Characteristic of chiral phase transition in QED_3 at zero density

Hong-tao Feng,[1,*] Bin Wang,[2] Wei-min Sun,[2,3] and Hong-shi Zong[2,3,†]

[1]*Department of Physics, Southeast University, Nanjing, People's Republic of China*
[2]*Department of Physics, Nanjing University, Nanjing, People's Republic of China*
[3]*Joint Center for Particle, Nuclear Physics and Cosmology, Nanjing, People's Republic of China*
(Received 30 September 2012; published 26 November 2012)

Based on the truncated Dyson-Schwinger equation for the fermion propagator, the Cornwall-Jackiw-Tomboulis effective potential near the critical point is investigated. We show that, at zero temperature, the system undergoes a continuous phase transition into the chiral symmetric phase at a critical number of fermion flavors, while undergoing a second-order phase transition at high temperature.

DOI: 10.1103/PhysRevD.86.105042 PACS numbers: 11.10.Kk, 11.15.Tk, 11.30.Qc

I. INTRODUCTION

Dynamical chiral symmetry breaking (DCSB) in quantum electrodynamics in $(2 + 1)$ dimensions (QED_3) has been investigated for a long time. Although it is an Abelian gauge theory, QED_3 is shown to exhibit DCSB [1–6] and confinement [7,8], which is similar to QCD. In addition, since the discovery of high-T_c superconductivity, QED_3 has attracted more attention from physicists. It is generally believed that QED_3 with N flavors can be regarded as a possible effective theory for high-T_c superconductivity in underdoped cuprates [9–11] and graphene [12–14]. Because of these features, QED_3 has been extensively studied in recent years.

A breakthrough in the study of chiral phase transition (*CPT*) in QED_3 was achieved in the paper of Appelquist *et al.* [1], who found that *CPT* happens when the number of flavors of massless fermions reaches a critical number $N_c \approx 3.24$. They arrived at this conclusion by analytically and numerically solving the lowest-order approximation for the Dyson-Schwinger equation (DSE) for the fermion propagator, where the wave reformulation $A(p^2) \equiv 1$ and the involved one-loop boson polarization are obtained by the free form of the fermion propagator. Later, some groups adopted improved schemes for DSE and obtained qualitatively similar results with $N_c \approx 3.3$ [6,15]. However, as far as we know, the nature of *CPT* at N_c has not been reported in the existing literature. One of the motivations in this paper is to study this issue.

At finite temperature, much of the literature shows that *CPT* in QCD with two massless quarks is likely to be of second order [16,17]. Then, two natural questions may be raised: how does one chart the phase transition of QED_3 at finite temperature and does this phenomenon exist in this Abelian system. Although the results from chiral and fermion number susceptibility of QED_3 reveal that this system exhibits a typical characteristic of second-order phase transition driven by chiral symmetry restoration [18], it is interesting to adopt an alternative method (more specifically, the Cornwall-Jackiw-Tomboulis (CJT) effective potential) to reanalyze the nature of this phase transition and see whether it is consistent with the results obtained using chiral and fermion number susceptibility.

In thermal QED_3, following the lowest-order DSE for the fermion propagator, Dorey investigated the DCSB of QED_3 and showed that QED_3 at $N = 1$ undergoes *CPT* into a chiral symmetric phase when the temperature reaches a critical value T_c. Later, the authors of Refs. [19,20] studied an improved truncated scheme for DSE to study the *CPT* and found that the correctional contribution to the factor only slightly changes the results qualitatively. These conclusions suggest that the lowest-order DSE for the fermion propagator is a suitable approximation for studying *CPT* in thermal QED_3.

In the rainbow approximation, the CJT effective potential provides us with a useful tool to analyze the phase structure of QED_3 [21]. Therefore, in this paper, we shall try to adopt the CJT effective potential and the truncated DSE for the fermion propagator at zero and finite temperature to study the nature of the phase transition in QED_3.

II. PHASE TRANSITION AT ZERO TEMPERATURE

A. CJT effective potential

In Euclidean space, the Lagrangian of QED_3 with N fermion flavors in the chiral limit reads

$$\mathcal{L} = \sum_j^N \bar{\psi}_j(\not{\partial} + ie\not{A})\psi_j + \frac{1}{4}F_{\sigma\nu}^2, \qquad (1)$$

where the four-component spinors are employed. At zero temperature and density, this Lagrangian is chiral symmetric, but DCSB occurs because of nonperturbative effects. The order parameter of *CPT* is defined by

$$\langle\bar{\psi}\psi\rangle = \mathrm{Tr}[S(x \equiv 0)] = \int \frac{d^3p}{(2\pi)^3}\frac{4B(p^2)}{A^2(p^2)p^2 + B(p^2)}. \qquad (2)$$

The two functions $A(p^2)$ and $B(p^2)$ in the above equation are related to the inverse fermion propagator as

*fenght@seu.edu.cn
†zonghs@chenwang.nju.edu.cn

$$S^{-1}(p) = i\gamma \cdot pA(p^2) + B(p^2). \tag{3}$$

Working in CJT's framework [21], we write the effective pressure as

$$\mathcal{P}(N) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \mathrm{Tr}\Big[\ln(1 - BS) - \frac{1}{2}BS\Big], \tag{4}$$

where the trace operation is over the flavor indices. After some algebra, the CJT effective potential in the truncated DSE for the fermion propagator reduces to

$$\mathcal{P}(N) = 2N \int \frac{\mathrm{d}^3 p}{(2\pi)^2}\Big[\ln\Big(1 + \frac{B^2}{p^2}\Big) - \frac{B^2}{p^2 + B^2}\Big]. \tag{5}$$

B. Order parameter

To indicate the order of the phase transition, we regard $\mathcal{P}(N)$ as a function of N and give the Taylor expansion for $\mathcal{P}(N)$ near $N = N_c$

$$\begin{aligned}\mathcal{P}(N) &= \mathcal{P}(N_c) + (N - N_c)\frac{\partial \mathcal{P}(N)}{\partial N}\Big|_{N=N_c} \\ &\quad + \frac{(N-N_c)^2}{2}\frac{\partial^2 \mathcal{P}(N)}{\partial N^2}\Big|_{N=N_c} + \cdots \\ &= \mathcal{P}(N_c) + (N - N_c)\mathcal{P}'(N_c) \\ &\quad + \frac{(N-N_c)^2}{2}\mathcal{P}''(N_c) + \ldots,\end{aligned} \tag{6}$$

with

$$\mathcal{P}'(N) \equiv \frac{\partial \mathcal{P}(N)}{\partial N}\Big|_{N}, \tag{7}$$

$$\mathcal{P}''(N) \equiv \frac{\partial^2 \mathcal{P}(N)}{\partial N^2}\Big|_{N}. \tag{8}$$

The singularity of $\mathcal{P}'(N_c)$ shows a first-order phase transition and its continuity might imply a second-order phase transition.

From Eq. (5), the two parameters $\mathcal{P}'$, $\mathcal{P}''$ are given as

$$\begin{aligned}\mathcal{P}'(N) = 2\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\Big[\ln\Big(1 + \frac{B^2}{p^2}\Big) - \frac{B^2}{p^2 + B^2} \\ + \frac{2NB^3B'}{(p^2 + B^2)^2}\Big],\end{aligned} \tag{9}$$

$$\begin{aligned}\mathcal{P}''(N) = 4\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\Big[\frac{2B^3B' + 3NB^2B'^2 + NB^3B''}{(p^2 + B^2)^2} \\ - \frac{4NB^4B'^2}{(p^2 + B^2)^3}\Big],\end{aligned} \tag{10}$$

where $B' = \frac{\partial B}{\partial N}$ and $B'' = \frac{\partial^2 B}{\partial N^2}$. According to the above two equations, once the function B is obtained, one can analyze the nature of the phase transition near the critical number of fermion flavors.

C. Numerical results

The next task is to calculate the three functions in the right-hand side of Eqs. (9) and (10), which can be obtained via the fermion self-energy:

$$B(p^2) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\frac{2B(k^2)}{[k^2 + B^2(k^2)][q^2 + \frac{Nq}{8}]}, \tag{11}$$

where $q = p - k$ and the coupling constant $e^2 = 1$. From the iterative equation, we also obtain B', B''. The typical behaviors of the three functions in breaking phase are shown in Fig. 1.

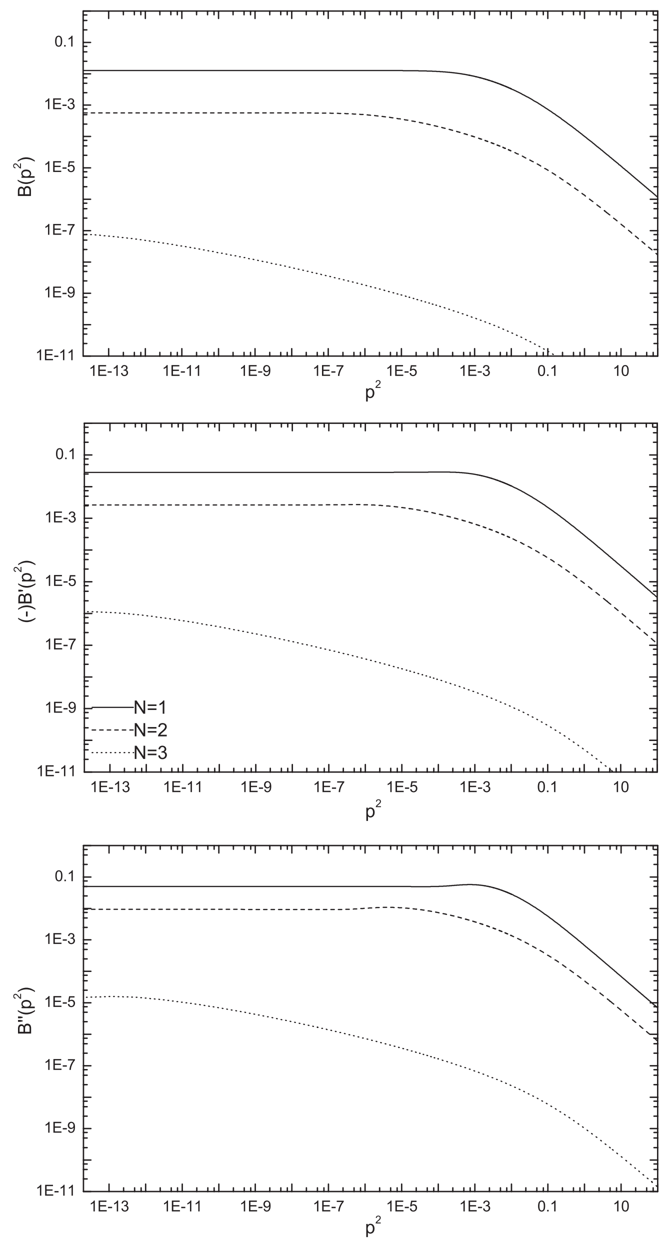

FIG. 1. The typical behavior of the three functions B, B'', and $-B'$ for several values of N.

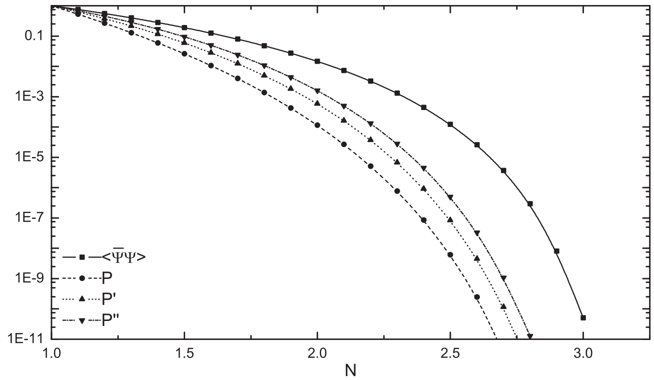

FIG. 2. The dependence of $\langle\bar{\psi}\psi\rangle$, $\mathcal{P}$, P', P'' on the number of fermion flavors, where each parameter is normalized by its value at $N = 1$ and hence is dimensionless (here we note that the original value of $\mathcal{P}'$ is negative).

From Fig. 1 it can be seen that each of the three functions almost keeps a constant value in the infrared region and diminishes at large p^2. Comparing the behaviors of the three functions, we also find that all of them $\propto 1/p^2$ in the high-energy region and hence the two parameters $\mathcal{P}'(N)$ and $\mathcal{P}''(N)$ are also convergent. Substituting Eq. (11) into Eqs. (5), (9), and (10), we immediately obtain the three parameters for the phase transition which are plotted in Fig. 2.

From Fig. 2 we see that, with the increasing N, all four parameters diminish and chiral symmetry gets restored because of the vanishing $\langle\bar{\psi}\psi\rangle$ at N_c. Moreover, it is found that the curves for $\mathcal{P}'$, P'' fall monotonously and *do not* show any singularity as the number of fermion flavors rises.

III. PHASE TRANSITION AT FINITE TEMPERATURE

Under rainbow-ladder approximation, the pressure at finite temperature is

$$\mathcal{P}(T) = \frac{1}{\mathcal{V}}\mathrm{Tr}\Big[\ln(\beta S^{-1}) - \frac{1}{2}BS\Big], \tag{12}$$

with $\beta = 1/T$. As a general discussion, we only investigate CPT in the case of $N = 1$. Before going to the model calculation of the pressure, let us first analyze its limiting behavior at high temperature. In the limit of high temperature, the dressed fermion propagator reduces to the free one and the pressure for a free gas is given as

$$\begin{aligned}\mathcal{P}_0(T) &= \frac{T}{V}\,\mathrm{Tr}\ln(\beta S_0^{-1}) = 2T\sum_n\int\frac{\mathrm{d}^2P}{(2\pi)^2}\ln\frac{P^2+\varpi_n^2}{T^2}\\ &= 2\int\frac{\mathrm{d}^2P}{(2\pi)^2}[\mathcal{E}_{p0} + 2T\ln(e^{-\beta\mathcal{E}_{p0}}+1)]\\ &= 4T\int\frac{\mathrm{d}^2P}{(2\pi)^2}\ln(e^{-\beta\mathcal{E}_{p0}}+1) = \frac{3\zeta(3)}{2\pi}T^3,\end{aligned} \tag{13}$$

where the term $\mathcal{E}_{p0} = \sqrt{P^2}$ has been dropped since it is independent of T and thus we are not interested in it.

A. Model for the pressure

In the case of zero chemical potential, the fermion propagator at finite temperature T can be written as

$$S^{-1}(P) = i\vec{\gamma}\cdot\vec{P}A(P^2) + i\varpi_n\gamma_3C(P^2) + B(P^2), \tag{14}$$

with $\varpi_n = (2n+1)\pi T$. Just as we mentioned in the Introduction, in this paper we work in the lowest-order DSE, so the fermion propagator reduces to

$$S^{-1}(P) = i\vec{\gamma}\cdot\vec{P} + i\varpi_n\gamma_3 + B(P^2), \tag{15}$$

and the pressure is totally determined by the fermion self-energy,

$$\begin{aligned}\mathcal{P}(T) &= T\sum_n\int\frac{\mathrm{d}^2P}{(2\pi)^2}\mathrm{Tr}\Big[\ln[\beta S^{-1}(P)] - \frac{1}{2}B(P^2)S(P)\Big]\\ &= 2T\sum_n\int\frac{\mathrm{d}^2P}{(2\pi)^2}\Big[\ln\frac{P^2+\varpi_n^2+B^2}{T^2} - \frac{B^2}{P^2+\varpi_n^2+B^2}\Big].\end{aligned} \tag{16}$$

Since the zero frequency approximation is widely adopted [18–20,22], we also work in this framework and then the pressure at finite temperature is obtained

$$\begin{aligned}\mathcal{P}(T) &= 2T\sum_n\int\frac{\mathrm{d}^2P}{(2\pi)^2}\Big[\ln\frac{\varpi_n^2+\mathcal{E}_p^2}{T^2} - \frac{B^2}{\varpi_n^2+\mathcal{E}_p^2}\Big]\\ &= 2\int\frac{\mathrm{d}^2P}{(2\pi)^2}\Big[\mathcal{E}_p + 2T\ln(1+e^{-\beta\mathcal{E}_p})\\ &\quad - \frac{B^2}{2\mathcal{E}_p}\tanh\frac{\mathcal{E}_p}{2T}\Big],\end{aligned} \tag{17}$$

where $\mathcal{E}_p = \sqrt{P^2+B^2(P^2)}$. It can be easily seen that the integral in Eq. (17) is divergent. We can use a small trick described in the equation below to treat this divergence,

$$\begin{aligned}\mathcal{P}(T) &= \mathcal{P}(T) - \mathcal{P}_0(T) + \mathcal{P}_0(T)\\ &= 2\int\frac{\mathrm{d}^2P}{(2\pi)^2}\Big[\mathcal{E}_p - \mathcal{E}_{p0} + 2T\ln\frac{e^{-\beta\mathcal{E}_p}+1}{e^{-\beta\mathcal{E}_{p0}}+1}\\ &\quad - \frac{B^2}{2\mathcal{E}_p}\tanh\frac{\mathcal{E}_p}{2T}\Big] + T^3\frac{3\zeta(3)}{2\pi}.\end{aligned} \tag{18}$$

From Eq. (18) it can be easily seen that $\mathcal{P}(T)$ reduces to Eq. (5) when $T \to 0$ and to its free value (13) in the high-temperature limit, respectively. This is what one expects in advance.

Once the fermion self-energy is known, we can immediately obtain the pressure. Then, from Eq. (11), the integral equation for the dynamically generated mass function at finite temperature reads [22]

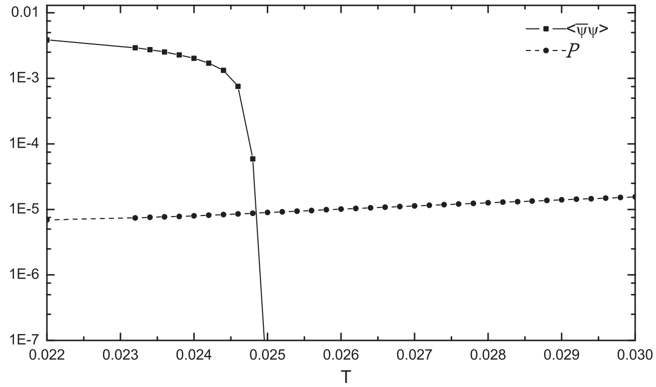

FIG. 3. The dependence of the fermion chiral condensate and the pressure on the temperature near the critical point T_c.

$$B(P^2) = 2T\sum_n \int \frac{d^2K}{(2\pi)^2} \frac{B(K^2)}{[\varpi_n^2 + \mathcal{E}_k^2][Q^2 + \Pi_0(Q)]}$$
$$= \int \frac{d^2K}{(2\pi)^2} \frac{B(K^2)\tanh\frac{\mathcal{E}_p}{2T}}{\mathcal{E}_p[Q^2 + \Pi_0(Q)]}, \tag{19}$$

where $Q = P - K$ and

$$\Pi_0(Q) = \frac{T}{\pi}\int_0^1 dx \ln\left[4\cosh^2\frac{\sqrt{x(1-x)Q^2}}{2T}\right]. \tag{20}$$

To numerically solve this equation, an iteration algorithm is employed. In the real numerical calculation, we adopt an ultraviolet cutoff $\Lambda = 10^4$, which is large enough to ensure that the calculated results are stable with respect to Λ. Then, we obtain the dependence of $\mathcal{P}$ on the temperature, and the results are plotted in Fig. 3. It can be seen that as the temperature rises, the fermion chiral condensate decreases and vanishes at $T_c = 2.5 \times 10^{-2}$, where the system undergoes CPT from the chiral symmetry breaking phase into the chiral symmetric phase, whereas the pressure increases monotonously around T_c.

B. Continuous entropy

With the increasing temperature, the entropy rises. It is generally believed that this parameter skips near the critical point when a first-order phase transition occurs, whereas it shows a continuous behavior at the critical point for a second-order phase transition. The definition of entropy is given trivially via the pressure,

$$s(T) = \frac{\partial\mathcal{P}(T)}{\partial T}. \tag{21}$$

From the expression for the pressure (18), we immediately obtain the dependence of the entropy on the temperature which is shown in Fig. 4. We see that s also increases continuously with the rise of temperature and shows an inflexion at T_c where CPT happens. Moreover, the slope of $s(T)$ in the chiral symmetry breaking phase is obviously larger than that in the chiral symmetric phase.

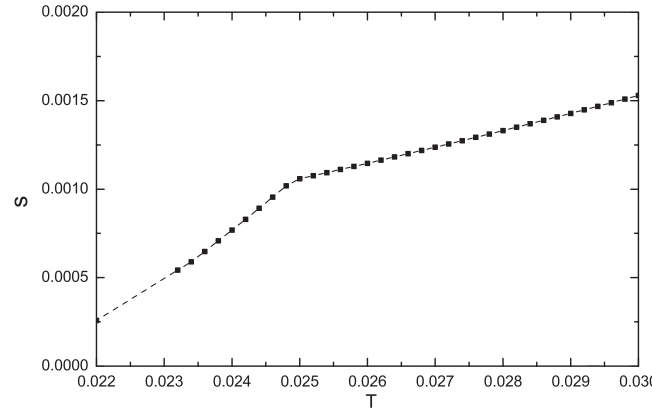

FIG. 4. Entropy as a function of temperature.

C. Signal for second-order phase transition

Analogous to the relation between the entropy and the first-order phase transition, the specific heat can be treated as a typical order parameter to indicate a second-order phase transition and exhibits its singular behavior at the critical point. It is defined by the entropy

$$C_V = \frac{\partial s(T)}{\partial T} = \frac{\partial^2\mathcal{P}(T)}{\partial T^2}. \tag{22}$$

Based on Eqs. (18) and (21), we can obtain the specific heat of QED_3 as the rise of temperature and the data near T_c are plotted in Fig. 5.

One sees that C_V in each phase exhibits a continuous behavior with the alteration of temperature but skips at T_c. Near the critical point, the value of C_V in the chiral symmetry breaking phase is about four times larger than that in the chiral symmetric phase. The jumping behavior of the specific heat at T_c shows that the system in the original chiral symmetry breaking phase undergoes a second-order phase transition into the chiral symmetric phase.

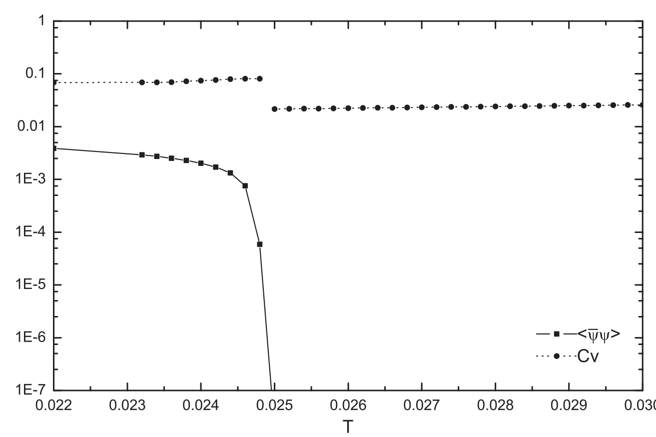

FIG. 5. The behavior of the specific heat near T_c.

IV. CONCLUSIONS

In this paper, we adopt the lowest-order approximation of Dyson-Schwinger equation to investigate the CJT effective potential and try to say something about the nature of the phase transition of QED_3 at zero density.

At zero temperature, we investigate the CJT effective potential as the rise of the number of fermion flavors and illustrate that, at the critical value N_c, the chiral phase transition is neither of first order nor of second order, and thus it should be a continuous phase transition of higher order.

With the increasing temperature, the behaviors of the entropy and the specific heat are investigated. It is found that the entropy shows a continuous behavior while the specific heat jumps at the critical temperature. This result implies that the chiral phase transition at finite temperature is of second order and is consistent with those obtained using the chiral and fermion number susceptibility as the order parameter [18]. Therefore, both of the two approaches show that the chiral phase transition at finite temperature is a second-order phase transition.

Of course, the adopted model in the present work is schematic and might be discrepant from reality, since the lowest-order DSE for the fermion propagator and the zero frequency approximation are adopted. To further confirm those observations, we need to study this problem in more realistic models.

ACKNOWLEDGMENTS

We would like to thank Dr. Y. Jiang for helpful discussions. This work was supported by the National Natural Science Foundation of China (under Grants No. 11047005, No. 11105029, No. 10935001, and No. 11075075) and by a project funded by the Priority Academic Program Development of Jiangsu Higher Education Institution.

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