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Superconducting phase transitions induced by chemical potential in (2 + 1)-dimensional four-fermion quantum field theory

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In the paper a generalization of the (1 + 1)-dimensional model by Chodos *et al.* [Phys. Rev. D **61**, 045011 (2000)] has been performed to the case of (2 + 1)-dimensional spacetime. The model includes four-fermion interactions both in the fermion-antifermion (or chiral) and fermion-fermion (or superconducting) channels. We study temperature T and chemical potential μ induced phase transitions in the leading order of large-N expansion technique, where N is a number of fermion fields. It is shown that at sufficiently large values of μ and arbitrary relations between coupling constants, the superconducting phase appears in the system both at $T = 0$ and $T > 0$. In particular, at $T = 0$ and sufficiently weak attractive interaction in the chiral channel, the Cooper pairing occurs for arbitrary couplings in the superconducting channel even at infinitesimal values of μ.

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I. INTRODUCTION

Last years, great attention has been paid to the investigation of (2 + 1)-dimensional quantum field theories (QFT) and, in particular, to models with four-fermion interactions of the Gross-Neveu (GN) [1] type. Partially, this interest is explained by a more simple structure of QFT in two, rather than in three spatial dimensions. As a result, it is much easier to investigate qualitatively such real physical phenomena as dynamical symmetry breaking [1–8] and color superconductivity [9], and to model phase diagrams of real quantum chromodynamics (QCD) [10], etc. in the framework of (2 + 1)-dimensional models. Another example of this kind is spontaneous chiral symmetry breaking induced by external magnetic or chromomagnetic fields. This effect was for the first time studied also in terms of the (2 + 1)-dimensional GN model [11]. Moreover, these theories are very useful in developing new QFT techniques like the optimized perturbation theory [10,12], and so on.

However, there is yet another more serious motivation for studying (2 + 1)-dimensional QFT. It is supported by the fact that there are many condensed matter systems which, firstly, have a (quasi)planar structure and, secondly, their excitation spectrum is described adequately by a relativistic Dirac-like equation rather than by a Schrödinger one. Among these systems are the high-T_c cuprate and iron superconductors [13], the one-atom thick layer of carbon atoms, or graphene [14,15], etc. Thus, many properties of such condensed matter systems can be explained in the framework of various (2 + 1)-dimensional QFTs, including the GN-type models (see, e.g., Refs. [16–24] and references therein).

In this paper we study phase transitions in a (2 + 1)-dimensional GN-type model which describes competition between two processes: chiral symmetry breaking (excitonic pairing) and superconductivity (Cooper pairing). Clearly, the model is suitable for qualitative analysis of superconducting phase transitions in quasiplanar condensed matter systems. The structure of our model is a direct generalization of known (1 + 1)-dimensional model of Chodos *et al.* [25,26], which remarkably mimics the temperature T and chemical potential μ phase diagram of real QCD, to the case of (2 + 1)-dimensional spacetime. Recall that in Ref. [25], in order to avoid the prohibition on Cooper pairing as well as spontaneous breaking of continuous symmetry in (1 + 1)-dimensional models (known as the Mermin-Wagner-Coleman no-go theorem [27]), the consideration was performed in the leading order of $1/N$-technique, i.e., in the large-N limit assumption, where N is the number of fermion fields. In this case quantum fluctuations, which would otherwise destroy a long-range order corresponding to spontaneous symmetry breaking, are suppressed by $1/N$ factors. By the same reason in (2 + 1)-dimensional spacetime and in the case of finite values of N, spontaneous breaking of continuous symmetry is allowed only at zero temperature, i.e., it is forbidden at $T > 0$. Hence, in order to make the investigation of superconducting phase transitions possible at $T > 0$, we suppose, as it was done in Ref. [25], that in the framework of our model $N \to \infty$.

So at $T = 0$ the results of our paper may be applied for the description of superconductivity in different N-layer condensed matter systems (N is finite and can even be equal to one), whereas at $T > 0$ it is better to use the results in the description of macroscopic systems composed of a very large number of layers, such as graphite, etc.

The paper is organized as follows. In Sec. II the GN-type model with four-fermion interactions in the fermion-antifermion (or chiral) and fermion-fermion (or superconducting) channels is presented. Here the unrenormalized thermodynamic potential (TDP) of the model is obtained in the leading order of the large-N expansion technique. In Sec. III a renormalization group invariant expression for the TDP is obtained whose global minimum point provides us with chiral and Cooper pairs condensates. In Sec. IV phase structure of the model is described at $T = 0$ both at $\mu = 0$ and $\mu \neq 0$. In particular, it is established in this section that infinitesimal chemical potential induces the superconductivity phenomenon in the case of a rather weak attractive interaction in the fermion-antifermion channel. Finally, in Sec. V the (μ, T)-phase diagrams are presented for some representative values of coupling constants. We show in this section that at arbitrary fixed $T > 0$ superconductivity is induced in the system at sufficiently large values of μ. Some related problems of our consideration are relegated to three appendixes.

II. THE MODEL AND ITS THERMODYNAMIC POTENTIAL

Our investigation is based on a $(2 + 1)$-dimensional GN-type model with massless fermions belonging to a fundamental multiplet of the auxiliary $O(N)$ flavor group. Its Lagrangian describes the interaction both in the scalar fermion-antifermion and scalar difermion channels:

$$L = \sum_{k=1}^{N} \bar{\psi}_k(\gamma^\nu i\partial_\nu + \mu\gamma^0)\psi_k + \frac{G_1}{N}\left(\sum_{k=1}^{N} \bar{\psi}_k\psi_k\right)^2 + \frac{G_2}{N}\left(\sum_{k=1}^{N} \psi_k^T C\psi_k\right)\left(\sum_{j=1}^{N} \bar{\psi}_j C\bar{\psi}_j^T\right), \quad (1)$$

where μ is the fermion number chemical potential [see also the comments after Eq. (3)]. As noted above, all fermion fields ψ_k $(k = 1, \ldots, N)$ form a fundamental multiplet of the $O(N)$ group. Moreover, each field ψ_k is a four-component Dirac spinor (the symbol T denotes the transposition operation). The quantities γ^ν $(\nu = 0, 1, 2)$ are matrices in the 4-dimensional spinor space. Moreover, $C \equiv \gamma^2$ is the charge conjugation matrix. The algebra of the γ^ν matrices as well as their particular representation are given in Appendix A. Clearly, the Lagrangian L is invariant under transformations from the internal auxiliary $O(N)$ group, which is introduced here in order to make it possible to perform all of the calculations in the framework of the nonperturbative large-N expansion method. Physically more interesting is that the model (1) is invariant under the discrete chiral transformation, $\psi_k \to \gamma^5\psi_k$ (the particular realization of the γ^5 matrix is presented in Appendix A), as well as with respect to the transformations from the continuous $U(1)$ fermion number group, $\psi_k \to \exp(i\alpha)\psi_k$ $(k = 1, \ldots, N)$, responsible for the fermion number conservation or, equivalently, for the electric charge conservation law in the system under consideration.

The linearized version of Lagrangian (1) that contains auxiliary bosonic fields $\sigma(x)$, $\Delta(x)$, and $\Delta^*(x)$ has the following form:

$$\mathcal{L} = -\frac{N\sigma^2}{4G_1} - \frac{N}{4G_2}\Delta^*\Delta + \sum_{k=1}^{N}\Big[\bar{\psi}_k(\gamma^\nu i\partial_\nu + \mu\gamma^0 - \sigma)\psi_k - \frac{\Delta^*}{2}\psi_k^T C\psi_k - \frac{\Delta}{2}\bar{\psi}_k C\bar{\psi}_k^T\Big]. \quad (2)$$

Clearly, the Lagrangians (1) and (2) are equivalent, as can be seen by using the Euler-Lagrange equations of motion for bosonic fields which take the form

$$\sigma(x) = -2\frac{G_1}{N}\sum_{k=1}^{N} \bar{\psi}_k\psi_k,$$
$$\Delta(x) = -2\frac{G_2}{N}\sum_{k=1}^{N} \psi_k^T C\psi_k, \quad (3)$$
$$\Delta^*(x) = -2\frac{G_2}{N}\sum_{k=1}^{N} \bar{\psi}_k C\bar{\psi}_k^T.$$

One can easily see from (3) that the neutral field $\sigma(x)$ is a real quantity, i.e., $(\sigma(x))^\dagger = \sigma(x)$ (the superscript symbol $\dagger$ denotes the Hermitian conjugation), but the (charged) difermion fields $\Delta(x)$ and $\Delta^*(x)$ are mutually Hermitian conjugated complex quantities, so $(\Delta(x))^\dagger = \Delta^*(x)$ and vice versa. Clearly, all of the fields (3) are singlets with respect to the auxiliary $O(N)$ group.[1] Moreover, with respect to parity transformation P (see also the comment in Appendix A),

$$P\colon\ \psi_k(t, x, y) \to \gamma^5\gamma^1\psi_k(t, -x, y), \qquad k = 1, \ldots, N, \quad (4)$$

the fields $\sigma(x)$, $\Delta(x)$, and $\Delta^*(x)$ are even quantities, i.e., they are scalars. If the difermion field $\Delta(x)$ has a nonzero ground state expectation value, i.e., $\langle\Delta(x)\rangle \neq 0$, the Abelian fermion number $U(1)$ symmetry of the model is spontaneously broken down and the superconducting phase is realized in the model. However, if $\langle\sigma(x)\rangle \neq 0$ then the discrete chiral symmetry of the model is spontaneously broken.

Let us now study the phase structure of the four-fermion model (1) starting from the equivalent semibosonized Lagrangian (2). In the leading order of the large-N approximation, the effective action $\mathcal{S}_{\rm eff}(\sigma, \Delta, \Delta^*)$ of the considered model is expressed by means of the path integral over fermion fields

$$\exp(i\mathcal{S}_{\rm eff}(\sigma, \Delta, \Delta^*)) = \int \prod_{l=1}^{N}[d\bar{\psi}_l][d\psi_l]\exp\Big(i\int \mathcal{L}d^3x\Big),$$

where

[1]Note that the $\Delta(x)$ field is a flavor $O(N)$ singlet, since the representations of this group are real.

$$\mathcal{S}_{\rm eff}(\sigma, \Delta, \Delta^*) = -\int d^3x\left[\frac{N}{4G_1}\sigma^2(x) + \frac{N}{4G_2}\Delta(x)\Delta^*(x)\right] + \tilde{\mathcal{S}}_{\rm eff}. \quad (5)$$

The fermion contribution to the effective action, i.e., the term $\tilde{\mathcal{S}}_{\rm eff}$ in (5), is given by

$$\exp(i\tilde{\mathcal{S}}_{\rm eff}) = \int \prod_{l=1}^{N}[d\bar{\psi}_l][d\psi_l] \times \exp\left\{i\int \sum_{k=1}^{N}\left[\bar{\psi}_k(\gamma^\nu i\partial_\nu + \mu\gamma^0 - \sigma)\psi_k - \frac{\Delta^*}{2}\psi_k^T C\psi_k - \frac{\Delta}{2}\bar{\psi}_k C\bar{\psi}_k^T\right]d^3x\right\}. \quad (6)$$

The ground state expectation values $\langle\sigma(x)\rangle$, $\langle\Delta(x)\rangle$, and $\langle\Delta^*(x)\rangle$ of the composite bosonic fields are determined by the saddle point equations,

$$\frac{\delta\mathcal{S}_{\rm eff}}{\delta\sigma(x)} = 0, \qquad \frac{\delta\mathcal{S}_{\rm eff}}{\delta\Delta(x)} = 0, \qquad \frac{\delta\mathcal{S}_{\rm eff}}{\delta\Delta^*(x)} = 0. \quad (7)$$

For simplicity, throughout the paper we suppose that the above mentioned ground state expectation values do not depend on spacetime coordinates, i.e.,

$$\langle\sigma(x)\rangle \equiv M, \qquad \langle\Delta(x)\rangle \equiv \Delta, \qquad \langle\Delta^*(x)\rangle \equiv \Delta^*, \quad (8)$$

where M, Δ, Δ^*, are constant quantities. In fact, they are coordinates of the global minimum point of the TDP $\Omega(M, \Delta, \Delta^*)$. In the leading order of the large-N expansion it is defined by the following expression:

$$\int d^3x\Omega(M, \Delta, \Delta^*) = -\frac{1}{N}\mathcal{S}_{\rm eff}\{\sigma(x), \Delta(x), \Delta^*(x)\}|_{\sigma(x)=M,\Delta(x)=\Delta,\Delta^*(x)=\Delta^*},$$

which gives

$$\int d^3x\Omega(M, \Delta, \Delta^*) = \int d^3x\left(\frac{M^2}{4G_1} + \frac{\Delta\Delta^*}{4G_2}\right) + \frac{i}{N}\ln\left(\int\prod_{l=1}^{N}[d\bar{\psi}_l][d\psi_l] \times \exp\left(i\int\sum_{k=1}^{N}\left[\bar{\psi}_k D\psi_k - \frac{\Delta^*}{2}\psi_k^T C\psi_k - \frac{\Delta}{2}\bar{\psi}_k C\bar{\psi}_k^T\right]d^3x\right)\right), \quad (9)$$

where $D = \gamma^\nu i\partial_\nu + \mu\gamma^0 - M$. To proceed, let us first point out that without loss of generality the quantities Δ, Δ^* might be considered as real ones.[2] So, in the following we will suppose that $\Delta = \Delta^* \equiv \Delta$, where Δ is already a real quantity. Then, in order to find a convenient expression for the TDP it is necessary to invoke Appendix B, where the path integral similar to (9) is evaluated.[3] So, taking into account in (9) the relation (B7) we obtain the following expression for the zero temperature, $T = 0$, TDP of the GN model (1):

$$\Omega(M, \Delta) = \frac{M^2}{4G_1} + \frac{\Delta^2}{4G_2} + i\int\frac{d^3p}{(2\pi)^3}\ln[(p_0^2 - (\mathcal{E}_\Delta^+)^2)(p_0^2 - (\mathcal{E}_\Delta^-)^2)], \quad (10)$$

where $(\mathcal{E}_\Delta^\pm)^2 = E^2 + \mu^2 + \Delta^2 \pm 2\sqrt{M^2\Delta^2 + \mu^2E^2}$ and $E = \sqrt{M^2 + |\vec{p}|^2}$. Obviously, the function $\Omega(M, \Delta)$ is invariant under each of the transformations $M \to -M$, $\Delta \to -\Delta$, and $\mu \to -\mu$. Hence, without loss of generality, we restrict ourselves to the constraints $M \geq 0$, $\Delta \geq 0$, and $\mu \geq 0$ and will investigate the global minimum point of the TDP (10) just on this region. Using in the expression (10) a rather general formula,

$$\int_{-\infty}^{\infty} dp_0 \ln(p_0 - A) = \mathrm{i}\pi|A|, \quad (11)$$

where A is a real quantity, it is possible to reduce it to the following one:

$$\Omega(M, \Delta) \equiv \Omega^{un}(M, \Delta) = \frac{M^2}{4G_1} + \frac{\Delta^2}{4G_2} - \int\frac{d^2p}{(2\pi)^2}(\mathcal{E}_\Delta^+ + \mathcal{E}_\Delta^-). \quad (12)$$

The integral term in (12) is an ultraviolet divergent one, hence to obtain any information from this expression we need to renormalize it.

III. THE RENORMALIZATION PROCEDURE AT $T = 0$

First of all, let us regularize the zero temperature TDP (12) by cutting momenta, i.e., we suppose that $|p_1| < \Lambda$, $|p_2| < \Lambda$ in (12). As a result we have the following regularized expression (which is finite at finite values of Λ):

$$\Omega^{\rm reg}(M, \Delta) = \frac{M^2}{4G_1} + \frac{\Delta^2}{4G_2} - \frac{1}{\pi^2}\int_0^\Lambda dp_1\int_0^\Lambda dp_2(\mathcal{E}_\Delta^+ + \mathcal{E}_\Delta^-). \quad (13)$$

Let us use in (13) the following asymptotic expansion:

[2] Otherwise, phases of the complex values Δ, Δ^* might be eliminated by an appropriate transformation of fermion fields in the path integral (9).

[3] In Appendix B we consider for simplicity the case $N = 1$, however the procedure is easily generalized to the case with $N > 1$.

$$\mathcal{E}_\Delta^+ + \mathcal{E}_\Delta^- = 2|\vec{p}| + \frac{M^2+\Delta^2}{|\vec{p}|} + \mathcal{O}(1/|\vec{p}|^3), \tag{14}$$

where $|\vec{p}| = \sqrt{p_1^2+p_2^2}$. Then, upon integration there term-by-term, it is possible to find

$$\Omega^{\rm reg}(M,\Delta) = M^2\left[\frac{1}{4G_1} - \frac{2\Lambda\ln(1+\sqrt{2})}{\pi^2}\right] + \Delta^2\left[\frac{1}{4G_2} - \frac{2\Lambda\ln(1+\sqrt{2})}{\pi^2}\right] - \frac{2\Lambda^3\left(\sqrt{2}+\ln\left(1+\sqrt{2}\right)\right)}{3\pi^2} + \mathcal{O}(\Lambda^0), \tag{15}$$

where $\mathcal{O}(\Lambda^0)$ denotes an expression which is finite in the limit $\Lambda\to\infty$. Secondly, we suppose that the bare coupling constants G_1 and G_2 depends on the cutoff parameter Λ in such a way that in the limit $\Lambda\to\infty$ one obtains finite expressions in the square brackets of (15). Clearly, to fulfill this requirement it is sufficient to require that

$$\frac{1}{4G_1} \equiv \frac{1}{4G_1(\Lambda)} = \frac{2\Lambda\ln(1+\sqrt{2})}{\pi^2} + \frac{1}{2\pi g_1},\qquad \frac{1}{4G_2} \equiv \frac{1}{4G_2(\Lambda)} = \frac{2\Lambda\ln(1+\sqrt{2})}{\pi^2} + \frac{1}{2\pi g_2}, \tag{16}$$

where $g_{1,2}$ are finite and Λ-independent model parameters with dimensionality of inverse mass. Moreover, since bare couplings G_1 and G_2 do not depend on a normalization point, the same property is also valid for $g_{1,2}$. Hence, taking into account in (13) and (15) the relations (16) and ignoring there an infinite M- and Δ-independent constant, one obtains the following *renormalized*, i.e., finite, expression for the TDP:

$$\Omega^{\rm ren}(M,\Delta) = \lim_{\Lambda\to\infty}\left\{\Omega^{\rm reg}(M,\Delta)|_{G_1=G_1(\Lambda),G_2=G_2(\Lambda)} + \frac{2\Lambda^3(\sqrt{2}+\ln(1+\sqrt{2}))}{3\pi^2}\right\}. \tag{17}$$

It should also be mentioned that the TDP (17) is a renormalization group invariant quantity.

The fact that it is possible to renormalize the effective potential of the initial model (1) in the leading order of the large N expansion is the reflection of a more general property of $(2+1)$-dimensional theories with four-fermion interactions. Indeed, it is well known that in the framework of the *naive* perturbation theory (over coupling constants) these models are not renormalizable. However, as it was proved in Ref. [3], in the framework of a nonperturbative large N technique these models are renormalizable in each order of $1/N$-expansion.

In vacuum, i.e., at $\mu=0$, the $\mathcal{O}(\Lambda^0)$ term in (15) can be calculated explicitly, so we have for the renormalized effective potential $V(M,\Delta)$ the expression[4]

$$V(M,\Delta) \equiv \Omega^{\rm ren}(M,\Delta)|_{\mu=0} = \frac{M^2}{2\pi g_1} + \frac{\Delta^2}{2\pi g_2} + \frac{(M+\Delta)^3}{6\pi} + \frac{|M-\Delta|^3}{6\pi}. \tag{18}$$

Now, let us obtain an alternative expression for the renormalized TDP (17) at $\mu\neq 0$. For this purpose one can rewrite the unrenormalized TDP $\Omega^{\rm un}(M,\Delta)$ (12) in the following way:

$$\Omega^{\rm un}(M,\Delta) = \frac{M^2}{4G_1} + \frac{\Delta^2}{4G_2} - \int\frac{d^2p}{(2\pi)^2}(\mathcal{E}_\Delta^+|_{\mu=0} + \mathcal{E}_\Delta^-|_{\mu=0}) - \int\frac{d^2p}{(2\pi)^2}(\mathcal{E}_\Delta^+ + \mathcal{E}_\Delta^- - \mathcal{E}_\Delta^+|_{\mu=0} - \mathcal{E}_\Delta^-|_{\mu=0}), \tag{19}$$

where

$$\mathcal{E}_\Delta^+|_{\mu=0} + \mathcal{E}_\Delta^-|_{\mu=0} = \sqrt{|\vec{p}|^2+(M+\Delta)^2} + \sqrt{|\vec{p}|^2+(M-\Delta)^2}.$$

Since the leading terms of the asymptotic expansion (14) do not depend on μ, it is clear that the last integral in (19) is a convergent one. Other terms in (19) form the unrenormalized TDP (effective potential) at $\mu=0$ which is reduced after renormalization procedure to the expression (18). Hence, after renormalization we obtain from (19) the following finite expression [evidently, it coincides with renormalized TDP (17)]:

$$\Omega^{\rm ren}(M,\Delta) = V(M,\Delta) - \int\frac{d^2p}{(2\pi)^2}\left(\mathcal{E}_\Delta^+ + \mathcal{E}_\Delta^- - \sqrt{|\vec{p}|^2+(M+\Delta)^2} - \sqrt{|\vec{p}|^2+(M-\Delta)^2}\right), \tag{20}$$

where $V(M,\Delta)$ is presented in (18). The integral term in (20) can be explicitly calculated. As a result, we have

$$12\pi\Omega^{\rm ren}(M,\Delta) = \frac{6M^2}{g_1} + \frac{6\Delta^2}{g_2} + 2(M+\sqrt{\mu^2+\Delta^2})^3 + 2|M-\sqrt{\mu^2+\Delta^2}|^3 - 3t_+(M+\sqrt{\mu^2+\Delta^2}) + 3t_-|M-\sqrt{\mu^2+\Delta^2}| - \frac{3(\mu^2-M^2)\Delta^2}{\mu}\ln\left|\frac{t_+ + \mu(M+\sqrt{\mu^2+\Delta^2})}{t_- + \mu|M-\sqrt{\mu^2+\Delta^2}|}\right|, \tag{21}$$

[4]Vacuum TDP is usually called effective potential.

where $t_\pm = M\sqrt{\mu^2+\Delta^2} \pm \mu^2$. It is not so evident, but at $\mu = 0$ the expression (21) for $\Omega^{\rm ren}(M, \Delta)$ coincides with $V(M, \Delta)$ (18).

IV. PHASE STRUCTURE OF THE MODEL AT $T = 0$

As was mentioned above, the coordinates of the global minimum point (M_0, Δ_0) of the TDP $\Omega^{\rm ren}(M, \Delta)$ define the ground state expectation values of auxiliary fields $\sigma(x)$ and $\Delta(x)$. Namely, $M_0 = \langle\sigma(x)\rangle$ and $\Delta_0 = \langle\Delta(x)\rangle$. The quantities M_0 and Δ_0 are usually called order parameters, or gaps, because they are responsible for the phase structure of the model or, in other words, for the properties of the model ground state [see also the comment after (4)]. Moreover, the gap M_0 is equal to the dynamical mass of one-fermionic excitations of the ground state. As a rule, gaps depend on model parameters as well as on various external factors. In our consideration the gaps M_0 and Δ_0 are certain functions of the free model parameters g_1 and g_2 and such external factors as chemical potential μ and temperature T.

A. The case $\mu = 0$

First of all, let us discuss the phase structure of the model (1) in the simplest case when $\mu = 0$ and $T = 0$. The corresponding TDP is given in (18) by the function $V(M, \Delta)$. Since the global minimum of this function was already investigated in Ref. [28], although in the framework of another (2 + 1)-dimensional GN model, we present at once the phase structure of the initial model (1) at $\mu = 0$ (see Fig. 1).

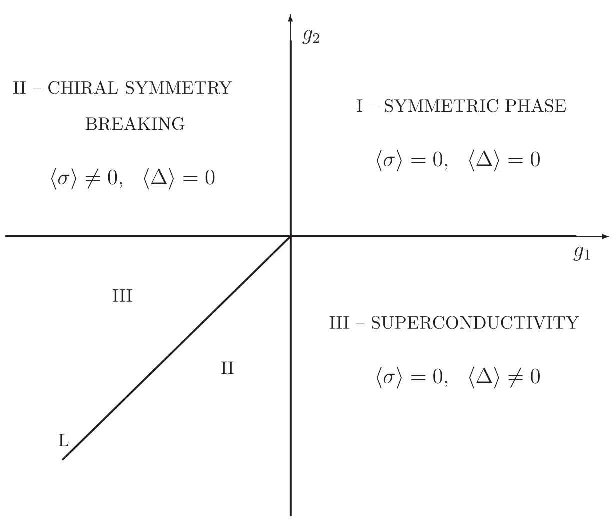

FIG. 1. The (g_1, g_2)-phase portrait of the model at $\mu = 0$. The labels I–III denote the symmetric, the chiral symmetry breaking, and the superconducting phases, respectively. In the phase II $\langle\sigma\rangle = -1/g_1$. In the phase III $\langle\Delta\rangle = -1/g_2$. On the line $L \equiv \{(g_1, g_2)\colon g_1 = g_2\}$, where $g_{1,2} < 0$, the TDP minima corresponding to the phase II and III are equivalent.

In Fig. 1 the phase portrait of the model is depicted depending on the values of the free model parameters g_1 and g_2. There the plane (g_1, g_2) is divided into several areas. In each area one of the phases I, II, or III is implemented. In the phase I, i.e., at $g_1 > 0$ and $g_2 > 0$, the global minimum of the effective potential $V(M, \Delta)$ is arranged at the origin. So in this case we have $M_0 = \langle\sigma(x)\rangle = 0$ and $\Delta_0 = \langle\Delta(x)\rangle = 0$. As a result, in the phase I both discrete chiral and continuous electromagnetic $U(1)$ symmetries remain intact and fermions are massless. Due to this reason the phase I is called symmetric. In the phase II, which is allowed only for $g_1 < 0$, at the global minimum point (M_0, Δ_0) the relations $M_0 = -1/g_1$ and $\Delta_0 = 0$ are valid. So in this phase chiral symmetry is spontaneously broken down and fermions acquire dynamically the mass M_0. Finally, in the superconducting phase III, where $g_2 < 0$, we have the following values for the gaps: $M_0 = 0$ and $\Delta_0 = -1/g_2$.

Note also that if $g_1 = g_2 \equiv g$ and, in addition, $g < 0$ (it is line L in Fig. 1), then the effective potential (18) has two equivalent global minima. The first one, the point $(M_0 = -1/g, \Delta_0 = 0)$, corresponds to a phase with chiral symmetry breaking. The second one, i.e., the point $(M_0 = 0, \Delta_0 = -1/g)$, corresponds to superconductivity.

Clearly, if the cutoff parameter Λ is fixed, then the phase structure of the model can be described in terms of bare coupling constants G_1, G_2 instead of finite quantities g_1, g_2. Indeed, let us first introduce a critical value of the couplings, $G_c = \frac{\pi^2}{8\Lambda\ln(1+\sqrt{2})}$. Then, as it follows from Fig. 1 and (16), at $G_1 < G_c$ and $G_2 < G_c$ the symmetric phase I of the model is located. If $G_1 > G_c$, $G_2 < G_c$ ($G_1 < G_c$, $G_2 > G_c$), then the chiral symmetry broken phase II (the superconducting phase III) is realized. Finally, let us suppose that both $G_1 > G_c$ and $G_2 > G_c$. In this case at $G_1 > G_2$ ($G_1 < G_2$) we have again the chiral symmetry broken phase II (the superconducting phase III).

B. Consideration of the chemical potential

In this section we study the influence of the chemical potential $\mu > 0$ on the phase structure of the model (1) (temperature is still vanishing). Numerical and analytical investigations of the TDP (21) show that its minimum points are of the form $(M \neq 0, \Delta = 0)$, $(M = 0, \Delta \neq 0)$, or $(M = 0, \Delta = 0)$ only. So to study the properties of the global minimum point of the function (21) it is enough to consider its reductions on the M- and Δ-axes, where the TDP (21) becomes

$$12\pi\Omega^{\rm ren}(M, \Delta)|_{\Delta=0} \equiv 12\pi\omega_1(M) = \frac{6M^2}{g_1} + 2(M+\mu)^3 + 2|M-\mu|^3 - 3\mu(M+\mu)^2 + 3\mu(M-\mu)|M-\mu|, \qquad (22)$$

$$12\pi\Omega^{\rm ren}(M,\Delta)|_{M=0} \equiv 12\pi\omega_2(\Delta) = \frac{6\Delta^2}{g_2} + 4(\mu^2+\Delta^2)^{3/2} - 6\mu^2\sqrt{\mu^2+\Delta^2} - 3\mu\Delta^2\ln\left(\frac{(\mu+\sqrt{\mu^2+\Delta^2})^2}{\Delta^2}\right), \tag{23}$$

respectively. Comparing the minima of the functions (22) and (23), it is possible to find the global minimum point of the whole TDP (21) and its dependence on the model parameters μ, g_1, and g_2, i.e., to determine the phase structure of the model. In addition, in the present section we will study the behavior of a particle density n in different phases when μ varies,

$$n = -\frac{\partial\Omega^{\rm ren}(M,\Delta)}{\partial\mu}|_{M=M_0,\Delta=\Delta_0}. \tag{24}$$

Since the global minimum point of the TDP (21) coincides with the global minimum point either of the function $\omega_1(M)$ (22) or the function $\omega_2(\Delta)$ (23), it is clear that in the chirally broken phase $\Delta_0 = 0$ and the gap M_0 does not depend on the parameter g_2. Correspondingly, in the superconducting phase we have $M_0 = 0$ and the gap Δ_0 does not depend on the parameter g_1. So, one can use the following expressions for the particle density in the chiral symmetry broken II and superconducting III phases:

$$n\Big|_{{\rm phase}II} = -\frac{\partial\omega_1(M)}{\partial\mu}\Big|_{M=M_0} = \frac{1}{2\pi}(\mu^2 - M_0^2)\theta(\mu - M_0), \tag{25}$$

$$n\Big|_{{\rm phase}III} = -\frac{\partial\omega_2(\Delta)}{\partial\mu}\Big|_{\Delta=\Delta_0} = \frac{1}{2\pi}\left[\mu\sqrt{\mu^2+\Delta_0^2} + \Delta_0^2\ln\frac{\mu+\sqrt{\mu^2+\Delta_0^2}}{\Delta_0}\right], \tag{26}$$

where $\theta(x)$ is the Heaviside step function.

The case $g_1 < 0$.—First of all, let us suppose that g_1 is fixed and negative, i.e., $g_1 < 0$. Then it is easy to show that for arbitrary value of g_2 there exists a critical chemical potential $\mu_{\rm crit}(g_2)$ (see Fig. 2)[5] such that at $\mu < \mu_{\rm crit}(g_2)$ the system is in the chiral symmetry breaking phase II (if $\mu_{\rm crit}(g_2) > 0$), and it is in the superconducting phase III at $\mu > \mu_{\rm crit}(g_2)$. In other words, if $\mu < \mu_{\rm crit}(g_2) \neq 0$, then

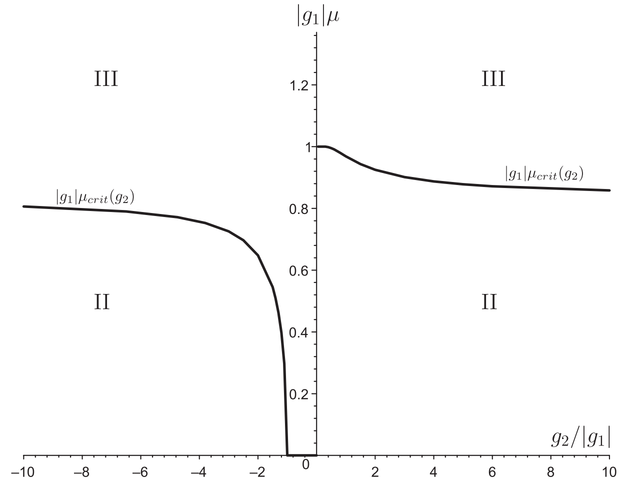

FIG. 2. The (μ, g_2)-phase portrait of the model and critical chemical potential $\mu_{\rm crit}(g_2)$ vs g_2 at arbitrary fixed $g_1 < 0$. At each point $\mu = \mu_{\rm crit}(g_2) \neq 0$ there is a first order phase transition from the chiral symmetry breaking phase II to the superconducting phase III.

the global minimum of the TDP (21) lies at the point $(M_0 = -1/g_1,\ \Delta_0 = 0)$ which does not depend on μ in the interval $0 < \mu < \mu_{\rm crit}(g_2)$. However, at $\mu = \mu_{\rm crit}(g_2)$ it jumps to the point $(M_0 = 0,\ \Delta_0 = \Delta_{\rm crit}(g_2))$, where $\Delta_{\rm crit}(g_2)$ vs g_2 is depicted in Fig. 3. Hence, at the critical point $\mu = \mu_{\rm crit}(g_2)$ a first order phase transition occurs and a superconducting gap $\Delta_0 = \Delta_{\rm crit}(g_2)$ is dynamically generated. It turns out that Δ_0 vs μ is an increasing function in the interval $\mu > \mu_{\rm crit}(g_2)$. In particular, the behavior Δ_0 vs μ is presented in Fig. 5 (at $g_2 = 0.5|g_1|$), Fig. 6 (at $g_2 = -1.5|g_1|$), and Fig. 7 (at $g_2 = -0.5|g_1|$) as the curve 1.

Moreover, it is clear from Fig. 2 that at $\mu < \mu_{\rm crit}(g_2)$, i.e., in the phase II, the particle density is equal to zero. To

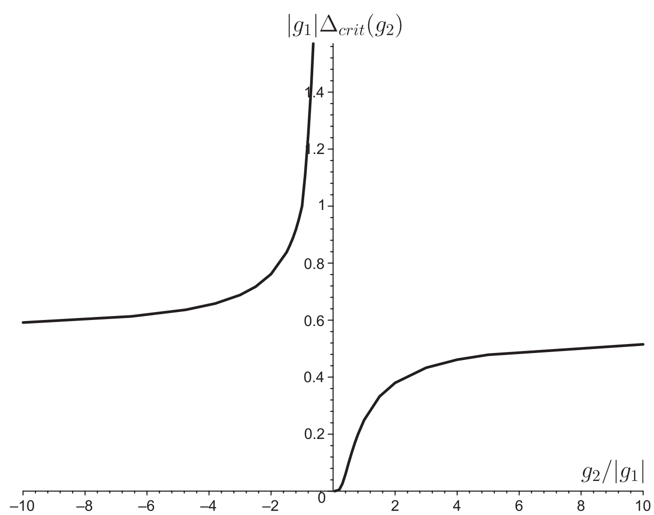

FIG. 3. Superconducting gap $\Delta_0 = \Delta_{\rm crit}(g_2)$ vs g_2 which is generated at the critical point, i.e., at $\mu = \mu_{\rm crit}(g_2)$, at arbitrary fixed $g_1 < 0$.

[5]All of the Figs. 2–10 are drawn in terms of dimensionless quantities which are obtained after multiplication of appropriate powers of $|g_1|$ with corresponding dimensional quantities. For example, there instead of μ, Δ_0, g_2 we use their dimensionless analogies $|g_1|\mu$, $|g_1|\Delta_0$, $g_2/|g_1|$. Instead of particle density n the dimensionless quantity $g_1^2 n$ is depicted there, etc.

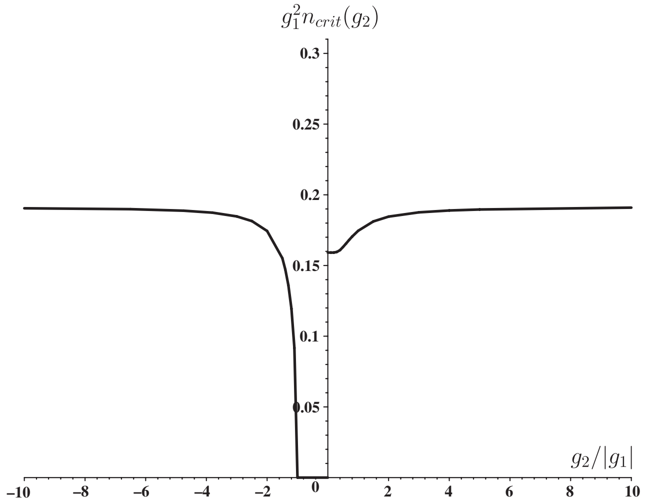

FIG. 4. Particle density $n = n_{\mathrm{crit}}(g_2)$ vs g_2 which is generated at the critical point, i.e., at $\mu = \mu_{\mathrm{crit}}(g_2)$, at arbitrary fixed $g_1 < 0$. At $\mu < \mu_{\mathrm{crit}}(g_2)$ the particle density n is equal to zero.

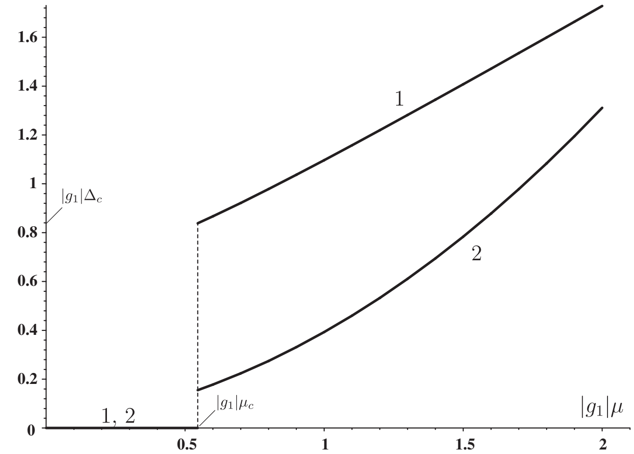

FIG. 6. Superconducting gap Δ_0 and particle density n vs μ at arbitrary fixed $g_1 < 0$ and $g_2 = -1.5|g_1|$. Curves 1 and 2 are the plots of the dimensionless quantities $|g_1|\Delta_0$ and $|g_1|^2 n$, respectively. Here $|g_1|\mu_c = |g_1|\mu_{\mathrm{crit}}(g_2 = -1.5|g_1|) \approx 0.545$ and $|g_1|\Delta_c = |g_1|\Delta_{\mathrm{crit}}(g_2 = -1.5|g_1|) \approx 0.838$.

explain this circumstance, recall that in the phase II the gap M_0 is equal to $1/|g_1|$. So, for all g_2 values the relation $\mu_{\mathrm{crit}}(g_2) < M_0$ is valid (see Fig. 2). As a result, throughout the phase II, where $\mu < \mu_{\mathrm{crit}}(g_2)$, we have $\mu < M_0$ and hence, as it follows from the relation (25), the zero particle density, $n = 0$. However, when μ reaches its critical value, $\mu = \mu_{\mathrm{crit}}(g_2)$, the nonzero particle density $n_{\mathrm{crit}}(g_2)$ is generated dynamically in the system (see Fig. 4). Further growth of the chemical potential is accompanied by increase of the particle density n vs μ. [Evidently, in this case the particle density must be calculated with the help of the expression (26).] For example, in Figs. 5–7 at the same representative relations between g_1 and g_2 the particle density n vs μ is depicted as a monotonically increasing curve 2.

Finally recall that at $\mu = 0$ the two phases, II and III, have equivalent minima of the TDP only at negative values of $g_1 = g_2$ (it is line L in Fig. 1). It turns out that for arbitrary fixed $g_1 < 0$ and at growing chemical potential, this property of the TDP is also allowed but in a much more extensive g_2 region. Indeed, as our analysis shows in this case, if $g_2 > 0$ or $g_2 < g_1$ then at $\mu = \mu_{\mathrm{crit}}(g_2)$ (see Fig. 2) the TDP has two equivalent minima, corresponding to these phases. As a result, for these values of g_1 and g_2 there is a coexistence of chirally broken and superconducting phases at $\mu = \mu_{\mathrm{crit}}(g_2)$. In this case, when viewed from

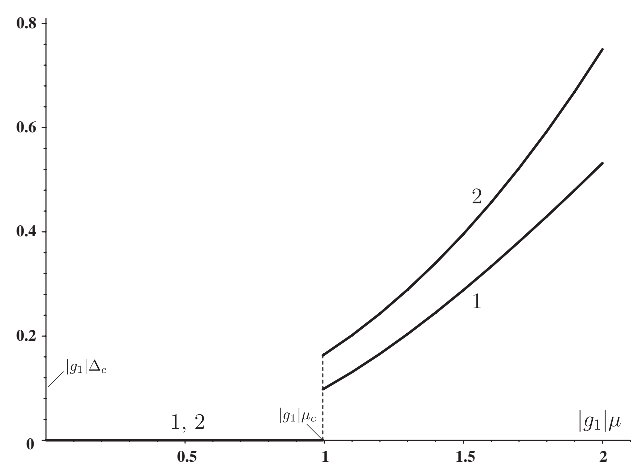

FIG. 5. Superconducting gap Δ_0 and particle density n vs μ at arbitrary fixed $g_1 < 0$ and $g_2 = 0.5|g_1|$. Curves 1 and 2 are the plots of the dimensionless quantities $|g_1|\Delta_0$ and $|g_1|^2 n$, correspondingly. Here $|g_1|\mu_c = |g_1|\mu_{crit}(g_2 = 0.5|g_1|) \approx 0.995$ and $|g_1|\Delta_c = |g_1|\Delta_{\mathrm{crit}}(g_2 = 0.5|g_1|) \approx 0.098$.

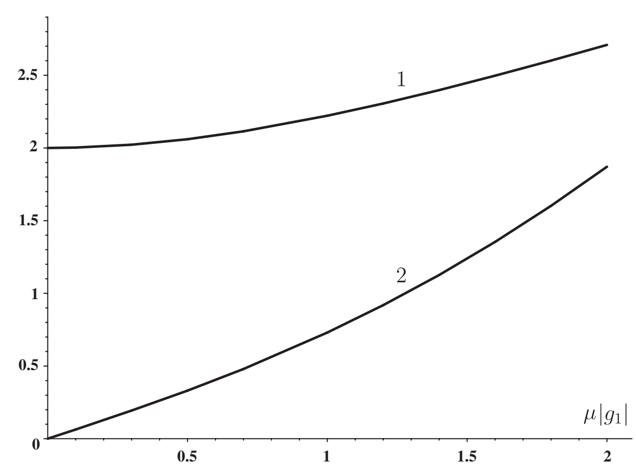

FIG. 7. Superconducting gap Δ_0 and particle density n vs μ at arbitrary fixed g_1 (both at $g_1 < 0$ and $g_1 > 0$) as well as at $g_2 = -0.5|g_1|$. Curves 1 and 2 are the plots of the dimensionless quantities $|g_1|\Delta_0$ and $6|g_1|^2 n$, respectively.

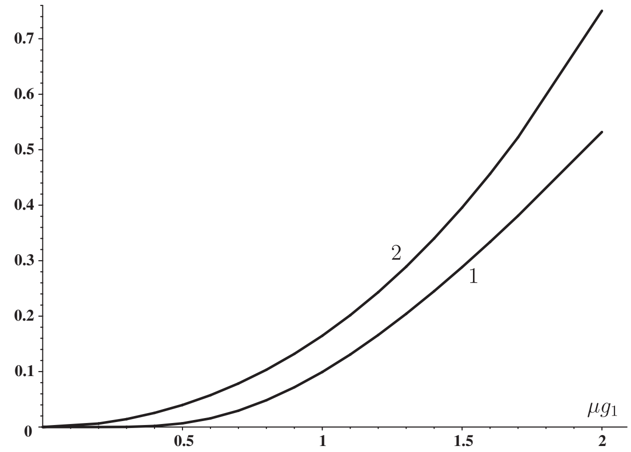

FIG. 8. Superconducting gap Δ_0 and particle density n vs μ at arbitrary fixed $g_1 > 0$ as well as at $g_2 = 0.5g_1$. Curves 1 and 2 are the plots of the dimensionless quantities $|g_1|\Delta_0$ and $|g_1|^2 n$, respectively.

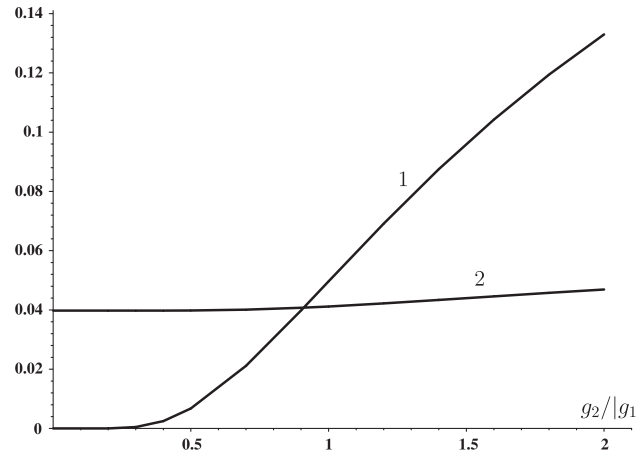

FIG. 10. Superconducting gap Δ_0 and particle density n vs $g_2 > 0$ at arbitrary fixed $g_1 > 0$ and $\mu = 0.5/g_1$. Curves 1 and 2 are the plots of the dimensionless quantities $g_1\Delta_0$ and $g_1^2 n$, respectively.

the side, we have the following picture of phase transitions in the system. At rather small values of μ the ground state of the system is an empty space (particle density is zero). If fermions are created in this state, they have a mass equal to $M_0 = -1/g_1$, i.e., the ground state corresponds to a chirally broken phase II. Then, if μ reaches the critical value $\mu = \mu_{\rm crit}(g_2)$, bubbles of a new phase III appear in the empty space. Inside each bubble the particle density n is nonzero and equal to $n_{\rm crit}(g_2)$ (see Fig. 4).

The case $g_1 > 0$.—Now the model phase structure consideration for a positive g_1 values is in order. Recall, in this case we have a rather weak attractive interaction in the chiral channel, i.e., $G_1 < G_c$. Evidently, if in addition $g_2 < 0$, then in this case the superconducting phase is realized for arbitrary values of $\mu \geq 0$. The behavior of the gap Δ_0 and particle density n vs μ in this branch of the superconducting phase is given in Fig. 7 in the particular case $g_2 = -0.5g_1$ for $g_1 > 0$. Moreover, as it is clear from Fig. 7, the same behavior for Δ_0 and n vs μ remains valid for the case $g_2 = -0.5|g_1|$ and negative values of g_1. To explain this fact, it is necessary to take into account the remark made after Eq. (24) that the superconducting gap does not depend on the coupling g_1 but only on the g_2 one. So, it is no wonder that the plots of Δ_0 and n are not changed when the parameter g_1 changes the sign.

Recall, if both $g_1 > 0$ and $g_2 > 0$, then we have at $\mu = 0$ the phase I without any symmetry breaking, where the gaps Δ_0 and M_0 vanishes (see Fig. 1). However, our analysis shows that at arbitrary small nonzero μ the global minimum point of the TDP (21) moves from the point $(M_0 = 0,\ \Delta_0 = 0)$ to the following one $(M_0 = 0,\ \Delta_0 \neq 0)$. Hence, at positive values of g_1 and g_2 a continuous second order phase transition occurs from symmetric phase I to superconducting one III when chemical potential acquires an arbitrary small nonzero value. The typical behavior of the gap Δ_0 and particle density n vs μ in this superconductivity region is depicted in Fig. 8. Comparing Figs. 7 and 8, we see that at the same value of μ the gap Δ_0 and particle density n are much greater in the case $g_1 > 0$, $g_2 < 0$, than in the case $g_1 > 0$, $g_2 > 0$. To support this statement we draw in Figs. 9 and 10 the plots of the gap Δ_0 and particle density n vs g_2 in two different regions $g_2 < 0$ and $g_2 > 0$, respectively, at the particular value of the chemical potential, $\mu = 0.5/g_1$.

We see that at $g_1 > 0$, i.e., at $G_1 < G_c$, the chiral symmetry breaking is absent but the Cooper pairing phase occurs at any $\mu > 0$. To explain this different behavior, one can use the following very naive physical arguments. Since at $\mu > 0$ we have a nonzero particle density (see, e.g., in Fig. 8), there is a Fermi sea of particles with

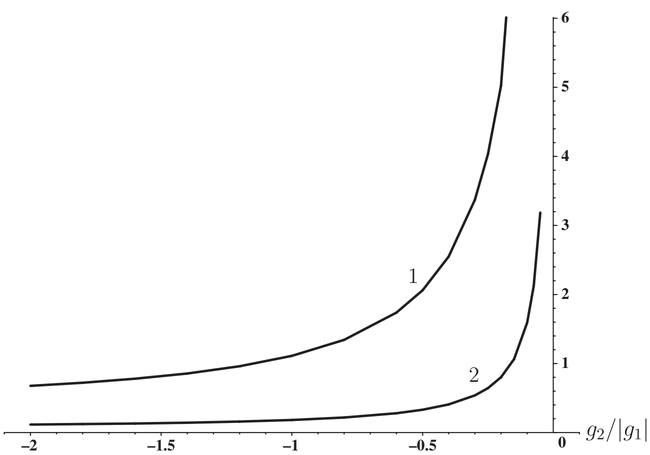

FIG. 9. Superconducting gap Δ_0 and particle density n vs $g_2 < 0$ at arbitrary fixed $g_1 > 0$ and $\mu = 0.5/g_1$. Curves 1 and 2 are the plots of the dimensionless quantities $g_1\Delta_0$ and $g_1^2 n$, respectively.

energies less or equal to μ (Fermi surface). Evidently, in this case there is no energy cost for creating a pair of particles with opposite momenta just over the Fermi surface. Then, due to an arbitrary weak attraction between these particles ($G_2 > 0$), the Cooper pair is formed and $U(1)$ symmetry is spontaneously broken, as a result of Bose-Einstein condensation of Cooper pairs. Note, since in the energy spectrum of fermions the gap $\Delta \neq 0$ appears (see in Fig. 8), rather small external forces are not able to destroy the superconducting condensate and it is a stable one.

Concerning the chiral symmetry breaking in this case, it is clear that a particle and a hole with opposite momenta can also be created without any energy cost in the system. Moreover, there is also an attraction between a particle and a hole. However, since the nonzero gap M does not appear in the energy spectrum at sufficiently small $G_1 < G_c$, the particle-hole pairing in this case is a rather weakly bounded resonance, which unlike a stable pair, could be easily destroyed by an arbitrary small external influence. So, no stable Bose-Einstein condensate of these pairs appears and chiral symmetry remains intact. (For a more detailed discussion on possible types of pairing in dense (quark) fermionic matter, see Ref. [29]).

In summary, we can say that at $T = 0$ chemical potential induces superconductivity in the model for arbitrary relations between coupling constants $g_{1,2}$ (or equivalently $G_{1,2}$).

V. FINITE TEMPERATURE

Now let us study the influence of both temperature T and chemical potential μ on the phase structure of the model. It is well known (see, e.g., in Ref. [30]) that in d space dimensions (in our case, evidently, $d = 2$) the transition probability from one degenerated minimum of the TDP to another is proportional to $\exp(-N\beta L^{d-2})$, where L is the linear size of the system and β is the inverse temperature, $\beta = 1/T$. It follows from this expression that at $d = 2$ the transition probability is zero even at finite N if $T = 0$. This leads to the fact that a continuous symmetry can be spontaneously broken in any planar systems at $T = 0$. (Hence, our consideration of superconducting phase transitions performed at $T = 0$ in the previous section is valid for arbitrary values of N). However, if $T \neq 0$, then transition probability in the above expression does not vanish at finite N. This circumstance ensures the vanishing of the order parameter and, as a result, might lead to a prohibition for spontaneous symmetry breaking in $d = 2$ spatial dimensions at finite N and $T \neq 0$. However, if $N \to \infty$ the transition probability vanishes and the spontaneous symmetry breaking is allowed. Just this assumption, i.e., the same as in Refs. [25,26], is used in the following consideration, where we study the temperature dependent superconducting phase transitions in the leading order of the large-N expansion technique.

In this case, in order to get the corresponding (unrenormalized) thermodynamic potential $\Omega_T(M, \Delta)$ one can simply start from the expression for the TDP at zero temperature (10) and perform the following standard replacements:

$$\int_{-\infty}^{\infty} \frac{dp_0}{2\pi}(\cdots) \to iT \sum_{n=-\infty}^{\infty}(\cdots),$$
$$p_0 \to p_{0n} \equiv i\omega_n \equiv i\pi T(2n+1), \qquad n = 0, \pm 1, \pm 2, \ldots, \tag{27}$$

i.e., the p_0 integration should be replaced by the summation over Matsubara frequencies ω_n. Summing over Matsubara frequencies in the obtained expression (the corresponding technique is presented, e.g., in Ref. [31]), one can find for the TDP

$$\Omega_T(M,\Delta) = \frac{M^2}{4G_1} + \frac{\Delta^2}{4G_2} - \int_{-\infty}^{\infty} \frac{d^2p}{(2\pi)^2}(\mathcal{E}_\Delta^+ + \mathcal{E}_\Delta^-) - 2T\int_{-\infty}^{\infty} \frac{d^2p}{(2\pi)^2} \ln([1+e^{-\beta\mathcal{E}_\Delta^+}][1+e^{-\beta\mathcal{E}_\Delta^-}]), \tag{28}$$

where $\beta = 1/T$ and $\mathcal{E}_\Delta^{\pm}$ are given in (10). Clearly, only the first integral in this expression (which is the same as in the zero temperature case) is responsible for ultraviolet divergency of the whole TDP (28). So, regularizing the TDP (28) in the way as it was done in (13) for zero temperature TDP and then replacing $G_{1,2} \to G_{1,2}(\Lambda)$ [see Eq. (16)], we can obtain in the limit $\Lambda \to \infty$ a finite expression denoted as $\Omega_T^{\rm ren}(M, \Delta)$,

$$\Omega_T^{\rm ren}(M,\Delta) = \Omega^{\rm ren}(M,\Delta) - 2T\int_{-\infty}^{\infty} \frac{d^2p}{(2\pi)^2} \ln([1+e^{-\beta\mathcal{E}_\Delta^+}][1+e^{-\beta\mathcal{E}_\Delta^-}]), \tag{29}$$

where $\Omega^{\rm ren}(M, \Delta)$ is the zero temperature TDP (21). Numerical investigations show that all possible local minima of the TDP $\Omega_T^{\rm ren}(M, \Delta)$ are located in the lines $M = 0$ or $\Delta = 0$. So it is sufficient to deal with corresponding restrictions of the TDP on these lines, i.e., with the following functions:

$$\begin{aligned} F_1(M) &\equiv \Omega_T^{\rm ren}(M,\Delta)|_{\Delta=0} \\ &= \omega_1(M) - 2T\int_{-\infty}^{\infty} \frac{d^2p}{(2\pi)^2} \ln([1+e^{-\beta(E+\mu)}] \\ &\quad \times [1+e^{-\beta|E-\mu|}]) \\ &= \frac{M^2}{2\pi g_1} + \frac{M^3}{3\pi} - 2T\int_{-\infty}^{\infty} \frac{d^2p}{(2\pi)^2} \ln([1+e^{-\beta(E+\mu)}] \\ &\quad \times [1+e^{-\beta(E-\mu)}]), \end{aligned} \tag{30}$$

$$F_2(\Delta) \equiv \Omega_T^{\rm ren}(M,\Delta)|_{M=0} = \omega_2(\Delta) - 2T\int_{-\infty}^{\infty}\frac{d^2p}{(2\pi)^2} \times \ln([1+e^{-\beta E_\Delta^+}][1+e^{-\beta E_\Delta^-}]), \quad (31)$$

where $E=\sqrt{|\vec{p}|^2+M^2}$, $(E_\Delta^\pm)^2=(|\vec{p}|\pm\mu)^2+\Delta^2$, and the functions $\omega_1(M)$, $\omega_2(\Delta)$ are presented in (22) and (23), respectively. The gaps M_0 and Δ_0 are the solutions of the following stationary (gap) equations:

$$\frac{\partial F_1(M)}{\partial M} \equiv \frac{M}{\pi}f_1(M)=0, \qquad \frac{\partial F_2(\Delta)}{\partial \Delta} \equiv \frac{\Delta}{\pi}f_2(\Delta)=0, \quad (32)$$

where

$$f_1(M)=\frac{1}{g_1}+M+T\ln\{[1+e^{-\beta(M+\mu)}][1+e^{-\beta(M-\mu)}]\}, \quad (33)$$

$$f_2(\Delta)=\frac{1}{g_2}+\sqrt{\mu^2+\Delta^2}+2T\ln\Big(1+e^{-\beta\sqrt{\mu^2+\Delta^2}}\Big) - \mu\int_0^\mu \tanh\left(\frac{\beta\sqrt{q^2+\Delta^2}}{2}\right)\frac{dq}{\sqrt{q^2+\Delta^2}}, \quad (34)$$

respectively (for details, see Appendix C). On the basis of these gap equations we will study the phase structure of the model at $T>0$.

The case $g_1>0$.—First of all let us consider the phase portrait of the model at $g_1>0$. It straightforwardly follows from (32) and (33) that the gap M_0 is always zero at $g_1>0$ (it is a nonzero quantity only at $g_1<0$). However, the gap Δ_0 is positive both at $g_2<0$ and $g_2>0$ if temperature is sufficiently small, i.e., when $f_2(0)<0$. So, at $g_1>0$ and for arbitrary values of μ the superconducting phase III is arranged at sufficiently small values of temperature $T<T_c(\mu)$. At $T>T_c(\mu)$ the gap equations (32) supply the $\Delta_0=0$ and $M_0=0$ gap values, i.e., the symmetric phase. The second order phase transition temperature $T_c(\mu)$ is the solution of the equation $f_2(0)=0$,

$$f_2(0)\equiv\frac{1}{g_2}+\mu+2T\ln(1+e^{-\beta\mu})-\mu\int_0^\mu\tanh\left(\frac{\beta q}{2}\right)\frac{dq}{q}=0. \quad (35)$$

Hence, in the (μ,T) plane the curve $T=T_c(\mu)$ is the boundary between symmetric and superconducting phases. A numerical investigation of Eq. (35) produces at $g_2=\pm0.5g_1$ the phase portraits of the model presented in Figs. 11 and 12.

If $g_2<0$, then it follows from (35) that $T_c(0)=-1/(2g_2\ln 2)$. Moreover, in this case the critical temperature can be given as a series over the small parameter μ,

$$T_c(\mu)=T_c(0)-\mu^2g_2/16+o(\mu^2g_2). \quad (36)$$

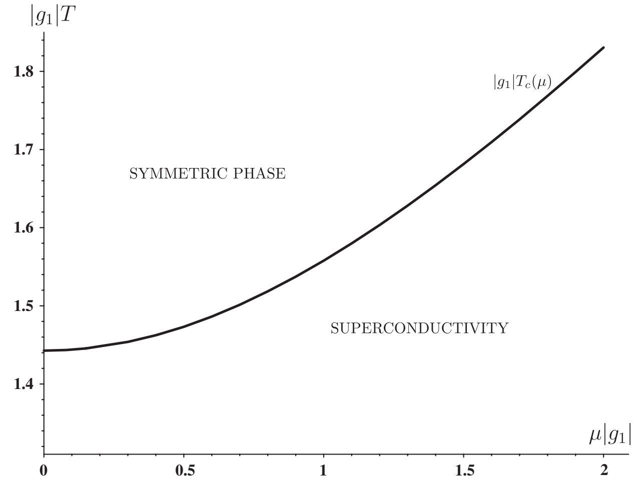

FIG. 11. (μ,T)-phase diagram of the model at $g_2=-0.5|g_1|$ and arbitrary fixed g_1 both at $g_1<0$ and $g_1>0$.

Comparing this expansion at $g_2=-0.5g_1$ with $T_c(\mu)$ of Fig. 11, we see that (36) supplies a rather good approximation for the critical temperature only in the interval $0<\mu g_1<0.2$.

Now let us try to present some analytical approximation for the $T_c(\mu)$ at $g_2>0$ (g_1 is still fixed and positive). For this purpose note first of all that for all points of the critical curve $T=T_c(\mu)$ of Fig. 12 the relation $\mu/T\equiv\mu\beta>>1$ is valid. Then, it is convenient to present Eq. (35) in the following equivalent form:

$$\frac{1}{2Tg_2}+\frac{\mu\beta}{2}+\ln(1+e^{-\beta\mu}) -\frac{\mu\beta}{2}\left\{C_1+\int_1^{\mu\beta/2}\frac{dz}{z}+C_2-\int_{\mu\beta/2}^{\infty}[\tanh z-1]\frac{dz}{z}\right\}=0, \quad (37)$$

where

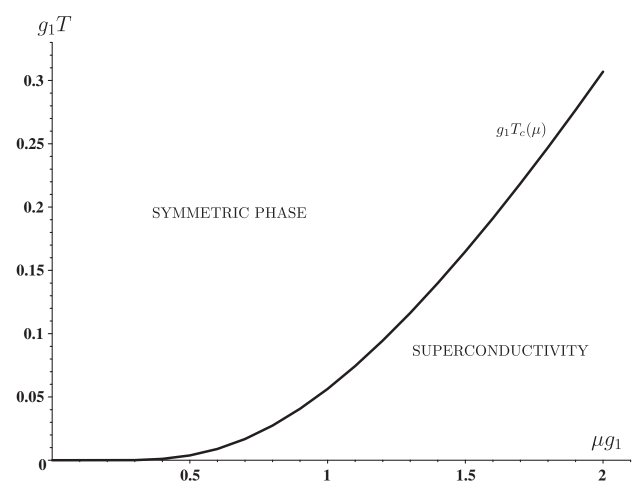

FIG. 12. (μ,T)-phase diagram of the model at arbitrary fixed $g_1>0$ and at $g_2=0.5g_1$.

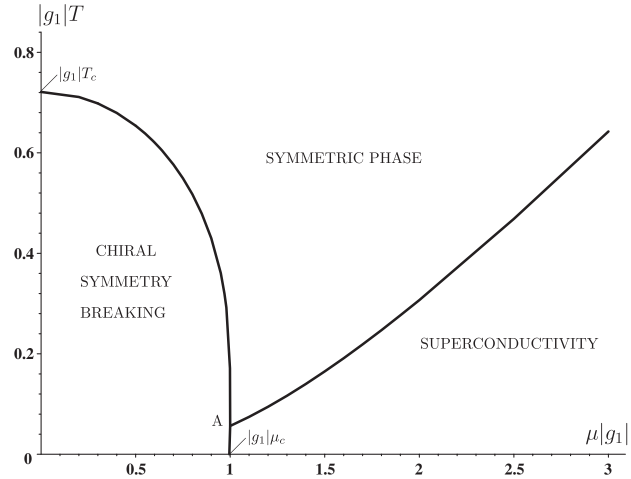

FIG. 13. (μ, T)-phase diagram of the model at $g_2 = 0.5|g_1|$ and arbitrary fixed $g_1 < 0$. All of the curves are the lines of second order phase transitions except the boundary between the superconducting and chiral symmetry breaking phases, where a first order phase transition is realized. The coordinates of the tricritical point A are the following ones: $|g_1|\mu_A \approx 0.999$ and $|g_1|T_A \approx 0.056$. Moreover, $|g_1|\mu_c \approx 0.995$ and $|g_1|T_c = 1/(2\ln 2) \approx 0.721$.

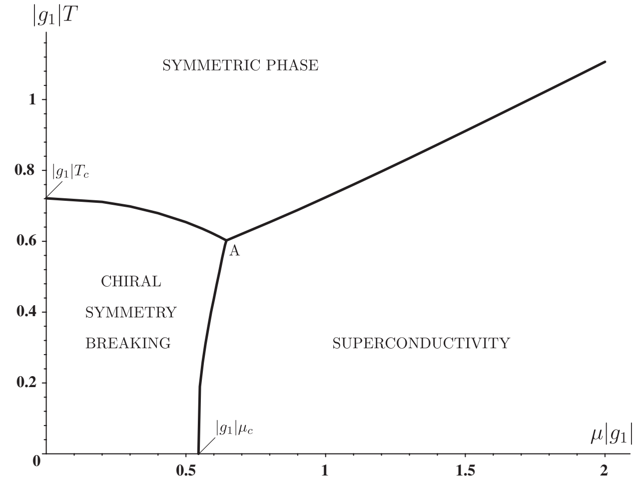

FIG. 14. (μ, T)-phase diagram of the model at $g_2 = -1.5|g_1|$ and arbitrary fixed $g_1 < 0$. All of the curves are the lines of second order phase transitions except the boundary between the superconducting and chiral symmetry breaking phases, where a first order phase transition is realized. The coordinates of the tricritical point A are the following ones: $|g_1|\mu_A \approx 0.645$ and $|g_1|T_A \approx 0.602$. Moreover, $|g_1|\mu_c \approx 0.545$ and $|g_1|T_c = 1/(2\ln 2) \approx 0.721$.

$$C_1 = \int_0^1 \tanh z \frac{dz}{z} \approx 0.910,$$
$$C_2 = \int_1^\infty [\tanh z - 1]\frac{dz}{z} \approx -0.091. \tag{38}$$

The third term in (37) as well as the last integral in the braces of (37) can be neglected in comparison with other terms. The obtained equation can be easily solved with respect to T. As a result we have

$$T_c(\mu) \approx \frac{\mu}{2}\exp[C_1 + C_2 - 1 - 1/(\mu g_2)]. \tag{39}$$

Note that at $g_2 = 0.5 g_1$ the plot of the expression (39) coincides with great accuracy with the critical temperature of Fig. 12 in the whole interval $0 < \mu g_1 < 2$.

The case $g_1 < 0$.—In this case we present three (μ, T)-phase portraits of the model for qualitatively distinct relations between g_1 and g_2. The first one for $g_2 = -0.5|g_1|$ (which is in Fig. 11) was already described above because it is the same as in the case $g_1 > 0$, $g_2 = -0.5g_1$. The other two phase portraits are represented in Figs. 13 and 14 for $g_2 = 0.5|g_1|$ and $g_2 = -1.5|g_1|$, respectively. There the points (μ, T) of the boundary between the symmetric and chiral symmetry breaking (or superconducting) phases are given implicitly by the equation $f_1(0) = 0$ (or $f_2(0) = 0$), where the functions $f_1(M)$ and $f_2(\Delta)$ are defined in (33) and (34), respectively. On these boundaries, the second order phase transitions occur. In contrast, the boundary between chiral symmetry breaking and superconducting phases is the curve of the first order phase transitions. So at the points (μ, T) of this boundary the two phases may coexist.

Analyzing the cited above (μ, T)-phase diagrams of Figs. 11–14, we see that for each arbitrary fixed value T of the temperature (and for all relations between coupling constants) there exist a definite value μ_T of the chemical potential such that for all $\mu > \mu_T$ the superconducting phase is realized in the system. This property is inherent only to a (2 + 1)-dimensional model (1) and it is absent in the two-dimensional analogue [25].

VI. SUMMARY AND CONCLUSIONS

In this paper we study the competition between chiral and superconducting condensations in the framework of the (2 + 1)-dimensional GN-type model (1) which is a direct generalization of the two-dimensional analogue by Chodos *et al.* [25]. So, the initial four-fermion model (1) describes interactions both in the fermion-antifermion (or chiral) and superconducting difermion (or Cooper pairing) channels with couplings G_1 and G_2, respectively. Moreover, it is chirally and $U(1)$ invariant one (the last group corresponds to conservation of the fermion number or electric charge of the system). To avoid the ban on the spontaneous breaking of continuous symmetry in (2 + 1)-dimensional field theories at $T > 0$, we consider, as it was done in Ref. [25], the phase structure of our model in the leading order of the large-N technique, i.e., in the limit $N \to \infty$, where N is a number of fermion fields.

The case $T = 0$, $\mu = 0$.—First of all we have investigated the thermodynamic potential of the model at $T = 0$,

$\mu = 0$. In this case the phase portrait is presented in Fig. 1 in terms of the renormalization group invariant finite coupling constants g_1 and g_2. Each point (g_1, g_2) of this diagram corresponds to a definite phase. For example, at $g_{1,2} > 0$, i.e., at sufficiently small values of the bare coupling constants $G_{1,2}$ (see the comment at the end of Sec. IVA), neither chiral nor $U(1)$ symmetries are violated and the system is in the symmetric phase, etc.

The case $T = 0$, $\mu \neq 0$.—In this case we select two qualitatively different situations, $g_1 < 0$ and $g_1 > 0$. If $g_1 < 0$ and fixed, then in Fig. 2 we draw the (g_2, μ)-phase diagram of the model. It means that at $g_2 > 0$ or at $g_2 < g_1$ the phase II with zero particle density is realized at sufficiently low values of μ. In this case the ground state of the system is an empty space. Then at some critical value $\mu = \mu_{\rm crit}(g_2)$ bubbles of the new phase III with particle density $n_{\rm crit}(g_2)$ (see Fig. 4) can appear in the space, and for all $\mu > \mu_{\rm crit}(g_2)$ the whole space is filled with superconducting phase, in which particle density n is not zero, $n > n_{\rm crit}(g_2)$. If $g_1 > 0$, then the system is in the superconducting phase even at arbitrary small values of μ. Hence, at $T = 0$ and at growing chemical potential, the system is transformed into a superconducting state.

The case $T > 0$, $\mu \neq 0$.—Phase portraits of the model are presented in this case in Figs. 11–14. It is clear from the figures that at fixed μ and increasing temperature the symmetric phase is restored. However, at arbitrary fixed T, growth of the chemical potential leads to appearing of superconductivity in the system at arbitrary relations between coupling constants g_1 and g_2.

The fact that chemical potential induces superconductivity phenomenon is the main result of our paper. Note that in general this property of the (2 + 1)-dimensional GN-type model (1) is not valid in the case of the two-dimensional model [25].

We hope that our investigations can shed new light on the superconducting phenomena in condensed matter systems with planar structures.

APPENDIX A: ALGEBRA OF THE γ-MATRICES IN THE CASE OF SO(2,1) GROUP

The two-dimensional irreducible representation of the 3-dimensional Lorentz group SO(2,1) is realized by the following 2×2 $\tilde{\gamma}$-matrices:

$$\tilde{\gamma}^0 = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \tilde{\gamma}^1 = i\sigma_1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad \tilde{\gamma}^2 = i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{A1}$$

acting on two-component Dirac spinors.

They have the properties

$$Tr(\tilde{\gamma}^\mu \tilde{\gamma}^\nu) = 2g^{\mu\nu}, \quad [\tilde{\gamma}^\mu, \tilde{\gamma}^\nu] = -2i\varepsilon^{\mu\nu\alpha}\tilde{\gamma}_\alpha, \quad \tilde{\gamma}^\mu\tilde{\gamma}^\nu = -i\varepsilon^{\mu\nu\alpha}\tilde{\gamma}_\alpha + g^{\mu\nu}, \tag{A2}$$

where $g^{\mu\nu} = g_{\mu\nu} = \mathrm{diag}(1, -1, -1)$, $\tilde{\gamma}_\alpha = g_{\alpha\beta}\tilde{\gamma}^\beta$, $\varepsilon^{012} = 1$. There is also the relation

$$Tr(\tilde{\gamma}^\mu \tilde{\gamma}^\nu \tilde{\gamma}^\alpha) = -2i\varepsilon^{\mu\nu\alpha}. \tag{A3}$$

Note that the definition of chiral symmetry is slightly unusual in three dimensions (spin is here a pseudoscalar rather than a (axial) vector). The formal reason is simply that there exists no other 2×2 matrix anticommuting with the Dirac matrices $\tilde{\gamma}^\nu$ which would allow the introduction of a γ^5 matrix in the irreducible representation. The important concept of *chiral* symmetries and their breakdown by mass terms can, nevertheless, be realized also in the framework of (2 + 1)-dimensional quantum field theories by considering a four-component reducible representation for Dirac fields. In this case the Dirac spinors ψ have the following form:

$$\psi(x) = \begin{pmatrix} \tilde{\psi}_1(x) \\ \tilde{\psi}_2(x) \end{pmatrix}, \tag{A4}$$

with $\tilde{\psi}_1$, $\tilde{\psi}_2$ being two-component spinors. In the reducible four-dimensional spinor representation one deals with (4×4) γ matrices $\gamma^\mu = \mathrm{diag}(\tilde{\gamma}^\mu, -\tilde{\gamma}^\mu)$, where $\tilde{\gamma}^\mu$ are given in (A1). One can easily show that $(\mu, \nu = 0, 1, 2)$

$$Tr(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}; \quad \gamma^\mu\gamma^\nu = \sigma^{\mu\nu} + g^{\mu\nu}, \quad \sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu] = \mathrm{diag}(-i\varepsilon^{\mu\nu\alpha}\tilde{\gamma}_\alpha, -i\varepsilon^{\mu\nu\alpha}\tilde{\gamma}_\alpha). \tag{A5}$$

In addition to the Dirac matrices γ^μ $(\mu = 0, 1, 2)$ there exist two other matrices γ^3, γ^5 which anticommute with all γ^μ $(\mu = 0, 1, 2)$ and with themselves,

$$\gamma^3 = \begin{pmatrix} 0, & I \\ I, & 0 \end{pmatrix}, \quad \gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3 = i\begin{pmatrix} 0, & -I \\ I, & 0 \end{pmatrix}, \tag{A6}$$

with I being the unit 2×2 matrix. Finally note that in terms of two-component spinors $\tilde{\psi}_1$, $\tilde{\psi}_2$ the parity transformation P, defined in the space of four-component spinors by the relation (4), looks like

$$P: \tilde{\psi}_1(t, x, y) \to i\tilde{\gamma}^1\tilde{\psi}_2(t, -x, y); \quad \tilde{\psi}_2(t, x, y) \to i\tilde{\gamma}^1\tilde{\psi}_1(t, -x, y). \tag{A7}$$

Such a definition of the space parity transformation is commonly used in (2 + 1)-dimensional theories with four-component representation for Dirac spinors (see, e.g., in Ref. [32]).

APPENDIX B: THE PATH INTEGRATION OVER ANTICOMMUTATING FIELDS

Let us calculate the following path integral over anticommutating four-component Dirac spinor fields $q(x)$, $\bar{q}(x)$:

$$I = \int [d\bar{q}][dq] \exp\Big(i \int d^3x \Big[\bar{q} D q - \frac{\Delta}{2}(q^T C q) - \frac{\Delta}{2}(\bar{q} C \bar{q}^T)\Big]\Big), \tag{B1}$$

where we use the notations of Sec. II and, in particular, the operator D is given in (9). Note in addition, the integral I is equal to the argument of the ln function in Eq. (9) in the particular case $N = 1$. Recall that there are general Gaussian path integrals [33],

$$\int [dq] \exp\Big(i \int d^3x \Big[-\frac{1}{2} q^T A q + \eta^T q\Big]\Big) = (\det(A))^{1/2} \exp\Big(-\frac{i}{2} \int d^3x [\eta^T A^{-1} \eta]\Big), \tag{B2}$$

$$\int [d\bar{q}] \exp\Big(i \int d^3x \Big[-\frac{1}{2} \bar{q} A \bar{q}^T + \bar{\eta} \bar{q}^T\Big]\Big) = (\det(A))^{1/2} \exp\Big(-\frac{i}{2} \int d^3x [\bar{\eta} A^{-1} \bar{\eta}^T]\Big), \tag{B3}$$

where A is an antisymmetric operator in coordinate and spinor spaces, and $\eta(x)$, $\bar{\eta}(x)$ are anticommutating spinor sources which also anticommute with q and $\bar{q}$. Firstly, let us integrate in (B1) over q fields with the help of the relation (B2) supposing there that $A = \Delta C$, $\bar{q} D = \eta^T$, i.e., $\eta = D^T \bar{q}^T$. Then,

$$I = (\det(\Delta C))^{1/2} \int [d\bar{q}] \exp\Big(-\frac{i}{2} \int d^3x \bar{q} \times [\Delta C + D(\Delta C)^{-1} D^T] \bar{q}^T\Big). \tag{B4}$$

Secondly, the integration over $\bar{q}$-fields in (B4) can be easily performed with the help of Eq. (B3), where one should put $A = \Delta C + D(\Delta C)^{-1} D^T$ and $\bar{\eta} = 0$. As a result, we have

$$I = (\det(\Delta C))^{1/2} (\det[\Delta C + D(\Delta C)^{-1} D^T])^{1/2} = (\det[\Delta^2 C^2 + D C^{-1} D^T C])^{1/2}. \tag{B5}$$

Taking into account the relations $(\partial_\nu)^T = -\partial_\nu$ and $C^{-1}(\gamma^\nu)^T C = -\gamma^\nu$ ($\nu = 0, 1, 2$), we obtain from (B5)

$$I = (\det[-\Delta^2 + D_+ D_-])^{1/2} \equiv (\det B)^{1/2}, \tag{B6}$$

where $D_\pm = \gamma^\nu i \partial_\nu - M \pm \mu \gamma^0$. Using the general relation $\det B = \exp(\mathrm{Tr} \ln B)$, we get from (B6)

$$\ln I = \frac{1}{2} \mathrm{Tr} \ln(B) = \sum_{i=1}^{2} \int \frac{d^3p}{(2\pi)^3} \ln(\lambda_i(p)) \int d^3x. \tag{B7}$$

(A more detailed consideration of operator traces is presented in Appendix A of the paper [34].) In this formula symbol Tr means the trace of an operator both in the coordinate and internal spaces. Moreover, $\lambda_i(p)$ ($i = 1, 2$) in (B7) are two twice degenerated eigenvalues of the 4×4 Fourier transformation matrix $\bar{B}(p)$ of the operator B, i.e.,

$$\lambda_{1,2}(p) = M^2 - p_1^2 - p_2^2 - \mu^2 + p_0^2 - \Delta^2 \pm 2\sqrt{-M^2 p_2^2 - M^2 p_1^2 + M^2 p_0^2 + \mu^2 p_2^2 + \mu^2 p_1^2}. \tag{B8}$$

APPENDIX C: GAP EQUATIONS

The equation for the gap M_0, i.e., the first one of equations (32), is obtained, e.g., in Ref. [4], where a phase structure of the initial model (1) was considered in the particular case of $G_2 = 0$.

To obtain a gap equation for the superconducting gap Δ_0, $\partial F_2(\Delta)/\partial\Delta = 0$, let us first transform the original expression (31) for the TDP $F_2(\Delta)$ using polar coordinates in the integral in (31). Integrating in the obtained expression over a polar angle, we have

$$F_2(\Delta) = \omega_2(\Delta) - \frac{T}{\pi} \int_0^\infty p dp \ln(1 + e^{-\beta\sqrt{(p+\mu)^2+\Delta^2}}) - \frac{T}{\pi} \int_0^\infty p dp \ln(1 + e^{-\beta\sqrt{(p-\mu)^2+\Delta^2}}). \tag{C1}$$

It is very convenient to change integration variables in (C1) (we use $q = p + \mu$ for the first integral and $q = p - \mu$ for the second one) and, after some manipulations, to get an equivalent expression,

$$F_2(\Delta) = \omega_2(\Delta) - \frac{2T}{\pi} \int_\mu^\infty q dq \ln(1 + e^{-\beta\sqrt{q^2+\Delta^2}}) - \frac{2T\mu}{\pi} \int_0^\mu dq \ln(1 + e^{-\beta\sqrt{q^2+\Delta^2}}). \tag{C2}$$

Starting from (C2) and taking into account the expression (23) for $\omega_2(\Delta)$, we have the following gap equation:

$$\frac{\partial F_2(\Delta)}{\partial \Delta} = \frac{\Delta}{\pi g_2} + \frac{\Delta}{\pi}\sqrt{\mu^2 + \Delta^2} - \frac{\mu\Delta}{\pi} \ln\Big(\frac{\mu + \sqrt{\mu^2 + \Delta^2}}{\Delta}\Big) + \frac{\Delta}{\pi} \int_\mu^\infty \frac{2q dq}{\sqrt{q^2 + \Delta^2}(1 + e^{\beta\sqrt{q^2+\Delta^2}})} + \frac{2\Delta\mu}{\pi} \int_0^\mu \frac{dq}{\sqrt{q^2 + \Delta^2}(1 + e^{\beta\sqrt{q^2+\Delta^2}})} = 0. \tag{C3}$$

The first integral in (C3) is a rather simple one, i.e.,

$$\int_{\mu}^{\infty} \frac{2q dq}{\sqrt{q^2+\Delta^2}\left(1+e^{\beta\sqrt{q^2+\Delta^2}}\right)} = \frac{2}{\beta}\ln\left(1+e^{-\beta\sqrt{\mu^2+\Delta^2}}\right). \tag{C4}$$

In contrast, let us present the third term in (C3) in the integral form, i.e.,

$$-\frac{\mu\Delta}{\pi}\ln\left(\frac{\mu+\sqrt{\mu^2+\Delta^2}}{\Delta}\right) = -\frac{\mu\Delta}{\pi}\int_0^{\mu}\frac{dq}{\sqrt{q^2+\Delta^2}}, \tag{C5}$$

which then can be combined with the last integral of (C3). As a result we obtain for the superconducting gap Δ_0 the second of equations (32).

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