Modified Higgs branching ratios versus CP and lepton flavor violation

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New physics thresholds which can modify the diphoton and dilepton Higgs branching ratios significantly may also provide new sources of CP and lepton flavor violation. We find that limits on electric dipole moments impose strong constraints on any \mathbb{CP} -odd contributions to Higgs diphoton decays unless there are degeneracies in the Higgs sector that enhance CP-violating mixing. We exemplify this point in the language of effective operators and in simple UV-complete models with vector-like fermions. In contrast, we find that electric dipole moments and lepton-flavor-violating observables provide less stringent constraints on new thresholds contributing to Higgs dilepton decays.

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I. INTRODUCTION

The recent discovery of a Higgs-like resonance at the LHC [\[1\]](#page-6-0), with a mass of approximately 125 GeV consistent with electroweak precision observables, has solidified the impressive verification of the Standard Model (SM) at the electroweak scale. At the present time, the couplings of this resonance agree on average rather well with those of the SM Higgs boson.

The lack of hints for new physics (NP) in other channels has focused attention on the detailed properties of the Higgs-like resonance, and deviations from the SM in its decays to various final states. Indeed, while the LHC now strongly constrains NP that can be produced either resonantly or in pairs from proton constituents with wellidentifiable final states—e.g., Z' bosons decaying to leptons, or squark/gluino decays to jets, leptons and missing energy—NP produced via electroweak interactions or other weakly coupled hidden sectors is far less constrained. The latter possibilities are now coming under additional scrutiny as possible explanations for small 2σ deviations from the SM in certain Higgs production/decay channels [\[1\]](#page-6-0), specifically, the apparent enhancement in the diphoton branching $Br(h \to \gamma \gamma)$ [\[2](#page-6-1)] and a possible suppression of decays to dileptons in Br($h \rightarrow \tau \tau$). Although these deviations are small and may well dissipate with more data, they motivate the exploration of viable models of NP that could provide an explanation. The recent literature has focused on $Br(h \to \gamma \gamma)$ and noted that relatively light (typically sub 300 GeV) electromagnetically charged fields that are vector-like (VL), i.e., with a contribution to their mass which does not come from electroweak symmetry breaking, can lead to the required enhancement while still being accessible with sufficient statistics at the LHC $[3-5]$ $[3-5]$ $[3-5]$.

Exploration of Higgs interactions in this way will be an important probe of NP in coming years, and thus it is important to clarify the full range of interactions that allow for measurable corrections to the Higgs branching rates, and the interplay with other precision data, particularly in

the Yukawa sector. In this paper, we ask whether new VL thresholds contributing to sizable deviations from SM Higgs branching can also provide new sources of CP and flavor violation $[6,7]$ $[6,7]$. In Sec. [II,](#page-0-0) building on $[7]$ $[7]$ $[7]$ we focus on the CP-odd operator $hF_{\mu\nu}\tilde{F}_{\mu\nu}$, and elucidate the connection between the CP-violating Higgs decay amplitude and the impressive constraints on the electric dipole moments (EDMs) of elementary particles [\[8](#page-6-6)–[11\]](#page-7-0). We find that the inferred bound on the EDM of the electron $[8,9]$ $[8,9]$ $[8,9]$ does not allow for significant CP-odd contributions to the Higgs diphoton decay at the level of this dimension-five operator. We then consider two UV completions involving VL fermions and/or singlets, and identify a special case where the Higgs is nearly degenerate with a singlet scalar that allows for large CP-odd contributions to the diphoton decay that can escape EDM bounds. In Sec. [III,](#page-4-0) we turn our attention to the $(H^{\dagger}H)\overline{L}_{L}^{i}He_{R}^{j}$ operators contributing to dilepton decays, and consider the benchmark sensitivity from lepton-flavor-violating (LFV) observables and EDMs. Section [IV](#page-6-8) contains some concluding remarks.

II. EDMS VERSUS DIPHOTON DECAYS

Consider new physics charged under $SU(2) \times U(1)$ only, so that the leading dimension-six operators which correct the diphoton branching ratio of the Higgs are

$$
\Delta \mathcal{L} = \frac{g_1^2}{e^2 \Lambda^2} H^{\dagger} H (a_h B_{\mu\nu} B^{\mu\nu} + \tilde{a}_h B_{\mu\nu} \tilde{B}^{\mu\nu})
$$

+
$$
\frac{g_2^2}{e^2 \tilde{\Lambda}^2} H^{\dagger} H (b_h W_{\mu\nu} W^{\mu\nu} + \tilde{b}_h W_{\mu\nu} \tilde{W}^{\mu\nu}) \quad (1)
$$

$$
\rightarrow \frac{c_h \nu}{\Lambda^2} h F_{\mu\nu} F^{\mu\nu} + \frac{\tilde{c}_h \nu}{\tilde{\Lambda}^2} h F_{\mu\nu} \tilde{F}^{\mu\nu} + \cdots \tag{2}
$$

Here $c_h = a_h + b_h$, $\tilde{c}_h = \tilde{a}_h + \tilde{b}_h$, $v = 246$ GeV and we have only retained the $h\gamma\gamma$ operators, disregarding couplings to Z and W. Since we focus on corrections that are sizable for loop-induced couplings to the photon, the associated corrections to the tree-level hZZ and hWW couplings can be consistently ignored.¹ For thresholds in the TeV range or above, measurement of the Higgs decay rate itself probably provides the best sensitivity to Λ . However, EDMs can provide sensitivity to the CP-odd threshold $\tilde{\Lambda}$.

The ensuing correction to the SM $h \to \gamma \gamma$ width,

$$
\Gamma_{\gamma\gamma}^{\rm SM} = \frac{m_h^3}{4\pi} \left(\frac{\alpha}{4\pi}\right)^2 \left|\frac{A_{\rm SM}}{2v}\right|^2 \approx 9.1 \text{ keV},\tag{3}
$$

takes the form

$$
R_{\gamma\gamma} = \frac{\Gamma_{\gamma\gamma}}{\Gamma_{\gamma\gamma}^{\text{SM}}} \simeq \left| 1 - c_h \frac{v^2}{\Lambda^2} \frac{8\pi}{\alpha A_{\text{SM}}} \right|^2 + \left| \tilde{c}_h \frac{v^2}{\tilde{\Lambda}^2} \frac{8\pi}{\alpha A_{\text{SM}}} \right|^2, \tag{4}
$$

where $A_{\text{SM}}(m_h = 125 \text{ GeV}) \simeq A_W + A_t \simeq -6.5 \text{ is propor-}$ tional to the SM amplitude [\[14\]](#page-7-1). The deviations in the width are of $\mathcal{O}(1)$ for $\Lambda/\sqrt{c_h} \sim 5$ TeV. Note that since the CP-odd operator does not interfere with the SM amplitude, the corresponding correction to the diphoton branching ratio is necessarily positive and scales as $\mathcal{O}(1/\tilde{\Lambda}^4)$.

A. EDM limit on contact operators

Current experiments [[8](#page-6-6)[–11\]](#page-7-0) already probe the EDMs of elementary particles at a level roughly commensurate with two-loop electroweak diagrams [\[15\]](#page-7-2), with the chirality of light particles protected by factors of $m_{e(q)}/v$. Thus it is useful to introduce the auxiliary quantity $d_f^{(2l)}$ that quantifies this two-loop benchmark EDM scale,

$$
d_f^{(2l)} = \frac{|e| \alpha m_f}{16\pi^3 v^2} \Rightarrow d_e^{(2l)} \approx 2.5 \times 10^{-27} e \cdot \text{cm.}
$$
 (5)

One observes that $d_e^{(2l)}$ has already been surpassed by the current electron EDM limits $[8,9]$ $[8,9]$ $[8,9]$ $[8,9]$, with the mercury $[10]$ and neutron [\[11\]](#page-7-0) EDMs not lagging far behind for $d_q^{(2l)}$ [[15\]](#page-7-2).

The CP-odd Higgs operator [\(2](#page-0-1)) generates fermionic EDMs via a Higgs-photon loop (as seen in Fig. [1\)](#page-1-0),

$$
d_i = \tilde{c}_h \frac{|e|m_f}{4\pi^2 \tilde{\Lambda}^2} \ln \left(\frac{\Lambda_{\text{UV}}^2}{m_h^2}\right) \tag{6}
$$

$$
= d_f^{(2l)} \times \frac{\tilde{c}_h}{\alpha/(4\pi)} \times \frac{\nu^2}{\tilde{\Lambda}^2} \ln\left(\frac{\Lambda_{\rm UV}^2}{m_h^2}\right),\tag{7}
$$

with explicit dependence on the UV scale Λ_{UV} . If this scale is identified with $\tilde{\Lambda}$, then using the current bound on the electron EDM, $|d_e|$ < 1.05 × 10⁻²⁷e cm [[8](#page-6-6)], we find

$$
\tilde{\Lambda} \approx 50\sqrt{\tilde{c}_h} \text{ TeV.}
$$
 (8)

FIG. 1. Left: the diagram that gives rise to fermionic EDMs via the insertion of the operator $h \tilde{F}$ from Eq. [\(2\)](#page-0-1). Right: the two-loop diagram that leads to fermion EDMs in the model involving a VL lepton, ψ , coupled to a singlet, S, that mixes with the Higgs. The cross on the scalar line indicates that this contribution is proportional to the mixing term, A, in the scalar potential.

Translating this to the Higgs diphoton branching ratio results in the conclusion that CP-odd corrections are limited by

$$
\Delta R_{\gamma\gamma}(\tilde{c}_h) \lesssim 1.6 \times 10^{-4}.\tag{9}
$$

However, this conclusion can be relaxed in specific UV completions. As we discuss in the next subsection, the logarithm $\ln(\tilde{\Lambda}^2/m_h^2) \sim 10$ cannot generally be stretched all the way to 50 TeV, as the loops of VL charged particles provide a much lower cutoff, while certain degeneracies may provide more significant qualitative changes to the implications of EDM limits.

B. UV-complete examples with VL fermions 1. Singlet scalar with pseudoscalar coupling to VL fermions

We will now consider a specific UV completion which allows the full two-loop function to be taken into account for the electron EDM. The addition of a (hyper)charged VL fermion ψ with mass m_{ψ} transforming as $(1, 1, Q_{\psi})$ under $SU(3) \times SU(2) \times U(1)$, and a singlet \hat{S} with a Higgs-portal interaction with the Higgs doublet H [\[16\]](#page-7-3), leads to the following Lagrangian:

$$
\mathcal{L}_{SH\psi} = \bar{\psi} i\gamma^{\mu} (i\partial_{\mu} - eQ_{\psi}A_{\mu})\psi \n+ \bar{\psi}[m_{\psi} + \hat{S}(Y_{S} + i\gamma_{S}\tilde{Y}_{S})]\psi + \mathcal{L}_{HS}.
$$
 (10)

The terms in \mathcal{L}_{HS} contain scalar kinetic terms and describe the Higgs-portal interaction between \hat{S} and H via the following potential:

$$
V_{HS} = -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^4 + \frac{1}{2} \hat{m}_S^2 \hat{S}^2
$$

$$
+ AH^\dagger H \hat{S} - B \hat{S} + \frac{\lambda_S}{4} \hat{S}^4.
$$
(11)

CP-odd couplings of the Higgs proportional to the combination $A\tilde{Y}_S$ are generated, while the term linear in \hat{S} can always be adjusted to ensure that $\langle \hat{S} \rangle = 0$. We retain only the photon contribution of the J_{μ}^{ψ} vector current, as the Z

¹For recent studies of the CP properties of the hZZ and hWW couplings, see, e.g., Refs. $[12,13]$ $[12,13]$ $[12,13]$ $[12,13]$ $[12,13]$ and references therein.

MODIFIED HIGGS BRANCHING RATIOS VERSUS CP ... PHYSICAL REVIEW D 86, 113004 (2012)

contribution is suppressed by the small value of g_V^e . After the breaking of $SU(2) \times U(1)$, the \hat{S} field mixes with what would be the SM Higgs boson \hat{h} to produce two mass eigenstates h and S,

$$
\begin{pmatrix} \hat{h} \\ \hat{S} \end{pmatrix} = \begin{pmatrix} c_{\theta} & s_{\theta} \\ -s_{\theta} & c_{\theta} \end{pmatrix} \begin{pmatrix} h \\ S \end{pmatrix}, \qquad \tan 2\theta = \frac{2Av}{\hat{m}_{S}^{2} - 2\lambda_{H}v^{2}},
$$
\n(12)

where s_{θ} (c_{θ}) stands for sin θ ($\cos\theta$). Both mass eigenstates inherit Higgs-like interactions with the SM fields and couplings to ψ fermions.

The dominant two-loop contribution to fermion EDMs is well-known [\[17\]](#page-7-6), and specializing to our case we arrive at the following result for the electron EDM as a function of Y_s , θ and m_{ψ} :

$$
d_f = d_f^{(2l)} \times Q_\psi^2 \tilde{Y}_S \frac{\nu}{m_\psi} \sin(2\theta) [g(m_\psi^2/m_h^2) - g(m_\psi^2/m_S^2)],
$$
\n(13)

where the loop function is given by

$$
g(z) = \frac{z}{2} \int_0^1 dx \frac{1}{x(1-x) - z} \ln\left(\frac{x(1-x)}{z}\right), \quad (14)
$$

which satisfies $g(1) \sim 1.17$ and $g \sim \frac{1}{2} \ln z$ for large z. We show the Feynman diagram responsible for this contribution on the right of Fig. [1](#page-1-0).

It is instructive to consider different limits of (13) . When $m_h \ll m_\psi$, m_S , to logarithmic accuracy $g(m_\psi^2/m_h^2)$ – $g(m_\psi^2/m_S^2) \rightarrow \frac{1}{2} \ln(m_{\min}^2/m_h^2)$, where m_{\min} is the smaller of m_S and m_w . In this limit, the heavy fields can be integrated out sequentially, with S and ψ first, and h second. The first step is simplified by the use of the chiral anomaly equation for ψ , $\partial_{\mu} \bar{\psi} \gamma_{\mu} \gamma_5 \psi = 2i \bar{\psi} \gamma_5 \psi +$
 $\frac{\alpha}{2} O^2 F$ F This leads to the following identification: $\frac{\alpha}{8\pi} Q^2_{\psi} F_{\mu\nu} \tilde{F}_{\mu\nu}$. This leads to the following identification:

$$
\frac{\tilde{c}_h}{\tilde{\Lambda}^2} = \frac{\alpha Q_\psi^2}{4\pi} \frac{\tilde{Y}_S A}{m_S^2 m_\psi}; \qquad \Lambda_{\text{UV}} \simeq \min(m_S, m_\psi). \tag{15}
$$

Apart from a smaller value for the logarithmic cutoff, the result in this limit differs little from the contact operator case above. Even if the value of the logarithm is not enhanced, $\ln(m_{\min}^2/m_h^2) \sim O(1)$, the corrections to the Higgs diphoton rate will be limited to at most the subpercent level unless a fine-tuned cancellation of d_e is arranged with some other CP-odd source.

We now consider a different near-degenerate limit, $|m_h - m_S| \ll m_h$, which turns out to be more interesting as it allows the EDM constraints to be bypassed. If the difference between the masses is small, we can approximate

$$
\sin(2\theta)(m_S^2 - m_h^2) \to 2Av,\tag{16}
$$

and the EDM becomes

$$
d_f = d_f^{(2l)} \times Q_\psi^2 \tilde{Y}_S \frac{2Av^2 m_\psi}{m_h^4} g'(m_\psi^2/m_h^2)
$$
 (17)

$$
\rightarrow d_f^{(2l)} \times Q_\psi^2 \tilde{Y}_S \frac{A \nu^2}{m_h^2 m_\psi},\tag{18}
$$

where in the final step we made use of the large m_{ψ} limit.

The limiting case ([18](#page-2-1)) receives no logarithmic enhancement. Moreover, the value of the A parameter can be very small, comparable to the mass splitting between h and S or less. An $O(1 \text{ GeV})$ mass splitting would naturally place $Av^2/(m_h^2 m_\psi)$ in the $O(10^{-2} - 10^{-3})$ range, suppressing the EDM safely below the bound.

At the same time, as explicitly shown in Ref. [[5\]](#page-6-3), modifications to the $h \rightarrow \gamma \gamma$ rate can be significant, and enhancement can come from the $F_{\mu\nu}\tilde{F}^{\mu\nu}$ amplitude. Unlike corrections to the $F_{\mu\nu}F^{\mu\nu}$ amplitudes that can enhance or suppress the effective rate, the CP-odd channel always adds to $R_{\gamma\gamma}$. Assuming that the mass difference between the singlet and the Higgs is small enough that they cannot be separately resolved (which requires $|m_S - m_h| \leq 3$ GeV with current statistics [[5\]](#page-6-3)), the apparent increase in the diphoton rate in this model is

$$
R_{\gamma\gamma}^{\text{eff}}(\tilde{Y}_S) = \cos^2\theta \times \frac{\text{Br}_{h\to\gamma\gamma}}{\text{Br}_{h\to\gamma\gamma}^{\text{SM}}} + \sin^2\theta \times \frac{\text{Br}_{S\to\gamma\gamma}}{\text{Br}_{h\to\gamma\gamma}^{\text{SM}}}.
$$
 (19)

If θ is in the range

$$
\sqrt{\frac{\Gamma_{\hat{S}\to\gamma\gamma}}{\Gamma_{\hat{h}\to\gamma\gamma}}\text{Br}_{h\to\gamma\gamma}^{\text{SM}}}\leq\theta\lesssim\sqrt{\frac{\Gamma_{\hat{h}\to\gamma\gamma}}{\Gamma_{\hat{S}\to\gamma\gamma}}}\tag{20}
$$

and $\Gamma_{\hat{h}\to\gamma\gamma}$ ~ $\Gamma_{\hat{S}\to\gamma\gamma}$, then $R_{\gamma\gamma}$ simplifies to a θ -independent expression,

$$
R_{\gamma\gamma}^{\text{eff}}(\tilde{Y}_S) \simeq 1 + \frac{\Gamma_{\hat{S}\to\gamma\gamma}}{\Gamma_{\hat{h}\to\gamma\gamma}}.
$$
 (21)

The rate for the weak eigenstate \hat{S} to decay to two photons via its pseudoscalar coupling to the VL fermions is

$$
\Gamma_{\hat{S}\to\gamma\gamma} = \frac{\alpha^2 Q_{\psi}^4 \tilde{Y}_s^2 m_S^3}{256 \pi^3 m_{\psi}^2} \left| A_{1/2}^P \left(\frac{m_S^2}{4m_{\psi}} \right) \right|^2, \qquad (22)
$$

with

$$
A_{1/2}^P(\tau) = \frac{2}{\tau} (\sin^{-1} \sqrt{\tau})^2.
$$
 (23)

For large m_{ψ} the apparent diphoton increase can then be expressed as

$$
R_{\gamma\gamma}^{\rm eff}(\tilde{Y}_S) \sim 1 + Q_{\psi}^4 \left(\frac{\tilde{Y}_S}{2}\right)^2 \left(\frac{150 \text{ GeV}}{m_{\psi}}\right)^2. \tag{24}
$$

A sizable increase in the apparent diphoton rate is seen to require rather large Yukawa couplings or light VL fermions. The VL leptons must be heavier than 105 GeV to avoid limits from LEP. Their decay channels are fairly modeldependent but they are well within the reach of the LHC if they are at all relevant for the $h \rightarrow \gamma \gamma$ rate. For more discussion on experimental searches for such VL fermions, see Ref. [[5](#page-6-3)].

In Fig. [2](#page-3-0) we show the relationship between the electron EDM and the enhancement to the Higgs diphoton rate that comes from the operator $hF_{\mu\nu}\tilde{F}^{\mu\nu}$ for both the contact operator and nearly degenerate singlet cases. In the case of the contact operator, we show two cutoffs, Λ_{UV} = 200 GeV and 1 TeV. As seen in Sec. [II A,](#page-1-1) it is apparent that in this simple situation, any appreciable increase in the $h \rightarrow \gamma \gamma$ rate must be accompanied by a value of the electron EDM that is in conflict with the present experimental limit. We also show the relationship between $R_{\gamma\gamma}^{\text{eff}}$ and d_e in the singlet case for two values of the mixing angle, $\theta = 0.1$ and $\pi/4$, fixing the pseudoscalar Yukawa to $\tilde{Y}_s = 2$ and choosing $Q_{\psi} = 1$. Different values of $R_{\gamma\gamma}^{\text{eff}}$ and d_e then correspond to different values of m_ψ . It is now apparent that a sizable increase in the effective diphoton rate can be obtained in this model without inducing a value of the electron EDM that is presently excluded, demonstrating a UV completion of the effective interaction that evades the constraints implied by a simple analysis of this contact operator. The reason that the EDM constraints are evaded in this case is clear: mixing of the two fields, \hat{h} and \hat{S} , due to the small mass difference can proceed rather efficiently even with a small value of A, while the EDM loop diagrams do not enjoy the same resonant enhancement. In this model, for fixed $R_{\gamma\gamma}^{\text{eff}}$, d_e increases with increasing ΔM and sin2 θ . The rough upper limit on ΔM of around 3 GeV with current data implies an upper limit on d_e of $\sim 10^{-28}e$ cm for $R_{\gamma\gamma}^{\text{eff}} \approx 1.5-2$. Separately resolving a degeneracy near 125 GeV or limiting the size of a potential mass splitting with more data clearly has important implications for EDM searches.

2. Full VL generation with CP-violating Higgs couplings.

Another simple UV completion is a full VL generation of SM-like fields $E_R \sim (1, 1, -1)$ and $L_L \sim (1, 2, -1/2)$, with their mirror image fields E_L and L_R ,

$$
-\mathcal{L}_{EL} \supset (\bar{E}_L, \bar{L}_L) \begin{pmatrix} M_E & y_1 H \\ y_2 H^* & M_L \end{pmatrix} \begin{pmatrix} E_R \\ L_R \end{pmatrix} + \text{H.c.} \quad (25)
$$

Every entry in this mass matrix, $M_{E(L)}$, $y_{1(2)}$, can be complex. However, there is only one physical CP-odd phase combination that cannot be removed by a field redefinition $\sim \phi_E + \phi_L - \phi_1 - \phi_2$, which will appear in Higgs-fermion CP-odd vertices. For the purposes of calculation, it is more convenient to switch to the mass eigenstate basis for the $Q = 1$ fermions (we denote the masses m_1 and m_2), related to the original basis ([25](#page-3-1)) by a unitary rotation of the left- and right-handed fields:

FIG. 2 (color online). The effective increase in the diphoton rate as a function of the electron EDM coming from a coupling of the Higgs to $F_{\mu\nu}\tilde{F}^{\mu\nu}$. The black dashed lines show the relationship in the case of the contact operator $hF_{\mu\nu}\tilde{F}^{\mu\nu}$ simply cut off at the scales Λ_{UV} = 200 GeV and 1 TeV. The solid lines show the relationship in the case of a scalar singlet, S, nearly degenerate with the Higgs coupled to a VL fermion, ψ . We choose a splitting between m_S and m_h of $\Delta M = 1$ GeV (left panel) and 3 GeV (right panel) and a CP-odd Yukawa coupling of the singlet to the VL fermions of $\tilde{Y}_s = 2$. The solid curve on the left of each panel (green or light gray) is for a mixing angle $\theta = 0.1$ and the solid curve on the right of each panel (blue or dark gray) for $\theta = \pi/4$. The dotted lines show the value of d_e implied for the two mixing angles for $m_{\psi} = 105$ GeV and 300 GeV. Values of the electron EDM that are excluded experimentally, $d_e > 1.05 \times$ 10^{-27} e cm, are in the shaded region. We observe that the degenerate scalar allows for a sizable apparent increase in the Higgs diphoton rate in the CP-odd channel while not conflicting with the electron EDM limit, unlike the simple contact operator case.

MODIFIED HIGGS BRANCHING RATIOS VERSUS CP ... PHYSICAL REVIEW D 86, 113004 (2012)

$$
\begin{pmatrix}\nM_E & y_1 \nu/\sqrt{2} \\
y_2 \nu/\sqrt{2} & M_L\n\end{pmatrix} = \begin{pmatrix}\n\cos \theta_L & \sin \theta_L e^{i\phi_L} \\
-\sin \theta_L e^{-i\phi_L} & \cos \theta_L\n\end{pmatrix} \begin{pmatrix}\nm_1 & 0 \\
0 & m_2\n\end{pmatrix} \begin{pmatrix}\n\cos \theta_R & -\sin \theta_R e^{-i\phi_R} \\
\sin \theta_R e^{i\phi_R} & \cos \theta_R\n\end{pmatrix}.
$$
\n(26)

In the mass eigenstate basis, the Higgs fields develops the following couplings to the ψ_1 and ψ_2 fermions:

$$
\mathcal{L} = \frac{h}{2v} m_1 \bar{\psi}_{1L} \bigg[1 - \cos(2\theta_L) \cos(2\theta_R) - \frac{m_2}{m_1} e^{-i(\phi_L - \phi_R)} \sin(2\theta_L) \sin(2\theta_R) \bigg] \psi_{1R} + \frac{h}{2v} m_2 \bar{\psi}_{2L} \bigg[1 - \cos(2\theta_L) \cos(2\theta_R) - \frac{m_1}{m_2} e^{i(\phi_L - \phi_R)} \sin(2\theta_L) \sin(2\theta_R) \bigg] \psi_{2R} + \text{H.c.} + \cdots
$$
(27)

The ellipsis denotes the off-diagonal $h\bar{\psi}_1\psi_2$ couplings, which will not affect the EDMs or Higgs decay phenomenology within our approximations. The CP-odd vertices from this Lagrangian can now be inserted directly into the two-loop formulas,

$$
d_e^{h\gamma} = d_e^{(2l)} \times \sin(\phi_L - \phi_R) \sin(2\theta_L) \sin(2\theta_R)
$$

$$
\times \frac{m_1 m_2}{m_h^2} \left[\frac{g(z_1)}{z_1} - \frac{g(z_2)}{z_2} \right],
$$
(28)

where $z_i = m_i^2/m_h^2$. In addition to the $h\gamma$ two-loop diagram, there is also a WW two-loop contribution, with the same topology. The mass of the neutral fermion that enters this diagram is given by M_L , where

$$
|M_L|^2 = m_2^2 \cos^2 \theta_L \cos^2 \theta_R + m_1^2 \sin^2 \theta_L \sin^2 \theta_R
$$

+
$$
\frac{m_1 m_2}{2} \cos(\phi_L - \phi_R) \sin(2\theta_L) \sin(2\theta_R).
$$
 (29)

 \mathcal{CP} violation enters the WW diagram via the relative phase of the left- and right-handed charged currents. Performing the calculation, we find

$$
d_e^{WW} = d_e^{(2l)} \times \sin(\phi_L - \phi_R) \sin(2\theta_L) \sin(2\theta_R)
$$

$$
\times \frac{m_1 m_2}{m_W^2} \frac{\alpha_W}{8\alpha} \left[\frac{j(z_1, z_L)}{z_1} - \frac{j(z_2, z_L)}{z_2} \right],
$$
(30)

where $z_i = m_i^2/m_W^2$, $z_L = |M_L|^2/m_W^2$, $\alpha_W = g_W^2/(4\pi)$, and the new loop function j is defined as

$$
j(z,r) = z \int_0^1 \frac{dx(1-x)}{(x-z)(1-x)-rx} \ln\left(\frac{x(1-x)}{z(1-x)+rx}\right).
$$
 (31)

Calculations of these two-loop effects closely resemble those for the two-loop chargino-neutralino EDM contributions in ''split SUSY'' models [[18](#page-7-7)] and the two-loop EDMs in theories with additional CP-violation in the top-Higgs coupling (see, e.g., Ref. [\[19\]](#page-7-8)).

In this model, the increase in the Higgs diphoton decay rate resulting from CP-violating couplings is

$$
R_{\gamma\gamma}(\phi_L - \phi_R)
$$

= $1 + \left(\frac{d_e}{d_e^{(2l)}}\right)^2 \frac{|m_2^2 A_{1/2}^P(m_h^2/4m_1^2) - m_1^2 A_{1/2}^P(m_h^2/4m_2^2)|^2}{4m_1 m_2 |A_{\text{SM}}|^2 D},$ (32)

where D is a [typically $\mathcal{O}(1)$] combination of two-loop functions,

$$
D = \frac{m_1 m_2}{m_h^2} \left[\frac{g(z_1^h)}{z_1^h} - \frac{g(z_2^h)}{z_2^h} \right] + \frac{m_1 m_2}{m_W^2} \frac{\alpha_W}{8\alpha} \times \left[\frac{j(z_1^W, z_L^W)}{z_1^W} - \frac{j(z_2^W, z_L^W)}{z_2^W} \right].
$$
\n(33)

A large enhancement of the diphoton rate through CP-violating effects would require large mass splittings between² ψ_1 and ψ_2 and for $d_e/d_e^{(2l)}$ to be at least a factor of a few. Since $d_e^{(2l)}$ is itself larger than the present limit on the electron EDM, a sizable CP-odd enhancement to the $h \rightarrow \gamma \gamma$ rate in this model will generate an electron EDM in conflict with experiment. Therefore, this model is an example of a UV completion that gives rise to the operator $hF_{\mu\nu}\tilde{F}^{\mu\nu}$ whose behavior aligns with that of the simple contact operator in Sec. [II A:](#page-1-1) a large CP-odd contribution to the Higgs diphoton rate conflicts with the experimental limit on the electron EDM.

III. CP AND FLAVOR OBSERVABLES VERSUS DILEPTON DECAYS

We now turn our attention to the leptonic branching ratio of the Higgs, and the interplay with sources of flavor violation in the Higgs couplings [[20,](#page-7-9)[21](#page-7-10)]. We will consider specific tree-level dimension-six threshold corrections to the lepton Yukawa couplings [[19](#page-7-8),[22](#page-7-11),[23](#page-7-12)], assumed to arise, for example, from a lepton-specific extension to the Higgs sector,

$$
- \mathcal{L} = Y^{ij} \bar{L}_L^i H e_R^j + \frac{Z^{ij}}{\Lambda^2} (H^\dagger H) \bar{L}_L^i H e_R^j + \text{h.c.}
$$
 (34)

$$
\rightarrow m_i \bar{l}_i l_i + \frac{m_i}{\nu} \bar{l}_i h(\delta_{ij} + \alpha_{ij} + i\beta_{ij}\gamma^5) l_j + \cdots, \quad (35)
$$

 2 We note that a large splitting is problematic for electroweak precision measurements but a detailed analysis of this issue lies outside the scope of this paper. Studies of electroweak precision and an increase in the Higgs diphoton rate have recently been undertaken in, e.g., Refs. [[4,](#page-6-10)[5\]](#page-6-3).

TABLE I. Limits on the flavor matrix elements $\gamma_{ij} = \sqrt{\alpha_{ij}^2 + \beta_{ij}^2}$ from various observables.

Observable	Source	Limit
$Br(\mu \rightarrow e\gamma)$ < 2.4 \times 10 ⁻¹²	MEG [25]	γ_{12} < 4.7 \times 10 ⁻³
$Br(\tau \rightarrow e\gamma)$ < 3.3 × 10 ⁻⁸	$BABAR$ [26]	γ_{13} < 1.4
$Br(\tau \rightarrow \mu \gamma) < 4.4 \times 10^{-8}$	$BABAR$ [26]	γ_{23} < 1.6
$\Gamma(\mu N \to eN)/\Gamma_{\text{capture}} < 7 \times 10^{-13}$	SINDRUM-II [27]	γ_{12} < 6 × 10 ⁻²
$Br(\tau \to \mu K^+ K^-)$ < 6.8 $\times 10^{-8}$	Belle $[28]$	$\gamma_{23} < 57$
$Br(\tau \to 3\mu) < 2.1 \times 10^{-8}$	Belle $[29]$	$\gamma_{23} < 67$

where the second line refers to the mass eigenstate basis, and the normalization assumes $m_i > m_j$. The flavor matrix is

$$
\alpha_{ij} + i\beta_{ij} = \frac{v^3}{\sqrt{2}\Lambda^2 m_i} (UZV^{\dagger})_{ij},\tag{36}
$$

where U and V rotate the left- and right-handed lepton fields from the weak basis to the mass basis, respectively.

The correction to the Higgs dilepton branching ratio takes the form,

$$
B_l = \frac{\Gamma_{l\bar{l}}}{\Gamma_{l\bar{l}}^{\text{SM}}} \simeq |1 + \alpha_{ll}|^2 + |\beta_{ll}|^2, \tag{37}
$$

so that any CP-odd correction is again necessarily positive.

A. Flavor sensitivity

We assume a generic flavor structure for the matrices α_{ij} and β_{ij} . Integrating out the Higgs, the operators with minimal Yukawa suppression are two-loop transition dipoles with top and W loops $[20]$,

$$
\mathcal{L}_{\text{dipole}} = \frac{\alpha e}{32\pi^3} \frac{m_i}{v^2} (C_t + C_W) \bar{l}_i F \sigma(\alpha_{ij} + i\beta_{ij} \gamma^5) l_j. \tag{38}
$$

The loop functions are^3

$$
C_t = 2N_c Q_t^2 f(z_t), \qquad (39)
$$

$$
C_W = -\left\{3f(z_W) + \frac{23}{4}g(z_W) + \frac{3}{4}h(z_W) + \frac{1}{2z_W}[f(z_W) - g(z_W)]\right\},\tag{40}
$$

where $z_t = m_t^2/m_h^2$, $z_W = m_W^2/m_h^2$, $g(z)$ is defined in Eq. ([14](#page-2-2)), and

$$
f(z) = \frac{z}{2} \int_0^1 dx \frac{1 - 2x(1 - x)}{x(1 - x) - z} \ln\left(\frac{x(1 - x)}{z}\right),\qquad(41)
$$

$$
h(z) = z^2 \frac{\partial}{\partial z} \left(\frac{g(z)}{z} \right). \tag{42}
$$

There are also one-loop contributions to the dipoles with an h - τ loop, proportional (with our normalization) to Y^2_τ , and Higgs-mediated four-fermion interactions that are further Yukawa-suppressed.

Using the effective interactions arising from (34) (34) (34) , various LFV transition rates are straightforwardly computed as discussed, e.g., in Refs. [[23](#page-7-12),[24](#page-7-13)], and we summarize some of the stronger limits in Table [I](#page-5-1). The transition dipoles generally lead to the strongest constraints, despite being generated at two-loop order, as they are not subject to additional Yukawa suppression. Indeed, the largest contribution to $\mu \rightarrow e$ conversion actually comes from the induced $\mu e\gamma$ vertex, despite being loop-suppressed relative to the Higgs-mediated four-fermion operator.

We observe that most of the LFV limits are relatively weak, particularly in the τ sector, and so thresholds that impact $BR(h \to \tau \tau)$ could still introduce new flavor structures in the τ sector. The most stringent limits apply to α_{12} and β_{12} , and a generic flavor structure in the muon sector would limit branching ratio corrections to the percent level.

B. CP sensitivity

There are analogous two-loop contributions to the electron EDM,

$$
d_e = \frac{\alpha e}{16\pi^3} \frac{m_e \beta_{11}}{v^2} (C_t + C_W).
$$
 (43)

The current electron EDM bound [[8](#page-6-6)] implies $|\beta_{11}| < 0.13$. Taken as a generic flavor-independent limit, this would restrict any CP-odd corrections to the branching ratio to $\mathcal{O}(2\%)$.

C. Comments on UV completions

A simple candidate model that can give rise to the effective Higgs-lepton interactions in [\(34\)](#page-5-0) is a two-Higgs doublet model [[30\]](#page-7-14) with one doublet, H_q , coupled to quarks and the other, H_{ℓ} , to leptons. H_{q} can be arranged to be SM-like, suppressing the branching of the SM-like Higgs to leptons. The charged Higgses, if light, could contribute to the branching rate to diphotons. However, substantially increasing this rate appears to require large, negative quartic couplings, with potential issues for vacuum stability. For further discussion of this point in the

³We include only the leading diagrams involving virtual W/Goldstone bosons.

context of colored particles contributing to the diphoton rate, see Ref. [\[31\]](#page-7-20).

The relatively weak limits on flavor-violating observables could allow for $\mathcal{O}(1)$ deviations in the Higgs sector with respect to leptons. Measuring the Higgs decay rate to taus would be a highly desirable step towards testing this possibility. Looking for lepton-flavor-violating decays with a large sample of taus, as could be obtained at a Super-B factory, would shed further light on the situation. Additionally, we note that it appears possible to check the CP properties of the $h-\tau-\tau$ coupling at a linear collider [[32](#page-7-21)].

IV. CONCLUDING REMARKS

With the discovery of a new Higgs-like resonance at the LHC, attention is turning to precision tests of its interactions. The variety of decay channels accessible at \sim 125 GeV is already providing important information about its couplings to vector bosons and fermions. Further tests of these production and decay channels in coming years will provide an important new probe of physics beyond the SM, and allow for a useful interplay with other precision data, particularly in the Yukawa sector. In this paper, we have studied the extent to which a generic new threshold with CP and lepton flavor violation can impact Higgs branchings to diphotons and dileptons. We find that precision constraints on EDMs and LFV decays restrict this possibility quite significantly in many cases. In particular, large CP-violating contributions to $h \rightarrow \gamma \gamma$ require an extended scalar sector with mass degeneracies. While there is currently limited information about $h \rightarrow \tau \tau$ decays, large corrections to the SM rate are possible with new flavor structures at relatively low scales. Progress in studies of rare τ decays, e.g., at a Super-B factory, could provide further constraints on this possibility.

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Note added:—As this work was being finalized, several related publications appeared on the arXiv. Reference [\[33\]](#page-7-22) discusses CP-odd contributions to Higgs digamma decays in models with VL fermions, while Ref. [[13](#page-7-5)] consider the CP properties of the Higgs-like resonance. We also thank P. Winslow for informing us of related work in progress with S. Tulin.

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