

Model-independent access to the structure of quark flavor mixing

Zhi-zhong Xing*

Institute of High Energy Physics and Theoretical Physics Center for Science Facilities, Chinese Academy of Sciences, Beijing 100049, China and Center for High Energy Physics, Peking University, Beijing 100871, China

(Received 13 November 2012; published 11 December 2012)

We show that the observed pattern of quark flavor mixing, such as $|V_{us}| \simeq |V_{cd}|$, $|V_{cb}| \simeq |V_{ts}|$, $|V_{cd}/V_{td}| \simeq |V_{cs}/V_{ts}| \simeq |V_{tb}/V_{cb}|$, and $|V_{ub}/V_{cb}| < |V_{td}/V_{ts}|$, can essentially be understood in the chiral and heavy quark mass limits. In particular, the phenomenologically successful relations $|V_{ub}/V_{cb}| \sim \sqrt{m_u/m_c}$ and $|V_{td}/V_{ts}| \simeq \sqrt{m_d/m_s}$ can be reasonably conjectured in the $m_b \rightarrow \infty$ and $m_t \rightarrow \infty$ limits, respectively. We stress that the strength of weak CP violation in the quark sector is determined by the moduli of the four corner elements of the Cabibbo-Kobayashi-Maskawa matrix. A comparison between strong and weak CP -violating effects in the standard model is also made.

DOI: 10.1103/PhysRevD.86.113006

PACS numbers: 14.60.Pq, 25.30.Pt

I. INTRODUCTION

The standard model (SM) has proved to be very successful in describing the fundamental properties of elementary particles and their interactions, but it has to be extended in the flavor sector by covering the facts that the three known neutrinos are not massless and different lepton flavors can mix [1]. In this case even a minimal extension of the SM involves twenty (or twenty-two) flavor parameters, among which twelve are masses, six are flavor mixing angles, and two (or four) are CP -violating phases if the massive neutrinos are of the Dirac (or Majorana) nature.¹ Determining the values of these parameters and understanding why they are what they are constitute a central part of today's particle physics. However, we have to confront some flavor puzzles revealed by current experimental data.

The flavor puzzles include why up-type quarks, down-type quarks, and charged leptons all have strong mass hierarchies (i.e., $m_u \ll m_c \ll m_t$, $m_d \ll m_s \ll m_b$, and $m_e \ll m_\mu \ll m_\tau$) at a given energy scale; why the masses of three neutrinos are extremely small in comparison with those of nine charged fermions; why the six off-diagonal elements of the Cabibbo-Kobayashi-Maskawa (CKM) quark flavor mixing matrix V [2] are strongly suppressed such that the three mixing angles are very small; why the Maki-Nakagawa-Sakata-Pontecorvo lepton flavor mixing matrix U [3] contains two relatively large mixing angles and the third one is not strongly suppressed either; why the patterns of V and U are so different but their smallest matrix elements are both located at the upper-right corner (i.e., V_{ub} and U_{e3}) [4]; how the origin of CP violation is correlated with the origin of fermion masses; and so on.

In the lack of a complete flavor theory capable of predicting the flavor structures of leptons and quarks or revealing possible symmetries behind them, it is a big challenge to answer even a part of the aforementioned questions. The great ideas like grand unifications, supersymmetries, and extra dimensions are still not very helpful to solve the flavor puzzles, and the exercises of various group languages or flavor symmetries turn out to be too divergent to converge to something unique [5].

In this paper, we shall follow a purely phenomenological way to speculate whether the observed pattern of quark flavor mixing can be partly understood in some reasonable limits of quark masses. This starting point of view is more or less motivated by two useful working symmetries in understanding the strong interactions of quarks and hadrons by means of quantum chromodynamics (QCD) or an effective field theory based on QCD [6]: the chiral quark symmetry (i.e., $m_u, m_d, m_s \rightarrow 0$) and the heavy quark symmetry (i.e., $m_c, m_b, m_t \rightarrow \infty$). The reason for the usefulness of these two symmetries is simply that the masses of the light quarks are far below the typical QCD scale $\Lambda_{\text{QCD}} \sim 0.2$ GeV, whereas the masses of the heavy quarks are far above it. Because the elements of the CKM matrix V are dimensionless and their magnitudes lie in the range of 0–1, they are in general expected to depend on the mass ratios of the lighter quarks to the heavier quarks. The mass limits corresponding to the chiral and heavy quark symmetries are therefore equivalent to setting the relevant mass ratios to zero, and they are possible to help reveal a part of the salient features of V . In this spirit, some preliminary attempts have been made to look at the quark flavor mixing pattern in the $m_u, m_d \rightarrow 0$ or $m_t, m_b \rightarrow \infty$ limits [7].

The present work aims to show that it is actually possible to gain an insight into the observed pattern of quark flavor mixing in the chiral and heavy quark mass limits. Such model-independent access to the underlying quark flavor structure can at least explain why $|V_{us}| \simeq |V_{cd}|$ and $|V_{cb}| \simeq |V_{ts}|$ hold to a good degree of accuracy, why $|V_{cd}/V_{td}| \simeq |V_{cs}/V_{ts}| \simeq |V_{tb}/V_{cb}|$ is a reasonable approximation, and

*xingzz@ihep.ac.cn

¹In this regard we have assumed the 3×3 lepton flavor mixing matrix to be unitary, regardless of the origin of tiny neutrino masses. Here the effective strong CP -violating parameter $\bar{\theta}$ is not taken into account, but it will be briefly discussed in Sec. IV.

why $|V_{ub}/V_{cb}|$ should be smaller than $|V_{td}/V_{ts}|$. In particular, the phenomenologically successful relations $|V_{ub}/V_{cb}| \sim \sqrt{m_u/m_c}$ and $|V_{td}/V_{ts}| \approx \sqrt{m_d/m_s}$ can be reasonably conjectured in the heavy quark mass limits. We also point out that the strength of CP violation in the quark sector is simply determined by the product of the four corner elements of V (i.e., $|V_{ud}|$, $|V_{ub}|$, $|V_{td}|$, and $|V_{tb}|$), based on the experimental fact that two of the CKM unitarity triangles are almost the right triangles. This interesting observation motivates us to pay more attention to a particular parametrization of V [8], in which the three flavor mixing angles are all comparable with the Cabibbo angle and the tiny CP -violating phase shows up only in the four corners of V . Finally, we make a brief comment on the effect of strong CP violation and then compare its strength with that of weak CP violation.

II. THE CKM MATRIX IN THE QUARK MASS LIMITS

We begin with the weak charged-current interactions of six quarks in their mass basis:

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \overline{(u \ c \ t)}_L \gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu^+ + \text{H.c.}, \quad (1)$$

where the CKM matrix V measures a nontrivial mismatch between the flavor and mass eigenstates and can be decomposed into $V = O_u^\dagger O_d$ with O_u and O_d being the unitary transformations responsible for the diagonalizations of the up- and down-type quark mass matrices in the flavor basis, respectively. Namely,

$$\begin{aligned} O_u^\dagger H_u O_u &= O_u^\dagger M_u M_u^\dagger O_u = \text{Diag}\{m_u^2, m_c^2, m_t^2\}, \\ O_d^\dagger H_d O_d &= O_d^\dagger M_d M_d^\dagger O_d = \text{Diag}\{m_d^2, m_s^2, m_b^2\}, \end{aligned} \quad (2)$$

where H_u and H_d are defined to be Hermitian. To be explicit, the nine elements of V read

$$V_{\alpha i} = \sum_{k=1}^3 (O_u)_{k\alpha}^* (O_d)_{ki}, \quad (3)$$

where α and i run over (u, c, t) and (d, s, b) , respectively. Because of Eqs. (2) and (3), the dimensionless $V_{\alpha i}$ elements are expected to be more or less dependent on the quark mass ratios and nontrivial phase differences between M_u and M_d . Given current experimental data and the unitarity of V , the magnitudes of all the nine CKM matrix elements have been determined to an impressively good degree of accuracy [1]:

$$|V| = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}. \quad (4)$$

We shall show that the strong mass hierarchies of up- and down-type quarks allow us to account for a part of the observed flavor mixing properties in a model-independent way.

A. H_u and H_d in the quark mass limits

In general, the mass limit $m_u \rightarrow 0$ (or $m_d \rightarrow 0$) does not correspond to a unique form of H_u (or H_d). The reason is simply that the form of a quark mass matrix is always basis dependent. Without loss of generality, one may choose a particular flavor basis such that H_u and H_d can be written as

$$\begin{aligned} \lim_{m_u \rightarrow 0} H_u &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}, \\ \lim_{m_d \rightarrow 0} H_d &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}, \end{aligned} \quad (5)$$

in which “ \times ” denotes an arbitrary nonzero element. To prove that Eq. (5) is the result of a basis choice instead of an assumption, we refer the reader to the Appendix.

When the mass of a given quark goes to infinity, we argue that it becomes decoupled from the masses of other quarks. In this sense we may choose a specific flavor basis where H_u and H_d can be written as

$$\begin{aligned} \lim_{m_t \rightarrow \infty} H_u &= \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \infty \end{pmatrix}, \\ \lim_{m_b \rightarrow \infty} H_d &= \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \infty \end{pmatrix}. \end{aligned} \quad (6)$$

In other words, the 3×3 Hermitian matrices H_u and H_d can be simplified to the effective 2×2 Hermitian matrices in either the chiral quark mass limit or the heavy quark mass limit. In view of the fact that $m_u \ll m_c \ll m_t$ and $m_d \ll m_s \ll m_b$ hold at an arbitrary energy scale [9], we believe that Eqs. (5) and (6) are phenomenologically reasonable and can help explain some of the observed properties of quark flavor mixing in a model-independent way. Let us go into detail in the following.

B. Why do $|V_{us}| \simeq |V_{cd}|$ and $|V_{cb}| \simeq |V_{ts}|$ hold?

A glance at Eq. (4) tells us that $|V_{us}| \simeq |V_{cd}|$ is an excellent approximation. Such an approximate equality can be well understood in the heavy quark mass limits, where Hermitian H_u and H_d may take the form of Eq. (6). In this case the unitary matrices O_u and O_d used to diagonalize H_u and H_d in Eq. (2) can be expressed as

$$\begin{aligned} \lim_{m_t \rightarrow \infty} O_u &= P_{12} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ \lim_{m_b \rightarrow \infty} O_d &= P'_{12} \begin{pmatrix} c'_{12} & s'_{12} & 0 \\ -s'_{12} & c'_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \end{aligned} \quad (7)$$

where $c_{12}^{(i)} \equiv \cos \vartheta_{12}^{(i)}$, $s_{12}^{(i)} \equiv \sin \vartheta_{12}^{(i)}$, and $P_{12}^{(i)} = \text{Diag}\{e^{i\phi_{12}^{(i)}}, 1, 1\}$. So Eq. (7) yields

$$|V_{us}| = |c_{12}s'_{12} - s_{12}c'_{12}e^{i\Delta_{12}}| = |V_{cd}| \quad (8)$$

in the $m_t \rightarrow \infty$ and $m_b \rightarrow \infty$ limits, where $\Delta_{12} \equiv \phi'_{12} - \phi_{12}$ denotes the nontrivial phase difference between the up- and down-quark sectors. Since $m_u/m_c \sim m_c/m_t \sim \lambda^4$ and $m_d/m_s \sim m_s/m_b \sim \lambda^2$ hold [9], where $\lambda \equiv \sin\theta_C \simeq 0.22$ with θ_C being the Cabibbo angle, the mass limits taken above are apparently a good approximation. Therefore, we conclude that the approximate equality $|V_{us}| \simeq |V_{cd}|$ is attributed to the fact that $m_t \gg m_u, m_c$ and $m_b \gg m_d, m_s$ hold.²

One may similarly consider the chiral quark mass limits $m_u \rightarrow 0$ and $m_d \rightarrow 0$ in order to understand why $|V_{ts}| \simeq |V_{cb}|$ holds. In this case, Eq. (5) leads us to

$$\begin{aligned} \lim_{m_u \rightarrow 0} O_u &= P_{23} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}, \\ \lim_{m_d \rightarrow 0} O_d &= P'_{23} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c'_{23} & s'_{23} \\ 0 & -s'_{23} & c'_{23} \end{pmatrix}, \end{aligned} \quad (9)$$

where $c_{23}^{(i)} \equiv \cos \vartheta_{23}^{(i)}$, $s_{23}^{(i)} \equiv \sin \vartheta_{23}^{(i)}$, and $P_{23}^{(i)} = \text{Diag}\{1, 1, e^{i\phi_{23}^{(i)}}\}$. We are therefore left with

$$|V_{cb}| = |c_{23}s'_{23} - s_{23}c'_{23}e^{i\Delta_{23}}| = |V_{ts}| \quad (10)$$

in the $m_u \rightarrow 0$ and $m_d \rightarrow 0$ limits, where $\Delta_{23} \equiv \phi'_{23} - \phi_{23}$ stands for the nontrivial phase difference between the up- and down-quark sectors. This model-independent result is also in good agreement with the experimental data $|V_{cb}| \simeq |V_{ts}|$ as given in Eq. (4). In other words, the approximate

²Quantitatively, $|V_{us}| \simeq |V_{cd}| \simeq \lambda$ holds. Hence, $s_{12} \simeq \sqrt{m_u/m_c} \simeq \lambda^2$ and $s'_{12} \simeq \sqrt{m_d/m_s} \simeq \lambda$ are often conjectured and can easily be derived from some *Ansätze* of quark mass matrices [10].

equality $|V_{cb}| \simeq |V_{ts}|$ is a natural consequence of $m_u \ll m_c$, m_t and $m_d \ll m_s$, m_b in no need of any specific assumptions.³

C. Why does $|V_{cd}/V_{td}| \simeq |V_{cs}/V_{ts}| \simeq |V_{tb}/V_{cb}|$ hold?

Given the magnitudes of the CKM matrix elements in Eq. (4), it is straightforward to obtain $|V_{cd}/V_{td}| \simeq 26.0$, $|V_{cs}/V_{ts}| \simeq 24.1$, and $|V_{tb}/V_{cb}| \simeq 24.3$. These numbers imply $|V_{cd}/V_{td}| \simeq |V_{cs}/V_{ts}| \simeq |V_{tb}/V_{cb}|$ as a reasonably good approximation, which has not come into notice in the literature. We find that such an approximate relation becomes exact in the mass limits $m_u \rightarrow 0$ and $m_b \rightarrow \infty$. To be explicit,

$$\begin{aligned} V &= \lim_{m_u \rightarrow 0} O_u^\dagger \lim_{m_b \rightarrow \infty} O_d \\ &= P'_{12} \begin{pmatrix} c'_{12} & s'_{12} & 0 \\ -c_{23}s'_{12} & c_{23}c'_{12} & -s_{23} \\ -s_{23}s'_{12} & s_{23}c'_{12} & c_{23} \end{pmatrix} P_{12}^\dagger, \end{aligned} \quad (11)$$

where Eqs. (7) and (9) have been used. Therefore,

$$\left| \frac{V_{cd}}{V_{td}} \right| = \left| \frac{V_{cs}}{V_{ts}} \right| = \left| \frac{V_{tb}}{V_{cb}} \right| = |\cot \vartheta_{23}| \quad (12)$$

holds in the chosen quark mass limits, which assure the smallest CKM matrix element V_{ub} to vanish. This simple result is essentially consistent with the experimental data if $\vartheta_{23} \simeq 2.35^\circ$ is taken.⁴ Note that the quark mass limits $m_t \rightarrow \infty$ and $m_d \rightarrow 0$ are less favored because they predict both $|V_{td}| = 0$ and $|V_{us}/V_{ub}| = |V_{cs}/V_{cb}| = |V_{tb}/V_{ts}|$, which are in conflict with current experimental data. In particular, the limit $|V_{ub}| = 0$ is apparently closer to reality than the limit $|V_{td}| = 0$.

But why V_{ub} is smaller in magnitude than all the other CKM matrix elements remains a puzzle, since it is difficult for us to judge that the quark mass limits $m_u \rightarrow \infty$ and $m_b \rightarrow 0$ should make more sense than the quark mass limits $m_t \rightarrow \infty$ and $m_d \rightarrow 0$ from a phenomenological point of view. The experimental data in Eq. (4) indicate $|V_{td}| \gtrsim 2|V_{ub}|$ and $|V_{ts}| \simeq |V_{cb}|$. So a comparison between the ratios $|V_{ub}/V_{cb}|$ and $|V_{td}/V_{ts}|$ might be able to tell us an acceptable reason for $|V_{td}| > |V_{ub}|$.

D. Why is $|V_{ub}/V_{cb}|$ smaller than $|V_{td}/V_{ts}|$?

With the help of Eqs. (3), (6), and (7), we can calculate the ratios $|V_{ub}/V_{cb}|$ and $|V_{td}/V_{ts}|$ in the respective heavy quark mass limits:

³It is possible to obtain the quantitative relationship $|V_{cb}| \simeq |V_{ts}| \simeq \lambda^2$ through $s_{23} \simeq m_c/m_t \simeq \lambda^4$ and $s'_{23} \simeq m_s/m_b \simeq \lambda^2$ from a number of *Ansätze* of quark mass matrices [11].

⁴This numerical estimate implies $\tan \vartheta_{23} \simeq \lambda^2 \simeq \sqrt{m_c/m_t}$, which can easily be derived from the Fritzsch *Ansatz* of quark mass matrices [12].

$$\lim_{m_b \rightarrow \infty} \left| \frac{V_{ub}}{V_{cb}} \right| = \left| \frac{(O_u)_{3u}}{(O_u)_{3c}} \right|, \quad (13)$$

$$\lim_{m_t \rightarrow \infty} \left| \frac{V_{td}}{V_{ts}} \right| = \left| \frac{(O_d)_{3d}}{(O_d)_{3s}} \right|.$$

This result is quite nontrivial in the sense that $|V_{ub}/V_{cb}|$ turns out to be independent of the mass ratios of three down-type quarks in the $m_b \rightarrow \infty$ limit, and $|V_{td}/V_{ts}|$ has nothing to do with the mass ratios of three up-type quarks in the $m_t \rightarrow \infty$ limit. In particular, the flavor indices showing up on the right-hand side of Eq. (13) are rather suggestive: $|V_{ub}/V_{cb}|$ is relevant to u and c quarks, and $|V_{td}/V_{ts}|$ depends on d and s quarks. We are therefore encouraged to conjecture that $|V_{ub}/V_{cb}|$ (or $|V_{td}/V_{ts}|$) should be a simple function of the mass ratio m_u/m_c (or m_d/m_s) in the $m_t \rightarrow \infty$ (or $m_b \rightarrow \infty$) limit.

Given the renormalized quark mass values $m_u = 1.38_{-0.41}^{+0.42}$ MeV, $m_d = 2.82 \pm 0.048$ MeV, $m_s = 57_{-12}^{+18}$ MeV, and $m_c = 0.638_{-0.084}^{+0.043}$ GeV at the energy scale $\mu = M_Z$ [9], the simplest phenomenological conjectures turn out to be

$$\lim_{m_b \rightarrow \infty} \left| \frac{V_{ub}}{V_{cb}} \right| \simeq c_1 \sqrt{\frac{m_u}{m_c}}, \quad \lim_{m_t \rightarrow \infty} \left| \frac{V_{td}}{V_{ts}} \right| \simeq c_2 \sqrt{\frac{m_d}{m_s}}, \quad (14)$$

where c_1 and c_2 are the $\mathcal{O}(1)$ coefficients. In view of $\sqrt{m_u/m_c} \simeq \lambda^2$ and $\sqrt{m_d/m_s} \simeq \lambda$, we expect that $|V_{ub}/V_{cb}|$ is naturally smaller than $|V_{td}/V_{ts}|$ in the heavy quark mass limits. Taking $c_1 = 2$ and $c_2 = 1$, for example, we obtain $|V_{ub}/V_{cb}| \simeq 0.093$ and $|V_{td}/V_{ts}| \simeq 0.222$ from Eq. (14), consistent with the experimental results $|V_{ub}/V_{cb}| \simeq 0.085$ and $|V_{td}/V_{ts}| \simeq 0.214$ as given in Eq. (4) [1]. Because $m_t = 172.1 \pm 1.2$ GeV and $m_b = 4.19_{-0.16}^{+0.18}$ GeV at $\mu = M_Z$ [9], one may argue that $m_t \rightarrow \infty$ is a much better limit and thus the relation $|V_{td}/V_{ts}| \simeq \sqrt{m_d/m_s}$ has a good chance to be true. In comparison, $|V_{ub}/V_{cb}| \simeq 2\sqrt{m_u/m_c}$ suffers from much bigger uncertainties associated with the values of m_u and m_c , and even its coefficient “2” is questionable.

It is well known that the Fritzsch Ansatz of quark mass matrices [12] predicts $c_1 \simeq c_2 \simeq 1$. A straightforward extension of the Fritzsch texture [13],

$$M_q = \begin{pmatrix} 0 & C_q & 0 \\ C_q^* & \tilde{B}_q & B_q \\ 0 & B_q^* & A_q \end{pmatrix} \quad (15)$$

with $|A_q| \gg |B_q| \sim |\tilde{B}_q| \gg |C_q|$ (for $q = u$ or d), can also lead us to $c_1 \simeq c_2 \simeq 1$. However, it is always possible to get $c_1 \simeq \sqrt{2}$ (or $\sqrt{3}, 2, \dots$) together with $c_2 \simeq 1$ if the matrix elements A_q and B_q have a quite weak hierarchy in magnitude [14]. This observation is interesting, as it implies that the phenomenological conjectures made in

Eq. (14) can be a good starting point of view for model building in order to understand a possible correlation between the quark mass spectrum and the flavor mixing structure.

III. IMPLICATIONS OF THE RIGHT UNITARITY TRIANGLES

A rephasing-invariant description of CP violation in the quark sector is the Jarlskog parameter \mathcal{J}_q defined through [15]

$$\text{Im}(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*) = \mathcal{J}_q \sum_{\gamma} \epsilon_{\alpha\beta\gamma} \sum_k \epsilon_{ijk}, \quad (16)$$

in which the greek and latin subscripts run over (u, c, t) and (d, s, b) , respectively. The unitarity of V leads us to six triangles in the complex plane, whose areas are all equal to $\mathcal{J}_q/2$ [11]. Among the six CKM unitarity triangles, Δ_s and Δ_c are defined, respectively, by the orthogonality relations

$$\begin{aligned} \Delta_s: V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* &= 0, \\ \Delta_c: V_{tb}V_{ub}^* + V_{ts}V_{us}^* + V_{td}V_{ud}^* &= 0, \end{aligned} \quad (17)$$

as illustrated in Fig. 1. They are especially interesting in the sense that they are essentially the right triangles with a common inner angle

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) = (89.0_{-4.2}^{+4.4})^\circ, \quad (18)$$

as determined by current experimental data [1]. Given $\alpha = 90^\circ$ exactly, it turns out that $\text{Re}(V_{tb}V_{ud}V_{td}^*V_{ub}^*) = 0$ holds, and thus the rephasing-invariant quartet $V_{tb}V_{ud}V_{td}^*V_{ub}^*$ is purely imaginary [16]. In this case,

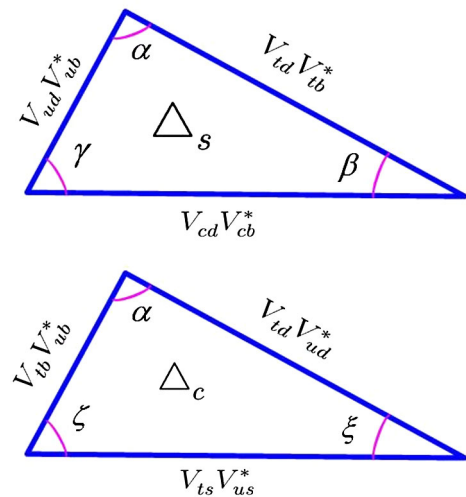


FIG. 1 (color online). The CKM unitarity triangles Δ_s and Δ_c in the complex plane. They share a common inner angle α , which is essentially equal to 90° as indicated by current experimental data.

Δ_s and Δ_c can be rescaled in such a way that they share a common side which is equal to⁵

$$\mathcal{J}_q = |V_{ud}| \cdot |V_{ub}| \cdot |V_{td}| \cdot |V_{tb}|, \quad (19)$$

as shown in Fig. 2, where Δ'_s and Δ'_c are the rescaled versions of Δ_s and Δ_c , respectively, with $\alpha = 90^\circ$ (i.e., $\Delta'_s: |V_{ud}V_{ub}^*|^2 + V_{ud}V_{cb}V_{ub}^*V_{cd}^* + i\mathcal{J}_q = 0$ and $\Delta'_c: |V_{tb}V_{ub}^*|^2 + V_{us}V_{tb}V_{ub}^*V_{ts}^* + i\mathcal{J}_q = 0$). This result is quite suggestive: The strength of weak CP violation in the quark sector is simply determined by the moduli of the four matrix elements at the four corners of V . Note that $|V_{tb}| > |V_{ud}| > |V_{cs}| \gg |V_{us}| > |V_{cd}| \gg |V_{cb}| > |V_{ts}| \gg |V_{td}| > |V_{ub}|$ holds for the nine CKM

matrix elements [17], so \mathcal{J}_q is actually equal to the product of the two largest and two smallest elements of V in magnitude. Because $|V_{ud}| \rightarrow 1$ and $|V_{ub}| \rightarrow 0$ (or $|V_{tb}| \rightarrow 1$ and $|V_{td}| \rightarrow 0$) can be achieved in the $m_u \rightarrow 0$ and $m_d \rightarrow 0$ limits (or in the $m_t \rightarrow \infty$ and $m_b \rightarrow \infty$ limits), we have $\mathcal{J}_q \rightarrow 0$ in both cases. With the help of Eqs. (4) and (19), we directly arrive at $\mathcal{J}_q \approx 2.96 \times 10^{-5}$, which is perfectly consistent with the result $\mathcal{J}_q = (2.96_{-0.16}^{+0.20}) \times 10^{-5}$ obtained by the Particle Data Group [1].

Given weak CP violation in the quark sector as a corner effect of the CKM matrix V , we have a good reason to consider the following parametrization of V [8]:

$$V = \begin{pmatrix} c_y & 0 & s_y \\ 0 & 1 & 0 \\ -s_y & 0 & c_y \end{pmatrix} \begin{pmatrix} c_x & s_x & 0 \\ -s_x & c_x & 0 \\ 0 & 0 & e^{-i\delta_q} \end{pmatrix} \begin{pmatrix} c_z & 0 & -s_z \\ 0 & 1 & 0 \\ s_z & 0 & c_z \end{pmatrix} \\ = \begin{pmatrix} c_x c_y c_z + s_y s_z e^{-i\delta_q} & s_x c_y & -c_x c_y s_z + s_y c_z e^{-i\delta_q} \\ -s_x c_z & c_x & s_x s_z \\ -c_x s_y c_z + c_y s_z e^{-i\delta_q} & -s_x s_y & c_x s_y s_z + c_y c_z e^{-i\delta_q} \end{pmatrix}, \quad (20)$$

where $c_x \equiv \cos\vartheta_x$ and $s_x \equiv \sin\vartheta_x$, and so on. One can see that the CP -violating phase δ_q just appears in the four corners of V . Confronting Eq. (20) with current experimental data leads us to $\vartheta_x \approx 13.2^\circ$, $\vartheta_y \approx 10.1^\circ$, $\vartheta_z \approx 10.3^\circ$, and $\delta_q \approx 1.1^\circ$ [8], consistent with $\alpha \approx 90^\circ$ or equivalently $\text{Re}(V_{tb}V_{ud}V_{td}^*V_{ub}^*) \approx 0$. Note that ϑ_x and δ_q are essentially stable when the energy scale changes, but ϑ_y and ϑ_z may slightly be modified due to radiative corrections. To see this point more clearly, let us take account of the approximate one-loop renormalization-group equations (RGEs) of nine CKM matrix elements [18]:

$$\frac{d}{dt} \ln|V_{ud}| \approx \frac{d}{dt} \ln|V_{cs}| \approx \frac{d}{dt} \ln|V_{tb}| \approx \frac{d}{dt} \ln|V_{us}| \approx \frac{d}{dt} \ln|V_{cd}| \approx 0, \\ \frac{d}{dt} \ln|V_{ub}| \approx \frac{d}{dt} \ln|V_{cb}| \approx \frac{d}{dt} \ln|V_{td}| \approx \frac{d}{dt} \ln|V_{ts}| \approx c(y_t^2 + y_b^2), \quad (21)$$

where $t \equiv (1/16\pi^2) \ln(\mu/M_Z)$ for a given energy scale μ above the electroweak scale, y_t and y_b standard for the Yukawa coupling eigenvalues of top and bottom quarks, respectively, and $c = -3$ (or -1) holds in the SM (or its supersymmetric extension). Combining Eq. (20) with Eq. (21), we obtain the approximate one-loop RGEs of ϑ_x , ϑ_y , ϑ_z , and δ_q as follows:

⁵One may also obtain $\mathcal{J}_q = |V_{ud}V_{ub}| \sqrt{|V_{cd}V_{cb}|^2 - |V_{ud}V_{ub}|^2}$ or $\mathcal{J}_q = |V_{ub}V_{tb}| \sqrt{|V_{us}V_{ts}|^2 - |V_{ub}V_{tb}|^2}$ from Fig. 1 by means of the Pythagorean theorem. In both cases, \mathcal{J}_q is proportional to the smallest CKM matrix element $|V_{ub}|$. Hence, $|V_{ub}| \neq 0$ is a necessary condition of CP violation.

$$\frac{d}{dt} \text{Insin}\vartheta_x \approx \frac{d}{dt} \text{Insin}\delta_q \approx 0, \\ \frac{d}{dt} \text{Insin}\vartheta_y \approx \frac{d}{dt} \text{Insin}\vartheta_z \approx c(y_t^2 + y_b^2). \quad (22)$$

Hence the mixing angle ϑ_x and the CP -violating phase δ_q are almost stable against the RGE running effects, and the mixing angles ϑ_y and ϑ_z are expected to have the same RGE running behaviors at the one-loop level. Because of $\mathcal{J}_q = c_x s_x^2 c_y s_y c_z s_z \sin\delta_q$, the RGE evolution of \mathcal{J}_q is mainly controlled by that of s_y and s_z .

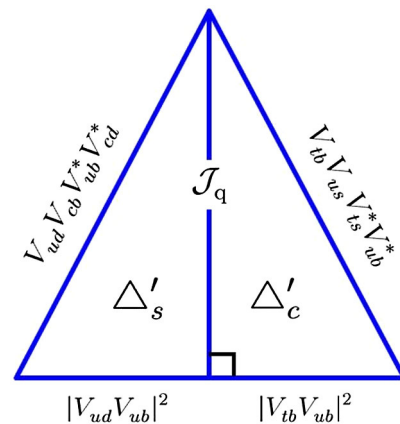


FIG. 2 (color online). The rescaled CKM unitarity triangles Δ'_s and Δ'_c , which share a common side equal to the Jarlskog invariant \mathcal{J}_q (due to $\alpha = 90^\circ$), in the complex plane.

IV. A BRIEF COMMENT ON THE STRONG CP PROBLEM

So far we have only paid attention to weak CP violation based on the CKM matrix in the SM. Here let us make a brief comment on the strong CP problem, because it is closely related to the quark masses and may naturally disappear if one of the six quark masses vanishes. It is well known that there exists a P - and T -violating term \mathcal{L}_θ , which comes from the instanton solution to the $U(1)_A$ problem [19], in the Lagrangian of QCD for strong interactions of quarks and gluons [20]. This CP -violating term can be compared with the mass term of six quarks, \mathcal{L}_m , as follows:

$$\mathcal{L}_\theta = \theta \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu},$$

$$\mathcal{L}_m = \overline{(u \ c \ t \ d \ s \ b)}_L \mathcal{M} \begin{pmatrix} u \\ c \\ t \\ d \\ s \\ b \end{pmatrix}_R + \text{H.c.}, \quad (23)$$

where θ is a free dimensionless parameter characterizing the presence of CP violation, α_s is the strong fine-structure constant, $G_{\mu\nu}^a$ (for $a = 1, 2, \dots, 8$) denote the $SU(3)_c$ gauge fields, $\tilde{G}^{a\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}^a / 2$, and \mathcal{M} is the overall 6×6 quark mass matrix. The chiral transformation of the quark fields $q \rightarrow \exp(i\phi_q \gamma_5) q$ (for $q = u, c, t; d, s, b$) leads to the changes

$$\theta \rightarrow \theta - 2 \sum_q \phi_q, \quad (24)$$

$$\arg(\det \mathcal{M}) \rightarrow \arg(\det \mathcal{M}) + 2 \sum_q \phi_q,$$

in which the change of θ follows from the chiral anomaly [21] in the chiral currents

$$\partial_\mu (\bar{q} \gamma^\mu \gamma_5 q) = 2im_q \bar{q} \gamma_5 q + \frac{\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}. \quad (25)$$

Then the effective CP -violating term in QCD, which is invariant under the above chiral transformation, turns out to be

$$\mathcal{L}_{\bar{\theta}} = \bar{\theta} \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}, \quad (26)$$

where $\bar{\theta} = \theta + \arg(\det \mathcal{M})$ is a sum of the QCD and electroweak contributions [22]. The latter depends on the phase structure of the quark mass matrix \mathcal{M} . Because of

$$|\det \mathcal{M}| = m_u m_c m_t m_d m_s m_b, \quad (27)$$

the determinant of \mathcal{M} becomes vanishing in the $m_u \rightarrow 0$ (or $m_d \rightarrow 0$) limit. In this case the phase of $\det \mathcal{M}$ is arbitrary, and thus it can be arranged to cancel out θ such

that $\bar{\theta} \rightarrow 0$. Namely, QCD would be a CP -conserving theory if one of the six quarks were massless. But current experimental data have definitely ruled out the possibility of $m_u = 0$ or $m_d = 0$. Moreover, the experimental upper limit on the neutron electric dipole moment yields $\bar{\theta} < 10^{-10}$ [23]. The strong CP problem is therefore a theoretical problem of how to explain why $\bar{\theta}$ is nonzero but so small [24].

A comparison between the weak and strong CP -violating effects might make sense, but it is difficult to choose a proper measure for either of them. The issue involves the reference energy scale and flavor parameters which may directly or indirectly determine the strength of CP violation. To illustrate, we consider the following preliminary measures of weak and strong CP -violating effects in the SM⁶:

$$\text{CP}_{\text{weak}} \sim \frac{1}{\Lambda_{\text{EW}}^6} (m_u - m_c)(m_c - m_t)(m_t - m_u)(m_d - m_s) \\ \times (m_s - m_b)(m_b - m_d) \mathcal{J}_q \sim 10^{-13},$$

$$\text{CP}_{\text{strong}} \sim \frac{1}{\Lambda_{\text{QCD}}^6} m_u m_c m_t m_d m_s m_b \sin \bar{\theta} \sim 10^4 \sin \bar{\theta} < 10^{-6}, \quad (28)$$

where $\Lambda_{\text{EW}} \sim 10^2$ GeV, $\Lambda_{\text{QCD}} \sim 0.2$ GeV, and the sine function of $\bar{\theta}$ has been adopted to take account of the periodicity in its values. So the effect of weak CP violation would vanish if the masses of any two quarks in the same (up or down) sector were equal,⁷ and the effect of strong CP violation would vanish if $m_u \rightarrow 0$ or $\sin \bar{\theta} \rightarrow 0$ held. The significant suppression of CP violation in the SM implies that an interpretation of the observed matter-antimatter asymmetry of the Universe [1] requires a new source of CP violation beyond the SM, such as leptonic CP violation in the decays of a heavy Majorana neutrino based on the seesaw and leptogenesis mechanisms [28] in neutrino physics.

V. SUMMARY

We have pointed out that it is possible to partly understand the observed pattern of quark flavor mixing in the chiral and heavy quark mass limits. Such model-independent access to the underlying quark flavor structure can help us explain $|V_{us}| \simeq |V_{cd}|$, $|V_{cb}| \simeq |V_{ts}|$, $|V_{cd}/V_{td}| \simeq |V_{cs}/V_{ts}| \simeq |V_{tb}/V_{cb}|$, and $|V_{ub}/V_{cb}| < |V_{td}/V_{ts}|$. In particular, we have argued that the phenomenologically successful relations $|V_{ub}/V_{cb}| \sim \sqrt{m_u/m_c}$

⁶We admit that running the heavy quark masses m_c, m_b , and m_t down to the QCD scale might not make sense [25]. One may only consider the masses of up and down quarks [26] and then propose $\text{CP}_{\text{strong}} \sim m_u m_d \sin \bar{\theta} / \Lambda_{\text{QCD}}^2$ as an alternative measure of strong CP violation.

⁷In this special case, one of the three mixing angles of V must vanish, leading to $\mathcal{J}_q = 0$, too [27].

and $|V_{td}/V_{ts}| \simeq \sqrt{m_d/m_s}$ can be reasonably conjectured in the heavy quark mass limits. In view of the experimental fact that two of the CKM unitarity triangles are almost the right triangles with $\alpha \simeq 90^\circ$, we have obtained $\mathcal{J}_q \simeq |V_{ud}| \cdot |V_{ub}| \cdot |V_{td}| \cdot |V_{tb}|$. A particular parametrization of V with the minimal CP -violating phase has been emphasized, and the RGE running behaviors of its parameters have been discussed. We have also made a very brief comment on the strong CP problem and compared between the preliminary measures of strong and weak CP -violating effects in the quark sector within the SM.

Although our present attempts in this regard remain quite limited, we have obtained some encouraging results. We hope that the underlying flavor theory, which might be related to a certain flavor symmetry and its spontaneous or explicit breaking mechanism, could provide us with a more convincing dynamical reason for what we have observed about the structure of quark flavor mixing and CP violation.

One may naturally ask whether the leptonic flavor mixing structure could similarly be understood in the reasonable mass limits of the charged leptons and neutrinos. While the charged leptons have a strong mass hierarchy, the neutrino mass spectrum remains unknown to us—we do not know whether the three neutrinos have a normal mass hierarchy $m_1 < m_2 < m_3$ or an inverted mass hierarchy $m_3 < m_1 < m_2$. On the other hand, our knowledge on the Maki-Nakagawa-Sakata-Pontecorvo matrix is still poor, because its CP -violating phases are all undetermined. Hence a lot more experimental and theoretical efforts are needed to make progress towards a better understanding of the flavor issues in the lepton sector.

ACKNOWLEDGMENTS

The author is deeply indebted to Shun Zhou for helpful comments in Oberwölz and useful discussions via e-mails. This work was supported in part by the Ministry of Science and Technology of China under Grant No. 2009CB825207 and in part by the National Natural Science Foundation of China under Grant No. 11135009.

APPENDIX

Given the 3×3 quark mass matrices M_u and M_d in an arbitrary flavor basis, it is always possible to make an appropriate basis transformation such that the resulting mass matrices \bar{M}_u and \bar{M}_d simultaneously have vanishing (1, 1), (2, 2), (1, 3), and (3, 1) elements [29]:

$$\bar{M}_u = \begin{pmatrix} 0 & X_u & 0 \\ X'_u & 0 & Y_u \\ 0 & Y'_u & Z_u \end{pmatrix}, \quad \bar{M}_d = \begin{pmatrix} 0 & X_d & 0 \\ X'_d & 0 & Y_d \\ 0 & Y'_d & Z_d \end{pmatrix}. \quad (\text{A1})$$

The determinants of \bar{M}_u and \bar{M}_d turn out to be

$$\begin{aligned} |\det \bar{M}_u| &= |X_u X'_u Z_u| = m_u m_c m_t, \\ |\det \bar{M}_d| &= |X_d X'_d Z_d| = m_d m_s m_b. \end{aligned} \quad (\text{A2})$$

In this basis, one may simply set $X_u \rightarrow 0$ (or $X_d \rightarrow 0$) to achieve the chiral quark mass limit $m_u \rightarrow 0$ (or $m_d \rightarrow 0$), or vice versa. Defining $\bar{H}_q \equiv \bar{M}_q \bar{M}_q^\dagger$ (for $q = u$ or d), we then obtain

$$\begin{aligned} \lim_{m_u \rightarrow 0} \bar{H}_u &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & |X'_u|^2 + |Y_u|^2 & Y_u Z_u^* \\ 0 & Y_u^* Z_u & |Y'_u|^2 + |Z_u|^2 \end{pmatrix}, \\ \lim_{m_d \rightarrow 0} \bar{H}_d &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & |X'_d|^2 + |Y_d|^2 & Y_d Z_d^* \\ 0 & Y_d^* Z_d & |Y'_d|^2 + |Z_d|^2 \end{pmatrix}. \end{aligned} \quad (\text{A3})$$

Hence, Eq. (5) is the result of a specific basis choice instead of a pure assumption. But we admit that a given quark mass limit does not uniquely correspond to a definite texture of the quark mass matrix, simply because the latter is basis dependent.

To illustrate the above point in a more transparent way, let us take a look at the following typical example of quark mass matrices in two different bases [11]:

$$M_q^{(H)} = A_q \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad M_q^{(D)} = \frac{A_q}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad (\text{A4})$$

where $q = u$ or d . It is well known that the democratic texture $M_q^{(D)}$ can be transformed into the hierarchical texture $M_q^{(H)}$ via $U_0 M_q^{(D)} U_0^\dagger = M_q^{(H)}$, where

$$U_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \quad (\text{A5})$$

which is actually the leading term of the democratic flavor mixing pattern [30]. We may obtain $m_u = m_c = 0$ (or $m_d = m_s = 0$) from either $M_u^{(D)}$ (or $M_d^{(D)}$) or $M_u^{(H)}$ (or $M_d^{(H)}$), but their textures are apparently different. If a diagonal perturbation of the form $\Delta M_q \propto \text{Diag}\{0, 0, A'_q\}$ (for $|A'_q| \ll |A_q|$) is simultaneously added to $M_q^{(D)}$ and $M_q^{(H)}$, one will arrive at $m_u = m_d = 0$ without any nontrivial quark flavor mixing in the hierarchical case, but $m_u = m_d = 0$ with a nontrivial flavor mixing effect between the second and third quark families in the democratic case. This observation clearly illustrates the point that a specific quark mass limit may correspond to quite different forms of the quark mass matrix in different flavor bases, leading to quite different flavor mixing effects.

- [1] J. Beringer *et al.* (Particle Data Group), *Phys. Rev. D* **86**, 010001 (2012).
- [2] N. Cabibbo, *Phys. Rev. Lett.* **10**, 531 (1963); M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
- [3] Z. Maki, M. Nakagawa, and S. Sakata, *Prog. Theor. Phys.* **28**, 870 (1962); B. Pontecorvo, *Sov. Phys. JETP* **26**, 984 (1968).
- [4] For a recent review, see Z.Z. Xing, *Chinese Phys. C* **36**, 281 (2012); S. Luo and Z.Z. Xing, *Int. J. Mod. Phys. A* **27**, 1230031 (2012).
- [5] A. Strumia and F. Vissani, [arXiv:hep-ph/0606054v3](https://arxiv.org/abs/hep-ph/0606054v3).
- [6] F. Wilczek, *Central Eur. J. Phys.* **10**, 1021 (2012).
- [7] See, e.g., H. Fritzsch, *Phys. Lett. B* **184**, 391 (1987); Z.Z. Xing, *J. Phys. G* **23**, 1563 (1997); H. Fritzsch and Z.Z. Xing, *Nucl. Phys. B* **556**, 49 (1999).
- [8] J. M. Gerard and Z.Z. Xing, *Phys. Lett. B* **713**, 29 (2012).
- [9] Z.Z. Xing, H. Zhang, and S. Zhou, *Phys. Rev. D* **77**, 113016 (2008); **86**, 013013 (2012).
- [10] S. Weinberg, *Trans. N.Y. Acad. Sci.* **38**, 185 (1977); F. Wilczek and A. Zee, *Phys. Lett. B* **70**, 418 (1977); H. Fritzsch, *Phys. Lett. B* **70**, 436 (1977).
- [11] H. Fritzsch and Z.Z. Xing, *Prog. Part. Nucl. Phys.* **45**, 1 (2000).
- [12] H. Fritzsch, *Phys. Lett. B* **73**, 317 (1978); *Nucl. Phys. B* **155**, 189 (1979).
- [13] D. S. Du and Z.Z. Xing, *Phys. Rev. D* **48**, 2349 (1993). For a review with extensive references, see Ref. [11].
- [14] H. Fritzsch and Z.Z. Xing, *Phys. Lett. B* **555**, 63 (2003); Z.Z. Xing and H. Zhang, *J. Phys. G* **30**, 129 (2004).
- [15] C. Jarlskog, *Phys. Rev. Lett.* **55**, 1039 (1985); D. D. Wu, *Phys. Rev. D* **33**, 860 (1986).
- [16] Z.Z. Xing, *Phys. Lett. B* **679**, 111 (2009).
- [17] Z.Z. Xing, *Nuovo Cimento A* **109**, 115 (1996).
- [18] K. S. Babu and Q. Shafi, *Phys. Rev. D* **47**, 5004 (1993); S. Luo and Z.Z. Xing, *J. Phys. G* **37**, 075018 (2010).
- [19] S. Weinberg, *Phys. Rev. D* **11**, 3583 (1975).
- [20] G.'t Hooft, *Phys. Rev. Lett.* **37**, 8 (1976); R. Jackiw and C. Rebbi, *Phys. Rev. Lett.* **37**, 172 (1976); C. G. Callan, R. F. Dashen, and D. J. Gross, *Phys. Lett. B* **63**, 334 (1976).
- [21] S. Adler, *Phys. Rev.* **177**, 2426 (1969); J. S. Bell and R. Jackiw, *Nuovo Cimento A* **60**, 47 (1969).
- [22] J. E. Kim, *Phys. Rep.* **150**, 1 (1987); H. Y. Cheng, *Phys. Rep.* **158**, 1 (1988); For a recent work, see J. M. Gerard and P. Mertens, *Phys. Lett. B* **716**, 316 (2012).
- [23] C. A. Baker *et al.*, *Phys. Rev. Lett.* **97**, 131801 (2006).
- [24] R. D. Peccei, [arXiv:hep-ph/9807516](https://arxiv.org/abs/hep-ph/9807516); P. Sikivie, *C.R. Physique* **13**, 176 (2012).
- [25] H. Fusaoka and Y. Koide, *Phys. Rev. D* **57**, 3986 (1998).
- [26] See, e.g., Z. Huang, *Phys. Rev. D* **48**, 270 (1993).
- [27] H. Fritzsch and Z.Z. Xing, in Ref. [7]; J. W. Mei and Z.Z. Xing, *J. Phys. G* **30**, 1243 (2004).
- [28] M. Fukugita and T. Yanagida, *Phys. Lett. B* **174**, 45 (1986).
- [29] G. C. Branco, L. Lavoura, and F. Mota, *Phys. Rev. D* **39**, 3443 (1989).
- [30] H. Fritzsch and Z.Z. Xing, *Phys. Lett. B* **372**, 265 (1996); **440**, 313 (1998); *Phys. Rev. D* **61**, 073016 (2000).