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Orientifold daughter of $\mathcal{N} = 4$ SYM theory and double-trace running

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We study the orientifold daughter of $\mathcal{N} = 4$ super-Yang-Mills as a candidate nonsupersymmetric large-N conformal field theory. In a theory with vanishing single-trace beta functions that contains scalars in the adjoint representation, conformal invariance might still be broken by renormalization of double-trace terms to leading order at large N. In this note we perform a diagrammatic analysis and argue that the orientifold daughter does not suffer from double-trace running. This implies an exact large-N equivalence between this theory and a subsector of $\mathcal{N} = 4$ super-Yang-Mills.

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I. INTRODUCTION

Conformal field theories (CFTs) play a prominent role in theoretical physics. In four dimensions, it is easy to find *supersymmetric* CFTs; however, constructing interacting conformal field theories in the absence of supersymmetry seems to be a harder task. An early attempt for the construction of such theories was to use the AdS/CFT correspondence [1–3] for orbifolds of $\mathcal{N} = 4$ super-Yang-Mills (SYM) [4,5]. These are constructed by placing a stack of N D3-branes at an orbifold singularity $\mathbb{R}^6/\Gamma$, where Γ is a discrete subgroup of the R symmetry. Inheritance principles [6,7], then, guarantee that the beta functions of marginal single-trace operators will vanish in the large-N limit. If the orbifold group $\Gamma \not\subset SU(3)$, supersymmetry is completely broken and a potential conformal field theory with reduced supersymmetry is obtained.

Now, whenever there are scalars in the adjoint or bifundamental representation, there is a logarithmic running of double-trace operators present in the quantum effective action [8–12]

$$\delta S = -f \int d^4x \mathcal{O}\bar{\mathcal{O}}. \tag{1}$$

This is a *leading* effect at large N. While for supersymmetric orbifolds one can always tune the double-trace couplings to their conformal fixed points, for nonsupersymmetric orbifolds the double-trace beta functions have complex zeros and conformal invariance is *always* broken [11,12].

The authors of Ref. [12] also found a nontrivial one-to-one correspondence between the breaking of conformal invariance in the field theory and the presence of closed string tachyons in the twisted sector of the dual string theory. This result is somewhat surprising, because the correspondence is between perturbative gauge theory (dual to strongly curved AdS) and *flat-space* tachyons, indicating that the breaking of conformal invariance may be read from string theory before taking the decoupling limit. These results were revisited in Ref. [13], where it was found that the double-trace beta functions are quadratic in the coupling to all orders in planar perturbation theory.

Here we will concentrate on the *orientifold* daughter of $\mathcal{N} = 4$ SYM [14–16]. This theory arises as the low-energy description of D3-branes in a nontachyonic orientifold of Type 0B, in which the tachyon in the original string theory has been projected out by a clever choice of the parity operator [17–19].

In Refs. [16,20] it was argued that this theory is planar equivalent to $\mathcal{N} = 4$ SYM. In light of the results obtained for orbifolds, the absence of a tachyon in the flat-space string theory is a good indication that the orientifold daughter should not suffer from double-trace running and, therefore, should be an example of a nonsupersymmetric conformal field theory.

In this note we perform a diagrammatic analysis to see if this theory suffers from double-trace running or not. One possible outcome is that perturbative renormalizability will force us to add double-trace couplings of the form of Eq. (1). If the double-trace beta functions have real zeros, conformal invariance can be recovered if we tune the new couplings to their fixed points. This would imply that we have a fixed line passing through the origin of the coupling constant space. On the other hand, if one or more zeros are complex, conformal invariance is broken and the theory is unstable.

Another possible outcome is that the are no leading double-trace contributions in the effective action. If this is the case, there will be no logarithmic running, and it would imply an exact equivalence between the orientifold and a subsector of $\mathcal{N} = 4$ SYM. Conformal invariance will be preserved, but in a rather trivial sense. Our results indicate that this last behavior is the one that characterizes the orientifold daughter. In Sec. II we briefly review how the orientifold theory is constructed. In Sec. III we perform a diagrammatic analysis and show that for each double-trace diagram in $\mathcal{N} = 4$, there is an analogous diagram in

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the orientifold daughter, and vice versa. Finally, we present our concluding remarks in Sec. IV.

II. ORIENTIFOLD CONSTRUCTION

The field theory in which we are interested is a nonsupersymmetric $SU(N)$ gauge theory that arises as the low-energy description of D3-branes in a nontachyonic orientifold of Type 0B.

A. Nontachyonic Ω' projection

The Type 0B modular invariant partition function is given by the following string states:

$$(\mathrm{NS}-, \mathrm{NS}-) \oplus (\mathrm{NS}+, \mathrm{NS}+) \oplus (\mathrm{R}-, \mathrm{R}-) \oplus (\mathrm{R}+, \mathrm{R}+).$$

As is well known, there is a tachyonic state coming from the $(\mathrm{NS}-, \mathrm{NS}-)$ sector. This theory admits more than one consistent orientifold projection characterized by different definitions of the parity operator Ω [17–19]. The parity operator that gives a nontachyonic orientifold is usually denoted by $\Omega' = \Omega(-1)^{f_R}$, where f_R is the right world-sheet fermion number. The projected theory $0\mathrm{B}/\Omega'$ is called $0'\mathrm{B}$. This theory has no tachyon and is similar to the bosonic sector of Type IIB. At the massless level it has a complete set of R-R fields, a graviton and a dilaton.

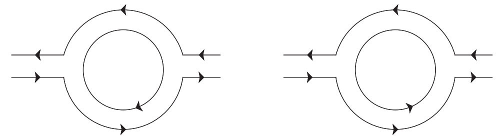
B. Orientifold daughter

If we T-dualize in six directions, we obtain the $\Omega(-1)^{f_R} I_6$ orientifold of Type 0B [14,15], where I_6 is an inversion operator ($x_i \to -x_i$, $i = 4, \ldots, 9$). This theory contains an orientifold $\mathrm{O}'3$ plane at $x_4 = \ldots = x_9 = 0$. The gauge theory describing N D3-branes in the presence of the $\mathrm{O}'3$ plane is the orientifold daughter of $\mathcal{N} = 4$ SYM we want to study. Its field content is given by Table I.

This theory is very similar to $\mathcal{N} = 4$ SYM; there are six real scalars in the adjoint representation and four Dirac fermions in the antisymmetric representation of the gauge group. Its planar diagrams are the same as those in the parent theory; using a double-line notation [21], it is clear that the only difference is in the orientation of the color arrows for diagrams involving fermions (see Fig. 1). This suggests that the orientifold daughter and the parent theory are equivalent in the large-N limit. However, we have to be careful with potential double-trace terms, as they might render the theory nonconformal.

TABLE I. Field content of the gauge theory describing N D3-branes in the presence of the $\mathrm{O}'3$ plane.

	$SU(N)$
Vector	adj.
Scalars	adj.
Weyl fermions	$\begin{smallmatrix}\square\\\square\end{smallmatrix} + \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$



(a) $\mathcal{N} = 4$ SYM (b) Orientifold

FIG. 1. Fermionic contribution to the scalar self-energy. The only difference is the orientation of the arrows.

We know that in geometric orbifolds of $\mathcal{N} = 4$ SYM, there is a one-to-one correspondence between the presence of tachyons in the flat-space string theory and the breaking of conformal invariance [12]. The absence of a tachyon in the flat-space construction of the orientifold field theory is then encouraging. Still, we feel an explicit analysis is necessary in order to check whether this theory suffers from double-trace running or not.

III. DIAGRAMMATIC ANALYSIS

Here we consider double-trace contributions to the effective action for the orientifold daughter. We will show that they cancel by comparing them with the respective diagrams in $\mathcal{N} = 4$, which we know does not suffer from double-trace running.

A. One-loop diagrams

The Lagrangian of $\mathcal{N} = 4$ SYM is well known:

$$\mathcal{L} = N\Big(-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}^a\bar{\sigma}^\mu D_\mu\psi_a - D_\mu X^I D^\mu X^I + 2\sqrt{\lambda}C^{Iab}\psi_a X^I\psi_b + 2\sqrt{\lambda}\bar{C}^I_{ab}\bar{\psi}^a X^I\bar{\psi}^b + \frac{\lambda}{2}[X^I, X^J]^2\Big). \tag{2}$$

The orientifold daughter is obtained through the replacement

$$\psi_i{}^j \to \{\xi_{[ij]}, \eta^{[ij]}\}, \tag{3}$$

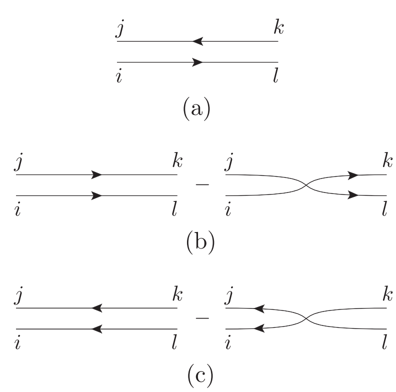


FIG. 2. Fermionic propagators in a double-line notation.

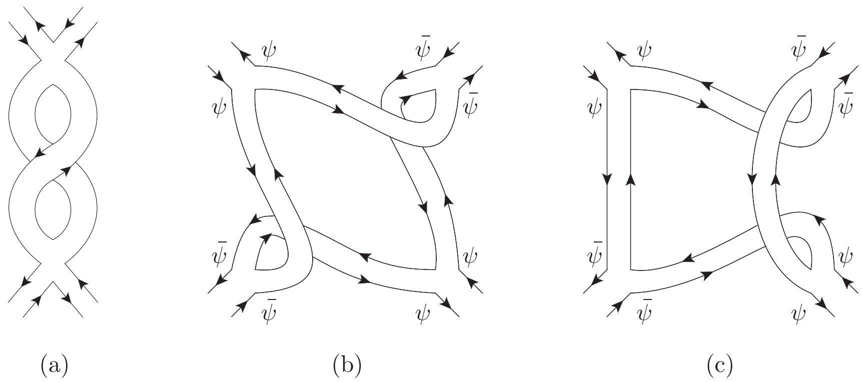


FIG. 3. Bosonic (a) and fermionic (b, c) contributions to the double-trace potential.

where i, j are color indices [22]. Its Lagrangian is

$$\mathcal{L} = N\Big(-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\bar{\eta}^a\bar{\sigma}^\mu D_\mu \eta_a + \frac{i}{2}\bar{\xi}^a\bar{\sigma}^\mu D_\mu \xi_a - D_\mu X^I D^\mu X^I + 2\sqrt{\lambda}C^{Iab}\xi_a X^I \eta_b + 2\sqrt{\lambda}\bar{C}^I_{ab}\bar{\eta}^a X^I \bar{\xi}^b + \frac{\lambda}{2}[X^I, X^J]^2\Big). \quad (4)$$

The color structure of the fermion propagators is as follows:

$$\langle \psi_i{}^j \bar{\psi}_k{}^l \rangle \sim \delta_i^l \delta_k^j, \quad (5)$$

$$\langle \xi_{[ij]}\bar{\xi}^{[kl]} \rangle \sim \delta_i^l \delta_j^k - \delta_i^k \delta_j^l, \quad (6)$$

$$\langle \eta^{[ij]}\bar{\eta}_{[kl]} \rangle \sim \delta_l^i \delta_k^j - \delta_k^i \delta_l^j. \quad (7)$$

From Fig. 2 we see that the orientifold daughter has an extra "nonplanar" term coming from the antisymmetry of the color indices.

We are only interested in double-trace contributions. In particular,

$$\delta\mathcal{L} = a(\lambda)\mathcal{O}^{IJ}\mathcal{O}^{IJ} + b(\lambda)\mathcal{O}^2, \quad (8)$$

where

$$\mathcal{O}^{IJ} = \mathrm{Tr}\Big(X^I X^J - \frac{\delta^{IJ}}{6}X^K X^K\Big), \quad (9)$$

$$\mathcal{O} = \mathrm{Tr}X^K X^K, \quad (10)$$

and $a(\lambda)$, $b(\lambda)$ are functions of the 't Hooft coupling.

These terms are easy to identify using the double-line notation; some sample contributions for $\mathcal{N} = 4$ SYM are shown in Fig. 3. The bosonic diagrams are identical in both theories, and we know that the $\mathcal{N} = 4$ SYM does not have double-trace running. Then we only need to concentrate on the fermions. There are two different sets of Wick contractions that give fermionic double-trace contributions in $\mathcal{N} = 4$, shown in Figs. 3(b) and 3(c).

To prove cancellation of double-trace terms in the orientifold, we need to find equivalent fermionic diagrams; these are shown in Fig. 4. These two diagrams will give identical contributions to those of the parent theory, and the cancellation of double-trace terms is then guaranteed. We see that the nonplanarities in the $\mathcal{N} = 4$ diagrams are here implemented by the nonplanar component of the fermion propagator. The extra minus sign in the nonplanar part could have presented a problem; however, both diagrams have an even number of them. This simple analysis confirms that the orientifold theory has no double-trace contributions at one loop.

B. Two-loop example

At two loops, we should have one power of N coming from a closed color loop; if not, the diagram is subleading. As before, we only need to concentrate on the diagrams with fermions. Because we will proceed with an all-loop analysis in the next section, we only consider one two-loop example, shown in Fig. 5.

From the color flows, it is clear that we have a factor of N coming from a closed color loop. Also, for the orientifold, we have an even number of nonplanar propagators, and so there is no extra minus sign.

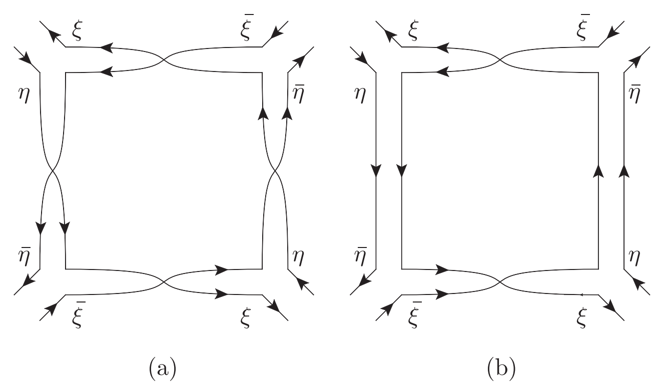


FIG. 4. Fermionic one-loop contributions from the orientifold daughter.

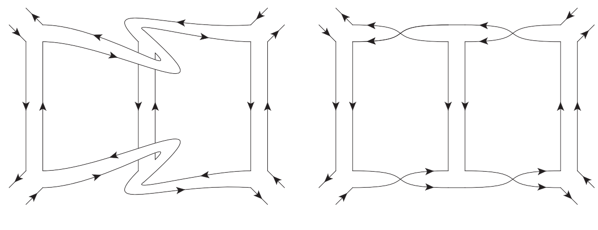


FIG. 5. Two-loop diagram in $\mathcal{N} = 4$ SYM (a); orientifold counterpart (b).

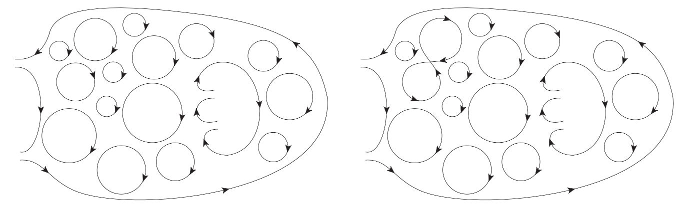


FIG. 6. Leading ℓ-loop double-trace diagram with $N^{\ell-1}$ coming from closed color loops (a); subleading diagram (b).

C. All-loop analysis

Here we will argue for the cancellation of double-trace terms at all loops. At ℓ loops, the leading single and double-trace contributions are of the form

$$\delta\mathcal{L}_{\mathrm{ST}} \sim N\lambda^{\ell+1}\,\mathrm{Tr}X^4 \quad \text{and} \quad \delta\mathcal{L}_{\mathrm{DT}} \sim \lambda^{\ell+1}\,\mathrm{Tr}X^2\,\mathrm{Tr}X^2. \tag{11}$$

The key thing to notice is that a leading double-trace diagram should have $N^{\ell-1}$ from closed color loops.[1] If we compare this with the N^{ℓ} of a leading single-trace diagram, it is clear that the double-trace contributions we are interested in are almost planar. The unique topology is shown in Fig. 6(a). It is not hard to see that the double-trace diagrams of the previous sections are of this form.

To prove that the orientifold daughter does not suffer from double-trace running, we will proceed as before. For each double-trace contribution in $\mathcal{N} = 4$ SYM, we will show that there is an equivalent diagram in the orientifold, and vice versa.

Consider an arbitrary leading double-trace diagram like the one shown in Fig. 6(a). Topologically, we have an inner and an outer boundary where the external bosonic legs sit and a number of color loops between them. Now, we connect the external legs belonging to the inner boundary using an "auxiliary" bosonic propagator, as shown in Fig. 7.

After this contraction, we have a *planar* diagram (the new topology consists of a single boundary with two external legs), but we know that there is a one-to-one correspondence between the planar diagrams of these two theories [16]. We also know that the bosonic fields are identical in both theories, and the only difference between Figs. 6(a) and 7 is a bosonic contraction. This implies that if there is a one-to-one correspondence between the diagrams of the form depicted in Fig. 7, then there is also a one-to-one correspondence between the class of diagrams depicted in Fig. 6(a). Let us rephrase this last statement: If we have a one-to-one correspondence between planar diagrams in $\mathcal{N} = 4$ SYM and its

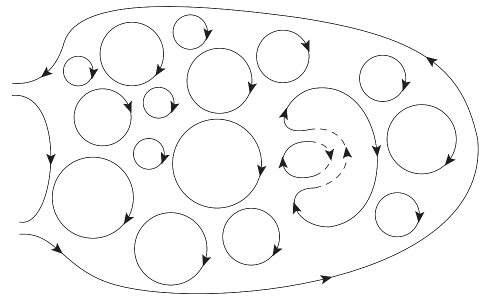


FIG. 7. Leading ℓ-loop double-trace diagram with an auxiliary bosonic propagator (dashed lines) contracting two external legs; compare with Fig. 6(a).

orientifold daughter, then we also have a one-to-one correspondence between the leading double-trace contributions to the scalar four-point function. This concludes our analysis and confirms that both theories have identical leading double-trace contributions at all loops.

IV. DISCUSSION

In this note we have shown by an explicit diagrammatic analysis that the orientifold daughter of $\mathcal{N} = 4$ SYM does not suffer from double-trace running. This is in agreement with the results of Refs. [12,20], where a one-to-one correspondence was found between the breaking of conformal invariance and the presence of tachyons in the flat-space string theory. Our calculation is yet another example that confirms this observation, namely that the flat-space theory seems to *know* about the stability of the field theory.

In the full 0B string theory, the calculation of Refs. [8,11] implies that there is a string state becoming tachyonic in the dual AdS background for sufficiently small λ (large curvature). The results of this paper confirm that the orientifold daughter is stable, and hence the AdS dual contains no tachyon. It would be interesting to understand more directly how the tachyon is projected out. Sadly, the AdS background dual to the orientifold daughter is not known. In Ref. [23] an outline was given of the main characteristics the dual theory should have. However, an explicit solution is yet to be found.

[1]In our normalization we have an overall factor of N multiplying the Lagrangian; this implies that each propagator goes as $\sim \frac{1}{N}$.

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