

Field propagation in a stochastic background space: The rate of light incoherence in stellar interferometry

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We present Hilbert space representation for a relatively broad class of minimum-length deformed quantum mechanical models obtained by incorporating a space-time uncertainty relation into quantum mechanics. The correspondingly modified field theory is used for estimating the deviation of the light incoherence rate from distant astrophysical sources from the standard case.

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I. INTRODUCTION

In the framework of general relativity, a gravitational field is described by the space-time metric ([1], Sec. 82). Thus, the measurement of the gravitational field is tantamount to the measurement of space-time distances. Taking into account the quantum-mechanical fluctuations, it was shown in a number of papers that there inevitably exists intrinsic uncertainties in measuring the space-time distances [2–10]. Otherwise speaking, a space-time metric undergoes inherent quantum-mechanical fluctuations. Simply put, on the dimensional grounds one could get an idea that in the Minkowski background, the rate of fluctuation of a length scale l should have the form $\delta l = \beta l_p^\alpha l^{1-\alpha}$, since l_p is the only quantity with dimensions of length that one can construct from the quantities c , G_N , \hbar [11] (here β is a numerical factor of order unity). For a relatively recent review we refer the reader to Ref. [12]. (In what follows we will use a natural system of units: $c = \hbar = 1$.) As to the parameter α , one cannot be very strict in defining its proper value. One can just require the values of α to be such that $\delta l \ll l$ for $l \gg l_p$. In what follows, we will be interested in length scales much greater than the Planck length. The purpose of this paper is (1) to develop a systematic way for incorporating the relation $\delta l = \beta l_p^\alpha l^{1-\alpha}$ into field theory and (2) to use the modified field theory for estimating the coherence rate for the light coming from distant astrophysical sources. The idea proposed in Ref. [13] to estimate the phase fluctuation accumulated by the plane wave $\exp(i[\omega t - \mathbf{k}\mathbf{x}])$ ($\omega = k$) over the time t as $t\delta\omega$, where $\delta\omega$ is calculated by means of the relations $\omega = 2\pi/\lambda$, $\delta\lambda = \beta l_p^\alpha \lambda^{1-\alpha}$, is clearly very dubious. So, an alternate idea is to assume that over the length scale l the path fluctuation for an electromagnetic wave caused by the background metric fluctuations should be estimated irrespective to its wavelength as $\delta l = \beta l_p^\alpha l^{1-\alpha}$ thus implying the phase fluctuation of the order of $\delta l/\lambda$ [14]. What one can say definitely on the bases of the above discussion is that the fluctuation in wavelength should be estimated as $\delta\lambda = \beta l_p^\alpha \lambda^{1-\alpha}$; however, the question of how

this fluctuation adds up over a length scale l requires a theoretical framework (one may naturally expect the wavelength to have some bearing on this question).

We will abandon the above-mentioned “imaginative” concepts and will try to incorporate the relation $\delta l = \beta l_p^\alpha l^{1-\alpha}$ into quantum mechanics (QM) and hence into the field theory.

II. INCORPORATING THE RELATION $\delta l = \beta l_p^\alpha l^{1-\alpha}$ INTO THE QUANTUM MECHANICS

We start off with a Minkowskian background, the spatial distance of which undergoes the fluctuations quantified by the relation $\delta l = \beta l_p^\alpha l^{1-\alpha}$. The question of the possible space-time structure at the Planck scale is beyond the scope of our discussion, so that from the outset we assume that $l \gg l_p$. As a concomitant of these fluctuations, the position uncertainty in the Heisenberg uncertainty relation gets increased as

$$\delta X \geq \frac{1}{2\delta P} + \beta l_p^\alpha \delta X^{1-\alpha}. \quad (1)$$

This modification can be understood as an immediate result of fluctuations (uncertainty) $\delta\lambda = \beta l_p^\alpha \lambda^{1-\alpha}$ in the de Broglie wavelength of the incident particle by means of which we are measuring the position of the observed particle. So long as $\delta X \gg l_p$ we can rewrite Eq. (1) in the form

$$\delta X \delta P \geq \frac{1}{2} + \beta l_p^\alpha \delta P^\alpha. \quad (2)$$

A. Hilbert space representation of Eq. (2)

To find a concrete Hilbert space representation of the \hat{X} , \hat{P} operators, we start off with the deformed QM

$$[\hat{X}, \hat{P}] = i(1 + \beta l_p^\alpha \hat{P}^\alpha), \quad (3)$$

which is dictated by the form of Eq. (2) (numerical factors of order unity are absorbed in β). This sort of QM is characterized with a minimum position uncertainty of the order of [15]

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$$\delta X \simeq \left[\int_0^\infty \frac{dP}{1 + \beta l_p^\alpha P^\alpha} \right]^{-1} = \frac{\beta^{1/\alpha} l_p}{\int_0^\infty \frac{dq}{1+q^\alpha}}.$$

So, in the case $\alpha < 1$ the position uncertainty can reach zero while for $\alpha > 1$ it exhibits a nonzero minimum uncertainty in a position that is seen immediately from Eq. (2) as well.

A multidimensional generalization of Eq. (3) can be written in the form

$$\begin{aligned} [\hat{X}_i, \hat{X}_j] &= 0, & [\hat{P}_i, \hat{P}_j] &= 0, \\ [\hat{X}_i, \hat{P}_j] &= i\{\Xi(\hat{P}^2)\delta_{ij} + \Theta(\hat{P}^2)\hat{P}_i\hat{P}_j\}, \end{aligned} \quad (4)$$

the Hilbert space representation of which can be constructed in terms of the standard \hat{x} , \hat{p} operators as

$$\hat{X}_i = \hat{x}_i, \quad \hat{P}_j = \hat{p}_j \xi(\hat{\mathbf{p}}^2). \quad (5)$$

Let us work in the eigen representation of operator $\hat{\mathbf{p}}$: $\hat{x}_i = i\partial/\partial p^i$, $\hat{p}_j = p_j$. The simplest ansatz would be to take

$$\Theta = \frac{2\beta l_p^\alpha}{\hat{\mathbf{p}}^{2-\alpha}};$$

thus, from Eqs. (4) and (5), we get

$$\begin{aligned} & \left(\frac{\partial}{\partial p^i} p_j \xi(\mathbf{p}^2) - p_j \xi(\mathbf{p}^2) \frac{\partial}{\partial p^i} \right) \psi(\mathbf{p}) \\ &= \left(\delta_{ij} \xi(\mathbf{p}^2) + 2p_i p_j \frac{d\xi(\mathbf{p}^2)}{d\mathbf{p}^2} \right) \psi(\mathbf{p}) \\ &= \left(\Xi(p^2 \xi^2) \delta_{ij} + 2\beta l_p^\alpha \frac{p_i p_j}{p^{2-\alpha}} \xi^\alpha \right) \psi(\mathbf{p}), \end{aligned}$$

that is,

$$\begin{aligned} \frac{d\xi(p^2)}{dp^2} &= \beta l_p^\alpha \frac{\xi^\alpha}{p^{2-\alpha}}, \Rightarrow \xi(p^2) \\ &= \left(1 - \frac{2\beta(\alpha-1)}{\alpha} l_p^\alpha p^\alpha \right)^{\frac{1}{1-\alpha}}. \end{aligned} \quad (6)$$

So we get

$$\hat{X}_i = \hat{x}_i, \quad \hat{P}_j = \hat{p}_j \left(1 - \frac{2\beta(\alpha-1)}{\alpha} l_p^\alpha \hat{p}^\alpha \right)^{\frac{1}{1-\alpha}}, \quad (7)$$

or in the eigen representation of operator $\hat{\mathbf{p}}$

$$\hat{X}_j = i \frac{\partial}{\partial p_j}, \quad \hat{P}_j = p_j \left(1 - \frac{2\beta(\alpha-1)}{\alpha} l_p^\alpha p^\alpha \right)^{\frac{1}{1-\alpha}}, \quad (8)$$

with a scalar product containing a cutoff on p when $\alpha > 1$,

$$\langle \psi_1 | \psi_2 \rangle = \int_{p^\alpha < \alpha/2\beta(\alpha-1)l_p^\alpha} d^3 p \psi_1^*(\mathbf{p}) \psi_2(\mathbf{p}),$$

and without a cutoff on p when $\alpha < 1$,

$$\langle \psi_1 | \psi_2 \rangle = \int d^3 p \psi_1^*(\mathbf{p}) \psi_2(\mathbf{p}).$$

In the case $\alpha = 2$ one recovers the well-known result; see Refs. [16,17]. Let us notice that the above-mentioned cut-off $p^\alpha < \alpha/2\beta(\alpha-1)l_p^\alpha$ when $\alpha > 1$ arises merely from the fact that when small p runs over this region then large P covers the whole region from 0 to ∞ ; see Eqs. (7) and (8). In what follows we will use the abbreviation PLQM for the Planck-length deformed quantum mechanics, Eqs. (4) and (7).

III. FIELD THEORY IN LIGHT OF THE PLQM

Before proceeding let us notice that for the stellar interferometry one usually deals with the natural light; there is no preferential polarization direction for the emitted field. So, we treat the light signal as a scalar quantity (that means to take account of the scalar potential only), and consider the scalar field instead of the electromagnetic one.

Let us first consider PLQM with $\alpha = 2$. In this case we have [16,17]

$$X^i = x^i, \quad P^i = \frac{p^i}{1 - \beta l_p^2 \mathbf{p}^2}.$$

Its Hilbert space realization in the \mathbf{p} representation has the form

$$X^i \psi(\mathbf{p}) = i \partial_{p_i} \psi(\mathbf{p}), \quad P^i \psi(\mathbf{p}) = \frac{p^i}{1 - \beta l_p^2 \mathbf{p}^2} \psi(\mathbf{p}),$$

with the scalar product

$$\langle \psi_1 | \psi_2 \rangle = \int_{\mathbf{p}^2 < 1/\beta l_p^2} d^3 p \psi_1^*(\mathbf{p}) \psi_2(\mathbf{p}).$$

The modified field theory,

$$\begin{aligned} \mathcal{A}[\Phi] &= - \int d^4 x \frac{1}{2} [\Phi \partial_t^2 \Phi + \Phi \mathbf{P}^2 \Phi] \\ &= - \int d^4 x \frac{1}{2} \left[\Phi \partial_t^2 \Phi + \Phi \frac{-\Delta}{(1 + \beta l_p^2 \Delta)^2} \Phi \right], \end{aligned} \quad (9)$$

results in the equation of motion of the form

$$\begin{aligned} (\partial_t^2 + \mathbf{P}^2) \Phi &= \left(\partial_t^2 - \frac{\Delta}{(1 + \beta l_p^2 \Delta)^2} \right) \Phi \\ &= \left(\partial_t^2 - \Delta \sum_{n=0}^{\infty} (n+1) (-\beta l_p^2 \Delta)^n \right) \Phi = 0. \end{aligned} \quad (10)$$

Let us look at the spherical solutions $\Phi(t, r)$. Recalling that the radial part of the Laplace operator in spherical coordinates has the form

$$\Delta \Phi(t, r) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \Phi}{\partial r} \right),$$

after making the substitution $\Phi = \varphi/r$ one finds

$$\Delta \frac{\varphi}{r} = \frac{1}{r} \frac{\partial^2 \varphi}{\partial r^2}, \quad \Delta^2 \frac{\varphi}{r} = \frac{1}{r} \frac{\partial^4 \varphi}{\partial r^4}, \quad \Delta^n \frac{\varphi}{r} = \frac{1}{r} \frac{\partial^{2n} \varphi}{\partial r^{2n}}.$$

Considering a monochromatic wave

$$\Phi(t, r) = \frac{e^{-i(\omega t - kr)}}{r},$$

from Eq. (10) we find the modified dispersion relation

$$\omega^2 = \frac{k^2}{(1 - \beta l_p^2 k^2)^2}, \Rightarrow k^2 = \frac{(\sqrt{1 + 4\beta l_p^2 \omega^2} - 1)^2}{4\beta^2 l_p^4 \omega^2}. \quad (11)$$

Now let us look at PLQM with $\alpha = 1/2$. In this particular case, from Eq. (7) one gets

$$\hat{X}_i = \hat{x}_i, \quad \hat{P}_j = \hat{p}_j(1 + 2\beta l_p^{1/2} \hat{p}^{1/2})^2. \quad (12)$$

Hence the field equation of motion gets modified as

$$(\partial_t^2 + \mathbf{P}^2)\Phi = (\partial_t^2 - \Delta[1 + 2\beta l_p^{1/2}(-\Delta)^{1/4}]^4)\Phi = 0.$$

Following the above discussion, now for the spherical, monochromatic-wave solution

$$\Phi(t, r) = \frac{e^{-i(\omega t - kr)}}{r},$$

we find

$$\omega = k[1 + 2\beta l_p^{1/2} k^{1/2}]^2, \Rightarrow k = \frac{(1 - \sqrt{1 + 8\beta \sqrt{l_p} \omega})^2}{16\beta^2 l_p}. \quad (13)$$

IV. THE DEGREE OF LIGHT COHERENCE FROM DISTANT CELESTIAL OBJECTS: VAN CITTERT-ZERNIKE FORMALISM

The light from the astrophysical source certainly cannot be strictly monochromatic for even the spectral lines for isolated atoms have finite widths. In addition, the broadening of the spectral lines are caused because of the motion of atoms (Doppler broadening) and also because of interaction (collisions) between the atoms. In the case of real sources it is appropriate to talk about the wave packet

$$\Phi(t, r) = \int d\omega a(\omega) \frac{e^{i[k(\omega)r - \omega t]}}{r}, \quad (14)$$

where $k(\omega)$ is defined by Eq. (11) and the function $a(\omega)$ is understood to differ appreciably from zero only within a narrow range around a mean frequency $\bar{\omega}$,

$$\bar{\omega} - \frac{\delta\omega}{2} \leq \omega \leq \bar{\omega} + \frac{\delta\omega}{2}, \quad \frac{\delta\omega}{\bar{\omega}} \ll 1.$$

If $\delta\omega$ is sufficiently small, the wave packet Eq. (14) can be interpreted as a plane wave with frequency $\bar{\omega}$, wave number $k(\bar{\omega})$, and variable amplitude

$$A(t, r) = \int_{\bar{\omega} - \delta\omega/2}^{\bar{\omega} + \delta\omega/2} d\omega a(\omega) e^{i[k(\omega)r - \omega t]}, \quad (15)$$

$$\Phi(t, r) = A(t, r) \frac{e^{i[k(\bar{\omega})r - \bar{\omega}t]}}{r}. \quad (16)$$

The width $\delta\omega$ determining the duration of the wave packet $\delta t \approx \delta\omega^{-1}$ is an important characteristic for the interference effect. Namely, the interference effect takes place when the path difference between the overlapping quasimonochromatic beams, Eq. (16), is less than the coherence length δt .

Now we are in a position to follow van Cittert-Zernike theory to the mutual coherence of light from an extended quasimonochromatic source [18]. A screen A is illuminated by an extended quasimonochromatic incoherent source Σ taken for simplicity to be a portion of a plane parallel to A ; see Fig. 1. The points P_1 and P_2 on the screen correspond to the interferometer slits. Before going on let us adopt the following natural assumptions. The linear dimensions of Σ are small compared to the distance OO' between the source and the screen, and the angles between OO' and the line joining a typical source point, S to P_1 and P_2 , are small. Dividing the source into elements $d\sigma_m$ centered on points S_m and denoted by $\Phi_{m1}(t)$ and $\Phi_{m2}(t)$, the disturbances at P_1 and P_2 due to element $d\sigma_m$, for total disturbances at these points, one finds

$$\Phi_{1,2}(t) = \sum_m \Phi_{m1,2}(t).$$

The correlation function between the light signals $\Phi_1(t)$ and $\Phi_2(t)$ takes the form (brackets $\langle \rangle$ denote time averaging)

$$\langle \Phi_1(t) \Phi_2^*(t) \rangle = \sum_m \langle \Phi_{m1}(t) \Phi_{m2}^*(t) \rangle + \sum_{m \neq n} \langle \Phi_{m1}(t) \Phi_{n2}^*(t) \rangle.$$

The light signals coming from different elements of the source are mutually incoherent; that is, there is no correlation between $\Phi_{m1}(t)$ and $\Phi_{n2}(t)$ when $m \neq n$,

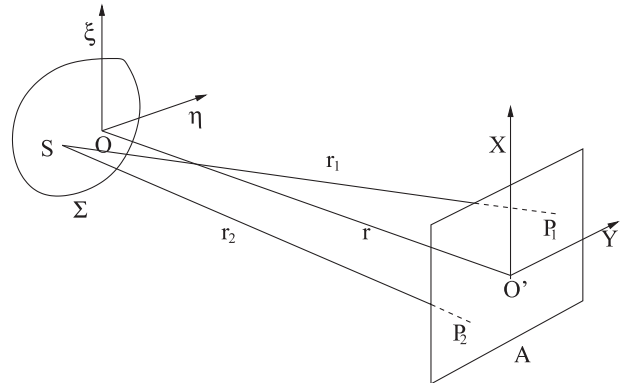


FIG. 1. A screen A illuminated by an extended quasimonochromatic incoherent source Σ .

$$\langle \Phi_{m_1}(t)\Phi_{n_2}^*(t) \rangle = 0 \quad \text{for } m \neq n.$$

Using an explicit expression

$$\Phi_{m_{1,2}}(t) = A_m(t, r_{m_{1,2}}) \frac{e^{-i[\bar{\omega}t - k(\bar{\omega})r_{m_{1,2}}]}}{r_{m_{1,2}}}, \quad (17)$$

where $r_{m_{1,2}}$ denote distances between the elements $d\sigma_m$ and the points $P_{1,2}$ for the correlation function, one finds

$$\langle \Phi_{m_1}(t)\Phi_{m_2}^*(t) \rangle = \langle A_m(t, r_{m_1})A_m^*(t, r_{m_2}) \rangle \frac{e^{ik(\bar{\omega})(r_{m_1} - r_{m_2})}}{r_{m_1}r_{m_2}}. \quad (18)$$

If the condition

$$|r_{m_2} - r_{m_1}| \ll |k(\bar{\omega} + \delta\omega) - k(\bar{\omega})|^{-1} \quad (19)$$

is satisfied, then one can write

$$\begin{aligned} A_m(t, r_{m_1})A_m^*(t, r_{m_2}) &\approx A_m(t, r_{m_1})A_m^*(t, r_{m_1}) \\ &\approx A_m(t, r_{m_2})A_m^*(t, r_{m_2}). \end{aligned} \quad (20)$$

Namely, using Eq. (15) one observes that ($\delta r \equiv r_2 - r_1$),

$$\begin{aligned} A_m(t, r_{m_1})A_m^*(t, r_{m_2}) &= \int_{\bar{\omega} - \delta\omega/2}^{\bar{\omega} + \delta\omega/2} d\omega d\omega' a(\omega)a^*(\omega') e^{-i(\omega - \omega')t} \\ &\quad \times e^{i[k(\omega) - k(\omega')]r_{m_1}} e^{-i[k(\omega') - k(\bar{\omega})]\delta r} \\ &\approx \int_{\bar{\omega} - \delta\omega/2}^{\bar{\omega} + \delta\omega/2} d\omega d\omega' a(\omega)a^*(\omega') e^{-i(\omega - \omega')t} e^{i[k(\omega) - k(\omega')]r_{m_1}} \\ &= A_m(t, r_{m_1})A_m^*(t, r_{m_1}), \end{aligned}$$

and analogously one finds that this expression is pretty much the same as $A_m(t, r_{m_2})A_m^*(t, r_{m_2})$. It is worth noticing that if Eq. (19) is satisfied in the case of the standard dispersion relation, then it is automatically satisfied for the dispersion relations, Eqs. (11) and (13), for $dk/d\omega < 1$ in both cases when $\beta > 0$. Thus, assuming the condition (19) is satisfied, we may write

$$\langle \Phi_1(t)\Phi_2^*(t) \rangle = \sum_m \langle A_m(t, r_{m_1})A_m^*(t, r_{m_1}) \rangle \frac{e^{ik(\bar{\omega})(r_{m_1} - r_{m_2})}}{r_{m_1}r_{m_2}}. \quad (21)$$

The quantity $\langle A_m(t, r_{m_1})A_m^*(t, r_{m_1}) \rangle$ characterizes the intensity of the radiation from the source element $d\sigma_m$. So the correlation function, Eq. (21), takes the form

$$\langle \Phi_1(t)\Phi_2^*(t) \rangle = \int_{\Sigma} d\sigma I(\sigma) \frac{e^{ik(\bar{\omega})(r_1 - r_2)}}{r_1 r_2}, \quad (22)$$

where $I(s_m)d\sigma_m = \langle A_m(t, r_{m_1})A_m^*(t, r_{m_1}) \rangle$. In most applications the intensity $I(\sigma)$ may be assumed to be uniform on Σ . To work out the integral, Eq. (22), let us denote by (ξ, η) the coordinates of a point S , referred to axes at O , and let (x_1, y_1) and (x_2, y_2) be the coordinates of P_1 and P_2 referred to parallel axes at O' ; see Fig. 1. Retaining only

leading terms in $x/r, y/r, \xi/r, \eta/r$, where r is the distance OO' , one finds

$$r_1 - r_2 \approx \frac{x_1^2 + y_1^2 - x_2^2 - y_2^2 + 2\xi(x_2 - x_1) + 2\eta(y_2 - y_1)}{2r}.$$

Denoting

$$\begin{aligned} p &= \frac{x_1 - x_2}{r}, & q &= \frac{y_1 - y_2}{r}, \\ \psi &= \frac{k(\bar{\omega})(x_1^2 + y_1^2 - x_2^2 - y_2^2)}{2r}, \end{aligned} \quad (23)$$

Eq. (22) takes the form

$$\langle \Phi_1(t)\Phi_2^*(t) \rangle \approx \frac{e^{i\psi}}{r^2} \int_{\Sigma} d\xi d\eta I(\xi, \eta) e^{ik(\bar{\omega})(p\xi - q\eta)}. \quad (24)$$

For a uniform circular source of radius ρ with its center at O , Eq. (24) reduces to

$$\langle \Phi_1(t)\Phi_2^*(t) \rangle \sim e^{i\psi} \frac{J_1(v)}{v}, \quad (25)$$

where $v = k(\bar{\omega})\rho\sqrt{p^2 + q^2}$ and J_1 stands for the first kind and first order Bessel function. In most applications, the quantity ψ is very small, so that one can neglect the corresponding phase factor in Eq. (25). The function $J_1(v)/v$ decreases steadily from the value 0.5 when $v = 0$ to the value zero when $v = 3.83$ indicating that the degree of coherence steadily decreases and approaches complete incoherence when P_1 and P_2 are separated by the distance

$$P_1 P_2 = \frac{0.61\bar{\lambda}r}{\rho},$$

where $\bar{\lambda} = 2\pi/k(\bar{\omega})$. In experiments on interference and diffraction a departure of 12% from the ideal value of coherence that occurs at $v = 1$ can be taken as a maximum permissible departure that gives the following for the separation of points P_1 and P_2 :

$$P_1 P_2 = \frac{0.16\bar{\lambda}r}{\rho}. \quad (26)$$

V. DISCUSSION

The present paper is a continuation of the discussion started in Ref. [19]. We have constructed the Hilbert space representation for a relatively broad class of minimum-length deformed position-momentum uncertainty relations (2), which in its turn can be understood as a modification arising because of quantum-gravitational fluctuations of the background Minkowski space. A particular case of this sort of PLQM is a well-known example studied in Refs. [16,17]. Following this construction, we have then used PLQM for constructing the correspondingly modified field theory, which can be used for estimating corrections to various quantities. We have applied the minimum-length

deformed field theory constructed this way for estimating the corrections to the light coherence rate from distant astrophysical objects. The rate of light (in)coherence is estimated on the basis of van Cittert-Zernike formalism (as pointed out in Ref. [20]). The picture arising from this consideration looks as follows.

If all the assumptions required for light coherence are satisfied in the case of standard dispersion relation, then those conditions are automatically satisfied for modified dispersion relations (11) and (13). Namely, if the requirement of the path difference $|r_{m2} - r_{m1}|$ to be smaller than the duration of the wave packet $\delta t \equiv \delta \omega^{-1}$ is satisfied in the standard case, that is, $|r_{m2} - r_{m1}| \ll \delta \omega^{-1}$, then Eq. (19) is automatically satisfied for dispersion relations (11) and (13) as $dk/d\omega < 1$ in those cases.

In the standard case, the phase ψ in Eq. (25) is usually neglected as it is usually small in most applications. In the cases of Eqs. (11) and (13), ψ becomes even smaller as $k(\bar{\omega}) < \bar{\omega}$.

And finally, the maximum separation of points P_1 and P_2 over which light is coherent, Eq. (26), becomes enlarged as compared to the standard case as the wave length $\bar{\lambda}$ is enlarged now: $1/k(\bar{\omega}) > 1/\bar{\omega}$.

One can simply estimate the rate of the effect in both cases: $\alpha = 2$ and $\alpha = 1/2$. Restricting ourselves to the leading order corrections, in the former case ($\alpha = 2$) one finds from Eq. (11) (as well as from the relation

$\delta \lambda = \beta l_p^\alpha \lambda^{1-\alpha}$ by which we started our discussion, see the Introduction) that the wave-length increment takes the form $\delta \bar{\lambda} \approx l_p^2 \bar{\omega}$. Let us see if the increment in separation $P_1 P_2$ can be made observationally perceptible, say of the order of 1 cm. Even if we take $\bar{\omega} \approx E_p$, from Eq. (26) one gets $r/\rho \sim 10^{33}$. To obtain such a huge ratio is absolutely impossible, for the present horizon radius is about 10^{28} cm and in its turn ρ is, by many orders of magnitude, greater than 1 cm. So for the realistic values of $\bar{\omega}$, r , and ρ one gets a minuscule effect.

Analogously, in the latter case $\alpha = 1/2$ one finds from Eq. (13) (as well as from the relation $\delta \lambda = \beta l_p^\alpha \lambda^{1-\alpha}$ by which we started our discussion, see the Introduction) that $\delta \bar{\lambda} \approx (l_p \bar{\lambda})^{1/2}$. Let us again consider an extreme case to demonstrate the smallness of the effect. Let us take $P_1 P_2 \approx 10^6$ cm and assume that we can measure this distance with accuracy ~ 1 cm. Taking $\bar{\lambda} \approx 10^{-9}$ cm, from Eq. (26) one finds

$$\delta(P_1 P_2) \approx P_1 P_2 \left(\frac{l_p}{\bar{\lambda}}\right)^{1/2} \approx 10^{-6} \text{ cm.}$$

So, in any realistic case we expect the effect to be tiny.

For other approaches describing the effect of quantum gravity on the light coherence from distant astrophysical objects (indicating that the effect is intangible), see Refs. [21–23].

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