Charged rotating Kaluza-Klein multiblack holes and multiblack strings in five-dimensional Einstein-Maxwell theory

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(Received 10 September 2012; published 26 November 2012)

We construct exact solutions, which represent regular charged rotating Kaluza-Klein multiblack holes in the five-dimensional pure Einstein-Maxwell theory. Quantization conditions between the mass, the angular momentum, and charges appear from the regularity condition of horizon. We also obtain multiblack string solutions by taking some limits in the solutions. We extend the black hole solutions to the five-dimensional Einstein-Maxwell-Chern-Simons theory with an arbitrary Chern-Simons coupling constant.

DOI: 10.1103/PhysRevD.86.104054 PACS numbers: 04.50.−h, 04.70.Bw

I. INTRODUCTION

Higher-dimensional black holes in asymptotically Kaluza-Klein spacetimes would play an important role in understanding basic properties of fundamental theories; since our real observable world is a macroscopically four-dimensional spacetime, then extra dimensions must be compactified. Exact solutions of the Kaluza-Klein black holes are constructed explicitly, and their properties are studied. For example, five-dimensional squashed Kaluza-Klein black hole solutions [1–23] behave as fully five-dimensional black holes in the vicinity of the squashed S^3 horizon, while they asymptote to four-dimensional flat spacetimes with a twisted S^1 as a compactified extra dimension. Then we can regard these squashed Kaluza-Klein black hole solutions as models of realistic higher-dimensional black holes.

Recently, we have investigated extremal rotating regular vacuum multiblack holes in the five-dimensional asymptotically Kaluza-Klein spacetimes [24,25]. We have shown that each black hole has a smooth horizon with the topology of the lens space in addition to the S^3. The compactness of the extra dimension plays an essential role for existence of multiblack holes in vacuum, and the mass of each black hole is quantized by the size of the compactified extra dimension.

In the present paper, we generalize these results to charged rotating Kaluza-Klein multi-black hole solutions in the five-dimensional Einstein-Maxwell theory. Until now, to our knowledge, any exact charged rotating black hole solutions have not been found in the five-dimensional pure Einstein-Maxwell theory. In contrast, the five-dimensional Einstein-Maxwell-Chern-Simons theory, which is suggested by the supergravity, admits regular charged rotating black hole solutions in closed form both in asymptotically flat spacetimes [3,26–32] and in asymptotically Kaluza-Klein spacetimes [3,4,8–14,17,23,33]. The Chern-Simons term is necessary for the solutions. Then, the solutions presented in this article are the first example of exact solutions that represent charged rotating multiblack holes in the five-dimensional pure Einstein-Maxwell theory.

This paper is organized as follows. We present explicit forms of solutions in Sec. II, and investigate asymptotic structures of the solutions, the regularity at the horizons, and the conserved charges in Sec. III. We show in Sec. IV that some known multiblack hole solutions are obtained by taking a limit in our solutions, and we also show in Sec. V that multiblack string solutions are obtained by another limit. In the Appendices, we generalize our solutions to the solutions of nondegenerate horizon case and solutions to the five-dimensional Einstein-Maxwell-Chern-Simons theory with an arbitrary value of the Chern-Simons coupling constant.

II. SOLUTIONS

We consider charged rotating multiblack hole solutions in the five-dimensional Einstein-Maxwell theory with the action

$$S = \frac{1}{16\pi}\int d^5x\sqrt{-g}(R - F_{\mu\nu}F^{\mu\nu}). \tag{1}$$

We assume that the metric and the Maxwell field are written as

$$ds^2 = -H^{-2}dt^2 + H^2(dx^2 + dy^2 + dz^2) + \left[\alpha(H^{-1} - 1)dt + \frac{L}{2}d\psi + \beta\boldsymbol{\omega}\right]^2, \tag{2}$$

$$A_\mu dx^\mu = \gamma H^{-1}dt + \delta\boldsymbol{\omega}, \tag{3}$$

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where

$$H = 1 + \sum_i \frac{m_i}{|\boldsymbol{r} - \boldsymbol{r}_i|} \tag{4}$$

is the harmonic function on the three-dimensional Euclid space with point sources located at $\boldsymbol{r} = \boldsymbol{r}_i := (x_i, y_i, z_i)$. The 1-form $\boldsymbol{\omega}$, which satisfies

$$\nabla \times \boldsymbol{\omega} = \nabla H, \tag{5}$$

has the explicit form

$$\boldsymbol{\omega} = \sum_i m_i \frac{z - z_i}{|\boldsymbol{r} - \boldsymbol{r}_i|} \frac{(x - x_i)dy - (y - y_i)dx}{(x - x_i)^2 + (y - y_i)^2}. \tag{6}$$

In the expressions (2)–(6), m_i and L are constants, and α, β, γ, δ are parameters to be fixed. As will be shown later, $\boldsymbol{r} = \boldsymbol{r}_i$ are black hole horizons.

The five-dimensional Einstein equations require that the parameters α, β, γ, δ should satisfy

$$2\alpha^2 - \beta^2 + 4\gamma^2 - 2 = 0, \tag{7}$$

$$\alpha^2 - 2\beta^2 - 4\delta^2 + 2 = 0, \tag{8}$$

and the Maxwell equations require

$$\alpha\gamma - \beta\delta = 0. \tag{9}$$

From (7)–(9) we find two possible cases:

$$\alpha^2 = \beta^2, \qquad \gamma^2 = \delta^2 = \frac{2 - \alpha^2}{4}, \tag{10}$$

or

$$\alpha^2 = \frac{4}{3}\delta^2, \qquad \beta^2 = \frac{4}{3}\gamma^2 = 1 - \alpha^2, \tag{11}$$

where sign of the parameters should keep (9).

First, we assume $\beta \neq 0$, where the solutions describe multiblack holes. After that, we consider the $\beta = 0$ case, where the solutions describe multiblack strings.

III. CHARGED ROTATING MULTIBLACK HOLES

We begin with the case $\beta \neq 0$. In this case, the solutions describe charged rotating multiblack holes in the Einstein-Maxwell system.

A. Asymptotic structure

First, we see the asymptotic behavior of the metric (2). In the limit $r \to \infty$, the metric behaves as

$$ds^2 \to -dt^2 + dr^2 + r^2 d\Omega_{S^2}^2 + \frac{L^2}{4}\left(d\psi + \frac{2\beta}{L}\sum_i m_i \cos\theta d\phi\right)^2, \tag{12}$$

where $d\Omega_{S^2}^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the metric of the unit two-sphere. Then, the spacetime with the metric (2) is asymptotically locally flat, i.e., the metric asymptotes to a twisted constant S^1 fiber bundle over the four-dimensional Minkowski spacetime, and the spatial infinity has the structure of an S^1 bundle over an S^2. We see that the size of a twisted S^1 fiber as an extra dimension takes the constant value L everywhere.

Next, we inspect the apparent singularities $\boldsymbol{r} = \boldsymbol{r}_i$ of the metric (2). The absence of a naked singularity on and outside the surfaces $\boldsymbol{r} = \boldsymbol{r}_i$ requires the parameters m_i should be

$$m_i > 0. \tag{13}$$

For simplicity, we restrict ourselves to the cases of two-black holes, i.e., $m_1 \neq 0$, $m_2 \neq 0$, otherwise $m_i = 0$. Without loss of generality, we can put the locations of two point sources as $\boldsymbol{r}_1 = (0, 0, 0)$ and $\boldsymbol{r}_2 = (0, 0, a)$, where the constant a denotes the separation between two black holes.

In this case, the metric is

$$ds^2 = -H^{-2}dt^2 + H^2(dr^2 + r^2 d\Omega_{S^2}^2) + \left[\alpha(H^{-1} - 1)dt + \frac{L}{2}d\psi + \beta\boldsymbol{\omega}\right]^2, \tag{14}$$

where H and $\boldsymbol{\omega}$ are given by

$$H = 1 + \frac{m_1}{r} + \frac{m_2}{\sqrt{r^2 + a^2 - 2ar\cos\theta}}, \tag{15}$$

$$\boldsymbol{\omega} = \left(m_1 \cos\theta + m_2 \frac{r\cos\theta - a}{\sqrt{r^2 + a^2 - 2ar\cos\theta}}\right)d\phi, \tag{16}$$

respectively. The coordinates run the ranges of $-\infty < t < \infty$, $0 < r < \infty$, $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$, and $0 \leq \psi \leq 4\pi$.

In the coordinate system $(t, r, \theta, \phi, \psi)$, the metric (14) diverges at the locations of two point sources, i.e., $\boldsymbol{r} = \boldsymbol{r}_1$ $(r = 0)$ and $\boldsymbol{r} = \boldsymbol{r}_2$ $(r = a,\ \theta = 0)$. In order to remove apparent divergences at $r = 0$, we introduce new coordinates (v, ψ') such that

$$dv = dt + H^2 dr + W d\theta, \tag{17}$$

$$d\psi' = d\psi - \frac{2\alpha}{L}(dt + H dr + V d\theta), \tag{18}$$

where the functions W and V are given by

$$W(r, \theta) = \int dr \frac{\partial}{\partial\theta}(H^2), \tag{19}$$

$$V(r, \theta) = \int dr \frac{\partial}{\partial\theta} H, \tag{20}$$

respectively. Then, the metric (14) takes the form of

$$ds^2 = -H^{-2}(dv - Wd\theta)^2 + 2dr(dv - Wd\theta) + H^2 r^2 d\Omega_{S^2}^2 + \Big[\alpha H^{-1} dv + \beta\boldsymbol{\omega} + \alpha(V - H^{-1}W)d\theta + \frac{L}{2}d\psi'\Big]^2. \quad (21)$$

In the neighborhood of $r = 0$, the metric (21) behaves as

$$ds^2 \simeq \frac{(\alpha^2-1)r^2}{m_1^2}dv^2 + 2dvdr + m_1^2\Big[d\Omega_{S^2}^2 + \beta^2\Big(\frac{L}{2\beta m_1}d\psi'' + \cos\theta d\phi\Big)^2\Big] + 2r\Big[\frac{2m_1m_2\sin\theta}{a^2}drd\theta + \alpha\beta\Big(dv + \frac{3m_1m_2\sin\theta}{2a^2}rd\theta\Big) \times\Big(\frac{L}{2\beta m_1}d\psi'' + \cos\theta d\phi\Big)\Big] + \mathcal{O}(r^3), \quad (22)$$

where we have used

$$d\psi'' = d\psi' - \frac{2\beta}{L}m_2 d\phi. \quad (23)$$

If the factor $2\beta m_1/L$ is a natural number, say n_1, the induced metric on the three-dimensional spatial cross section of $r = 0$ with a $t = \text{const}$ surface is

$$ds^2|_{r=0} = \frac{n_1^2 L^2}{4\beta^2}\Big[d\Omega_{S^2}^2 + \beta^2\Big(\frac{d\psi''}{n_1} + \cos\theta d\phi\Big)^2\Big]. \quad (24)$$

That is, the $r = 0$ surface admits the smooth metric of the lens space $L(n_1;1) = S^3/\mathbb{Z}_{n_1}$.

In this case, from (22) we see that $r = 0$ is a null surface where the metric (21) is regular and each component is an analytic function of r. Therefore, the metric (21) gives an analytic extension across the surface $r = 0$. By the same discussion, we see that the metric (14) also admits an analytic extension across the surface $\boldsymbol{r} = \boldsymbol{r}_2$ if $2\beta m_2/L$ is equal to a natural number n_2.

We also see that $\eta = \partial_v$ is a Killing vector field that becomes null at $r = 0$. Furthermore, η is hypersurface orthogonal to the surface $r = 0$, i.e., $\eta_\mu dx^\mu = g_{vr}dr = dr$ there. These mean that the null hypersurface $r = 0$ is a Killing horizon. Similarly, $\boldsymbol{r} = \boldsymbol{r}_2$ is also a Killing horizon. Hence, we can see that the solutions (14) with (15) and (16) describe Kaluza-Klein multiblack holes, which have smooth Killing horizons without a singularity on and outside the black hole horizons. Since the ϕ-ψ part of the metric is positive definite, it is clear that no closed timelike curve exists. Near each horizon limit, the metric (14) approaches the $L(n_i;1)$ bundle over the AdS_2 space at the horizon [34,35].

We can generalize the discussion to the metric (2). The metrics (2) describe multiblack holes of the lens spaces $L(n_i;1)$ topology if the parameters m_i are quantized as

$$m_i = \frac{L}{2\beta}n_i, \quad (25)$$

where n_i are natural numbers. The area of the horizons are given by

$$\mathcal{A}|_{\boldsymbol{r}=\boldsymbol{r}_i} = \frac{n_i^2 L^3}{\beta^2}\mathcal{A}_{S^3}, \quad (26)$$

where $\mathcal{A}_{S^3}$ denotes the area of a unit S^3.

B. Mass and charges

We define the Komar mass M associated with the timelike Killing vector $\xi_{(t)} = \partial/\partial t$, and obtain as

$$M = \frac{-3}{32\pi}\int_\infty dS_{\mu\nu}\nabla^\mu \xi_{(t)}^\nu = \frac{3L\sum_i m_i}{4\pi}\mathcal{A}_{S^3}. \quad (27)$$

We also obtain the angular momentum J^ψ associated with the spacelike Killing vector $\xi_{(\psi)} = \partial/\partial\psi$ as

$$J^\psi = \frac{1}{16\pi}\int_\infty dS_{\mu\nu}\nabla^\mu \xi_{(\psi)}^\nu = \frac{\alpha L^2\sum_i m_i}{8\pi}\mathcal{A}_{S^3}. \quad (28)$$

We see that the spacetime (2) is rotating along the extra dimension.

We can obtain the electric charge Q_i and the magnetic flux Ψ_i,

$$Q_i = \frac{1}{4\pi}\int_{\Sigma^3} {}^*\boldsymbol{F} = \frac{\gamma L m_i}{\pi}\mathcal{A}_{S^3}, \quad (29)$$

$$\Psi_i = \frac{1}{4\pi}\int_{\Sigma^2}\boldsymbol{F} = -\delta m_i, \quad (30)$$

respectively, where Σ^3 denotes a closed surface on a time slice surrounding each black hole, and Σ^2 denotes a closed surface surrounding each black hole on the base space.

In these calculations, we see that m_i characterize the mass of black holes, and α, γ, δ are related with the angular momentum, the electric charge, the magnetic flux, respectively. The parameter L is related to the size of the extra dimension at infinity as seen before. The parameter β controls the twist of the extra dimension, and gives the unit of quantization of mass in cooperation with L. If none of these parameters vanish, the solutions describe electrically and magnetically charged rotating Kaluza-Klein multiblack holes to the Einstein-Maxwell equations.

From conditions (7) and (8), we have

$$\frac{1}{4}(\alpha^2 + \beta^2) + \gamma^2 + \delta^2 - 1 = 0. \quad (31)$$

Substituting the Komar mass (27), the angular momentum (28), the electric charge (29), and the magnetic flux (30) into (31), we obtain the extremal condition as

$$\Big(\frac{4J^\psi}{L}\Big)^2 + Q^2 + (2\pi L\Psi)^2 + \Big(\frac{\pi n L^2}{2}\Big)^2 = \Big(\frac{4M}{3}\Big)^2, \quad (32)$$

where $Q = \sum_i Q_i$, $\Psi = \sum_i \Psi_i$, and $n = \sum_i n_i$. We should note that the size of extra dimension L cannot be infinitely large if the mass M is finite. The Kaluza-Klein structure is a critical property for the present solutions. We can rewrite the conditions (10) and (11) for two possible cases in terms of M, J^ψ, Q, Ψ, and L. In the first case (10) we have

$$\begin{aligned} Q^2 &= (2\pi L\Psi)^2, \\ \left(\frac{4J^\psi}{L}\right)^2 &= \frac{1}{2}\left(\frac{4M}{3}\right)^2 - Q^2 = \left(\frac{\pi n L^2}{2}\right)^2. \end{aligned} \tag{33}$$

The angular momentum and the mass square minus the electric charge square are quantized by the size of extra dimension. In the second case (11) we have

$$\begin{aligned} \left(\frac{4J^\psi}{L}\right)^2 &= \frac{1}{3}(2\pi L\Psi)^2, \\ \frac{1}{3}Q^2 &= \frac{1}{4}\left(\frac{4M}{3}\right)^2 - \left(\frac{4J^\psi}{L}\right)^2 = \left(\frac{\pi n L^2}{2}\right)^2. \end{aligned} \tag{34}$$

The electric charge and the mass square minus the angular momentum square are quantized in this case. If $\Psi = 0$, one of Q or J^ψ vanishes. It is clear that nonvanishing magnetic flux is a key for the charged rotating Kaluza-Klein multiblack hole solutions.

With respect to the timelike Killing vector $\xi_{(t)}$, we define the ergosurfaces where the Killing vector becomes null, i.e.,

$$g_{tt} = -H^{-2} + \alpha^2(H^{-1} - 1)^2 = 0. \tag{35}$$

Since g_{tt} is a continuous function on the spacetime outside the horizons for the range of parameters (13), and $g_{tt}(\boldsymbol{r} = \boldsymbol{r}_i) = \alpha^2 > 0$ and $g_{tt}(\infty) = -1 < 0$, then there always exist ergoregions around the black hole horizons. The topology of the ergosurface depends on the location of black holes [10].

In the same manner, we can construct charged rotating single Kaluza-Klein black hole solutions with nondegenerate horizons to the Einstein-Maxwell equations. This case is discussed briefly in Appendix A. In Appendix B, we also generalize our solutions to the solutions in the five-dimensional Einstein-Maxwell-Chern-Simons theory with an arbitrary value of the Chern-Simons coupling constant.

IV. BLACK HOLES WITH $\Psi_i = 0$

We consider the limiting solutions with $\Psi_i = 0$. There are two subcases: $\delta = \gamma = 0$, $\alpha^2 = \beta^2 = 2$ and $\delta = \alpha = 0$, $\beta^2 = \frac{4}{3}\gamma^2 = 1$.

In the first case, $Q_i = \Psi_i = 0$ then the Maxwell field (3) vanishes. Then the metric (2) coincides with the Kaluza-Klein vacuum multiblack holes [24,25]:

$$\begin{aligned} ds^2 &= -H^{-2}dt^2 + H^2(dx^2 + dy^2 + dz^2) \\ &\quad + 2\left[(H^{-1} - 1)dt + \frac{L}{2\sqrt{2}}d\psi \pm \boldsymbol{\omega}\right]^2. \end{aligned} \tag{36}$$

In the second case, $J^\psi = \Psi_i = 0$ then the metric (2) and the Maxwell field (3) reduce to

$$\begin{aligned} ds^2 &= -H^{-2}dt^2 + H^2(dx^2 + dy^2 + dz^2) \\ &\quad + \left(\frac{L}{2}d\psi + \boldsymbol{\omega}\right)^2, \end{aligned} \tag{37}$$

$$A_\mu dx^\mu = \pm\frac{\sqrt{3}}{2}H^{-1}dt, \tag{38}$$

which describe charged static Kaluza-Klein multiblack holes with a twisted constant S^1 [3,36].

V. MULTIBLACK STRINGS

Here, we consider the case $\beta = 0$. In this case, the fiber-bundle structure reduces to the direct product of the S^1 fiber as the extra dimension and base space. Then, the metrics (2) describe multiblack strings.

We have two subcases. In the first case, $\beta = \alpha = 0$ and $\gamma^2 = \delta^2 = 1/2$, the metric (2) and the Maxwell field (3) become

$$ds^2 = -H^{-2}dt^2 + H^2(dx^2 + dy^2 + dz^2) + \frac{L^2}{4}d\psi^2, \tag{39}$$

$$A_\mu dx^\mu = \pm\frac{1}{\sqrt{2}}(H^{-1}dt + \boldsymbol{\omega}), \tag{40}$$

which describe dyonically charged static multiblack strings.

In the second case, $\beta = \gamma = 0$ and $\alpha^2 = \frac{4}{3}\delta^2 = 1$, the metric (2) and the Maxwell field (3) reduce to

$$\begin{aligned} ds^2 &= -H^{-2}dt^2 + H^2(dx^2 + dy^2 + dz^2) \\ &\quad + \left((H^{-1} - 1)dt + \frac{L}{2}d\psi\right)^2, \end{aligned} \tag{41}$$

$$A_\mu dx^\mu = \pm\frac{\sqrt{3}}{2}\boldsymbol{\omega}, \tag{42}$$

which describe magnetically charged boosted multiblack strings.

In the single black string case, the solution (39) with (40) was obtained in the context of the ten-dimensional type-IIA string theory in Ref. [37], and the solution (41) with (42) was obtained in the context of the five-dimensional Einstein-Maxwell-dilaton theory in Refs. [38,39]. Analytic extensions across the horizons for (39) and (41) are given by (21) with $\beta = 0$.

VI. SUMMARY

We have constructed exact charged rotating Kaluza-Klein multiblack hole solutions in the five-dimensional pure Einstein-Maxwell theory. The metric asymptotes to the effectively four-dimensional spacetime at infinity, and the size of the compactified extra dimension takes the constant value everywhere. We have shown that each black hole has a smooth horizon and its topology is the three-dimensional sphere or lens space $L(n_i; 1)$ with an arbitrary n_i. The positions of black holes on the three-dimensional flat base space are free parameters.

The solutions are characterized by the size of the extra dimension, the mass, the angular momentum along the extra circular dimension, the electric charge, and the magnetic flux. These quantities are related with three conditions which come from the Einstein-Maxwell equations. Regularity of horizons requires that some of these quantities are quantized by the size of extra dimension L. Then, the minimum size of black hole which is comparable to L exists. By this reason, we cannot obtain asymptotically flat solutions from the present solutions by taking the limit $L \to \infty$ keeping the black hole mass finite. This is consistent with the fact that any exact charged rotating black hole solutions in asymptotically flat spacetimes have not been found yet in the five-dimensional pure Einstein-Maxwell theory.[1] The Kaluza-Klein spacetime structure with a compact extra dimension plays a crucial role in constructions of exact charged rotating black hole solutions in the five-dimensional pure Einstein-Maxwell theory.

We have also obtained multiblack string solutions by taking limits. Furthermore, we can easily generalize the solutions in the five-dimensional Einstein-Maxwell-Chern-Simons theory with an arbitrary value of the Chern-Simons coupling constant (see Appendix B).

ACKNOWLEDGMENTS

This work is supported by the Grant-in-Aid for Scientific Research No. 19540305. M. K. is supported by the JSPS Grant-in-Aid for Scientific Research No. 11J02182.

APPENDIX A: CHARGED ROTATING KALUZA-KLEIN BLACK HOLES WITH NONDEGENERATE HORIZONS

We consider the metric and the Maxwell field of the charged rotating Kaluza-Klein black holes with nondegenerate horizons in five dimensions as

$$ds^2 = -\left(1-\frac{2m}{R}+\frac{q^2}{R^2}\right)dt^2 + \left(1-\frac{2m}{R}+\frac{q^2}{R^2}\right)^{-1} dR^2 + R^2 d\Omega_{S^2}^2 + \left(\frac{L}{2}d\psi - \alpha\frac{q}{R}dt + \beta q\cos\theta d\phi\right)^2, \quad \text{(A1)}$$

$$A_\mu dx^\mu = -\gamma\frac{q}{R}dt + \delta q\cos\theta d\phi. \quad \text{(A2)}$$

The Einstein equations require the conditions (7) and (8) between the parameters α, β, γ, δ and the Maxwell equations require (9), the same as the multiblack hole case.

For the absence of naked singularity $q \leq m$ then we have

$$\left(\frac{4J^\psi}{L}\right)^2 + Q^2 + (2\pi L\Psi)^2 + \left(\frac{\pi n L^2}{2}\right)^2 \leq \left(\frac{4M}{3}\right)^2, \quad \text{(A3)}$$

instead of (32). If $m = q$, by the use of $r = R - m$, we recover the single black hole case of the metric (2) and the Maxwell field (3) with $m_1 = m$ and $m_i = 0$ $(i \geq 2)$.

The metric (A1) and the Maxwell field (A2) are discussed as a special solution, where the size of the extra dimension is constant, to the Einstein-Maxwell-Chern-Simons equations in Ref. [8]. The parameters in their solution are different from the present solution.

APPENDIX B: CHARGED ROTATING BLACK HOLES IN EINSTEIN-MAXWELL-CHERN-SIMONS SYSTEM

We consider the five-dimensional Einstein-Maxwell-Chern-Simons theory with the action

$$S = \frac{1}{16\pi}\int d^5x\left[\sqrt{-g}(R - F_{\mu\nu}F^{\mu\nu}) - \frac{2\lambda}{3\sqrt{3}}\epsilon^{\mu\nu\rho\sigma\zeta}A_\mu F_{\nu\rho}F_{\sigma\zeta}\right], \quad \text{(B1)}$$

where λ is the Chern-Simons coupling constant [41–43,47–49]. For vanishing λ, pure Einstein-Maxwell theory is recovered, and $\lambda = 1$ is suggested by the minimal supergravity.

We assume the same forms of the metric and the Maxwell field (A1) and (A2) for single black holes or (2) and (3) for multiblack holes. Since the Chern-Simons term is free from the metric, the Einstein equations require the same conditions (7) and (8). On the other hand, the Maxwell equations modified by the Chern-Simons term require

$$3(\alpha\gamma - \beta\delta) - 4\sqrt{3}\gamma\delta\lambda = 0. \quad \text{(B2)}$$

If the parameters α, β, γ, δ satisfy (7), (8), and (B2), the sets of the metrics and the Maxwell fields are solutions which represent charged rotating Kaluza-Klein black holes in the Einstein-Maxwell-Chern-Simons theory with arbitrary λ. In the $\lambda = 1$ case, the single black hole solution coincides with a special case of solutions obtained in Refs. [8,17].

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