta_q + \eta_s \qquad \zeta = \zeta_g + \zeta_q + \zeta_s. \tag{38}$$

The additivity of various components here is attributed to the fact that all of them belong to the same process, viz., the anomalous transport. Viscosity contributions from distinct processes (e.g., anomalous and collisional) are inverse additive due to the fact that various rates [12,18] are additive.

Let us define the relative quantities of interest. First, we shall define the ratios of various components of η to that for the ideal system of quarks and gluons [denoted as η^{Id} and displayed in Eq. (25)], which are defined as follows,

$$R_{gi} \equiv \frac{\eta_g}{\eta_g^{Id}}; \qquad R_{qi,si} \equiv \frac{\eta_{q,s}}{\eta_{q,s}^{Id}}$$
$$R_i \equiv \frac{(\eta_g + \eta_q + \eta_s)}{(\eta_g^{Id} + \eta_q^{Id} + \eta_s^{Id})}. \tag{39}$$

Similarly, to compare the relative contributions among various components of η, we define the following ratios,

$$R_{gq} \equiv \frac{\eta_g}{\eta_q}; \qquad R_{gs} \equiv \frac{\eta_g}{\eta_s}; \qquad R_{sq} \equiv \frac{\eta_s}{\eta_q}. \tag{40}$$

On the other hand, to compare the relative contributions among the various components of ζ, the following quantities have been defined,

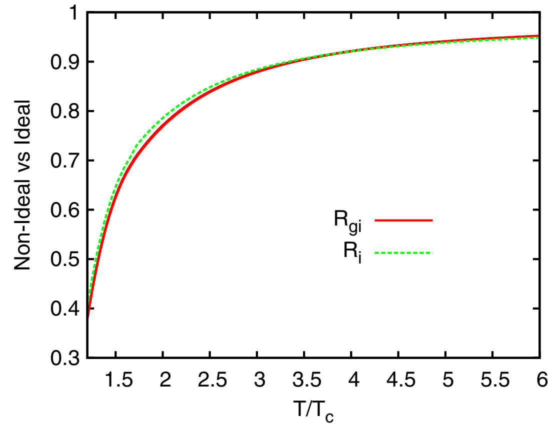

FIG. 1 (color online). η relative to that obtained using the ideal EOS for QGP, in the gluonic sector, and the (2 + 1) flavor is plotted as a function of T/T_c. The solid curve denotes the gluonic sector and dashed line denotes the (2 + 1) flavor. Both R_{gi} and R_i approach the ideal limit asymptotically.

$$R^{gq} \equiv \frac{\zeta_g}{\zeta_q}; \qquad R^{gs} \equiv \frac{\zeta_g}{\zeta_s}; \qquad R^{sq} \equiv \frac{\zeta_s}{\zeta_q}. \tag{41}$$

The quantities defined in Eqs. (39)–(41) have been shown as functions of T/T_c in Figs. 1–4. The ratios R_{gi} and R_i are shown as a function of temperature in Fig. 1. The parameter $\hat{q}$ is assumed to be the same in the interacting and ideal sector. We have considered temperature dependence beyond $1.2T_c$. Both R_{gi} and R_i show that interactions significantly modify the shear viscosity in the gluonic sector and the (2 + 1)-flavor QCD at lower temperatures. Both of them lie within the range $\{0.40, 0.97\}$ for the temperature $\{T/T_c = 1.2, 6.0\}$. R_{qi} and R_{si} are shown in Fig. 2 as a function of temperature. Both of them sit on top of each other. This is not surprising since

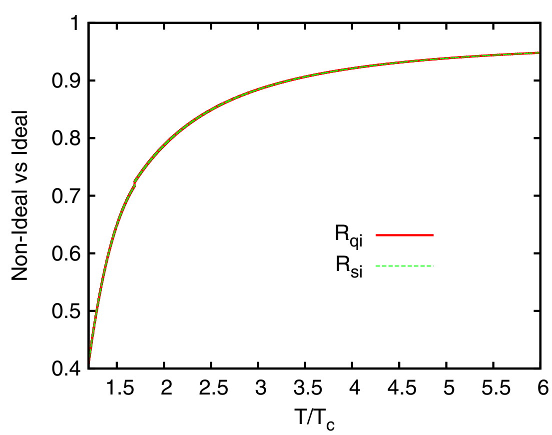

FIG. 2 (color online). η relative to that obtained using the ideal EOS for the QGP, in the matter sector. The R_{qi} is η relative to η^{Id} in the light-quark sector, and similarly R_{si} is for the strange-quark sector. Both the curves sit on top of each other since the mass effects from the strange-quark sector do not play a significant role here.

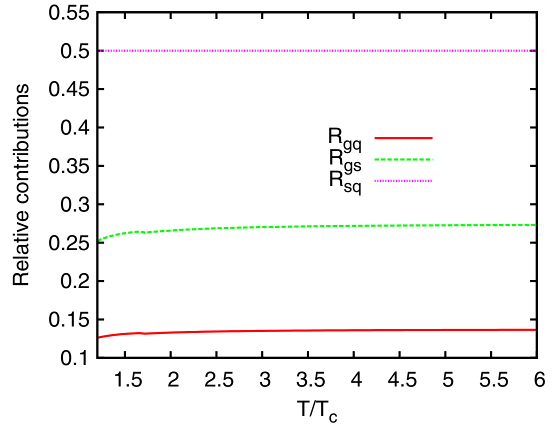

FIG. 3 (color online). Shear viscosity in the effective gluonic sector relative to the matter sector. The solid lines denotes η_g relative to η_q, thin dashed lines (middle) represent η_g relative to η_s, and upper thick dashed line represents η_s relative to η_q, as a function of T/T_c.

the mass effects coming from the strange-quark sector contribute negligibly in the temperature range considered here. The light-quark sector and strange quarks differ from each other by a factor of 2 coming from the degrees of freedom. From Fig. 2, it is evident that the hot QCD interactions significantly modify the shear viscosity in the matter sector the same as in the gluonic sector as compared to the ideal counter parts. All of them approach asymptotically the ideal limit which is nothing but *unity.* These observations suggest that η could be thought of as a good diagnostic tool to distinguish various equations of state at the RHIC and the LHC.

Next, we investigate the gluonic shear and bulk viscosities relative to that of the matter sector. The relevant quantities in this context of η are R_{gq}, R_{gs}, and R_{sq}, given in Eq. (40). These are shown as a function of temperature in Fig. 3. On the other hand, for ζ, R^{qg}, R^{qs}, and R^{sq} are shown as a function of temperature in Fig. 4. It can be observed from Figs. 3 and 4 that the matter sector contributions significantly dominate over the gluonic contributions as far as the η and ζ are concerned. This could perhaps be understood by the following facts, viz., the higher transport rates in the gluonic sector as compared to the quark sector as encoded in $\hat{q}$ and the interactions entering through the effective fugacities z_g and z_q. Quantitatively, η_g is $\sim 0.125\eta_q$, and $0.250\eta_s$ at $T = 1.20T_c$, and increases quite slowly as a function of T/T_c reaching around $0.135\eta_q$ around $6T_c$ (see Fig. 3). The η_s almost stays $0.5\eta_q$ for the considered range of temperature (contribution from the strange-quark mass is almost negligible). From Fig. 4, it can be observed that R^{gq} and R^{gs} have the same qualitative behavior as a function of temperature. The quantitative difference is because of a factor ~ 2, since $\zeta_s \sim 0.5\zeta_q$. Again the mass effects in the strange-quark-sector play an almost negligible role. The ratio R^{gq} initially increases and attains a peak around $T/T_c \sim 1.37$ and then decreases sharply until $T/T_c = 1.6$ and slightly increases beyond 1.6, tending toward saturation at higher temperatures. Quantitatively, $\zeta_g \approx 0.27\zeta_q$ around $1.2T_c$, and $0.13\zeta_q$ at around $3.0T_c$. These observations are very crucial in deciding the temperature dependence of η and ζ, and the ratios $\eta/\mathcal{S}$, $\zeta/\mathcal{S}$, and ζ/η. Most of the recent studies devoted to the η and ζ draw inferences for the QGP, which are purely based on the study of the pure SU(3) sector of QCD only. The matter sector has largely been ignored. In light of the above observations, it is not desirable to exclude the matter sector since the dominant contributions are from there. Finally, we can obtain the exact value of the ratios $\eta/\mathcal{S}$ and $\zeta/\mathcal{S}$ by employing the values of $\hat{q}$ quoted earlier ($\hat{q} = 4.5\ \mathrm{GeV}^2/\mathrm{fm}$ for gluons and $2.0\ \mathrm{GeV}^2/\mathrm{fm}$ for quarks at $T = 400$ MeV). The ratio $\eta/\mathcal{S}$ thus obtained as 0.570 and $\zeta/\mathcal{S}$ came out to be 0.057 at $T = 400$ MeV. As discussed earlier, to obtain the exact temperature dependence of η and ζ, one must fix the temperature dependence of $\hat{q}$ within the quasiparticle model employed here. This will be taken up separately in the near future. The quantity which can be determined unambiguously is the ratio ζ/η which is very crucial in deciding when the hot QCD becomes conformal. In other words, up to what temperature value are the effects coming from ζ important while studying the QGP? We shall now proceed to discuss these issues next.

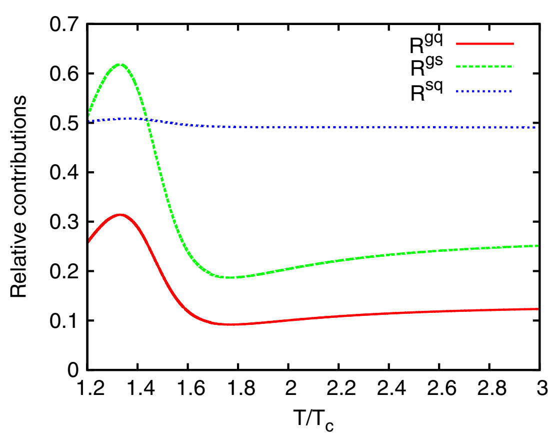

FIG. 4 (color online). Temperature dependence of the relative quasiparton bulk viscosities. The solid line shows the behavior of ζ_g relative to ζ_q, the thin dashed line shows the behavior of ζ_g relative to ζ_s, and the thick dashed line shows the behavior of ζ_s relative to ζ_q, as a function of T/T_c.

B. The ratio ζ/η

The behavior of the ratio ζ/η as a function of temperature is shown in Fig. 5, and the temperature dependence of the ratios $\frac{\eta}{\mathcal{S}} \equiv \frac{\eta\hat{q}}{T^3\mathcal{S}}$ and $\frac{\zeta}{\mathcal{S}} \equiv \frac{\zeta\hat{q}}{T^3\mathcal{S}}$ is shown in Fig. 6.

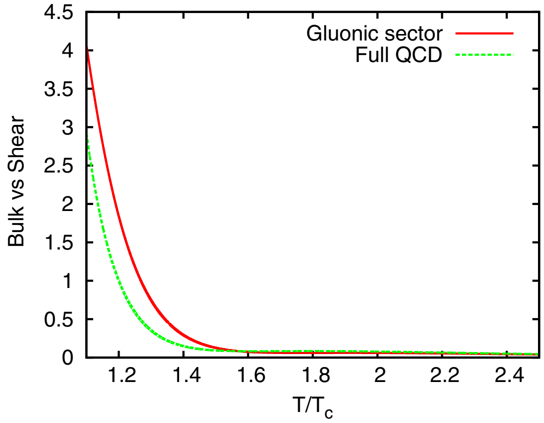

FIG. 5 (color online). Temperature dependence of the ratio ζ/η in the effective gluonic sector and the $(2 + 1)$-flavor QCD. The solid line represents the gluonic sector, and the dashed line represents the $(2 + 1)$-flavor case.

Most importantly, from Fig. 5, there are clear indications that ζ in the gluonic sector and the $(2 + 1)$-flavor QCD diverge as we approach closer to T_c (the results are not shown around T_c, since such a quasiparticle picture may not be valid there). The quantity ζ/η shows a sharp decrease until one reaches up to $1.4T_c$ in the gluonic sector and $1.6T_c$ in the $(2 + 1)$-flavor QCD sector. Beyond that, the decrease becomes slow and the ratio slowly approaches zero. Such a behavior of ζ/η as a function of temperature could mainly be described in the formal expressions in Eqs. (22) and (23) and decided not only by the temperature dependence of c_s^2 but also by the energy-dispersion relations, $\omega_{g,q,s}$, and the temperature dependence of the effective fugacities, $z_{g,q}$. It is evident that there is no way to obtain a $(c_s^2 - \frac{1}{3})^2$ factor out from the expression while performing the integration. However, such a scaling could be realized whenever $p \ll T^2\partial_T(\ln(z_{g,q}))$ and $\omega_{g,q,s}$ happen to be independent of z_g and z_q, and the thermal distribution of quasipartons show near-ideal behavior. It may perhaps be realized at a very high temperature which is not relevant to the study of QGP in the RHIC and the LHC. Therefore, ζ/η obtained here does not follow either a quadratic scaling or a linear scaling with the conformal measure $(c_s^2 - \frac{1}{3})$. The same conclusions were obtained in the case of the pure gauge theory recently [18]. Note that for the scalar field theories, $\zeta/\eta = 15(c_s^2 - \frac{1}{3})^2$ (quadratic scaling) [71], and it has been found to be true for a photon gas coupled with the matter [72]. The quadratic scaling is also valid in the case of perturbative QCD with a proportionality factor different from 15 [73]. Furthermore, in the case of near-conformal theories with gravity duals, ζ/η shows linear dependence on $(c_s^2 - \frac{1}{3})$ [74].

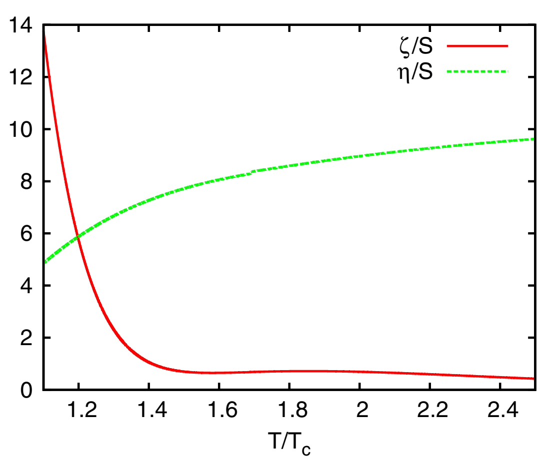

FIG. 6 (color online). The temperature dependence of the quantities η/S and ζ/S in the $(2 + 1)$-flavor QCD. Here $S = \frac{\mathcal{S}T^3}{\hat{q}}$, where $\mathcal{S}$ denotes the entropy density.

Finally, in Fig. 6, $\frac{\eta}{S}$ and $\frac{\zeta}{S}$ are plotted as a function of T/T_c (here the quantity S is related to the entropy density ($\mathcal{S}$) as $S = \frac{\mathcal{S}\hat{q}}{T^3}$. For the entropy density, we utilize the quasiparticle results which are shown to be consistent with the predictions of lattice QCD, and in all the plots, c_s^2 has been obtained from the quasiparticle model employing the method quoted in Ref. [65]. Interestingly, these are of the same order at $T = 1.2T_c$. Below that temperature, the latter dominates over the former and vice versa for $T \geq 1.2T_c$. The former increases, in contrast to the latter, as a function of T/T_c. There is a sharp increase shown by the latter until one reaches $1.4T_c$, and beyond that the decrease is slower and one is quite close to the conformal limit of QCD. The important inference that could be drawn from here is that while studying the QGP, one needs to incorporate the effects of both shear and bulk viscosities until approximately $1.5T_c$. This confirms our viewpoint that both η and ζ have a significant impact on the properties of the QGP at the RHIC and the LHC.

IV. CONCLUSIONS AND FUTURE PROSPECTS

In conclusion, the shear and bulk viscosities of the hot QCD are estimated by combining a semiclassical transport equation with a quasiparticle realization of the $(2 + 1)$-flavor lattice QCD. The effective gluonic sector contributes an order of magnitude lower as compared to the matter sector while determining the transport coefficients of the hot QCD and the QGP. This could perhaps be understood in terms of transport cross sections of gluons and quark-antiquarks. Since transport coefficients are inversely proportional to the cross sections. The bulk viscosity of the $(2 + 1)$-flavor QCD is found to be equally significant as the shear viscosity while modeling the QGP. Indications are seen regarding a blow-up in the bulk viscosity as we go closer to T_c.

The temperature dependence of the ratio ζ/η suggests that the QGP becomes almost conformal around 1.4–$1.5T_c$. The ratio sharply decreases from $T = 1.1$–$1.4T_c$, and beyond that it slowly approaches zero. Therefore, in this regime we can ignore the effects of ζ while studying the hydrodynamic evolution and properties of the QGP. We further found that η and ζ are of same order around $T = 1.2T_c$. For temperatures lower than that, ζ is dominant, and for higher temperatures, η is dominant. Importantly, both η and ζ came out to be highly sensitive to the presence of interactions. This can be visualized from the modulation of η, as compared to its ideal counter part, and large and rising value of ζ for the temperatures that are closer to T_c (due to the large interaction measure there). The above conclusions are based on the fact that the ratio $\hat{q}_g/\hat{q}_q$ is temperature independent, which is approximately true with the definition of $\hat{q}$ considered in the present analysis (leading order in perturbative QCD). A generalization of the definition of the $\hat{q}$ in view of the quasiparticle picture may induce both qualitative and quantitative modifications to the ratio ζ/η and will be investigated in the near future.

It would be a matter of immediate future investigation to utilize the more recent lattice data and compare the predictions for the data from the HOTQCD collaboration [9] and the Budapest-Marseille-Wuppertal Collaboration [10]. This would indeed be helpful in understating the impact of lattice artifacts and uncertainties on the transport properties of the QGP.

The investigations on the other contributions to the shear and bulk viscosities (collisional, etc.), and their interplay with the corresponding anomalous transport coefficients will be a matter of future investigations. It will be interesting to include the effects of nonvanishing baryon density on the transport coefficients of the QGP. Moreover, one could include the anomalous transport coefficients in the Boltzmann-transport theory approach and study the impact on the response functions and quarkonia physics along the lines of Refs. [75,76], as well as dilepton production at the RHIC and the LHC. These ideas will be studied in the near future.

ACKNOWLEDGMENTS

V. C. is extremely grateful to R. S. Bhalerao and V. Ravishankar for encouragement and helpful discussions and to F. Karsch and S. Datta for providing the lattice QCD data in the past. For hospitality, he sincerely acknowledges M. Maggiore and the Department of Theoretical Physics, University of Geneva Switzerland, where a significant part of this work was completed. He is thankful to B. N. Tiwari for reading the manuscript and helping with the language. He would like to acknowledge the financial support of the Tata Institute of Fundamental Research, Mumbai, India, in the form of a Visiting Fellow position, and of the INFN, Italy, for awarding an INFN postdoctoral fellowship. He would further like to acknowledge the hospitality and financial support of the CERN-Theory division, Switzerland, through the visitor program, and he is indebted to the people of India for their invaluable support of research in basic science in the country.

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